Computer algebra independent integration tests

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/1.2.2.6-P-x-d-x-^m-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [145]. This is test number [43].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (145)	% 0.00 (0)
Mathematica	% 100.00 (145)	% 0.00 (0)
Maple	% 98.62 (143)	% 1.38 (2)
Maxima	% 50.34 (73)	% 49.66 (72)
Fricas	% 79.31 (115)	% 20.69 (30)
Sympy	% 54.48 (79)	% 45.52 (66)
Giac	% 95.86 (139)	% 4.14 (6)
Mupad	% 98.62 (143)	% 1.38 (2)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

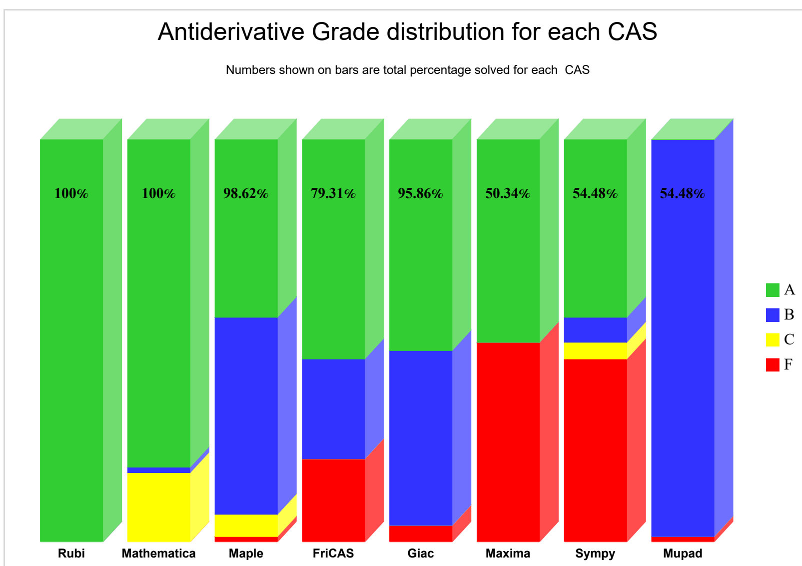
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

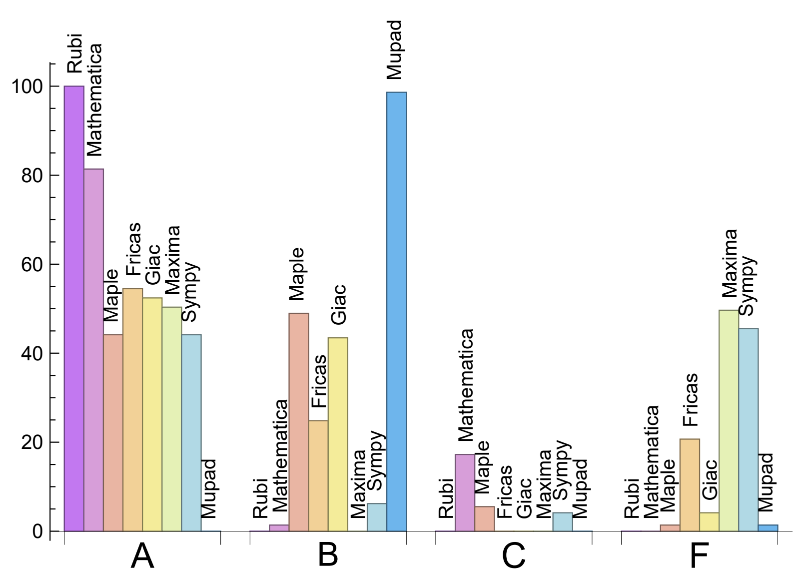
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	81.38	1.38	17.24	0.00
Maple	44.14	48.97	5.52	1.38
Maxima	50.34	0.00	0.00	49.66
Fricas	54.48	24.83	0.00	20.69
Sympy	44.14	6.21	4.14	45.52
Giac	52.41	43.45	0.00	4.14
Mupad	0.00	98.62	0.00	1.38

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	2	100.00 %	0.00 %	0.00 %
Maxima	72	77.78 %	0.00 %	22.22 %
Fricas	30	6.67 %	93.33 %	0.00 %
Sympy	66	1.52 %	98.48 %	0.00 %
Giac	6	33.33 %	0.00 %	66.67 %
Mupad	2	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

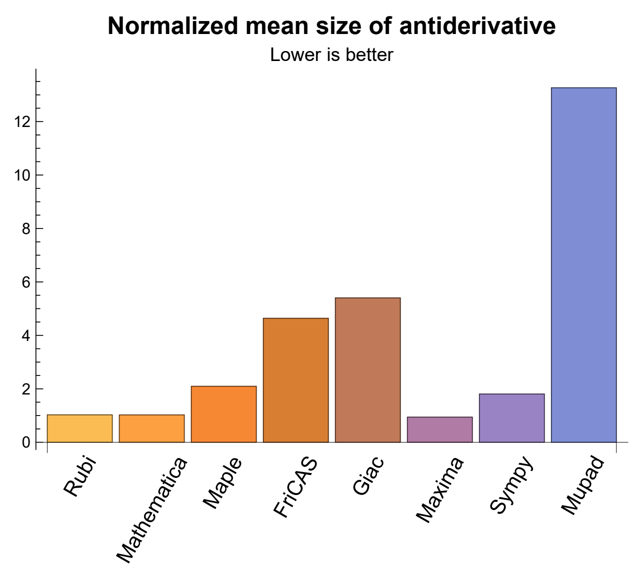
1.3 Performance

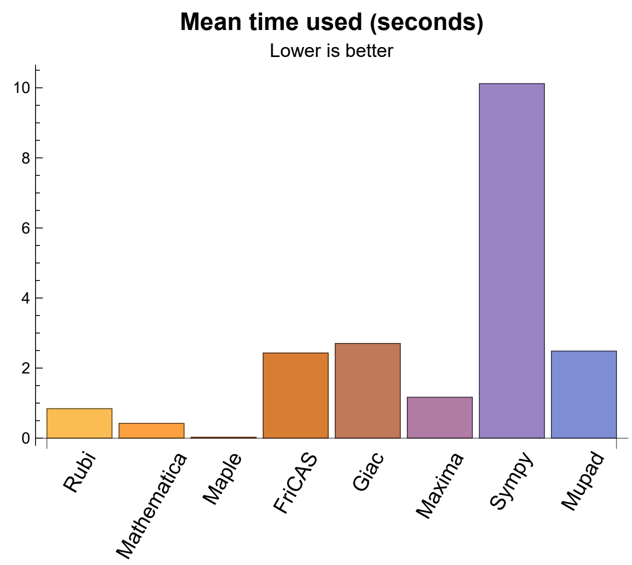
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.84	207.48	1.03	179.00	1.00
Mathematica	0.42	208.92	1.02	144.00	1.00
Maple	0.03	585.73	2.09	227.00	1.68
Maxima	1.17	104.12	0.94	65.00	0.88
Fricas	2.43	1240.70	4.64	137.00	1.37
Sympy	10.12	261.04	1.81	71.00	0.98
Giac	2.70	1805.94	5.40	227.00	1.22
Mupad	2.48	4593.26	13.26	176.00	0.96

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {40, 41}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
```

```
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

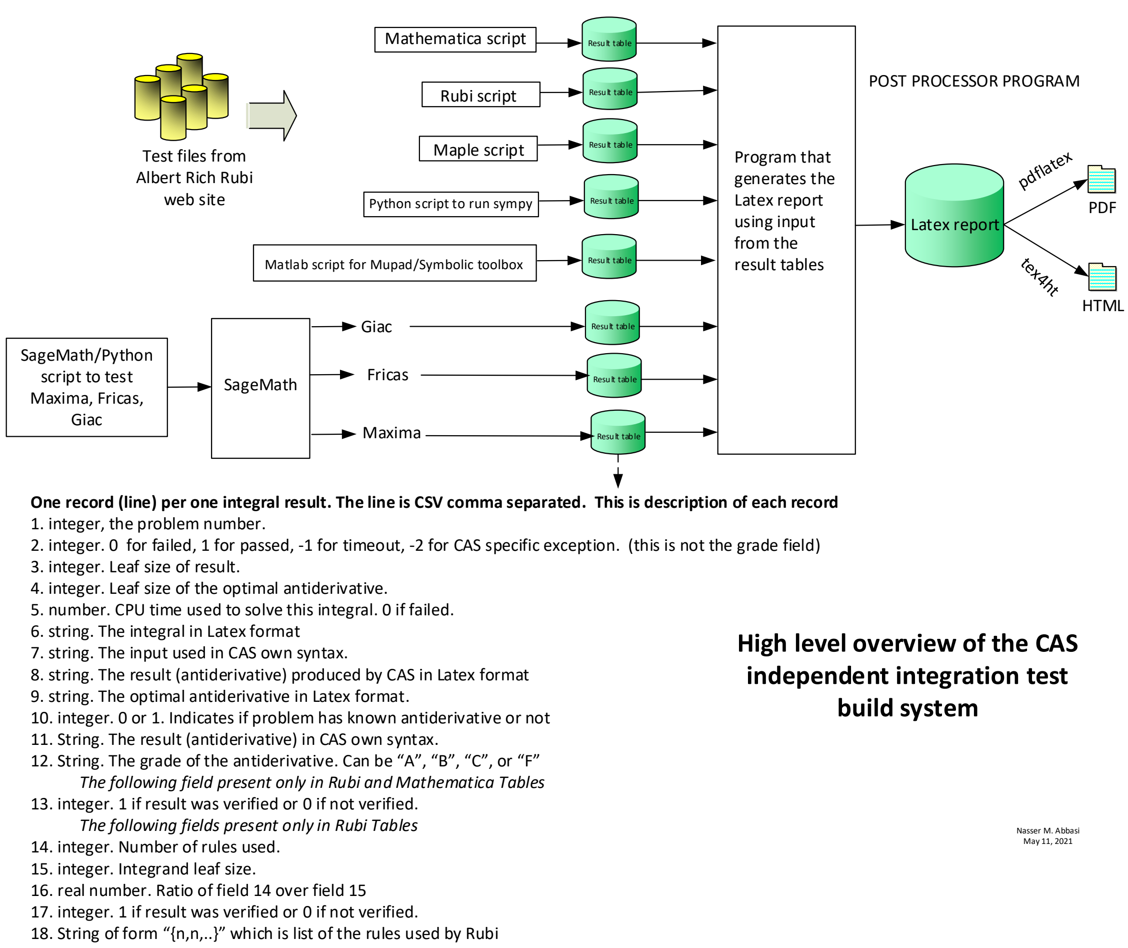
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 125, 126, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { 135, 136 }

C grade: { 40, 41, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 132, 133, 134 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 51, 64, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 131, 132, 133, 134, 143, 144, 145 }

B grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130 }

C grade: { 135, 136, 137, 138, 139, 140, 141, 142 }

F grade: { 40, 41 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 37, 38, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

B grade: { 37, 38, 39, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 69, 70, 71, 72, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 40, 41, 42, 43, 44, 45, 46, 68, 73, 126, 127, 128, 129, 130 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 39, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 115, 118, 119, 123, 124 }

B grade: { 64, 110, 113, 114, 116, 117, 120, 121, 122 }

C grade: { 133, 134, 135, 136, 141, 142 }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 71, 72, 73, 125, 126, 127, 128, 129, 130, 131, 132, 137, 138, 139, 140, 143, 144, 145 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51, 52, 53, 54, 61, 62, 63, 64, 65, 66, 67, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 125, 132, 133, 134, 139, 140 }

B grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 55, 56, 57, 58, 59, 60, 68, 69, 70, 71, 72, 73, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 126, 127, 128, 129, 130, 131, 141, 142, 143, 144, 145 }

C grade: { }

F grade: { 40, 41, 135, 136, 137, 138 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145 }

C grade: { }

F grade: { 40, 41 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	64	68	64	62
normalized size	1	1.00	1.00	0.82	0.81	0.86	0.92	0.86	0.84
time (sec)	N/A	0.082	0.015	0.000	0.452	0.663	0.071	0.286	0.036

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	61	60	64	68	64	62
normalized size	1	1.00	1.00	0.82	0.81	0.86	0.92	0.86	0.84
time (sec)	N/A	0.057	0.011	0.003	0.465	0.607	0.072	0.227	0.030

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	58	57	61	65	61	59
normalized size	1	1.00	1.00	0.84	0.83	0.88	0.94	0.88	0.86
time (sec)	N/A	0.036	0.012	0.000	0.455	0.519	0.069	0.377	0.029

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	60	55	55	63	60	57
normalized size	1	1.00	1.00	0.92	0.85	0.85	0.97	0.92	0.88
time (sec)	N/A	0.040	0.015	0.002	0.604	0.749	0.157	0.372	0.036

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	57	55	62	58	57	56
normalized size	1	1.00	1.00	0.90	0.87	0.98	0.92	0.90	0.89
time (sec)	N/A	0.051	0.022	0.006	0.680	0.822	0.175	0.299	0.037

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	58	58	55	62	61	58	56
normalized size	1	1.00	0.92	0.92	0.87	0.98	0.97	0.92	0.89
time (sec)	N/A	0.048	0.038	0.010	0.752	0.502	0.294	0.290	0.035

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	60	57	56	62	63	56	55
normalized size	1	1.00	0.95	0.90	0.89	0.98	1.00	0.89	0.87
time (sec)	N/A	0.051	0.045	0.007	0.659	0.742	0.521	0.369	0.033

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	58	56	62	63	57	56
normalized size	1	1.00	0.98	0.92	0.89	0.98	1.00	0.90	0.89
time (sec)	N/A	0.051	0.027	0.006	0.752	0.722	1.755	0.288	0.048

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	63	60	56	62	66	57	56
normalized size	1	1.00	1.00	0.95	0.89	0.98	1.05	0.90	0.89
time (sec)	N/A	0.052	0.054	0.007	0.739	0.571	5.698	0.259	0.777

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	68	63	59	62	70	60	60
normalized size	1	1.00	1.00	0.93	0.87	0.91	1.03	0.88	0.88
time (sec)	N/A	0.048	0.045	0.006	0.874	0.659	15.378	0.390	0.790

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	154	168	154	141
normalized size	1	1.00	1.00	0.89	0.90	0.97	1.06	0.97	0.89
time (sec)	N/A	0.214	0.044	0.001	1.134	0.469	0.093	0.406	0.817

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	142	143	154	163	154	141
normalized size	1	1.00	1.00	0.89	0.90	0.97	1.03	0.97	0.89
time (sec)	N/A	0.143	0.036	0.001	1.182	0.549	0.094	0.306	0.068

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	139	140	151	165	151	138
normalized size	1	1.00	1.00	0.90	0.91	0.98	1.07	0.98	0.90
time (sec)	N/A	0.111	0.029	0.002	0.629	0.554	0.093	0.300	0.070

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	150	149	138	138	156	149	135
normalized size	1	1.00	1.00	0.99	0.92	0.92	1.04	0.99	0.90
time (sec)	N/A	0.107	0.038	0.004	0.843	0.582	0.306	0.364	0.799

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	145	147	137	145	156	147	135
normalized size	1	1.00	1.00	1.01	0.94	1.00	1.08	1.01	0.93
time (sec)	N/A	0.121	0.091	0.007	0.734	0.647	0.322	0.282	0.797

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	139	148	139	145	153	148	135
normalized size	1	1.00	0.93	0.99	0.93	0.97	1.03	0.99	0.91
time (sec)	N/A	0.123	0.094	0.007	0.622	0.684	0.460	0.395	0.792

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	151	146	140	145	160	146	137
normalized size	1	1.00	1.01	0.98	0.94	0.97	1.07	0.98	0.92
time (sec)	N/A	0.137	0.077	0.009	0.681	0.752	0.719	0.284	0.059

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	130	144	139	145	153	142	134
normalized size	1	1.00	0.88	0.97	0.94	0.98	1.03	0.96	0.91
time (sec)	N/A	0.142	0.081	0.008	0.624	0.919	2.349	0.377	0.058

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	142	144	138	145	155	140	136
normalized size	1	1.00	0.99	1.01	0.97	1.01	1.08	0.98	0.95
time (sec)	N/A	0.147	0.077	0.009	0.612	0.760	7.809	0.292	0.054

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	144	148	140	145	158	141	136
normalized size	1	1.00	0.97	0.99	0.94	0.97	1.06	0.95	0.91
time (sec)	N/A	0.143	0.092	0.009	0.688	0.594	27.402	0.396	0.057

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	460	1622	0	0	0	5304	2588
normalized size	1	1.00	1.36	4.78	0.00	0.00	0.00	15.65	7.63
time (sec)	N/A	1.856	0.572	0.065	0.000	0.000	0.000	5.749	0.958

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	377	1171	0	0	0	3519	2696
normalized size	1	1.00	1.36	4.21	0.00	0.00	0.00	12.66	9.70
time (sec)	N/A	0.466	0.420	0.054	0.000	0.000	0.000	5.025	1.527

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	360	1327	0	0	0	3843	1890
normalized size	1	1.00	1.33	4.91	0.00	0.00	0.00	14.23	7.00
time (sec)	N/A	0.835	0.366	0.050	0.000	0.000	0.000	5.566	2.001

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	240	728	0	0	0	2369	5594
normalized size	1	1.00	1.08	3.26	0.00	0.00	0.00	10.62	25.09
time (sec)	N/A	0.213	0.359	0.040	0.000	0.000	0.000	5.359	1.885

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	234	616	0	0	0	1616	3942
normalized size	1	1.00	1.11	2.92	0.00	0.00	0.00	7.66	18.68
time (sec)	N/A	0.266	0.209	0.025	0.000	0.000	0.000	4.372	2.306

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	285	488	0	0	0	2336	2258
normalized size	1	1.00	1.24	2.13	0.00	0.00	0.00	10.20	9.86
time (sec)	N/A	0.259	0.445	0.037	0.000	0.000	0.000	5.075	1.493

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	315	811	0	0	0	3507	2588
normalized size	1	1.00	1.21	3.12	0.00	0.00	0.00	13.49	9.95
time (sec)	N/A	0.471	1.080	0.040	0.000	0.000	0.000	5.393	1.022

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	377	1054	0	0	0	3353	3563
normalized size	1	1.00	1.31	3.66	0.00	0.00	0.00	11.64	12.37
time (sec)	N/A	0.474	0.889	0.056	0.000	0.000	0.000	5.872	1.172

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	444	1429	0	0	0	5219	4754
normalized size	1	1.00	1.08	3.47	0.00	0.00	0.00	12.67	11.54
time (sec)	N/A	1.334	1.382	0.062	0.000	0.000	0.000	8.471	1.774

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	358	831	0	0	0	3228	3278
normalized size	1	1.00	1.03	2.39	0.00	0.00	0.00	9.30	9.45
time (sec)	N/A	0.618	0.894	0.044	0.000	0.000	0.000	5.369	1.613

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	4440	3835
normalized size	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.902	1.039	0.055	0.000	0.000	0.000	7.049	1.671

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	335	1344	0	0	0	3013	3198
normalized size	1	1.00	1.06	4.24	0.00	0.00	0.00	9.50	10.09
time (sec)	N/A	0.415	1.249	0.177	0.000	0.000	0.000	5.173	1.595

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	393	1813	0	0	0	5158	4707
normalized size	1	1.00	1.07	4.93	0.00	0.00	0.00	14.02	12.79
time (sec)	N/A	0.867	1.224	0.151	0.000	0.000	0.000	7.849	1.675

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	458	1603	0	0	0	6022	8129
normalized size	1	1.00	1.14	3.98	0.00	0.00	0.00	14.94	20.17
time (sec)	N/A	0.932	1.473	0.063	0.000	0.000	0.000	6.553	1.838

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	559	2398	0	0	0	9015	8684
normalized size	1	1.00	1.09	4.67	0.00	0.00	0.00	17.54	16.89
time (sec)	N/A	1.486	2.027	0.085	0.000	0.000	0.000	11.545	2.468

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	655	2512	0	0	0	6938	10595
normalized size	1	1.00	1.23	4.70	0.00	0.00	0.00	12.99	19.84
time (sec)	N/A	1.992	2.470	0.096	0.000	0.000	0.000	7.565	2.773

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	296	5520	611	3898	0	7808	2443
normalized size	1	1.00	0.74	13.83	1.53	9.77	0.00	19.57	6.12
time (sec)	N/A	0.425	0.919	0.013	1.706	1.909	0.000	1.131	3.280

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	185	2187	344	1603	0	3203	1314
normalized size	1	1.00	0.71	8.41	1.32	6.17	0.00	12.32	5.05
time (sec)	N/A	0.223	0.277	0.010	1.731	1.041	0.000	0.725	1.810

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	90	585	155	444	3735	914	527
normalized size	1	1.00	0.66	4.27	1.13	3.24	27.26	6.67	3.85
time (sec)	N/A	0.088	0.104	0.005	0.815	1.338	2.575	0.533	1.075

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	438	0	0	0	0	0	-1
normalized size	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.622	0.474	0.035	0.000	1.068	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	685	670	242	0	0	0	0	0	-1
normalized size	1	0.98	0.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.378	0.327	0.031	0.000	1.106	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	4440	3835
normalized size	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.924	1.038	0.000	0.000	0.000	0.000	7.038	0.004

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	4440	3835
normalized size	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.371	0.425	0.036	0.000	0.000	0.000	7.135	1.551

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	4439	3835
normalized size	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.370	0.154	0.039	0.000	0.000	0.000	6.896	1.474

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	4439	3835
normalized size	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.356	0.154	0.034	0.000	0.000	0.000	7.311	1.389

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	378	1119	0	0	0	4439	3835
normalized size	1	1.00	1.06	3.14	0.00	0.00	0.00	12.47	10.77
time (sec)	N/A	0.359	0.153	0.033	0.000	0.000	0.000	6.836	1.414

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	260	622	0	900	0	306	2972
normalized size	1	1.00	0.95	2.28	0.00	3.30	0.00	1.12	10.89
time (sec)	N/A	0.854	0.199	0.007	0.000	1.846	0.000	1.868	1.604

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	193	474	0	677	0	214	2295
normalized size	1	1.00	0.95	2.33	0.00	3.33	0.00	1.05	11.31
time (sec)	N/A	0.424	0.143	0.006	0.000	1.957	0.000	2.005	1.626

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	136	321	0	473	0	141	1689
normalized size	1	1.00	0.94	2.23	0.00	3.28	0.00	0.98	11.73
time (sec)	N/A	0.272	0.099	0.006	0.000	1.504	0.000	1.987	1.300

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	100	211	0	318	0	99	1081
normalized size	1	1.00	0.97	2.05	0.00	3.09	0.00	0.96	10.50
time (sec)	N/A	0.179	0.066	0.004	0.000	1.337	0.000	1.780	1.830

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	178	165	0	309	0	97	3927
normalized size	1	1.00	1.84	1.70	0.00	3.19	0.00	1.00	40.48
time (sec)	N/A	0.200	0.137	0.009	0.000	1.410	0.000	1.903	8.881

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	203	227	0	399	0	135	4437
normalized size	1	1.00	1.72	1.92	0.00	3.38	0.00	1.14	37.60
time (sec)	N/A	0.285	0.150	0.010	0.000	1.643	0.000	1.777	7.857

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	314	356	0	609	0	212	6187
normalized size	1	1.00	1.80	2.05	0.00	3.50	0.00	1.22	35.56
time (sec)	N/A	0.407	0.345	0.012	0.000	2.538	0.000	1.719	9.917

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	416	523	0	834	0	313	9141
normalized size	1	1.00	1.70	2.14	0.00	3.42	0.00	1.28	37.46
time (sec)	N/A	0.573	0.348	0.013	0.000	5.313	0.000	1.941	13.829

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	369	369	456	1450	0	15467	0	7243	23332
normalized size	1	1.00	1.24	3.93	0.00	41.92	0.00	19.63	63.23
time (sec)	N/A	4.577	0.509	0.035	0.000	35.653	0.000	5.025	4.912

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	365	1035	0	9364	0	5461	15674
normalized size	1	1.00	1.29	3.67	0.00	33.21	0.00	19.37	55.58
time (sec)	N/A	3.590	0.492	0.030	0.000	8.048	0.000	4.755	3.359

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	258	676	0	5788	0	4086	10209
normalized size	1	1.00	1.18	3.09	0.00	26.43	0.00	18.66	46.62
time (sec)	N/A	0.637	0.326	0.027	0.000	4.487	0.000	3.908	3.360

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	253	563	0	5930	0	3988	10170
normalized size	1	1.00	1.19	2.64	0.00	27.84	0.00	18.72	47.75
time (sec)	N/A	0.839	0.302	0.025	0.000	2.258	0.000	5.936	3.515

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	284	727	0	9850	0	3813	15505
normalized size	1	1.00	1.06	2.72	0.00	36.89	0.00	14.28	58.07
time (sec)	N/A	1.065	0.339	0.027	0.000	10.542	0.000	3.442	4.763

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	394	1121	0	15830	0	6718	23019
normalized size	1	1.00	1.20	3.41	0.00	48.12	0.00	20.42	69.97
time (sec)	N/A	1.942	0.551	0.033	0.000	38.588	0.000	7.015	6.247

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	309	1167	0	2111	0	424	3499
normalized size	1	1.00	0.97	3.65	0.00	6.60	0.00	1.32	10.93
time (sec)	N/A	1.233	0.497	0.024	0.000	1.656	0.000	1.952	1.333

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	236	832	0	1455	0	279	2450
normalized size	1	1.00	1.00	3.53	0.00	6.17	0.00	1.18	10.38
time (sec)	N/A	0.440	0.356	0.017	0.000	1.014	0.000	1.865	1.811

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	175	336	0	970	0	195	1651
normalized size	1	1.00	1.06	2.04	0.00	5.88	0.00	1.18	10.01
time (sec)	N/A	0.287	0.249	0.015	0.000	1.056	0.000	1.838	2.717

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	130	205	0	650	474	140	342
normalized size	1	1.00	1.06	1.67	0.00	5.28	3.85	1.14	2.78
time (sec)	N/A	0.184	0.102	0.012	0.000	0.890	38.035	2.167	0.378

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	268	462	0	1103	0	227	8706
normalized size	1	1.00	1.61	2.78	0.00	6.64	0.00	1.37	52.45
time (sec)	N/A	0.394	0.445	0.017	0.000	3.256	0.000	2.001	11.849

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	403	722	0	1764	0	287	11879
normalized size	1	1.00	1.72	3.09	0.00	7.54	0.00	1.23	50.76
time (sec)	N/A	0.725	0.658	0.023	0.000	7.130	0.000	1.846	12.979

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	592	1078	0	2567	0	535	15905
normalized size	1	1.00	1.80	3.28	0.00	7.80	0.00	1.63	48.34
time (sec)	N/A	1.157	1.216	0.029	0.000	16.734	0.000	1.888	21.016

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	550	550	648	2558	0	0	0	8957	33799
normalized size	1	1.00	1.18	4.65	0.00	0.00	0.00	16.29	61.45
time (sec)	N/A	13.227	2.131	0.056	0.000	0.000	0.000	9.044	4.104

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	511	1977	0	12597	0	7496	25862
normalized size	1	1.00	1.17	4.53	0.00	28.89	0.00	17.19	59.32
time (sec)	N/A	5.541	1.542	0.052	0.000	17.362	0.000	8.250	2.648

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	414	1300	0	8951	0	6208	19494
normalized size	1	1.00	1.14	3.59	0.00	24.73	0.00	17.15	53.85
time (sec)	N/A	2.498	1.103	0.043	0.000	8.443	0.000	6.811	6.543

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	382	1182	0	8991	0	6356	19589
normalized size	1	1.00	1.10	3.42	0.00	25.99	0.00	18.37	56.62
time (sec)	N/A	1.896	1.081	0.040	0.000	8.489	0.000	6.973	6.552

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	444	1575	0	13111	0	7182	28164
normalized size	1	1.00	1.11	3.95	0.00	32.86	0.00	18.00	70.59
time (sec)	N/A	2.203	1.319	0.048	0.000	19.288	0.000	7.093	6.862

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	575	548	2180	0	0	0	8660	36097
normalized size	1	1.00	0.95	3.79	0.00	0.00	0.00	15.06	62.78
time (sec)	N/A	9.906	1.797	0.065	0.000	0.000	0.000	8.591	7.370

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	56	58	82	61	63	57
normalized size	1	1.00	0.91	0.82	0.85	1.21	0.90	0.93	0.84
time (sec)	N/A	0.126	0.028	0.017	0.602	0.785	0.172	0.362	0.056

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	51	53	77	56	58	53
normalized size	1	1.00	1.00	0.84	0.87	1.26	0.92	0.95	0.87
time (sec)	N/A	0.118	0.027	0.016	0.719	0.829	0.171	0.323	0.039

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	46	48	72	48	53	47
normalized size	1	1.00	1.00	0.85	0.89	1.33	0.89	0.98	0.87
time (sec)	N/A	0.108	0.024	0.016	1.075	0.708	0.173	0.367	0.897

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	41	43	67	44	45	43
normalized size	1	1.00	1.00	0.84	0.88	1.37	0.90	0.92	0.88
time (sec)	N/A	0.086	0.022	0.015	0.514	0.901	0.174	0.390	0.038

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	36	38	57	36	40	37
normalized size	1	1.00	1.00	0.86	0.90	1.36	0.86	0.95	0.88
time (sec)	N/A	0.049	0.017	0.016	0.522	1.045	0.166	0.354	0.049

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	38	44	71	41	47	40
normalized size	1	1.00	1.00	0.86	1.00	1.61	0.93	1.07	0.91
time (sec)	N/A	0.077	0.022	0.016	0.724	0.901	0.183	0.377	0.041

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	45	53	92	51	53	50
normalized size	1	1.00	0.91	0.82	0.96	1.67	0.93	0.96	0.91
time (sec)	N/A	0.104	0.025	0.017	0.792	0.980	0.205	0.374	0.044

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	50	56	97	56	66	55
normalized size	1	1.00	0.88	0.78	0.88	1.52	0.88	1.03	0.86
time (sec)	N/A	0.111	0.027	0.019	0.679	1.053	0.213	0.341	0.919

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	56	58	79	68	58	58
normalized size	1	1.00	1.01	0.80	0.83	1.13	0.97	0.83	0.83
time (sec)	N/A	0.085	0.044	0.012	1.639	1.087	0.210	0.331	0.952

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	58	49	51	74	54	51	50
normalized size	1	1.00	1.02	0.86	0.89	1.30	0.95	0.89	0.88
time (sec)	N/A	0.082	0.048	0.012	1.621	0.865	0.207	0.314	0.054

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	57	46	48	69	54	48	48
normalized size	1	1.00	1.02	0.82	0.86	1.23	0.96	0.86	0.86
time (sec)	N/A	0.073	0.042	0.013	1.635	0.909	0.210	0.306	0.918

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	50	41	43	64	48	43	42
normalized size	1	1.00	1.02	0.84	0.88	1.31	0.98	0.88	0.86
time (sec)	N/A	0.066	0.038	0.011	1.633	0.914	0.207	0.340	0.068

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	46	38	40	59	46	40	40
normalized size	1	1.00	0.96	0.79	0.83	1.23	0.96	0.83	0.83
time (sec)	N/A	0.028	0.040	0.012	1.529	1.075	0.203	0.339	0.072

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	51	43	45	68	49	45	45
normalized size	1	1.00	0.96	0.81	0.85	1.28	0.92	0.85	0.85
time (sec)	N/A	0.073	0.049	0.013	1.554	0.916	0.219	0.384	0.070

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	56	48	52	79	56	52	51
normalized size	1	1.00	0.90	0.77	0.84	1.27	0.90	0.84	0.82
time (sec)	N/A	0.084	0.051	0.015	1.597	0.906	0.240	0.386	0.923

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	61	53	57	84	61	57	57
normalized size	1	1.00	0.88	0.77	0.83	1.22	0.88	0.83	0.83
time (sec)	N/A	0.090	0.058	0.017	1.764	0.873	0.254	0.343	0.916

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	77	58	62	89	66	62	61
normalized size	1	1.00	1.01	0.76	0.82	1.17	0.87	0.82	0.80
time (sec)	N/A	0.100	0.055	0.016	1.528	0.877	0.276	0.450	0.074

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	71	64	71	114	75	61	70
normalized size	1	1.00	0.88	0.79	0.88	1.41	0.93	0.75	0.86
time (sec)	N/A	0.112	0.056	0.013	1.584	0.877	0.258	0.317	0.059

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	66	62	68	109	76	58	68
normalized size	1	1.00	0.82	0.78	0.85	1.36	0.95	0.72	0.85
time (sec)	N/A	0.100	0.054	0.013	1.914	0.862	0.256	0.414	0.052

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	56	63	104	70	53	63
normalized size	1	1.00	0.80	0.75	0.84	1.39	0.93	0.71	0.84
time (sec)	N/A	0.091	0.061	0.013	1.850	0.642	0.255	0.324	0.928

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	55	54	60	99	65	50	59
normalized size	1	1.00	0.76	0.75	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.068	0.058	0.011	1.527	0.851	0.262	0.345	0.928

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	60	99	66	50	60
normalized size	1	1.00	0.78	0.74	0.83	1.38	0.92	0.69	0.83
time (sec)	N/A	0.066	0.061	0.013	1.744	0.686	0.251	0.379	0.071

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	53	60	99	65	50	59
normalized size	1	1.00	0.78	0.74	0.83	1.38	0.90	0.69	0.82
time (sec)	N/A	0.037	0.059	0.013	1.671	0.897	0.248	0.328	0.070

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	63	58	65	108	71	55	65
normalized size	1	1.00	0.80	0.73	0.82	1.37	0.90	0.70	0.82
time (sec)	N/A	0.103	0.065	0.014	1.797	0.856	0.276	0.338	0.920

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	64	72	119	76	62	71
normalized size	1	1.00	0.91	0.74	0.84	1.38	0.88	0.72	0.83
time (sec)	N/A	0.119	0.059	0.016	1.527	1.013	0.293	0.361	0.923

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	73	68	77	124	82	67	77
normalized size	1	1.00	0.78	0.73	0.83	1.33	0.88	0.72	0.83
time (sec)	N/A	0.134	0.076	0.015	1.737	1.224	0.312	0.356	0.935

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	78	74	71	95	87	76	75
normalized size	1	1.00	0.91	0.86	0.83	1.10	1.01	0.88	0.87
time (sec)	N/A	0.135	0.045	0.010	1.333	1.051	0.183	1.018	0.901

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	73	69	66	90	80	71	69
normalized size	1	1.00	0.90	0.85	0.81	1.11	0.99	0.88	0.85
time (sec)	N/A	0.127	0.030	0.009	1.824	1.011	0.185	1.169	0.051

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	66	64	59	85	73	66	65
normalized size	1	1.00	0.89	0.86	0.80	1.15	0.99	0.89	0.88
time (sec)	N/A	0.121	0.029	0.009	1.585	1.057	0.180	1.090	0.045

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	59	54	80	68	54	60
normalized size	1	1.00	0.94	0.91	0.83	1.23	1.05	0.83	0.92
time (sec)	N/A	0.105	0.026	0.010	1.718	0.973	0.181	1.184	0.919

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	54	49	70	60	49	69
normalized size	1	1.00	1.00	0.93	0.84	1.21	1.03	0.84	1.19
time (sec)	N/A	0.067	0.022	0.010	1.406	0.787	0.179	1.133	0.048

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	93	58	55	84	65	62	59
normalized size	1	1.00	1.41	0.88	0.83	1.27	0.98	0.94	0.89
time (sec)	N/A	0.108	0.062	0.012	1.685	0.775	0.198	1.103	0.907

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	97	63	66	105	76	66	68
normalized size	1	1.00	1.37	0.89	0.93	1.48	1.07	0.93	0.96
time (sec)	N/A	0.134	0.050	0.013	1.427	0.582	0.212	1.072	0.061

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	105	68	71	110	80	79	72
normalized size	1	1.00	1.31	0.85	0.89	1.38	1.00	0.99	0.90
time (sec)	N/A	0.137	0.060	0.014	2.381	0.804	0.226	1.165	0.060

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	110	73	76	115	85	84	78
normalized size	1	1.00	1.26	0.84	0.87	1.32	0.98	0.97	0.90
time (sec)	N/A	0.149	0.066	0.014	2.474	0.841	0.241	1.165	0.065

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	145	427	0	519	71	585	171
normalized size	1	1.00	0.58	1.72	0.00	2.09	0.29	2.36	0.69
time (sec)	N/A	0.345	0.172	0.119	0.000	0.810	0.611	1.887	0.106

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	132	419	0	476	1205	576	164
normalized size	1	1.00	0.56	1.77	0.00	2.01	5.08	2.43	0.69
time (sec)	N/A	0.293	0.158	0.032	0.000	0.757	1.360	1.851	0.943

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	129	416	0	508	60	573	162
normalized size	1	1.00	0.56	1.79	0.00	2.19	0.26	2.47	0.70
time (sec)	N/A	0.292	0.159	0.030	0.000	1.091	0.615	1.854	0.095

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	121	412	0	459	51	566	156
normalized size	1	1.00	0.54	1.83	0.00	2.04	0.23	2.52	0.69
time (sec)	N/A	0.297	0.164	0.034	0.000	0.845	0.602	1.825	0.958

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	115	408	0	454	1185	565	153
normalized size	1	1.00	0.51	1.82	0.00	2.03	5.29	2.52	0.68
time (sec)	N/A	0.215	0.264	0.029	0.000	0.601	1.291	1.818	0.127

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	126	414	0	471	1192	572	159
normalized size	1	1.00	0.55	1.81	0.00	2.06	5.21	2.50	0.69
time (sec)	N/A	0.310	0.176	0.033	0.000	0.832	1.323	1.937	0.136

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	131	419	0	528	60	579	165
normalized size	1	1.00	0.55	1.76	0.00	2.22	0.25	2.43	0.69
time (sec)	N/A	0.335	0.291	0.035	0.000	0.852	0.653	1.851	0.137

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	140	424	0	496	1202	584	171
normalized size	1	1.00	0.57	1.73	0.00	2.02	4.91	2.38	0.70
time (sec)	N/A	0.329	0.290	0.034	0.000	0.790	1.332	1.785	0.143

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	156	429	0	561	1204	588	184
normalized size	1	1.00	0.64	1.77	0.00	2.31	4.95	2.42	0.76
time (sec)	N/A	0.360	0.216	0.035	0.000	0.760	1.350	2.694	0.111

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	155	426	0	557	82	585	182
normalized size	1	1.00	0.64	1.76	0.00	2.30	0.34	2.42	0.75
time (sec)	N/A	0.310	0.204	0.033	0.000	0.840	0.680	2.588	0.942

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	138	422	0	551	71	580	176
normalized size	1	1.00	0.59	1.80	0.00	2.34	0.30	2.47	0.75
time (sec)	N/A	0.300	0.317	0.035	0.000	0.946	0.656	2.612	0.992

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	129	418	0	546	1198	577	173
normalized size	1	1.00	0.54	1.76	0.00	2.29	5.03	2.42	0.73
time (sec)	N/A	0.290	0.297	0.032	0.000	0.878	1.306	2.600	0.146

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	133	418	0	570	1200	577	174
normalized size	1	1.00	0.54	1.70	0.00	2.32	4.88	2.35	0.71
time (sec)	N/A	0.284	0.297	0.032	0.000	0.872	1.332	2.688	1.012

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	129	418	0	576	1195	577	173
normalized size	1	1.00	0.52	1.69	0.00	2.32	4.82	2.33	0.70
time (sec)	N/A	0.254	0.291	0.032	0.000	0.857	1.337	2.618	1.008

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	140	424	0	630	75	582	179
normalized size	1	1.00	0.55	1.68	0.00	2.49	0.30	2.30	0.71
time (sec)	N/A	0.343	0.366	0.034	0.000	0.909	0.672	3.266	0.993

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	139	429	0	652	80	589	185
normalized size	1	1.00	0.53	1.64	0.00	2.49	0.31	2.25	0.71
time (sec)	N/A	0.366	0.315	0.036	0.000	0.866	0.689	2.991	1.022

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	142	357	0	486	0	146	1834
normalized size	1	1.00	0.95	2.40	0.00	3.26	0.00	0.98	12.31
time (sec)	N/A	0.295	0.116	0.006	0.000	1.071	0.000	1.898	1.685

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	594	594	721	3028	0	0	0	10761	47339
normalized size	1	1.00	1.21	5.10	0.00	0.00	0.00	18.12	79.70
time (sec)	N/A	14.113	2.641	0.060	0.000	0.000	0.000	9.953	4.731

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	575	2300	0	0	0	9170	36589
normalized size	1	1.00	1.22	4.88	0.00	0.00	0.00	19.47	77.68
time (sec)	N/A	6.662	1.946	0.047	0.000	0.000	0.000	8.921	4.218

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	512	1760	0	0	0	8913	32587
normalized size	1	1.00	1.14	3.92	0.00	0.00	0.00	19.85	72.58
time (sec)	N/A	2.867	1.651	0.053	0.000	0.000	0.000	8.514	5.821

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	529	2045	0	0	0	9176	40860
normalized size	1	1.00	1.15	4.45	0.00	0.00	0.00	19.95	88.83
time (sec)	N/A	2.791	2.434	0.055	0.000	0.000	0.000	8.303	7.761

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	542	542	612	2503	0	0	0	10422	51386
normalized size	1	1.00	1.13	4.62	0.00	0.00	0.00	19.23	94.81
time (sec)	N/A	7.265	2.153	0.067	0.000	0.000	0.000	9.055	8.467

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	21	31	31	0	58	31
normalized size	1	1.00	1.00	1.05	1.55	1.55	0.00	2.90	1.55
time (sec)	N/A	0.036	0.141	0.010	1.028	1.059	0.000	0.609	1.098

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	278	265	145	295	138	0	269	287
normalized size	1	1.32	1.26	0.69	1.40	0.66	0.00	1.28	1.37
time (sec)	N/A	0.315	1.379	0.009	1.026	1.019	0.000	0.894	1.645

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	213	232	109	217	104	367	194	215
normalized size	1	1.34	1.46	0.69	1.36	0.65	2.31	1.22	1.35
time (sec)	N/A	0.189	1.069	0.007	1.062	0.857	135.140	0.749	1.494

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	149	194	73	139	71	350	121	143
normalized size	1	1.37	1.78	0.67	1.28	0.65	3.21	1.11	1.31
time (sec)	N/A	0.122	0.691	0.006	1.032	1.031	90.666	0.626	1.377

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	151	217	143	105	80	304	0	161
normalized size	1	1.62	2.33	1.54	1.13	0.86	3.27	0.00	1.73
time (sec)	N/A	0.165	0.880	0.042	1.004	0.963	91.279	0.000	2.949

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	C	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	155	233	163	123	98	270	0	422
normalized size	1	1.57	2.35	1.65	1.24	0.99	2.73	0.00	4.26
time (sec)	N/A	0.252	0.213	0.024	1.017	0.801	133.790	0.000	5.151

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	182	134	222	193	102	0	0	932
normalized size	1	1.44	1.06	1.76	1.53	0.81	0.00	0.00	7.40
time (sec)	N/A	0.277	0.161	0.030	1.034	0.798	0.000	0.000	10.816

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	248	173	306	271	137	0	0	1621
normalized size	1	1.17	0.82	1.44	1.28	0.65	0.00	0.00	7.65
time (sec)	N/A	0.373	0.192	0.036	1.033	0.815	0.000	0.000	20.051

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	245	202	273	190	134	0	208	1132
normalized size	1	1.13	0.94	1.26	0.88	0.62	0.00	0.96	5.24
time (sec)	N/A	0.205	0.791	0.037	1.106	0.850	0.000	0.749	23.121

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	179	157	191	113	100	0	135	651
normalized size	1	1.40	1.23	1.49	0.88	0.78	0.00	1.05	5.09
time (sec)	N/A	0.091	0.562	0.020	1.018	0.947	0.000	0.595	12.861

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	155	135	148	73	90	287	257	306
normalized size	1	1.52	1.32	1.45	0.72	0.88	2.81	2.52	3.00
time (sec)	N/A	0.122	0.555	0.023	1.052	0.959	104.024	1.046	7.002

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	81	146	85	90	257	555	138
normalized size	1	1.00	0.52	0.93	0.54	0.57	1.64	3.54	0.88
time (sec)	N/A	0.125	0.124	0.026	1.005	0.832	116.433	1.467	2.268

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	87	82	148	76	0	1103	146
normalized size	1	1.00	0.54	0.51	0.92	0.48	0.00	6.89	0.91
time (sec)	N/A	0.145	0.118	0.005	1.019	0.858	0.000	2.546	1.732

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	124	118	226	110	0	1517	218
normalized size	1	1.00	0.55	0.52	1.00	0.49	0.00	6.71	0.96
time (sec)	N/A	0.178	0.142	0.007	1.010	0.972	0.000	4.731	1.817

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	158	154	304	144	0	1931	290
normalized size	1	1.00	0.54	0.53	1.04	0.49	0.00	6.61	0.99
time (sec)	N/A	0.242	0.176	0.007	1.031	1.024	0.000	7.348	1.873

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [.4643]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	26	0.038
2	A	2	1	1.00	24	0.042
3	A	2	1	1.00	23	0.043
4	A	2	1	1.00	26	0.038
5	A	2	1	1.00	26	0.038
6	A	2	1	1.00	26	0.038
7	A	2	1	1.00	26	0.038
8	A	2	1	1.00	26	0.038
9	A	2	1	1.00	26	0.038
10	A	2	1	1.00	26	0.038
11	A	2	1	1.00	28	0.036
12	A	2	1	1.00	26	0.038
13	A	2	1	1.00	25	0.040

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	2	1	1.00	28	0.036
15	A	2	1	1.00	28	0.036
16	A	2	1	1.00	28	0.036
17	A	2	1	1.00	28	0.036
18	A	2	1	1.00	28	0.036
19	A	2	1	1.00	28	0.036
20	A	2	1	1.00	28	0.036
21	A	13	11	1.00	28	0.393
22	A	12	11	1.00	28	0.393
23	A	11	10	1.00	28	0.357
24	A	10	9	1.00	26	0.346
25	A	8	7	1.00	25	0.280
26	A	12	10	1.00	28	0.357
27	A	13	12	1.00	28	0.429
28	A	13	11	1.00	28	0.393
29	A	11	10	1.00	28	0.357
30	A	10	9	1.00	28	0.321
31	A	10	9	1.00	28	0.321
32	A	10	9	1.00	26	0.346
33	A	10	9	1.00	25	0.360
34	A	14	12	1.00	28	0.429
35	A	15	13	1.00	28	0.464
36	A	15	13	1.00	28	0.464
37	A	2	1	1.00	30	0.033
38	A	2	1	1.00	30	0.033
39	A	2	1	1.00	28	0.036
40	A	8	5	1.00	30	0.167
41	A	10	6	0.98	30	0.200
42	A	10	9	1.00	28	0.321
43	A	11	10	1.00	30	0.333
44	A	11	10	1.00	31	0.323
45	A	11	10	1.00	34	0.294
46	A	11	10	1.00	34	0.294
47	A	7	6	1.00	30	0.200
48	A	7	6	1.00	30	0.200
49	A	7	6	1.00	30	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	7	6	1.00	28	0.214
51	A	7	6	1.00	30	0.200
52	A	7	6	1.00	30	0.200
53	A	7	6	1.00	30	0.200
54	A	7	6	1.00	30	0.200
55	A	5	3	1.00	30	0.100
56	A	5	3	1.00	30	0.100
57	A	5	3	1.00	27	0.111
58	A	5	3	1.00	30	0.100
59	A	5	3	1.00	30	0.100
60	A	5	3	1.00	30	0.100
61	A	8	7	1.00	30	0.233
62	A	7	7	1.00	30	0.233
63	A	6	6	1.00	30	0.200
64	A	5	5	1.00	28	0.179
65	A	8	7	1.00	30	0.233
66	A	8	7	1.00	30	0.233
67	A	8	7	1.00	30	0.233
68	A	6	4	1.00	30	0.133
69	A	6	4	1.00	30	0.133
70	A	4	3	1.00	30	0.100
71	A	4	3	1.00	27	0.111
72	A	6	4	1.00	30	0.133
73	A	6	4	1.00	30	0.133
74	A	7	5	1.00	31	0.161
75	A	7	5	1.00	31	0.161
76	A	7	5	1.00	31	0.161
77	A	7	5	1.00	31	0.161
78	A	5	4	1.00	29	0.138
79	A	4	3	1.00	31	0.097
80	A	4	3	1.00	31	0.097
81	A	4	3	1.00	31	0.097
82	A	6	4	1.00	31	0.129
83	A	6	4	1.00	31	0.129
84	A	6	4	1.00	31	0.129
85	A	6	4	1.00	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	1.00	28	0.107
87	A	5	3	1.00	31	0.097
88	A	5	3	1.00	31	0.097
89	A	5	3	1.00	31	0.097
90	A	5	3	1.00	31	0.097
91	A	7	5	1.00	31	0.161
92	A	7	5	1.00	31	0.161
93	A	7	5	1.00	31	0.161
94	A	5	4	1.00	31	0.129
95	A	5	4	1.00	31	0.129
96	A	5	4	1.00	28	0.143
97	A	6	3	1.00	31	0.097
98	A	6	3	1.00	31	0.097
99	A	6	3	1.00	31	0.097
100	A	8	7	1.00	31	0.226
101	A	8	7	1.00	31	0.226
102	A	8	7	1.00	31	0.226
103	A	8	7	1.00	31	0.226
104	A	6	6	1.00	29	0.207
105	A	8	7	1.00	31	0.226
106	A	8	7	1.00	31	0.226
107	A	8	7	1.00	31	0.226
108	A	8	7	1.00	31	0.226
109	A	12	7	1.00	31	0.226
110	A	12	7	1.00	31	0.226
111	A	12	7	1.00	31	0.226
112	A	12	7	1.00	31	0.226
113	A	10	6	1.00	28	0.214
114	A	12	7	1.00	31	0.226
115	A	12	7	1.00	31	0.226
116	A	12	7	1.00	31	0.226
117	A	13	8	1.00	31	0.258
118	A	13	8	1.00	31	0.258
119	A	13	8	1.00	31	0.258
120	A	11	7	1.00	31	0.226
121	A	11	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	11	7	1.00	28	0.250
123	A	13	7	1.00	31	0.226
124	A	13	7	1.00	31	0.226
125	A	7	6	1.00	33	0.182
126	A	6	4	1.00	35	0.114
127	A	6	4	1.00	35	0.114
128	A	4	3	1.00	32	0.094
129	A	6	4	1.00	35	0.114
130	A	6	4	1.00	35	0.114
131	A	1	1	1.00	42	0.024
132	A	5	4	1.32	35	0.114
133	A	4	3	1.34	35	0.086
134	A	4	3	1.37	33	0.091
135	A	6	5	1.62	35	0.143
136	A	6	6	1.57	35	0.171
137	A	6	6	1.44	35	0.171
138	A	7	7	1.17	35	0.200
139	A	6	6	1.13	35	0.171
140	A	5	5	1.40	32	0.156
141	A	5	5	1.52	35	0.143
142	A	5	5	1.00	35	0.143
143	A	4	4	1.00	35	0.114
144	A	5	5	1.00	35	0.143
145	A	6	5	1.00	35	0.143

Chapter 3

Listing of integrals

3.1 $\int x^2 \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=74

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8
*B*c*x^8+1/9*c*C*x^9
```

Rubi [A] time = 0.08, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)dx &= \int \left(aAx^2+aBx^3+(Ab+aC)x^4+bBx^5+(Ac+bC)x^6+Bcx^7+\right.\\ &= \frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{5}(Ab+aC)x^5+\frac{1}{6}bBx^6+\frac{1}{7}(Ac+bC)x^7+\frac{1}{8}B\end{aligned}$$

Mathematica [A] time = 0.02, size = 74, normalized size = 1.00

$$\frac{1}{5}x^5(aC+Ab)+\frac{1}{3}aAx^3+\frac{1}{4}aBx^4+\frac{1}{7}x^7(Ac+bC)+\frac{1}{6}bBx^6+\frac{1}{8}Bcx^8+\frac{1}{9}cCx^9$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^3)/3 + (a*B*x^4)/4 + ((A*b + a*C)*x^5)/5 + (b*B*x^6)/6 + ((A*c + b*C
)*x^7)/7 + (B*c*x^8)/8 + (c*C*x^9)/9
```

fricas [A] time = 0.66, size = 64, normalized size = 0.86

$$\frac{1}{9}x^9cC + \frac{1}{8}x^8cB + \frac{1}{7}x^7bC + \frac{1}{7}x^7cA + \frac{1}{6}x^6bB + \frac{1}{5}x^5aC + \frac{1}{5}x^5bA + \frac{1}{4}x^4aB + \frac{1}{3}x^3aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c*C + 1/8*x^8*c*B + 1/7*x^7*b*C + 1/7*x^7*c*A + 1/6*x^6*b*B + 1/5*x
^5*a*C + 1/5*x^5*b*A + 1/4*x^4*a*B + 1/3*x^3*a*A
```

giac [A] time = 0.29, size = 64, normalized size = 0.86

$$\frac{1}{9}\,Ccx^9 + \frac{1}{8}\,Bcx^8 + \frac{1}{7}\,Cbx^7 + \frac{1}{7}\,Acx^7 + \frac{1}{6}\,Bbx^6 + \frac{1}{5}\,Cax^5 + \frac{1}{5}\,Abx^5 + \frac{1}{4}\,Bax^4 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/7*C*b*x^7 + 1/7*A*c*x^7 + 1/6*B*b*x^6 + 1/5*C
*a*x^5 + 1/5*A*b*x^5 + 1/4*B*a*x^4 + 1/3*A*a*x^3
```

maple [A] time = 0.00, size = 61, normalized size = 0.82

$$\frac{Cc\,x^9}{9} + \frac{Bc\,x^8}{8} + \frac{Bb\,x^6}{6} + \frac{(Ac + bC)\,x^7}{7} + \frac{Ba\,x^4}{4} + \frac{Aa\,x^3}{3} + \frac{(Ab + aC)\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*a*A*x^3+1/4*a*B*x^4+1/5*(A*b+C*a)*x^5+1/6*b*B*x^6+1/7*(A*c+C*b)*x^7+1/8
*B*c*x^8+1/9*c*C*x^9
```

maxima [A] time = 0.45, size = 60, normalized size = 0.81

$$\frac{1}{9}\,Ccx^9 + \frac{1}{8}\,Bcx^8 + \frac{1}{6}\,Bbx^6 + \frac{1}{7}\,(Cb + Ac)x^7 + \frac{1}{4}\,Bax^4 + \frac{1}{5}\,(Ca + Ab)x^5 + \frac{1}{3}\,Aax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*C*c*x^9 + 1/8*B*c*x^8 + 1/6*B*b*x^6 + 1/7*(C*b + A*c)*x^7 + 1/4*B*a*x^4
 + 1/5*(C*a + A*b)*x^5 + 1/3*A*a*x^3
```

mupad [B] time = 0.04, size = 62, normalized size = 0.84

$$\frac{C\,c\,x^9}{9} + \frac{B\,c\,x^8}{8} + \left(\frac{A\,c}{7} + \frac{C\,b}{7}\right)\,x^7 + \frac{B\,b\,x^6}{6} + \left(\frac{A\,b}{5} + \frac{C\,a}{5}\right)\,x^5 + \frac{B\,a\,x^4}{4} + \frac{A\,a\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((A*b)/5 + (C*a)/5) + x^7*((A*c)/7 + (C*b)/7) + (A*a*x^3)/3 + (B*a*x^4)
/4 + (B*b*x^6)/6 + (B*c*x^8)/8 + (C*c*x^9)/9
```

sympy [A] time = 0.07, size = 68, normalized size = 0.92

$$\frac{Aax^3}{3} + \frac{Bax^4}{4} + \frac{Bbx^6}{6} + \frac{Bcx^8}{8} + \frac{Ccx^9}{9} + x^7\left(\frac{Ac}{7} + \frac{Cb}{7}\right) + x^5\left(\frac{Ab}{5} + \frac{Ca}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**3/3 + B*a*x**4/4 + B*b*x**6/6 + B*c*x**8/8 + C*c*x**9/9 + x**7*(A*c/
7 + C*b/7) + x**5*(A*b/5 + C*a/5)
```

3.2 $\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=74

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/7
*B*c*x^7+1/8*c*C*x^8
```

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {1628}

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx &= \int \left(aAx+aBx^2+(Ab+aC)x^3+bBx^4+(Ac+bC)x^5+Bcx^6+cCx^7\right. \\ &= \frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{4}(Ab+aC)x^4+\frac{1}{5}bBx^5+\frac{1}{6}(Ac+bC)x^6+\frac{1}{7}Bcx^7\end{aligned}$$

Mathematica [A] time = 0.01, size = 74, normalized size = 1.00

$$\frac{1}{4}x^4(aC+Ab)+\frac{1}{2}aAx^2+\frac{1}{3}aBx^3+\frac{1}{6}x^6(Ac+bC)+\frac{1}{5}bBx^5+\frac{1}{7}Bcx^7+\frac{1}{8}cCx^8$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*x^2)/2 + (a*B*x^3)/3 + ((A*b + a*C)*x^4)/4 + (b*B*x^5)/5 + ((A*c + b*C
)*x^6)/6 + (B*c*x^7)/7 + (c*C*x^8)/8
```

fricas [A] time = 0.61, size = 64, normalized size = 0.86

$$\frac{1}{8}x^8cC+\frac{1}{7}x^7cB+\frac{1}{6}x^6bC+\frac{1}{6}x^6cA+\frac{1}{5}x^5bB+\frac{1}{4}x^4aC+\frac{1}{4}x^4bA+\frac{1}{3}x^3aB+\frac{1}{2}x^2aA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*x^8*c*C + 1/7*x^7*c*B + 1/6*x^6*b*C + 1/6*x^6*c*A + 1/5*x^5*b*B + 1/4*x
^4*a*C + 1/4*x^4*b*A + 1/3*x^3*a*B + 1/2*x^2*a*A
```

giac [A] time = 0.23, size = 64, normalized size = 0.86

$$\frac{1}{8}\,Ccx^{8} + \frac{1}{7}\,Bcx^{7} + \frac{1}{6}\,Cbx^{6} + \frac{1}{6}\,Acx^{6} + \frac{1}{5}\,Bbx^{5} + \frac{1}{4}\,Cax^{4} + \frac{1}{4}\,Abx^{4} + \frac{1}{3}\,Bax^{3} + \frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/6*C*b*x^6 + 1/6*A*c*x^6 + 1/5*B*b*x^5 + 1/4*C
*a*x^4 + 1/4*A*b*x^4 + 1/3*B*a*x^3 + 1/2*A*a*x^2
```

maple [A] time = 0.00, size = 61, normalized size = 0.82

$$\frac{Cc\,x^{8}}{8}+\frac{Bc\,x^{7}}{7}+\frac{Bb\,x^{5}}{5}+\frac{\left(Ac+bC\right)x^{6}}{6}+\frac{Ba\,x^{3}}{3}+\frac{Aa\,x^{2}}{2}+\frac{\left(Ab+aC\right)x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*a*A*x^2+1/3*a*B*x^3+1/4*(A*b+C*a)*x^4+1/5*b*B*x^5+1/6*(A*c+C*b)*x^6+1/7
*B*c*x^7+1/8*c*C*x^8
```

maxima [A] time = 0.47, size = 60, normalized size = 0.81

$$\frac{1}{8}\,Ccx^{8} + \frac{1}{7}\,Bcx^{7} + \frac{1}{5}\,Bbx^{5} + \frac{1}{6}\,{\left(Cb + Ac\right)}x^{6} + \frac{1}{3}\,Bax^{3} + \frac{1}{4}\,{\left(Ca + Ab\right)}x^{4} + \frac{1}{2}\,Aax^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/8*C*c*x^8 + 1/7*B*c*x^7 + 1/5*B*b*x^5 + 1/6*(C*b + A*c)*x^6 + 1/3*B*a*x^3
 + 1/4*(C*a + A*b)*x^4 + 1/2*A*a*x^2
```

mupad [B] time = 0.03, size = 62, normalized size = 0.84

$$\frac{C\,c\,x^8}{8}+\frac{B\,c\,x^7}{7}+\left(\frac{A\,c}{6}+\frac{C\,b}{6}\right)\,x^6+\frac{B\,b\,x^5}{5}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^4+\frac{B\,a\,x^3}{3}+\frac{A\,a\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4*((A*b)/4 + (C*a)/4) + x^6*((A*c)/6 + (C*b)/6) + (A*a*x^2)/2 + (B*a*x^3)
/3 + (B*b*x^5)/5 + (B*c*x^7)/7 + (C*c*x^8)/8
```

sympy [A] time = 0.07, size = 68, normalized size = 0.92

$$\frac{Aax^{2}}{2} + \frac{Bax^{3}}{3} + \frac{Bbx^{5}}{5} + \frac{Bcx^{7}}{7} + \frac{Ccx^{8}}{8} + x^{6} \left(\frac{Ac}{6} + \frac{Cb}{6}\right) + x^{4} \left(\frac{Ab}{4} + \frac{Ca}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x**2/2 + B*a*x**3/3 + B*b*x**5/5 + B*c*x**7/7 + C*c*x**8/8 + x**6*(A*c/
6 + C*b/6) + x**4*(A*b/4 + C*a/4)
```

3.3 $\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=69

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*x
^6+1/7*c*C*x^7
```

Rubi [A] time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {1657}

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)dx=\int \left(aA+aBx+(Ab+aC)x^2+bBx^3+(Ac+bC)x^4+Bcx^5+cCx^6\right)dx$$
$$=aAx+\frac{1}{2}aBx^2+\frac{1}{3}(Ab+aC)x^3+\frac{1}{4}bBx^4+\frac{1}{5}(Ac+bC)x^5+\frac{1}{6}Bcx^6+\frac{1}{7}$$

Mathematica [A] time = 0.01, size = 69, normalized size = 1.00

$$\frac{1}{3}x^3(aC+Ab)+aAx+\frac{1}{2}aBx^2+\frac{1}{5}x^5(Ac+bC)+\frac{1}{4}bBx^4+\frac{1}{6}Bcx^6+\frac{1}{7}cCx^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*A*x + (a*B*x^2)/2 + ((A*b + a*C)*x^3)/3 + (b*B*x^4)/4 + ((A*c + b*C)*x^5)
/5 + (B*c*x^6)/6 + (c*C*x^7)/7
```

fricas [A] time = 0.52, size = 61, normalized size = 0.88

$$\frac{1}{7}x^7cC+\frac{1}{6}x^6cB+\frac{1}{5}x^5bC+\frac{1}{5}x^5cA+\frac{1}{4}x^4bB+\frac{1}{3}x^3aC+\frac{1}{3}x^3bA+\frac{1}{2}x^2aB+xaA$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c*C + 1/6*x^6*c*B + 1/5*x^5*b*C + 1/5*x^5*c*A + 1/4*x^4*b*B + 1/3*x
^3*a*C + 1/3*x^3*b*A + 1/2*x^2*a*B + x*a*A
```

giac [A] time = 0.38, size = 61, normalized size = 0.88

$$\frac{1}{7}\,Ccx^{7}+\frac{1}{6}\,Bcx^{6}+\frac{1}{5}\,Cbx^{5}+\frac{1}{5}\,Acx^{5}+\frac{1}{4}\,Bbx^{4}+\frac{1}{3}\,Cax^{3}+\frac{1}{3}\,Abx^{3}+\frac{1}{2}\,Bax^{2}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/5*C*b*x^5 + 1/5*A*c*x^5 + 1/4*B*b*x^4 + 1/3*C
*a*x^3 + 1/3*A*b*x^3 + 1/2*B*a*x^2 + A*a*x
```

maple [A] time = 0.00, size = 58, normalized size = 0.84

$$\frac{Cc\,x^{7}}{7}+\frac{Bc\,x^{6}}{6}+\frac{Bb\,x^{4}}{4}+\frac{(Ac+bC)\,x^{5}}{5}+\frac{Ba\,x^{2}}{2}+Aax+\frac{(Ab+aC)\,x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] a*A*x+1/2*a*B*x^2+1/3*(A*b+C*a)*x^3+1/4*b*B*x^4+1/5*(A*c+C*b)*x^5+1/6*B*c*x
^6+1/7*c*C*x^7
```

maxima [A] time = 0.46, size = 57, normalized size = 0.83

$$\frac{1}{7}\,Ccx^{7}+\frac{1}{6}\,Bcx^{6}+\frac{1}{4}\,Bbx^{4}+\frac{1}{5}\,(Cb+Ac)x^{5}+\frac{1}{2}\,Bax^{2}+\frac{1}{3}\,(Ca+Ab)x^{3}+Aax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/7*C*c*x^7 + 1/6*B*c*x^6 + 1/4*B*b*x^4 + 1/5*(C*b + A*c)*x^5 + 1/2*B*a*x^2
 + 1/3*(C*a + A*b)*x^3 + A*a*x
```

mupad [B] time = 0.03, size = 59, normalized size = 0.86

$$\frac{C\,c\,x^{7}}{7}+\frac{B\,c\,x^{6}}{6}+\left(\frac{A\,c}{5}+\frac{C\,b}{5}\right)\,x^{5}+\frac{B\,b\,x^{4}}{4}+\left(\frac{A\,b}{3}+\frac{C\,a}{3}\right)\,x^{3}+\frac{B\,a\,x^{2}}{2}+A\,a\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*((A*b)/3 + (C*a)/3) + x^5*((A*c)/5 + (C*b)/5) + A*a*x + (B*a*x^2)/2 + (
B*b*x^4)/4 + (B*c*x^6)/6 + (C*c*x^7)/7
```

sympy [A] time = 0.07, size = 65, normalized size = 0.94

$$Aax+\frac{Bax^{2}}{2}+\frac{Bbx^{4}}{4}+\frac{Bcx^{6}}{6}+\frac{Ccx^{7}}{7}+x^{5}\left(\frac{Ac}{5}+\frac{Cb}{5}\right)+x^{3}\left(\frac{Ab}{3}+\frac{Ca}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] A*a*x + B*a*x**2/2 + B*b*x**4/4 + B*c*x**6/6 + C*c*x**7/7 + x**5*(A*c/5 + C
*b/5) + x**3*(A*b/3 + C*a/3)
```

3.4 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx$

Optimal. Leaf size=65

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

```
[Out] a*B*x+1/2*(A*b+C*a)*x^2+1/3*b*B*x^3+1/4*(A*c+C*b)*x^4+1/5*B*c*x^5+1/6*c*C*x
^6+a*A*ln(x)
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x}\,dx &= \int \left(aB+\frac{aA}{x}+(Ab+aC)x+bBx^2+(Ac+bC)x^3+Bcx^4+cCx^5\right)dx\\ &= aBx+\frac{1}{2}(Ab+aC)x^2+\frac{1}{3}bBx^3+\frac{1}{4}(Ac+bC)x^4+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6+aA\end{aligned}$$

Mathematica [A] time = 0.01, size = 65, normalized size = 1.00

$$\frac{1}{2}x^2(aC+Ab)+aA\log(x)+aBx+\frac{1}{4}x^4(Ac+bC)+\frac{1}{3}bBx^3+\frac{1}{5}Bcx^5+\frac{1}{6}cCx^6$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] a*B*x + ((A*b + a*C)*x^2)/2 + (b*B*x^3)/3 + ((A*c + b*C)*x^4)/4 + (B*c*x^5)
/5 + (c*C*x^6)/6 + a*A*Log[x]
```

fricas [A] time = 0.75, size = 55, normalized size = 0.85

$$\frac{1}{6}\,Ccx^6+\frac{1}{5}\,Bcx^5+\frac{1}{3}\,Bbx^3+\frac{1}{4}\,(Cb+Ac)x^4+Bax+\frac{1}{2}\,(Ca+Ab)x^2+Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

giac [A] time = 0.37, size = 60, normalized size = 0.92

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{4}Cbx^4 + \frac{1}{4}Acx^4 + \frac{1}{3}Bbx^3 + \frac{1}{2}Cax^2 + \frac{1}{2}Abx^2 + Bax + Aa\log(|x|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/4*C*b*x^4 + 1/4*A*c*x^4 + 1/3*B*b*x^3 + 1/2*C
*a*x^2 + 1/2*A*b*x^2 + B*a*x + A*a*log(abs(x))
```

maple [A] time = 0.00, size = 60, normalized size = 0.92

$$\frac{Cc\,x^6}{6} + \frac{Bc\,x^5}{5} + \frac{Ac\,x^4}{4} + \frac{Cb\,x^4}{4} + \frac{Bb\,x^3}{3} + \frac{Ab\,x^2}{2} + \frac{Ca\,x^2}{2} + Aa\ln(x) + Bax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x)
```

```
[Out] 1/6*c*C*x^6+1/5*B*c*x^5+1/4*A*x^4*c+1/4*C*x^4*b+1/3*b*B*x^3+1/2*A*x^2*b+1/2
*C*x^2*a+a*B*x+a*A*ln(x)
```

maxima [A] time = 0.60, size = 55, normalized size = 0.85

$$\frac{1}{6}Ccx^6 + \frac{1}{5}Bcx^5 + \frac{1}{3}Bbx^3 + \frac{1}{4}(Cb + Ac)x^4 + Bax + \frac{1}{2}(Ca + Ab)x^2 + Aa\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/6*C*c*x^6 + 1/5*B*c*x^5 + 1/3*B*b*x^3 + 1/4*(C*b + A*c)*x^4 + B*a*x + 1/2
*(C*a + A*b)*x^2 + A*a*log(x)
```

mupad [B] time = 0.04, size = 57, normalized size = 0.88

$$x^2\left(\frac{A\,b}{2} + \frac{C\,a}{2}\right) + x^4\left(\frac{A\,c}{4} + \frac{C\,b}{4}\right) + B\,a\,x + \frac{B\,b\,x^3}{3} + \frac{B\,c\,x^5}{5} + \frac{C\,c\,x^6}{6} + A\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x,x)
```

```
[Out] x^2*((A*b)/2 + (C*a)/2) + x^4*((A*c)/4 + (C*b)/4) + B*a*x + (B*b*x^3)/3 + (
B*c*x^5)/5 + (C*c*x^6)/6 + A*a*log(x)
```

sympy [A] time = 0.16, size = 63, normalized size = 0.97

$$Aa\log(x) + Bax + \frac{Bbx^3}{3} + \frac{Bcx^5}{5} + \frac{Ccx^6}{6} + x^4\left(\frac{Ac}{4} + \frac{Cb}{4}\right) + x^2\left(\frac{Ab}{2} + \frac{Ca}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] A*a*log(x) + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*c*x**6/6 + x**4*(A*c/4 + C
*b/4) + x**2*(A*b/2 + C*a/2)
```

3.5 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^2} \, dx$

Optimal. Leaf size=63

$$x(aC + Ab) - \frac{aA}{x} + aB\log(x) + \frac{1}{3}x^3(Ac + bC) + \frac{1}{2}bBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

```
[Out] -a*A/x+(A*b+C*a)*x+1/2*b*B*x^2+1/3*(A*c+C*b)*x^3+1/4*B*c*x^4+1/5*c*C*x^5+a*
B*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$x(aC + Ab) - \frac{aA}{x} + aB\log(x) + \frac{1}{3}x^3(Ac + bC) + \frac{1}{2}bBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A + Bx + Cx^2)(a + bx^2 + cx^4)}{x^2} \, dx &= \int \left(Ab\left(1 + \frac{aC}{Ab}\right) + \frac{aA}{x^2} + \frac{aB}{x} + bBx + (Ac + bC)x^2 + Bcx^3 + cCx^4\right) dx \\ &= -\frac{aA}{x} + (Ab + aC)x + \frac{1}{2}bBx^2 + \frac{1}{3}(Ac + bC)x^3 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5 + aB1\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 1.00

$$x(aC + Ab) - \frac{aA}{x} + aB\log(x) + \frac{1}{3}x^3(Ac + bC) + \frac{1}{2}bBx^2 + \frac{1}{4}Bcx^4 + \frac{1}{5}cCx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*A)/x) + (A*b + a*C)*x + (b*B*x^2)/2 + ((A*c + b*C)*x^3)/3 + (B*c*x^4)/
4 + (c*C*x^5)/5 + a*B*Log[x]
```

fricas [A] time = 0.82, size = 62, normalized size = 0.98

$$\frac{12\,Ccx^6 + 15\,Bcx^5 + 30\,Bbx^3 + 20\,(Cb + Ac)x^4 + 60\,Bax\log(x) + 60\,(Ca + Ab)x^2 - 60\,Aa}{60\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c*x^6 + 15*B*c*x^5 + 30*B*b*x^3 + 20*(C*b + A*c)*x^4 + 60*B*a*x*
log(x) + 60*(C*a + A*b)*x^2 - 60*A*a)/x
```

giac [A] time = 0.30, size = 57, normalized size = 0.90

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{3}Cbx^3 + \frac{1}{3}Acx^3 + \frac{1}{2}Bbx^2 + Cax + Abx + Ba\log(|x|) - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/3*C*b*x^3 + 1/3*A*c*x^3 + 1/2*B*b*x^2 + C*a*x
 + A*b*x + B*a*log(abs(x)) - A*a/x
```

maple [A] time = 0.01, size = 57, normalized size = 0.90

$$\frac{Cc\,x^5}{5} + \frac{Bc\,x^4}{4} + \frac{Ac\,x^3}{3} + \frac{Cb\,x^3}{3} + \frac{Bb\,x^2}{2} + Abx + Ba\ln(x) + Cax - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x)
```

```
[Out] 1/5*c*C*x^5+1/4*B*c*x^4+1/3*A*x^3*c+1/3*C*x^3*b+1/2*b*B*x^2+A*b*x+a*C*x+a*B
*ln(x)-a*A/x
```

maxima [A] time = 0.68, size = 55, normalized size = 0.87

$$\frac{1}{5}Ccx^5 + \frac{1}{4}Bcx^4 + \frac{1}{2}Bbx^2 + \frac{1}{3}(Cb + Ac)x^3 + Ba\log(x) + (Ca + Ab)x - \frac{Aa}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*C*c*x^5 + 1/4*B*c*x^4 + 1/2*B*b*x^2 + 1/3*(C*b + A*c)*x^3 + B*a*log(x)
+ (C*a + A*b)*x - A*a/x
```

mupad [B] time = 0.04, size = 56, normalized size = 0.89

$$x\,(A\,b + C\,a) + x^3\left(\frac{A\,c}{3} + \frac{C\,b}{3}\right) - \frac{A\,a}{x} + \frac{B\,b\,x^2}{2} + \frac{B\,c\,x^4}{4} + \frac{C\,c\,x^5}{5} + B\,a\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x*(A*b + C*a) + x^3*((A*c)/3 + (C*b)/3) - (A*a)/x + (B*b*x^2)/2 + (B*c*x^4)
/4 + (C*c*x^5)/5 + B*a*log(x)
```

sympy [A] time = 0.18, size = 58, normalized size = 0.92

$$-\frac{Aa}{x} + Ba\log(x) + \frac{Bbx^2}{2} + \frac{Bcx^4}{4} + \frac{Ccx^5}{5} + x^3\left(\frac{Ac}{3} + \frac{Cb}{3}\right) + x\,(Ab + Ca)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -A*a/x + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*c*x**5/5 + x**3*(A*c/3 +
C*b/3) + x*(A*b + C*a)
```

3.6 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx$

Optimal. Leaf size=63

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}-\frac{aB}{x}+\frac{1}{2}x^2(Ac+bC)+bBx+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4$$

```
[Out] -1/2*a*A/x^2-a*B/x+b*B*x+1/2*(A*c+C*b)*x^2+1/3*B*c*x^3+1/4*c*C*x^4+(A*b+C*a
)*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\log(x)(aC+Ab)-\frac{aA}{2x^2}-\frac{aB}{x}+\frac{1}{2}x^2(Ac+bC)+bBx+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -(a*A)/(2*x^2) - (a*B)/x + b*B*x + ((A*c + b*C)*x^2)/2 + (B*c*x^3)/3 + (c*C
*x^4)/4 + (A*b + a*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^3}\,dx &= \int \left(bB+\frac{aA}{x^3}+\frac{aB}{x^2}+\frac{Ab+aC}{x}+(Ac+bC)x+Bcx^2+cCx^3\right)dx\\ &= -\frac{aA}{2x^2}-\frac{aB}{x}+bBx+\frac{1}{2}(Ac+bC)x^2+\frac{1}{3}Bcx^3+\frac{1}{4}cCx^4+(Ab+aC)\log(\end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.92

$$\log(x)(aC+Ab)-\frac{a(A+2Bx)}{2x^2}+\frac{1}{12}x\left(cx\left(6A+4Bx+3Cx^2\right)+6b(2B+Cx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -1/2*(a*(A + 2*B*x))/x^2 + (x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C*x^2
)))/12 + (A*b + a*C)*Log[x]
```

fricas [A] time = 0.50, size = 62, normalized size = 0.98

$$\frac{3\,Ccx^6+4\,Bcx^5+12\,Bbx^3+6\,(Cb+Ac)x^4+12\,(Ca+Ab)x^2\log(x)-12\,Bax-6\,Aa}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*C*c*x^6 + 4*B*c*x^5 + 12*B*b*x^3 + 6*(C*b + A*c)*x^4 + 12*(C*a + A*
b)*x^2*log(x) - 12*B*a*x - 6*A*a)/x^2
```

giac [A] time = 0.29, size = 58, normalized size = 0.92

$$\frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+\frac{1}{2}\,Cbx^2+\frac{1}{2}\,Acx^2+Bbx+(Ca+Ab)\log\left(|x|\right)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + 1/2*C*b*x^2 + 1/2*A*c*x^2 + B*b*x + (C*a + A*b)
*log(abs(x)) - 1/2*(2*B*a*x + A*a)/x^2
```

maple [A] time = 0.01, size = 58, normalized size = 0.92

$$\frac{Cc\,x^4}{4}+\frac{Bc\,x^3}{3}+\frac{Ac\,x^2}{2}+\frac{Cb\,x^2}{2}+Ab\ln(x)+Bbx+Ca\ln(x)-\frac{Ba}{x}-\frac{Aa}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x)
```

```
[Out] 1/4*c*C*x^4+1/3*B*c*x^3+1/2*A*x^2*c+1/2*C*x^2*b+b*B*x+A*ln(x)*b+C*ln(x)*a-a
*B/x-1/2*a*A/x^2
```

maxima [A] time = 0.75, size = 55, normalized size = 0.87

$$\frac{1}{4}\,Ccx^4+\frac{1}{3}\,Bcx^3+Bbx+\frac{1}{2}\,(Cb+Ac)x^2+(Ca+Ab)\log(x)-\frac{2\,Bax+Aa}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*C*c*x^4 + 1/3*B*c*x^3 + B*b*x + 1/2*(C*b + A*c)*x^2 + (C*a + A*b)*log(x
) - 1/2*(2*B*a*x + A*a)/x^2
```

mupad [B] time = 0.03, size = 56, normalized size = 0.89

$$x^2\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)-\frac{\frac{A\,a}{2}+B\,a\,x}{x^2}+\ln(x)\,\left(A\,b+C\,a\right)+B\,b\,x+\frac{B\,c\,x^3}{3}+\frac{C\,c\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^3,x)
```

```
[Out] x^2*((A*c)/2 + (C*b)/2) - ((A*a)/2 + B*a*x)/x^2 + log(x)*(A*b + C*a) + B*b*
x + (B*c*x^3)/3 + (C*c*x^4)/4
```

sympy [A] time = 0.29, size = 61, normalized size = 0.97

$$Bbx+\frac{Bcx^3}{3}+\frac{Ccx^4}{4}+x^2\left(\frac{Ac}{2}+\frac{Cb}{2}\right)+(Ab+Ca)\log\left(x\right)+\frac{-Aa-2Bax}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] B*b*x + B*c*x**3/3 + C*c*x**4/4 + x**2*(A*c/2 + C*b/2) + (A*b + C*a)*log(x)
 + (-A*a - 2*B*a*x)/(2*x**2)
```

3.7 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

```
[Out] -1/3*a*A/x^3-1/2*a*B/x^2+(-A*b-C*a)/x+(A*c+C*b)*x+1/2*B*c*x^2+1/3*c*C*x^3+b
*B*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$-\frac{aC+Ab}{x}-\frac{aA}{3x^3}-\frac{aB}{2x^2}+x(Ac+bC)+bB\log(x)+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -(a*A)/(3*x^3) - (a*B)/(2*x^2) - (A*b + a*C)/x + (A*c + b*C)*x + (B*c*x^2)/
2 + (c*C*x^3)/3 + b*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^4}\,dx &= \int \left(Ac\left(1+\frac{bC}{Ac}\right)+\frac{aA}{x^4}+\frac{aB}{x^3}+\frac{Ab+aC}{x^2}+\frac{bB}{x}+Bcx+cCx^2\right)dx\\ &= -\frac{aA}{3x^3}-\frac{aB}{2x^2}-\frac{Ab+aC}{x}+(Ac+bC)x+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3+bB\log(x)\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.95

$$-\frac{a(2A+3x(B+2Cx))}{6x^3}-\frac{Ab}{x}+Acx+bB\log(x)+bCx+\frac{1}{2}Bcx^2+\frac{1}{3}cCx^3$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x]
```

```
[Out] -((A*b)/x) + A*c*x + b*C*x + (B*c*x^2)/2 + (c*C*x^3)/3 - (a*(2*A + 3*x*(B +
 2*C*x)))/(6*x^3) + b*B*Log[x]
```

fricas [A] time = 0.74, size = 62, normalized size = 0.98

$$\frac{2\,Ccx^6+3\,Bcx^5+6\,Bbx^3\log(x)+6\,(Cb+Ac)x^4-3\,Bax-6\,(Ca+Ab)x^2-2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*C*c*x^6 + 3*B*c*x^5 + 6*B*b*x^3*log(x) + 6*(C*b + A*c)*x^4 - 3*B*a*x
 - 6*(C*a + A*b)*x^2 - 2*A*a)/x^3
```

giac [A] time = 0.37, size = 56, normalized size = 0.89

$$\frac{1}{3}\,Ccx^3 + \frac{1}{2}\,Bcx^2 + Cbx + Acx + Bb\log\left(|x|\right) - \frac{3\,Bax + 6\,(Ca + Ab)x^2 + 2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="giac")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + C*b*x + A*c*x + B*b*log(abs(x)) - 1/6*(3*B*a*x
+ 6*(C*a + A*b)*x^2 + 2*A*a)/x^3
```

maple [A] time = 0.01, size = 57, normalized size = 0.90

$$\frac{Cc\,x^3}{3} + \frac{Bc\,x^2}{2} + Acx + Bb\ln(x) + Cbx - \frac{Ab}{x} - \frac{Ca}{x} - \frac{Ba}{2x^2} - \frac{Aa}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x)
```

```
[Out] 1/3*c*C*x^3+1/2*B*c*x^2+A*c*x+b*C*x+b*B*ln(x)-1/x*A*b-1/x*a*C-1/3*a*A/x^3-1
/2*a*B/x^2
```

maxima [A] time = 0.66, size = 56, normalized size = 0.89

$$\frac{1}{3}\,Ccx^3 + \frac{1}{2}\,Bcx^2 + Bb\log(x) + (Cb + Ac)x - \frac{3\,Bax + 6\,(Ca + Ab)x^2 + 2\,Aa}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*C*c*x^3 + 1/2*B*c*x^2 + B*b*log(x) + (C*b + A*c)*x - 1/6*(3*B*a*x + 6*(
C*a + A*b)*x^2 + 2*A*a)/x^3
```

mupad [B] time = 0.03, size = 55, normalized size = 0.87

$$x\,\left(A\,c+C\,b\right)-\frac{\left(A\,b+C\,a\right)\,x^2+\frac{B\,a\,x}{2}+\frac{A\,a}{3}}{x^3}+\frac{B\,c\,x^2}{2}+\frac{C\,c\,x^3}{3}+B\,b\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^4,x)
```

```
[Out] x*(A*c + C*b) - ((A*a)/3 + x^2*(A*b + C*a) + (B*a*x)/2)/x^3 + (B*c*x^2)/2 +
 (C*c*x^3)/3 + B*b*log(x)
```

sympy [A] time = 0.52, size = 63, normalized size = 1.00

$$Bb\log\left(x\right) + \frac{Bcx^2}{2} + \frac{Ccx^3}{3} + x\left(Ac + Cb\right) + \frac{-2Aa - 3Bax + x^2\left(-6Ab - 6Ca\right)}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**4,x)
```

```
[Out] B*b*log(x) + B*c*x**2/2 + C*c*x**3/3 + x*(A*c + C*b) + (-2*A*a - 3*B*a*x +
x**2*(-6*A*b - 6*C*a))/(6*x**3)
```

3.8 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

```
[Out] -1/4*a*A/x^4-1/3*a*B/x^3+1/2*(-A*b-C*a)/x^2-b*B/x+B*c*x+1/2*c*C*x^2+(A*c+C*
b)*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {1628}

$$-\frac{aC+Ab}{2x^2}-\frac{aA}{4x^4}-\frac{aB}{3x^3}+\log(x)(Ac+bC)-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -(a*A)/(4*x^4) - (a*B)/(3*x^3) - (A*b + a*C)/(2*x^2) - (b*B)/x + B*c*x + (c
*C*x^2)/2 + (A*c + b*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^5}\,dx &= \int \left(Bc+\frac{aA}{x^5}+\frac{aB}{x^4}+\frac{Ab+aC}{x^3}+\frac{bB}{x^2}+\frac{Ac+bC}{x}+cCx\right)dx\\ &= -\frac{aA}{4x^4}-\frac{aB}{3x^3}-\frac{Ab+aC}{2x^2}-\frac{bB}{x}+Bcx+\frac{1}{2}cCx^2+(Ac+bC)\log(x)\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.98

$$-\frac{a(3A+4Bx+6Cx^2)}{12x^4}+\frac{-Ab-2bBx+cx^3(2B+Cx)}{2x^2}+\log(x)(Ac+bC)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x]
```

```
[Out] -1/12*(a*(3*A + 4*B*x + 6*C*x^2))/x^4 + (-(A*b) - 2*b*B*x + c*x^3*(2*B + C*
x))/(2*x^2) + (A*c + b*C)*Log[x]
```

fricas [A] time = 0.72, size = 62, normalized size = 0.98

$$\frac{6\,Ccx^6+12\,Bcx^5+12\,(Cb+Ac)x^4\log(x)-12\,Bbx^3-4\,Bax-6\,(Ca+Ab)x^2-3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(6*C*c*x^6 + 12*B*c*x^5 + 12*(C*b + A*c)*x^4*log(x) - 12*B*b*x^3 - 4*B
*a*x - 6*(C*a + A*b)*x^2 - 3*A*a)/x^4
```

giac [A] time = 0.29, size = 57, normalized size = 0.90

$$\frac{1}{2}\,Ccx^2 + Bcx + (Cb + Ac)\log\left(|x|\right) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca + Ab)x^2 + 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="giac")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(abs(x)) - 1/12*(12*B*b*x^3 + 4*B*a*x
+ 6*(C*a + A*b)*x^2 + 3*A*a)/x^4
```

maple [A] time = 0.01, size = 58, normalized size = 0.92

$$\frac{Cc\,x^2}{2} + Ac\ln(x) + Bcx + Cb\ln(x) - \frac{Bb}{x} - \frac{Ab}{2x^2} - \frac{Ca}{2x^2} - \frac{Ba}{3x^3} - \frac{Aa}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x)
```

```
[Out] 1/2*c*C*x^2+B*c*x+A*ln(x)*c+C*ln(x)*b-b*B/x-1/3*a*B/x^3-1/4*a*A/x^4-1/2/x^2
*A*b-1/2/x^2*a*C
```

maxima [A] time = 0.75, size = 56, normalized size = 0.89

$$\frac{1}{2}\,Ccx^2 + Bcx + (Cb + Ac)\log(x) - \frac{12\,Bbx^3 + 4\,Bax + 6\,(Ca + Ab)x^2 + 3\,Aa}{12\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^5,x, algorithm="maxima")
```

```
[Out] 1/2*C*c*x^2 + B*c*x + (C*b + A*c)*log(x) - 1/12*(12*B*b*x^3 + 4*B*a*x + 6*(
C*a + A*b)*x^2 + 3*A*a)/x^4
```

mupad [B] time = 0.05, size = 56, normalized size = 0.89

$$\ln(x)\,\left(A\,c+C\,b\right)-\frac{B\,b\,x^3+\left(\frac{A\,b}{2}+\frac{C\,a}{2}\right)\,x^2+\frac{B\,a\,x}{3}+\frac{A\,a}{4}}{x^4}+B\,c\,x+\frac{C\,c\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^5,x)
```

```
[Out] log(x)*(A*c + C*b) - ((A*a)/4 + x^2*((A*b)/2 + (C*a)/2) + (B*a*x)/3 + B*b*x
^3)/x^4 + B*c*x + (C*c*x^2)/2
```

sympy [A] time = 1.76, size = 63, normalized size = 1.00

$$Bcx + \frac{Ccx^2}{2} + (Ac + Cb)\log(x) + \frac{-3Aa - 4Bax - 12Bbx^3 + x^2(-6Ab - 6Ca)}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**5,x)
```

```
[Out] B*c*x + C*c*x**2/2 + (A*c + C*b)*log(x) + (-3*A*a - 4*B*a*x - 12*B*b*x**3 +
 x**2*(-6*A*b - 6*C*a))/(12*x**4)
```

3.9 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx$

Optimal. Leaf size=63

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

```
[Out] -1/5*a*A/x^5-1/4*a*B/x^4+1/3*(-A*b-C*a)/x^3-1/2*b*B/x^2+(-A*c-C*b)/x+c*C*x+
B*c*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$-\frac{aC+Ab}{3x^3}-\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ac+bC}{x}-\frac{bB}{2x^2}+Bc\log(x)+cCx$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -(a*A)/(5*x^5) - (a*B)/(4*x^4) - (A*b + a*C)/(3*x^3) - (b*B)/(2*x^2) - (A*c
 + b*C)/x + c*C*x + B*c*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^6}\,dx &= \int \left(cC+\frac{aA}{x^6}+\frac{aB}{x^5}+\frac{Ab+aC}{x^4}+\frac{bB}{x^3}+\frac{Ac+bC}{x^2}+\frac{Bc}{x}\right)dx\\ &= -\frac{aA}{5x^5}-\frac{aB}{4x^4}-\frac{Ab+aC}{3x^3}-\frac{bB}{2x^2}-\frac{Ac+bC}{x}+cCx+Bc\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 63, normalized size = 1.00

$$Bc\log(x)-\frac{12aA+5ax(3B+4Cx)+20Ax^2\left(b+3cx^2\right)+30bx^3(B+2Cx)-60cCx^6}{60x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x]
```

```
[Out] -1/60*(12*a*A - 60*c*C*x^6 + 30*b*x^3*(B + 2*C*x) + 5*a*x*(3*B + 4*C*x) + 2
0*A*x^2*(b + 3*c*x^2))/x^5 + B*c*Log[x]
```

fricas [A] time = 0.57, size = 62, normalized size = 0.98

$$\frac{60\,Ccx^6+60\,Bcx^5\log(x)-30\,Bbx^3-60\,(Cb+Ac)x^4-15\,Bax-20\,(Ca+Ab)x^2-12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6 + 60*B*c*x^5*log(x) - 30*B*b*x^3 - 60*(C*b + A*c)*x^4 - 15
*B*a*x - 20*(C*a + A*b)*x^2 - 12*A*a)/x^5
```

giac [A] time = 0.26, size = 57, normalized size = 0.90

$$Ccx + Bc\log(|x|) - \frac{30\,Bbx^3 + 60\,(Cb + Ac)x^4 + 15\,Bax + 20\,(Ca + Ab)x^2 + 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="giac")
```

```
[Out] C*c*x + B*c*log(abs(x)) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x
+ 20*(C*a + A*b)*x^2 + 12*A*a)/x^5
```

maple [A] time = 0.01, size = 60, normalized size = 0.95

$$Bc\ln(x) + Ccx - \frac{Ac}{x} - \frac{Cb}{x} - \frac{Bb}{2x^2} - \frac{Ab}{3x^3} - \frac{Ca}{3x^3} - \frac{Ba}{4x^4} - \frac{Aa}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x)
```

```
[Out] c*C*x+B*c*ln(x)-1/x*A*c-1/x*b*C-1/3/x^3*A*b-1/3/x^3*a*C-1/5*a*A/x^5-1/4*a*B
/x^4-1/2*b*B/x^2
```

maxima [A] time = 0.74, size = 56, normalized size = 0.89

$$Ccx + Bc\log(x) - \frac{30\,Bbx^3 + 60\,(Cb + Ac)x^4 + 15\,Bax + 20\,(Ca + Ab)x^2 + 12\,Aa}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^6,x, algorithm="maxima")
```

```
[Out] C*c*x + B*c*log(x) - 1/60*(30*B*b*x^3 + 60*(C*b + A*c)*x^4 + 15*B*a*x + 20*
(C*a + A*b)*x^2 + 12*A*a)/x^5
```

mupad [B] time = 0.78, size = 56, normalized size = 0.89

$$C\,c\,x - \frac{(A\,c + C\,b)\,x^4 + \frac{B\,b\,x^3}{2} + \left(\frac{A\,b}{3} + \frac{C\,a}{3}\right)\,x^2 + \frac{B\,a\,x}{4} + \frac{A\,a}{5}}{x^5} + B\,c\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^6,x)
```

```
[Out] C*c*x - ((A*a)/5 + x^2*((A*b)/3 + (C*a)/3) + x^4*(A*c + C*b) + (B*a*x)/4 +
(B*b*x^3)/2)/x^5 + B*c*log(x)
```

sympy [A] time = 5.70, size = 66, normalized size = 1.05

$$Bc\log(x) + Ccx + \frac{-12Aa - 15Bax - 30Bbx^3 + x^4(-60Ac - 60Cb) + x^2(-20Ab - 20Ca)}{60x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**6,x)
```

```
[Out] B*c*log(x) + C*c*x + (-12*A*a - 15*B*a*x - 30*B*b*x**3 + x**4*(-60*A*c - 60
*C*b) + x**2*(-20*A*b - 20*C*a))/(60*x**5)
```

3.10 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx$

Optimal. Leaf size=68

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

```
[Out] -1/6*a*A/x^6-1/5*a*B/x^5+1/4*(-A*b-C*a)/x^4-1/3*b*B/x^3+1/2*(-A*c-C*b)/x^2-
B*c/x+c*C*ln(x)
```

Rubi [A] time = 0.05, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$-\frac{aC+Ab}{4x^4}-\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ac+bC}{2x^2}-\frac{bB}{3x^3}-\frac{Bc}{x}+cC\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -(a*A)/(6*x^6) - (a*B)/(5*x^5) - (A*b + a*C)/(4*x^4) - (b*B)/(3*x^3) - (A*c
 + b*C)/(2*x^2) - (B*c)/x + c*C*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)}{x^7}\,dx &= \int \left(\frac{aA}{x^7}+\frac{aB}{x^6}+\frac{Ab+aC}{x^5}+\frac{bB}{x^4}+\frac{Ac+bC}{x^3}+\frac{Bc}{x^2}+\frac{cC}{x}\right)dx\\ &= -\frac{aA}{6x^6}-\frac{aB}{5x^5}-\frac{Ab+aC}{4x^4}-\frac{bB}{3x^3}-\frac{Ac+bC}{2x^2}-\frac{Bc}{x}+cC\log(x)\end{aligned}$$

Mathematica [A] time = 0.05, size = 68, normalized size = 1.00

$$cC\log(x)-\frac{a(10A+3x(4B+5Cx))+5x^2\left(3A\left(b+2cx^2\right)+2x\left(2bB+3bCx+6Bcx^2\right)\right)}{60x^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x]
```

```
[Out] -1/60*(a*(10*A + 3*x*(4*B + 5*C*x)) + 5*x^2*(3*A*(b + 2*c*x^2) + 2*x*(2*b*B
 + 3*b*C*x + 6*B*c*x^2)))/x^6 + c*C*Log[x]
```

fricas [A] time = 0.66, size = 62, normalized size = 0.91

$$\frac{60\,Ccx^6\log(x)-60\,Bcx^5-20\,Bbx^3-30\,(Cb+Ac)x^4-12\,Bax-15\,(Ca+Ab)x^2-10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(60*C*c*x^6*log(x) - 60*B*c*x^5 - 20*B*b*x^3 - 30*(C*b + A*c)*x^4 - 12
*B*a*x - 15*(C*a + A*b)*x^2 - 10*A*a)/x^6
```

giac [A] time = 0.39, size = 60, normalized size = 0.88

$$Cc\log\left(|x|\right)-\frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="giac")
```

```
[Out] C*c*log(abs(x)) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B
*a*x + 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

maple [A] time = 0.01, size = 63, normalized size = 0.93

$$Cc\ln(x)-\frac{Bc}{x}-\frac{Ac}{2x^2}-\frac{Cb}{2x^2}-\frac{Bb}{3x^3}-\frac{Ab}{4x^4}-\frac{Ca}{4x^4}-\frac{Ba}{5x^5}-\frac{Aa}{6x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x)
```

```
[Out] c*C*ln(x)-B*c/x-1/3*b*B/x^3-1/5*a*B/x^5-1/4/x^4*A*b-1/4/x^4*a*C-1/2/x^2*A*c
-1/2/x^2*b*C-1/6*a*A/x^6
```

maxima [A] time = 0.87, size = 59, normalized size = 0.87

$$Cc\log(x)-\frac{60\,Bcx^5+20\,Bbx^3+30\,(Cb+Ac)x^4+12\,Bax+15\,(Ca+Ab)x^2+10\,Aa}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)/x^7,x, algorithm="maxima")
```

```
[Out] C*c*log(x) - 1/60*(60*B*c*x^5 + 20*B*b*x^3 + 30*(C*b + A*c)*x^4 + 12*B*a*x
+ 15*(C*a + A*b)*x^2 + 10*A*a)/x^6
```

mupad [B] time = 0.79, size = 60, normalized size = 0.88

$$C\,c\,\ln(x)-\frac{B\,c\,x^5+\left(\frac{A\,c}{2}+\frac{C\,b}{2}\right)\,x^4+\frac{B\,b\,x^3}{3}+\left(\frac{A\,b}{4}+\frac{C\,a}{4}\right)\,x^2+\frac{B\,a\,x}{5}+\frac{A\,a}{6}}{x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4))/x^7,x)
```

```
[Out] C*c*log(x) - ((A*a)/6 + x^2*((A*b)/4 + (C*a)/4) + x^4*((A*c)/2 + (C*b)/2) +
 (B*a*x)/5 + (B*b*x^3)/3 + B*c*x^5)/x^6
```

sympy [A] time = 15.38, size = 70, normalized size = 1.03

$$Cc\log\left(x\right)+\frac{-10Aa-12Bax-20Bbx^3-60Bcx^5+x^4\left(-30Ac-30Cb\right)+x^2\left(-15Ab-15Ca\right)}{60x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)/x**7,x)
```

```
[Out] C*c*log(x) + (-10*A*a - 12*B*a*x - 20*B*b*x**3 - 60*B*c*x**5 + x**4*(-30*A*
c - 30*C*b) + x**2*(-15*A*b - 15*C*a))/(60*x**6)
```

3.11 $\int x^2 (A + Bx + Cx^2) (a + bx^2 + cx^4)^2 \, dx$

Optimal. Leaf size=159

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}$$

```
[Out] 1/3*a^2*A*x^3+1/4*a^2*B*x^4+1/5*a*(2*A*b+C*a)*x^5+1/3*a*b*B*x^6+1/7*(A*(2*a
*c+b^2)+2*a*b*C)*x^7+1/8*B*(2*a*c+b^2)*x^8+1/9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+
1/5*b*B*c*x^10+1/11*c*(A*c+2*C*b)*x^11+1/12*B*c^2*x^12+1/13*c^2*C*x^13
```

Rubi [A] time = 0.21, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{7}x^7\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*(b^2 + 2*a*c) + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
(b^2 + 2*a*c)*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2Ax^2+a^2Bx^3+a(2Ab+aC)x^4+2abBx^5+\left(A\left(b^2+2ac\right)+2a\right.\right.\\ &= \frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{5}a(2Ab+aC)x^5+\frac{1}{3}abBx^6+\frac{1}{7}\left(A\left(b^2+2ac\right)\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 159, normalized size = 1.00

$$\frac{1}{3}a^2Ax^3+\frac{1}{4}a^2Bx^4+\frac{1}{9}x^9\left(2acC+2Abc+b^2C\right)+\frac{1}{7}x^7\left(2aAc+2abC+Ab^2\right)+\frac{1}{5}ax^5(aC+2Ab)+\frac{1}{8}Bx^8\left(2ac+b^2\right)+\frac{1}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^3)/3 + (a^2*B*x^4)/4 + (a*(2*A*b + a*C)*x^5)/5 + (a*b*B*x^6)/3 + (
(A*b^2 + 2*a*A*c + 2*a*b*C)*x^7)/7 + (B*(b^2 + 2*a*c)*x^8)/8 + ((2*A*b*c +
b^2*C + 2*a*c*C)*x^9)/9 + (b*B*c*x^10)/5 + (c*(A*c + 2*b*C)*x^11)/11 + (B*c
^2*x^12)/12 + (c^2*C*x^13)/13
```

fricas [A] time = 0.47, size = 154, normalized size = 0.97

$$\frac{1}{13}x^{13}c^2C+\frac{1}{12}x^{12}c^2B+\frac{2}{11}x^{11}cbC+\frac{1}{11}x^{11}c^2A+\frac{1}{5}x^{10}cbB+\frac{1}{9}x^9b^2C+\frac{2}{9}x^9caC+\frac{2}{9}x^9cbA+\frac{1}{8}x^8b^2B+\frac{1}{4}x^8caB+\frac{2}{7}x^7baC+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/13*x^13*c^2*C + 1/12*x^12*c^2*B + 2/11*x^11*c*b*C + 1/11*x^11*c^2*A + 1/5
*x^10*c*b*B + 1/9*x^9*b^2*C + 2/9*x^9*c*a*C + 2/9*x^9*c*b*A + 1/8*x^8*b^2*B
 + 1/4*x^8*c*a*B + 2/7*x^7*b*a*C + 1/7*x^7*b^2*A + 2/7*x^7*c*a*A + 1/3*x^6*
b*a*B + 1/5*x^5*a^2*C + 2/5*x^5*b*a*A + 1/4*x^4*a^2*B + 1/3*x^3*a^2*A
```

giac [A] time = 0.41, size = 154, normalized size = 0.97

$$\frac{1}{13}\,Cc^2x^{13}+\frac{1}{12}\,Bc^2x^{12}+\frac{2}{11}\,Cbcx^{11}+\frac{1}{11}\,Ac^2x^{11}+\frac{1}{5}\,Bbcx^{10}+\frac{1}{9}\,Cb^2x^9+\frac{2}{9}\,Cacx^9+\frac{2}{9}\,Abcx^9+\frac{1}{8}\,Bb^2x^8+\frac{1}{4}\,Bacx^8+\frac{2}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 2/11*C*b*c*x^11 + 1/11*A*c^2*x^11 + 1/5
*B*b*c*x^10 + 1/9*C*b^2*x^9 + 2/9*C*a*c*x^9 + 2/9*A*b*c*x^9 + 1/8*B*b^2*x^8
 + 1/4*B*a*c*x^8 + 2/7*C*a*b*x^7 + 1/7*A*b^2*x^7 + 2/7*A*a*c*x^7 + 1/3*B*a*
b*x^6 + 1/5*C*a^2*x^5 + 2/5*A*a*b*x^5 + 1/4*B*a^2*x^4 + 1/3*A*a^2*x^3
```

maple [A] time = 0.00, size = 142, normalized size = 0.89

$$\frac{C\,c^2x^{13}}{13}+\frac{B\,c^2x^{12}}{12}+\frac{Bbc\,x^{10}}{5}+\frac{\left(A\,c^2+2Cbc\right)x^{11}}{11}+\frac{Bab\,x^6}{3}+\frac{\left(2ac+b^2\right)B\,x^8}{8}+\frac{\left(2Abc+\left(2ac+b^2\right)C\right)x^9}{9}+\frac{B\,a^2x^4}{4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/13*c^2*C*x^13+1/12*B*c^2*x^12+1/11*(A*c^2+2*C*b*c)*x^11+1/5*b*B*c*x^10+1/
9*(2*A*b*c+(2*a*c+b^2)*C)*x^9+1/8*B*(2*a*c+b^2)*x^8+1/7*(A*(2*a*c+b^2)+2*a*
b*C)*x^7+1/3*a*b*B*x^6+1/5*(2*A*a*b+C*a^2)*x^5+1/4*a^2*B*x^4+1/3*a^2*A*x^3
```

maxima [A] time = 1.13, size = 143, normalized size = 0.90

$$\frac{1}{13}\,Cc^2x^{13}+\frac{1}{12}\,Bc^2x^{12}+\frac{1}{5}\,Bbcx^{10}+\frac{1}{11}\left(2\,Cbc+Ac^2\right)x^{11}+\frac{1}{9}\left(Cb^2+2\,(Ca+Ab)c\right)x^9+\frac{1}{3}\,Babx^6+\frac{1}{8}\left(Bb^2+2\,Bac\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/13*C*c^2*x^13 + 1/12*B*c^2*x^12 + 1/5*B*b*c*x^10 + 1/11*(2*C*b*c + A*c^2)
*x^11 + 1/9*(C*b^2 + 2*(C*a + A*b)*c)*x^9 + 1/3*B*a*b*x^6 + 1/8*(B*b^2 + 2*
B*a*c)*x^8 + 1/7*(2*C*a*b + A*b^2 + 2*A*a*c)*x^7 + 1/4*B*a^2*x^4 + 1/3*A*a^
2*x^3 + 1/5*(C*a^2 + 2*A*a*b)*x^5
```

mupad [B] time = 0.82, size = 141, normalized size = 0.89

$$x^5\left(\frac{C\,a^2}{5}+\frac{2\,A\,b\,a}{5}\right)+x^{11}\left(\frac{A\,c^2}{11}+\frac{2\,C\,b\,c}{11}\right)+x^7\left(\frac{A\,b^2}{7}+\frac{2\,C\,a\,b}{7}+\frac{2\,A\,a\,c}{7}\right)+x^9\left(\frac{C\,b^2}{9}+\frac{2\,A\,c\,b}{9}+\frac{2\,C\,a\,c}{9}\right)+\frac{A}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^5*((C*a^2)/5 + (2*A*a*b)/5) + x^11*((A*c^2)/11 + (2*C*b*c)/11) + x^7*((A*
b^2)/7 + (2*A*a*c)/7 + (2*C*a*b)/7) + x^9*((C*b^2)/9 + (2*A*b*c)/9 + (2*C*a
*c)/9) + (A*a^2*x^3)/3 + (B*a^2*x^4)/4 + (B*c^2*x^12)/12 + (C*c^2*x^13)/13
 + (B*x^8*(2*a*c + b^2))/8 + (B*a*b*x^6)/3 + (B*b*c*x^10)/5
```

sympy [A] time = 0.09, size = 168, normalized size = 1.06

$$\frac{Aa^2x^3}{3}+\frac{Ba^2x^4}{4}+\frac{Babx^6}{3}+\frac{Bbcx^{10}}{5}+\frac{Bc^2x^{12}}{12}+\frac{Cc^2x^{13}}{13}+x^{11}\left(\frac{Ac^2}{11}+\frac{2Cbc}{11}\right)+x^9\left(\frac{2Abc}{9}+\frac{2Cac}{9}+\frac{Cb^2}{9}\right)+x^8\left(\frac{Bac}{4}+\frac{B}{8}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**3/3 + B*a**2*x**4/4 + B*a*b*x**6/3 + B*b*c*x**10/5 + B*c**2*x**12
/12 + C*c**2*x**13/13 + x**11*(A*c**2/11 + 2*C*b*c/11) + x**9*(2*A*b*c/9 +
2*C*a*c/9 + C*b**2/9) + x**8*(B*a*c/4 + B*b**2/8) + x**7*(2*A*a*c/7 + A*b**
2/7 + 2*C*a*b/7) + x**5*(2*A*a*b/5 + C*a**2/5)
```

3.12 $\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=159

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)$$

```
[Out] 1/2*a^2*A*x^2+1/3*a^2*B*x^3+1/4*a*(2*A*b+C*a)*x^4+2/5*a*b*B*x^5+1/6*(A*(2*a
*c+b^2)+2*a*b*C)*x^6+1/7*B*(2*a*c+b^2)*x^7+1/8*(2*A*b*c+(2*a*c+b^2)*C)*x^8+
2/9*b*B*c*x^9+1/10*c*(A*c+2*C*b)*x^10+1/11*B*c^2*x^11+1/12*c^2*C*x^12
```

Rubi [A] time = 0.14, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {1628}

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{6}x^6\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ (b^2 + 2*a*c)*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int x\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx &= \int \left(a^2Ax+a^2Bx^2+a(2Ab+aC)x^3+2abBx^4+\left(A\left(b^2+2ac\right)+2\right.\right.\\ &= \frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{4}a(2Ab+aC)x^4+\frac{2}{5}abBx^5+\frac{1}{6}\left(A\left(b^2+2ac\right.\right.\end{aligned}$$

Mathematica [A] time = 0.04, size = 159, normalized size = 1.00

$$\frac{1}{2}a^2Ax^2+\frac{1}{3}a^2Bx^3+\frac{1}{8}x^8\left(2acC+2Abc+b^2C\right)+\frac{1}{6}x^6\left(2aAc+2abC+Ab^2\right)+\frac{1}{4}ax^4(aC+2Ab)+\frac{1}{7}Bx^7\left(2ac+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*x^2)/2 + (a^2*B*x^3)/3 + (a*(2*A*b + a*C)*x^4)/4 + (2*a*b*B*x^5)/5 +
 ((A*b^2 + 2*a*A*c + 2*a*b*C)*x^6)/6 + (B*(b^2 + 2*a*c)*x^7)/7 + ((2*A*b*c
+ b^2*C + 2*a*c*C)*x^8)/8 + (2*b*B*c*x^9)/9 + (c*(A*c + 2*b*C)*x^10)/10 + (
B*c^2*x^11)/11 + (c^2*C*x^12)/12
```

fricas [A] time = 0.55, size = 154, normalized size = 0.97

$$\frac{1}{12}x^{12}c^2C+\frac{1}{11}x^{11}c^2B+\frac{1}{5}x^{10}cbC+\frac{1}{10}x^{10}c^2A+\frac{2}{9}x^9cbB+\frac{1}{8}x^8b^2C+\frac{1}{4}x^8caC+\frac{1}{4}x^8cbA+\frac{1}{7}x^7b^2B+\frac{2}{7}x^7caB+\frac{1}{3}x^6baC$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*x^12*c^2*C + 1/11*x^11*c^2*B + 1/5*x^10*c*b*C + 1/10*x^10*c^2*A + 2/9*
x^9*c*b*B + 1/8*x^8*b^2*C + 1/4*x^8*c*a*C + 1/4*x^8*c*b*A + 1/7*x^7*b^2*B +
 2/7*x^7*c*a*B + 1/3*x^6*b*a*C + 1/6*x^6*b^2*A + 1/3*x^6*c*a*A + 2/5*x^5*b*
a*B + 1/4*x^4*a^2*C + 1/2*x^4*b*a*A + 1/3*x^3*a^2*B + 1/2*x^2*a^2*A
```

giac [A] time = 0.31, size = 154, normalized size = 0.97

$$\frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{1}{5}Cbcx^{10}+\frac{1}{10}Ac^2x^{10}+\frac{2}{9}Bbcx^9+\frac{1}{8}Cb^2x^8+\frac{1}{4}Cacx^8+\frac{1}{4}Abcx^8+\frac{1}{7}Bb^2x^7+\frac{2}{7}Bacx^7+\frac{1}{3}Cabx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 1/5*C*b*c*x^10 + 1/10*A*c^2*x^10 + 2/9*
B*b*c*x^9 + 1/8*C*b^2*x^8 + 1/4*C*a*c*x^8 + 1/4*A*b*c*x^8 + 1/7*B*b^2*x^7 +
 2/7*B*a*c*x^7 + 1/3*C*a*b*x^6 + 1/6*A*b^2*x^6 + 1/3*A*a*c*x^6 + 2/5*B*a*b*
x^5 + 1/4*C*a^2*x^4 + 1/2*A*a*b*x^4 + 1/3*B*a^2*x^3 + 1/2*A*a^2*x^2
```

maple [A] time = 0.00, size = 142, normalized size = 0.89

$$\frac{C\,c^2x^{12}}{12}+\frac{B\,c^2x^{11}}{11}+\frac{2Bbc\,x^9}{9}+\frac{\left(A\,c^2+2Cbc\right)x^{10}}{10}+\frac{2Bab\,x^5}{5}+\frac{\left(2ac+b^2\right)B\,x^7}{7}+\frac{\left(2Abc+\left(2ac+b^2\right)C\right)x^8}{8}+\frac{B\,a^2x^3}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/12*c^2*C*x^12+1/11*B*c^2*x^11+1/10*(A*c^2+2*C*b*c)*x^10+2/9*b*B*c*x^9+1/8
*(2*A*b*c+(2*a*c+b^2)*C)*x^8+1/7*B*(2*a*c+b^2)*x^7+1/6*(2*C*a*b+(2*a*c+b^2)
*A)*x^6+2/5*a*b*B*x^5+1/4*(2*A*a*b+C*a^2)*x^4+1/3*a^2*B*x^3+1/2*a^2*A*x^2
```

maxima [A] time = 1.18, size = 143, normalized size = 0.90

$$\frac{1}{12}Cc^2x^{12}+\frac{1}{11}Bc^2x^{11}+\frac{2}{9}Bbcx^9+\frac{1}{10}\left(2\,Cbc+Ac^2\right)x^{10}+\frac{1}{8}\left(Cb^2+2\,(Ca+Ab)c\right)x^8+\frac{2}{5}Babx^5+\frac{1}{7}\left(Bb^2+2\,Bac\right)x^7+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/12*C*c^2*x^12 + 1/11*B*c^2*x^11 + 2/9*B*b*c*x^9 + 1/10*(2*C*b*c + A*c^2)*
x^10 + 1/8*(C*b^2 + 2*(C*a + A*b)*c)*x^8 + 2/5*B*a*b*x^5 + 1/7*(B*b^2 + 2*B
*a*c)*x^7 + 1/6*(2*C*a*b + A*b^2 + 2*A*a*c)*x^6 + 1/3*B*a^2*x^3 + 1/2*A*a^2
*x^2 + 1/4*(C*a^2 + 2*A*a*b)*x^4
```

mupad [B] time = 0.07, size = 141, normalized size = 0.89

$$x^4\left(\frac{C\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^{10}\left(\frac{A\,c^2}{10}+\frac{C\,b\,c}{5}\right)+x^6\left(\frac{A\,b^2}{6}+\frac{C\,a\,b}{3}+\frac{A\,a\,c}{3}\right)+x^8\left(\frac{C\,b^2}{8}+\frac{A\,c\,b}{4}+\frac{C\,a\,c}{4}\right)+\frac{A\,a^2\,x^2}{2}+\frac{B\,a^2\,x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^4*((C*a^2)/4 + (A*a*b)/2) + x^10*((A*c^2)/10 + (C*b*c)/5) + x^6*((A*b^2)/
6 + (A*a*c)/3 + (C*a*b)/3) + x^8*((C*b^2)/8 + (A*b*c)/4 + (C*a*c)/4) + (A*a
^2*x^2)/2 + (B*a^2*x^3)/3 + (B*c^2*x^11)/11 + (C*c^2*x^12)/12 + (B*x^7*(2*a
*c + b^2))/7 + (2*B*a*b*x^5)/5 + (2*B*b*c*x^9)/9
```

sympy [A] time = 0.09, size = 163, normalized size = 1.03

$$\frac{Aa^2x^2}{2}+\frac{Ba^2x^3}{3}+\frac{2Babx^5}{5}+\frac{2Bbcx^9}{9}+\frac{Bc^2x^{11}}{11}+\frac{Cc^2x^{12}}{12}+x^{10}\left(\frac{Ac^2}{10}+\frac{Cbc}{5}\right)+x^8\left(\frac{Abc}{4}+\frac{Cac}{4}+\frac{Cb^2}{8}\right)+x^7\left(\frac{2Bac}{7}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x**2/2 + B*a**2*x**3/3 + 2*B*a*b*x**5/5 + 2*B*b*c*x**9/9 + B*c**2*x*
*11/11 + C*c**2*x**12/12 + x**10*(A*c**2/10 + C*b*c/5) + x**8*(A*b*c/4 + C*
a*c/4 + C*b**2/8) + x**7*(2*B*a*c/7 + B*b**2/7) + x**6*(A*a*c/3 + A*b**2/6
+ C*a*b/3) + x**4*(A*a*b/2 + C*a**2/4)
```

3.13 $\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=154

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}ab$$

```
[Out] a^2*A*x+1/2*a^2*B*x^2+1/3*a*(2*A*b+C*a)*x^3+1/2*a*b*B*x^4+1/5*(A*(2*a*c+b^2
)+2*a*b*C)*x^5+1/6*B*(2*a*c+b^2)*x^6+1/7*(2*A*b*c+(2*a*c+b^2)*C)*x^7+1/4*b*
B*c*x^8+1/9*c*(A*c+2*C*b)*x^9+1/10*B*c^2*x^10+1/11*c^2*C*x^11
```

Rubi [A] time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1657}

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{5}x^5\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}ab$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*(b^
2 + 2*a*c) + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + (b^2 +
 2*a*c)*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx = \int \left(a^2A+a^2Bx+a(2Ab+aC)x^2+2abBx^3+\left(A\left(b^2+2ac\right)+2abC\right)x\right.$$
$$= a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{3}a(2Ab+aC)x^3+\frac{1}{2}abBx^4+\frac{1}{5}\left(A\left(b^2+2ac\right)+2ab\right.$$

Mathematica [A] time = 0.03, size = 154, normalized size = 1.00

$$a^2Ax+\frac{1}{2}a^2Bx^2+\frac{1}{7}x^7\left(2acC+2Abc+b^2C\right)+\frac{1}{5}x^5\left(2aAc+2abC+Ab^2\right)+\frac{1}{3}ax^3(aC+2Ab)+\frac{1}{6}Bx^6\left(2ac+b^2\right)+\frac{1}{2}abB$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] a^2*A*x + (a^2*B*x^2)/2 + (a*(2*A*b + a*C)*x^3)/3 + (a*b*B*x^4)/2 + ((A*b^2
 + 2*a*A*c + 2*a*b*C)*x^5)/5 + (B*(b^2 + 2*a*c)*x^6)/6 + ((2*A*b*c + b^2*C
+ 2*a*c*C)*x^7)/7 + (b*B*c*x^8)/4 + (c*(A*c + 2*b*C)*x^9)/9 + (B*c^2*x^10)/
10 + (c^2*C*x^11)/11
```

fricas [A] time = 0.55, size = 151, normalized size = 0.98

$$\frac{1}{11}x^{11}c^2C+\frac{1}{10}x^{10}c^2B+\frac{2}{9}x^9cbC+\frac{1}{9}x^9c^2A+\frac{1}{4}x^8cbB+\frac{1}{7}x^7b^2C+\frac{2}{7}x^7caC+\frac{2}{7}x^7cbA+\frac{1}{6}x^6b^2B+\frac{1}{3}x^6caB+\frac{2}{5}x^5baC+\frac{1}{5}x^5b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/11*x^11*c^2*C + 1/10*x^10*c^2*B + 2/9*x^9*c*b*C + 1/9*x^9*c^2*A + 1/4*x^8
*c*b*B + 1/7*x^7*b^2*C + 2/7*x^7*c*a*C + 2/7*x^7*c*b*A + 1/6*x^6*b^2*B + 1/
3*x^6*c*a*B + 2/5*x^5*b*a*C + 1/5*x^5*b^2*A + 2/5*x^5*c*a*A + 1/2*x^4*b*a*B
 + 1/3*x^3*a^2*C + 2/3*x^3*b*a*A + 1/2*x^2*a^2*B + x*a^2*A
```

giac [A] time = 0.30, size = 151, normalized size = 0.98

$$\frac{1}{11}Cc^2x^{11}+\frac{1}{10}Bc^2x^{10}+\frac{2}{9}Cbcx^9+\frac{1}{9}Ac^2x^9+\frac{1}{4}Bbcx^8+\frac{1}{7}Cb^2x^7+\frac{2}{7}Cacx^7+\frac{2}{7}Abcx^7+\frac{1}{6}Bb^2x^6+\frac{1}{3}Bacx^6+\frac{2}{5}Cabx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 2/9*C*b*c*x^9 + 1/9*A*c^2*x^9 + 1/4*B*b
*c*x^8 + 1/7*C*b^2*x^7 + 2/7*C*a*c*x^7 + 2/7*A*b*c*x^7 + 1/6*B*b^2*x^6 + 1/
3*B*a*c*x^6 + 2/5*C*a*b*x^5 + 1/5*A*b^2*x^5 + 2/5*A*a*c*x^5 + 1/2*B*a*b*x^4
 + 1/3*C*a^2*x^3 + 2/3*A*a*b*x^3 + 1/2*B*a^2*x^2 + A*a^2*x
```

maple [A] time = 0.00, size = 139, normalized size = 0.90

$$\frac{C\,c^2x^{11}}{11}+\frac{B\,c^2x^{10}}{10}+\frac{Bbc\,x^8}{4}+\frac{\left(A\,c^2+2Cbc\right)x^9}{9}+\frac{Bab\,x^4}{2}+\frac{\left(2ac+b^2\right)B\,x^6}{6}+\frac{\left(2Abc+\left(2ac+b^2\right)C\right)x^7}{7}+\frac{B\,a^2x^2}{2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/11*c^2*C*x^11+1/10*B*c^2*x^10+1/9*(A*c^2+2*C*b*c)*x^9+1/4*b*B*c*x^8+1/7*(
2*A*b*c+(2*a*c+b^2)*C)*x^7+1/6*B*(2*a*c+b^2)*x^6+1/5*(2*C*a*b+(2*a*c+b^2)*A
)*x^5+1/2*a*b*B*x^4+1/3*(2*A*a*b+C*a^2)*x^3+1/2*a^2*B*x^2+a^2*A*x
```

maxima [A] time = 0.63, size = 140, normalized size = 0.91

$$\frac{1}{11}Cc^2x^{11}+\frac{1}{10}Bc^2x^{10}+\frac{1}{4}Bbcx^8+\frac{1}{9}\left(2\,Cbc+Ac^2\right)x^9+\frac{1}{7}\left(Cb^2+2\,(Ca+Ab)c\right)x^7+\frac{1}{2}Babx^4+\frac{1}{6}\left(Bb^2+2\,Bac\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/11*C*c^2*x^11 + 1/10*B*c^2*x^10 + 1/4*B*b*c*x^8 + 1/9*(2*C*b*c + A*c^2)*x
^9 + 1/7*(C*b^2 + 2*(C*a + A*b)*c)*x^7 + 1/2*B*a*b*x^4 + 1/6*(B*b^2 + 2*B*a
*c)*x^6 + 1/5*(2*C*a*b + A*b^2 + 2*A*a*c)*x^5 + 1/2*B*a^2*x^2 + A*a^2*x + 1
/3*(C*a^2 + 2*A*a*b)*x^3
```

mupad [B] time = 0.07, size = 138, normalized size = 0.90

$$x^3\left(\frac{C\,a^2}{3}+\frac{2\,A\,b\,a}{3}\right)+x^9\left(\frac{A\,c^2}{9}+\frac{2\,C\,b\,c}{9}\right)+x^5\left(\frac{A\,b^2}{5}+\frac{2\,C\,a\,b}{5}+\frac{2\,A\,a\,c}{5}\right)+x^7\left(\frac{C\,b^2}{7}+\frac{2\,A\,c\,b}{7}+\frac{2\,C\,a\,c}{7}\right)+\frac{B\,a^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^3*((C*a^2)/3 + (2*A*a*b)/3) + x^9*((A*c^2)/9 + (2*C*b*c)/9) + x^5*((A*b^2
)/5 + (2*A*a*c)/5 + (2*C*a*b)/5) + x^7*((C*b^2)/7 + (2*A*b*c)/7 + (2*C*a*c)
/7) + (B*a^2*x^2)/2 + (B*c^2*x^10)/10 + (C*c^2*x^11)/11 + (B*x^6*(2*a*c + b
^2))/6 + A*a^2*x + (B*a*b*x^4)/2 + (B*b*c*x^8)/4
```

sympy [A] time = 0.09, size = 165, normalized size = 1.07

$$Aa^2x+\frac{Ba^2x^2}{2}+\frac{Babx^4}{2}+\frac{Bbcx^8}{4}+\frac{Bc^2x^{10}}{10}+\frac{Cc^2x^{11}}{11}+x^9\left(\frac{Ac^2}{9}+\frac{2Cbc}{9}\right)+x^7\left(\frac{2Abc}{7}+\frac{2Cac}{7}+\frac{Cb^2}{7}\right)+x^6\left(\frac{Bac}{3}+\frac{Bb^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] A*a**2*x + B*a**2*x**2/2 + B*a*b*x**4/2 + B*b*c*x**8/4 + B*c**2*x**10/10 +
C*c**2*x**11/11 + x**9*(A*c**2/9 + 2*C*b*c/9) + x**7*(2*A*b*c/7 + 2*C*a*c/7
 + C*b**2/7) + x**6*(B*a*c/3 + B*b**2/6) + x**5*(2*A*a*c/5 + A*b**2/5 + 2*C
*a*b/5) + x**3*(2*A*a*b/3 + C*a**2/3)
```

3.14 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x} \, dx$

Optimal. Leaf size=150

$$a^2 A \log(x)+a^2 Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)$$

```
[Out] a^2*B*x+1/2*a*(2*A*b+C*a)*x^2+2/3*a*b*B*x^3+1/4*(A*(2*a*c+b^2)+2*a*b*C)*x^4
+1/5*B*(2*a*c+b^2)*x^5+1/6*(2*A*b*c+(2*a*c+b^2)*C)*x^6+2/7*b*B*c*x^7+1/8*c*
(A*c+2*C*b)*x^8+1/9*B*c^2*x^9+1/10*c^2*C*x^10+a^2*A*ln(x)
```

Rubi [A] time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$a^2 A \log(x)+a^2 Bx+\frac{1}{6}x^6\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{4}x^4\left(A\left(2ac+b^2\right)+2abC\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*(b^2 + 2*a*c) + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x}\,dx &= \int \left(a^2B+\frac{a^2A}{x}+a(2Ab+aC)x+2abBx^2+\left(A\left(b^2+2ac\right)+2abC\right)\right.\\ &= a^2Bx+\frac{1}{2}a(2Ab+aC)x^2+\frac{2}{3}abBx^3+\frac{1}{4}\left(A\left(b^2+2ac\right)+2abC\right)x^4+\end{aligned}$$

Mathematica [A] time = 0.04, size = 150, normalized size = 1.00

$$a^2 A \log(x)+a^2 Bx+\frac{1}{6}x^6\left(2acC+2Abc+b^2C\right)+\frac{1}{4}x^4\left(2aAc+2abC+Ab^2\right)+\frac{1}{2}ax^2(aC+2Ab)+\frac{1}{5}Bx^5\left(2ac+b^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x]
```

```
[Out] a^2*B*x + (a*(2*A*b + a*C)*x^2)/2 + (2*a*b*B*x^3)/3 + ((A*b^2 + 2*a*A*c + 2
*a*b*C)*x^4)/4 + (B*(b^2 + 2*a*c)*x^5)/5 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^6
)/6 + (2*b*B*c*x^7)/7 + (c*(A*c + 2*b*C)*x^8)/8 + (B*c^2*x^9)/9 + (c^2*C*x^
10)/10 + a^2*A*Log[x]
```

fricas [A] time = 0.58, size = 138, normalized size = 0.92

$$\frac{1}{10}Cc^2x^{10}+\frac{1}{9}Bc^2x^9+\frac{2}{7}Bbcx^7+\frac{1}{8}\left(2Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2(Ca+Ab)c\right)x^6+\frac{2}{3}Babx^3+\frac{1}{5}\left(Bb^2+2Bac\right)x^5+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

giac [A] time = 0.36, size = 149, normalized size = 0.99

$$\frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{1}{4}\,Cbcx^8+\frac{1}{8}\,Ac^2x^8+\frac{2}{7}\,Bbcx^7+\frac{1}{6}\,Cb^2x^6+\frac{1}{3}\,Cacx^6+\frac{1}{3}\,Abcx^6+\frac{1}{5}\,Bb^2x^5+\frac{2}{5}\,Bacx^5+\frac{1}{2}\,Cabx^4+\frac{1}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="giac")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 1/4*C*b*c*x^8 + 1/8*A*c^2*x^8 + 2/7*B*b*c
*x^7 + 1/6*C*b^2*x^6 + 1/3*C*a*c*x^6 + 1/3*A*b*c*x^6 + 1/5*B*b^2*x^5 + 2/5*
B*a*c*x^5 + 1/2*C*a*b*x^4 + 1/4*A*b^2*x^4 + 1/2*A*a*c*x^4 + 2/3*B*a*b*x^3 +
 1/2*C*a^2*x^2 + A*a*b*x^2 + B*a^2*x + A*a^2*log(abs(x))
```

maple [A] time = 0.00, size = 149, normalized size = 0.99

$$\frac{C\,c^2x^{10}}{10}+\frac{B\,c^2x^9}{9}+\frac{A\,c^2x^8}{8}+\frac{Cbc\,x^8}{4}+\frac{2Bbc\,x^7}{7}+\frac{Abc\,x^6}{3}+\frac{Cac\,x^6}{3}+\frac{C\,b^2x^6}{6}+\frac{2Bac\,x^5}{5}+\frac{B\,b^2x^5}{5}+\frac{Aac\,x^4}{2}+\frac{A\,b^2x^4}{4}+\frac{Cab}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x)
```

```
[Out] 1/10*c^2*C*x^10+1/9*B*c^2*x^9+1/8*A*x^8*c^2+1/4*C*x^8*b*c+2/7*b*B*c*x^7+1/3
*A*x^6*b*c+1/3*C*x^6*a*c+1/6*C*x^6*b^2+2/5*B*x^5*a*c+1/5*B*x^5*b^2+1/2*A*x^
4*a*c+1/4*A*x^4*b^2+1/2*C*x^4*a*b+2/3*a*b*B*x^3+A*x^2*a*b+1/2*C*x^2*a^2+a^2
*B*x+a^2*A*ln(x)
```

maxima [A] time = 0.84, size = 138, normalized size = 0.92

$$\frac{1}{10}\,Cc^2x^{10}+\frac{1}{9}\,Bc^2x^9+\frac{2}{7}\,Bbcx^7+\frac{1}{8}\left(2\,Cbc+Ac^2\right)x^8+\frac{1}{6}\left(Cb^2+2\,(Ca+Ab)c\right)x^6+\frac{2}{3}\,Babx^3+\frac{1}{5}\left(Bb^2+2\,Bac\right)x^5+\frac{1}{4}\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/10*C*c^2*x^10 + 1/9*B*c^2*x^9 + 2/7*B*b*c*x^7 + 1/8*(2*C*b*c + A*c^2)*x^8
 + 1/6*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2/3*B*a*b*x^3 + 1/5*(B*b^2 + 2*B*a*c
)*x^5 + 1/4*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + B*a^2*x + A*a^2*log(x) + 1/2*
(C*a^2 + 2*A*a*b)*x^2
```

mupad [B] time = 0.80, size = 135, normalized size = 0.90

$$x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right)+x^8\left(\frac{A\,c^2}{8}+\frac{C\,b\,c}{4}\right)+x^4\left(\frac{A\,b^2}{4}+\frac{C\,a\,b}{2}+\frac{A\,a\,c}{2}\right)+x^6\left(\frac{C\,b^2}{6}+\frac{A\,c\,b}{3}+\frac{C\,a\,c}{3}\right)+\frac{B\,c^2\,x^9}{9}+\frac{C\,c^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x,x)
```

```
[Out] x^2*((C*a^2)/2 + A*a*b) + x^8*((A*c^2)/8 + (C*b*c)/4) + x^4*((A*b^2)/4 + (A
*a*c)/2 + (C*a*b)/2) + x^6*((C*b^2)/6 + (A*b*c)/3 + (C*a*c)/3) + (B*c^2*x^9
)/9 + (C*c^2*x^10)/10 + A*a^2*log(x) + (B*x^5*(2*a*c + b^2))/5 + B*a^2*x +
(2*B*a*b*x^3)/3 + (2*B*b*c*x^7)/7
```

sympy [A] time = 0.31, size = 156, normalized size = 1.04

$$Aa^2\log(x)+Ba^2x+\frac{2Babx^3}{3}+\frac{2Bbcx^7}{7}+\frac{Bc^2x^9}{9}+\frac{Cc^2x^{10}}{10}+x^8\left(\frac{Ac^2}{8}+\frac{Cbc}{4}\right)+x^6\left(\frac{Abc}{3}+\frac{Cac}{3}+\frac{Cb^2}{6}\right)+x^5\left(\frac{2Bac}{5}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x,x)
```

```
[Out] A*a**2*log(x) + B*a**2*x + 2*B*a*b*x**3/3 + 2*B*b*c*x**7/7 + B*c**2*x**9/9
+ C*c**2*x**10/10 + x**8*(A*c**2/8 + C*b*c/4) + x**6*(A*b*c/3 + C*a*c/3 + C
*b**2/6) + x**5*(2*B*a*c/5 + B*b**2/5) + x**4*(A*a*c/2 + A*b**2/4 + C*a*b/2
) + x**2*(A*a*b + C*a**2/2)
```

3.15 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^2}\,dx$

Optimal. Leaf size=145

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+ab$$

```
[Out] -a^2*A/x+a*(2*A*b+C*a)*x+a*b*B*x^2+1/3*(A*(2*a*c+b^2)+2*a*b*C)*x^3+1/4*B*(2
*a*c+b^2)*x^4+1/5*(2*A*b*c+(2*a*c+b^2)*C)*x^5+1/3*b*B*c*x^6+1/7*c*(A*c+2*C*
b)*x^7+1/8*B*c^2*x^8+1/9*c^2*C*x^9+a^2*B*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{3}x^3\left(A\left(2ac+b^2\right)+2abC\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+ab$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*(b^2 + 2*a*c) + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^2}\,dx &= \int \left(a(2Ab+aC)+\frac{a^2A}{x^2}+\frac{a^2B}{x}+2abBx+\left(A\left(b^2+2ac\right)+2abC\right)x^2+\right.\\ &= -\frac{a^2A}{x}+a(2Ab+aC)x+abBx^2+\frac{1}{3}\left(A\left(b^2+2ac\right)+2abC\right)x^3+\frac{1}{4}B\left(\right.\end{aligned}$$

Mathematica [A] time = 0.09, size = 145, normalized size = 1.00

$$-\frac{a^2A}{x}+a^2B\log(x)+\frac{1}{5}x^5\left(2acC+2Abc+b^2C\right)+\frac{1}{3}x^3\left(2aAc+2abC+Ab^2\right)+ax(aC+2Ab)+\frac{1}{4}Bx^4\left(2ac+b^2\right)+abB$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x]
```

```
[Out] -((a^2*A)/x) + a*(2*A*b + a*C)*x + a*b*B*x^2 + ((A*b^2 + 2*a*A*c + 2*a*b*C)
*x^3)/3 + (B*(b^2 + 2*a*c)*x^4)/4 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^5)/5 + (
b*B*c*x^6)/3 + (c*(A*c + 2*b*C)*x^7)/7 + (B*c^2*x^8)/8 + (c^2*C*x^9)/9 + a^
2*B*Log[x]
```

fricas [A] time = 0.65, size = 145, normalized size = 1.00

$$\frac{280\,Cc^2x^{10}+315\,Bc^2x^9+840\,Bbcx^7+360\left(2\,Cbc+Ac^2\right)x^8+504\left(Cb^2+2\,(Ca+Ab)c\right)x^6+2520\,Babx^3+630\,(}{2520\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] 1/2520*(280*C*c^2*x^10 + 315*B*c^2*x^9 + 840*B*b*c*x^7 + 360*(2*C*b*c + A*c
^2)*x^8 + 504*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 2520*B*a*b*x^3 + 630*(B*b^2 +
 2*B*a*c)*x^5 + 840*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 2520*B*a^2*x*log(x) -
 2520*A*a^2 + 2520*(C*a^2 + 2*A*a*b)*x^2)/x
```

giac [A] time = 0.28, size = 147, normalized size = 1.01

$$\frac{1}{9}Cc^2x^9+\frac{1}{8}Bc^2x^8+\frac{2}{7}Cbcx^7+\frac{1}{7}Ac^2x^7+\frac{1}{3}Bbcx^6+\frac{1}{5}Cb^2x^5+\frac{2}{5}Cacx^5+\frac{2}{5}Abcx^5+\frac{1}{4}Bb^2x^4+\frac{1}{2}Bacx^4+\frac{2}{3}Cabx^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="giac")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 2/7*C*b*c*x^7 + 1/7*A*c^2*x^7 + 1/3*B*b*c*x
^6 + 1/5*C*b^2*x^5 + 2/5*C*a*c*x^5 + 2/5*A*b*c*x^5 + 1/4*B*b^2*x^4 + 1/2*B*
a*c*x^4 + 2/3*C*a*b*x^3 + 1/3*A*b^2*x^3 + 2/3*A*a*c*x^3 + B*a*b*x^2 + C*a^2
*x + 2*A*a*b*x + B*a^2*log(abs(x)) - A*a^2/x
```

maple [A] time = 0.01, size = 147, normalized size = 1.01

$$\frac{C\,c^2x^9}{9}+\frac{B\,c^2x^8}{8}+\frac{A\,c^2x^7}{7}+\frac{2Cbc\,x^7}{7}+\frac{Bbc\,x^6}{3}+\frac{2Abc\,x^5}{5}+\frac{2Cac\,x^5}{5}+\frac{C\,b^2x^5}{5}+\frac{Bac\,x^4}{2}+\frac{B\,b^2x^4}{4}+\frac{2Aac\,x^3}{3}+\frac{A\,b^2x^3}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x)
```

```
[Out] 1/9*c^2*C*x^9+1/8*B*c^2*x^8+1/7*A*x^7*c^2+2/7*C*x^7*b*c+1/3*b*B*c*x^6+2/5*A
*x^5*b*c+2/5*C*x^5*a*c+1/5*C*x^5*b^2+1/2*B*x^4*a*c+1/4*B*x^4*b^2+2/3*A*x^3*
a*c+1/3*A*x^3*b^2+2/3*C*x^3*a*b+a*b*B*x^2+2*A*a*b*x+C*a^2*x+a^2*B*ln(x)-a^2
*A/x
```

maxima [A] time = 0.73, size = 137, normalized size = 0.94

$$\frac{1}{9}Cc^2x^9+\frac{1}{8}Bc^2x^8+\frac{1}{3}Bbcx^6+\frac{1}{7}\left(2\,Cbc+Ac^2\right)x^7+\frac{1}{5}\left(Cb^2+2\,(Ca+Ab)c\right)x^5+Babx^2+\frac{1}{4}\left(Bb^2+2\,Bac\right)x^4+\frac{1}{3}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] 1/9*C*c^2*x^9 + 1/8*B*c^2*x^8 + 1/3*B*b*c*x^6 + 1/7*(2*C*b*c + A*c^2)*x^7 +
 1/5*(C*b^2 + 2*(C*a + A*b)*c)*x^5 + B*a*b*x^2 + 1/4*(B*b^2 + 2*B*a*c)*x^4
+ 1/3*(2*C*a*b + A*b^2 + 2*A*a*c)*x^3 + B*a^2*log(x) - A*a^2/x + (C*a^2 + 2
*A*a*b)*x
```

mupad [B] time = 0.80, size = 135, normalized size = 0.93

$$x^7\left(\frac{A\,c^2}{7}+\frac{2\,C\,b\,c}{7}\right)+x^3\left(\frac{A\,b^2}{3}+\frac{2\,C\,a\,b}{3}+\frac{2\,A\,a\,c}{3}\right)+x^5\left(\frac{C\,b^2}{5}+\frac{2\,A\,c\,b}{5}+\frac{2\,C\,a\,c}{5}\right)+x\left(C\,a^2+2\,A\,b\,a\right)-\frac{A\,a^2}{x}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^2,x)
```

```
[Out] x^7*((A*c^2)/7 + (2*C*b*c)/7) + x^3*((A*b^2)/3 + (2*A*a*c)/3 + (2*C*a*b)/3)
 + x^5*((C*b^2)/5 + (2*A*b*c)/5 + (2*C*a*c)/5) + x*(C*a^2 + 2*A*a*b) - (A*a
^2)/x + (B*c^2*x^8)/8 + (C*c^2*x^9)/9 + B*a^2*log(x) + (B*x^4*(2*a*c + b^2)
)/4 + B*a*b*x^2 + (B*b*c*x^6)/3
```

<u>**sympy**</u> **[A]** time = 0.32, size = 156, normalized size = 1.08

$$-\frac{Aa^2}{x}+Ba^2\log{(x)}+Babx^2+\frac{Bbcx^6}{3}+\frac{Bc^2x^8}{8}+\frac{Cc^2x^9}{9}+x^7\left(\frac{Ac^2}{7}+\frac{2Cbc}{7}\right)+x^5\left(\frac{2Abc}{5}+\frac{2Cac}{5}+\frac{Cb^2}{5}\right)+x^4\left(\frac{Bac}{2}+\frac{Bb^2}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**2,x)
```

```
[Out] -A*a**2/x + B*a**2*log(x) + B*a*b*x**2 + B*b*c*x**6/3 + B*c**2*x**8/8 + C*c
**2*x**9/9 + x**7*(A*c**2/7 + 2*C*b*c/7) + x**5*(2*A*b*c/5 + 2*C*a*c/5 + C*
b**2/5) + x**4*(B*a*c/2 + B*b**2/4) + x**3*(2*A*a*c/3 + A*b**2/3 + 2*C*a*b/
3) + x*(2*A*a*b + C*a**2)
```

3.16 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^3} \, dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2$$

```
[Out] -1/2*a^2*A/x^2-a^2*B/x+2*a*b*B*x+1/2*(A*(2*a*c+b^2)+2*a*b*C)*x^2+1/3*B*(2*a
*c+b^2)*x^3+1/4*(2*A*b*c+(2*a*c+b^2)*C)*x^4+2/5*b*B*c*x^5+1/6*c*(A*c+2*C*b)
*x^6+1/7*B*c^2*x^7+1/8*c^2*C*x^8+a*(2*A*b+C*a)*ln(x)
```

Rubi [A] time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{2x^2}-\frac{a^2B}{x}+\frac{1}{4}x^4\left(C\left(2ac+b^2\right)+2Abc\right)+\frac{1}{2}x^2\left(A\left(2ac+b^2\right)+2abC\right)+a\log(x)(aC+2Ab)+\frac{1}{3}Bx^3\left(2ac+b^2\right)+2$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -(a^2*A)/(2*x^2) - (a^2*B)/x + 2*a*b*B*x + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^2
)/2 + (B*(b^2 + 2*a*c)*x^3)/3 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^4)/4 + (2*b*
B*c*x^5)/5 + (c*(A*c + 2*b*C)*x^6)/6 + (B*c^2*x^7)/7 + (c^2*C*x^8)/8 + a*(2
*A*b + a*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^3}\,dx &= \int \left(2abB+\frac{a^2A}{x^3}+\frac{a^2B}{x^2}+\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2abC\right)x\right.\\ &= -\frac{a^2A}{2x^2}-\frac{a^2B}{x}+2abBx+\frac{1}{2}\left(A\left(b^2+2ac\right)+2abC\right)x^2+\frac{1}{3}B\left(b^2+2a\right.\end{aligned}$$

Mathematica [A] time = 0.09, size = 139, normalized size = 0.93

$$-\frac{a^2(A+2Bx)}{2x^2}+\frac{1}{6}ax\left(cx\left(6A+4Bx+3Cx^2\right)+6b(2B+Cx)\right)+a\log(x)(aC+2Ab)+\frac{1}{840}x^2\left(140A\left(3b^2+3bcx^2\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x]
```

```
[Out] -1/2*(a^2*(A + 2*B*x))/x^2 + (a*x*(6*b*(2*B + C*x) + c*x*(6*A + 4*B*x + 3*C
*x^2)))/6 + (x^2*(70*b^2*x*(4*B + 3*C*x) + 56*b*c*x^3*(6*B + 5*C*x) + 15*c^
2*x^5*(8*B + 7*C*x) + 140*A*(3*b^2 + 3*b*c*x^2 + c^2*x^4)))/840 + a*(2*A*b
+ a*C)*Log[x]
```

fricas [A] time = 0.68, size = 145, normalized size = 0.97

$$\frac{105\,Cc^2x^{10}+120\,Bc^2x^9+336\,Bbcx^7+140\left(2\,Cbc+Ac^2\right)x^8+210\left(Cb^2+2\left(Ca+Ab\right)c\right)x^6+1680\,Babx^3+28}{840\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*C*c^2*x^10 + 120*B*c^2*x^9 + 336*B*b*c*x^7 + 140*(2*C*b*c + A*c^
2)*x^8 + 210*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 1680*B*a*b*x^3 + 280*(B*b^2 +
2*B*a*c)*x^5 + 420*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 840*B*a^2*x + 840*(C*a
^2 + 2*A*a*b)*x^2*log(x) - 420*A*a^2)/x^2
```

giac [A] time = 0.40, size = 148, normalized size = 0.99

$$\frac{1}{8}Cc^2x^8+\frac{1}{7}Bc^2x^7+\frac{1}{3}Cbcx^6+\frac{1}{6}Ac^2x^6+\frac{2}{5}Bbcx^5+\frac{1}{4}Cb^2x^4+\frac{1}{2}Cacx^4+\frac{1}{2}Abcx^4+\frac{1}{3}Bb^2x^3+\frac{2}{3}Bacx^3+Cabx^2+\frac{1}{2}Ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="giac")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 1/3*C*b*c*x^6 + 1/6*A*c^2*x^6 + 2/5*B*b*c*x
^5 + 1/4*C*b^2*x^4 + 1/2*C*a*c*x^4 + 1/2*A*b*c*x^4 + 1/3*B*b^2*x^3 + 2/3*B*
a*c*x^3 + C*a*b*x^2 + 1/2*A*b^2*x^2 + A*a*c*x^2 + 2*B*a*b*x + (C*a^2 + 2*A*
a*b)*log(abs(x)) - 1/2*(2*B*a^2*x + A*a^2)/x^2
```

maple [A] time = 0.01, size = 148, normalized size = 0.99

$$\frac{C\,c^2x^8}{8}+\frac{B\,c^2x^7}{7}+\frac{A\,c^2x^6}{6}+\frac{Cbc\,x^6}{3}+\frac{2Bbc\,x^5}{5}+\frac{Abc\,x^4}{2}+\frac{Cac\,x^4}{2}+\frac{C\,b^2x^4}{4}+\frac{2Bac\,x^3}{3}+\frac{B\,b^2x^3}{3}+Aac\,x^2+\frac{A\,b^2x^2}{2}+Cab\,x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x)
```

```
[Out] 1/8*c^2*C*x^8+1/7*B*c^2*x^7+1/6*A*x^6*c^2+1/3*C*x^6*b*c+2/5*b*B*c*x^5+1/2*A
*x^4*b*c+1/2*C*x^4*a*c+1/4*C*x^4*b^2+2/3*B*x^3*a*c+1/3*B*x^3*b^2+A*x^2*a*c+
1/2*A*x^2*b^2+C*x^2*a*b+2*a*b*B*x+2*A*ln(x)*a*b+C*ln(x)*a^2-a^2*B/x-1/2*a^2
*A/x^2
```

maxima [A] time = 0.62, size = 139, normalized size = 0.93

$$\frac{1}{8}Cc^2x^8+\frac{1}{7}Bc^2x^7+\frac{2}{5}Bbcx^5+\frac{1}{6}\left(2\,Cbc+Ac^2\right)x^6+\frac{1}{4}\left(Cb^2+2\,(Ca+Ab)c\right)x^4+2\,Babx+\frac{1}{3}\left(Bb^2+2\,Bac\right)x^3+\frac{1}{2}\left(2\,Ca\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^3,x, algorithm="maxima")
```

```
[Out] 1/8*C*c^2*x^8 + 1/7*B*c^2*x^7 + 2/5*B*b*c*x^5 + 1/6*(2*C*b*c + A*c^2)*x^6 +
 1/4*(C*b^2 + 2*(C*a + A*b)*c)*x^4 + 2*B*a*b*x + 1/3*(B*b^2 + 2*B*a*c)*x^3
+ 1/2*(2*C*a*b + A*b^2 + 2*A*a*c)*x^2 + (C*a^2 + 2*A*a*b)*log(x) - 1/2*(2*B
*a^2*x + A*a^2)/x^2
```

mupad [B] time = 0.79, size = 135, normalized size = 0.91

$$x^6\left(\frac{A\,c^2}{6}+\frac{C\,b\,c}{3}\right)+\ln(x)\,\left(C\,a^2+2\,A\,b\,a\right)+x^2\,\left(\frac{A\,b^2}{2}+C\,a\,b+A\,a\,c\right)+x^4\,\left(\frac{C\,b^2}{4}+\frac{A\,c\,b}{2}+\frac{C\,a\,c}{2}\right)-\frac{\frac{A\,a^2}{2}+B\,a^2\,x}{x^2}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^3,x)
```

```
[Out] x^6*((A*c^2)/6 + (C*b*c)/3) + log(x)*(C*a^2 + 2*A*a*b) + x^2*((A*b^2)/2 + A
*a*c + C*a*b) + x^4*((C*b^2)/4 + (A*b*c)/2 + (C*a*c)/2) - ((A*a^2)/2 + B*a^
```

```
2*x)/x^2 + (B*c^2*x^7)/7 + (C*c^2*x^8)/8 + (B*x^3*(2*a*c + b^2))/3 + (2*B*b
*c*x^5)/5 + 2*B*a*b*x
```

sympy [A] time = 0.46, size = 153, normalized size = 1.03

$$2Babx+\frac{2Bbcx^5}{5}+\frac{Bc^2x^7}{7}+\frac{Cc^2x^8}{8}+a\left(2Ab+Ca\right)\log\left(x\right)+x^6\left(\frac{Ac^2}{6}+\frac{Cbc}{3}\right)+x^4\left(\frac{Abc}{2}+\frac{Cac}{2}+\frac{Cb^2}{4}\right)+x^3\left(\frac{2Bac}{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**3,x)
```

```
[Out] 2*B*a*b*x + 2*B*b*c*x**5/5 + B*c**2*x**7/7 + C*c**2*x**8/8 + a*(2*A*b + C*a
)*log(x) + x**6*(A*c**2/6 + C*b*c/3) + x**4*(A*b*c/2 + C*a*c/2 + C*b**2/4)
+ x**3*(2*B*a*c/3 + B*b**2/3) + x**2*(A*a*c + A*b**2/2 + C*a*b) + (-A*a**2
- 2*B*a**2*x)/(2*x**2)
```

3.17 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^4}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)$$

```
[Out] -1/3*a^2*A/x^3-1/2*a^2*B/x^2-a*(2*A*b+C*a)/x+(A*(2*a*c+b^2)+2*a*b*C)*x+1/2*
B*(2*a*c+b^2)*x^2+1/3*(2*A*b*c+(2*a*c+b^2)*C)*x^3+1/2*b*B*c*x^4+1/5*c*(A*c+
2*C*b)*x^5+1/6*B*c^2*x^6+1/7*c^2*C*x^7+2*a*b*B*ln(x)
```

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(C\left(2ac+b^2\right)+2Abc\right)+x\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{x}+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -(a^2*A)/(3*x^3) - (a^2*B)/(2*x^2) - (a*(2*A*b + a*C))/x + (A*(b^2 + 2*a*c)
 + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^3)
/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)/
7 + 2*a*b*B*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^4}\,dx = \int \left(Ab^2\left(1+\frac{2a(Ac+bC)}{Ab^2}\right)+\frac{a^2A}{x^4}+\frac{a^2B}{x^3}+\frac{a(2Ab+aC)}{x^2}+\frac{2abB}{x}+B\right.$$
$$=-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}-\frac{a(2Ab+aC)}{x}+\left(A\left(b^2+2ac\right)+2abC\right)x+\frac{1}{2}B\left(b^2+2a\right.$$

Mathematica [A] time = 0.08, size = 151, normalized size = 1.01

$$\frac{a^2(-C)-2aAb}{x}-\frac{a^2A}{3x^3}-\frac{a^2B}{2x^2}+\frac{1}{3}x^3\left(2acC+2Abc+b^2C\right)+x\left(2aAc+2abC+Ab^2\right)+\frac{1}{2}Bx^2\left(2ac+b^2\right)+2abB\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x]
```

```
[Out] -1/3*(a^2*A)/x^3 - (a^2*B)/(2*x^2) + (-2*a*A*b - a^2*C)/x + (A*b^2 + 2*a*A*
c + 2*a*b*C)*x + (B*(b^2 + 2*a*c)*x^2)/2 + ((2*A*b*c + b^2*C + 2*a*c*C)*x^3
)/3 + (b*B*c*x^4)/2 + (c*(A*c + 2*b*C)*x^5)/5 + (B*c^2*x^6)/6 + (c^2*C*x^7)
/7 + 2*a*b*B*Log[x]
```

fricas [A] time = 0.75, size = 145, normalized size = 0.97

$$\frac{30\,Cc^2x^{10}+35\,Bc^2x^9+105\,Bbcx^7+42\left(2\,Cbc+Ac^2\right)x^8+70\left(Cb^2+2\,(Ca+Ab)c\right)x^6+420\,Babx^3\log(x)+105}{210\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="fricas")
```

```
[Out] 1/210*(30*C*c^2*x^10 + 35*B*c^2*x^9 + 105*B*b*c*x^7 + 42*(2*C*b*c + A*c^2)*
x^8 + 70*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 420*B*a*b*x^3*log(x) + 105*(B*b^2
+ 2*B*a*c)*x^5 + 210*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 105*B*a^2*x - 70*A*a
^2 - 210*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

giac [A] time = 0.28, size = 146, normalized size = 0.98

$$\frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{2}{5}\,Cbcx^5+\frac{1}{5}\,Ac^2x^5+\frac{1}{2}\,Bbcx^4+\frac{1}{3}\,Cb^2x^3+\frac{2}{3}\,Cacx^3+\frac{2}{3}\,Abcx^3+\frac{1}{2}\,Bb^2x^2+Bacx^2+2\,Cabx+Ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="giac")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 2/5*C*b*c*x^5 + 1/5*A*c^2*x^5 + 1/2*B*b*c*x
^4 + 1/3*C*b^2*x^3 + 2/3*C*a*c*x^3 + 2/3*A*b*c*x^3 + 1/2*B*b^2*x^2 + B*a*c*
x^2 + 2*C*a*b*x + A*b^2*x + 2*A*a*c*x + 2*B*a*b*log(abs(x)) - 1/6*(3*B*a^2*
x + 2*A*a^2 + 6*(C*a^2 + 2*A*a*b)*x^2)/x^3
```

maple [A] time = 0.01, size = 146, normalized size = 0.98

$$\frac{C\,c^2x^7}{7}+\frac{B\,c^2x^6}{6}+\frac{A\,c^2x^5}{5}+\frac{2Cbc\,x^5}{5}+\frac{Bbc\,x^4}{2}+\frac{2Abc\,x^3}{3}+\frac{2Cac\,x^3}{3}+\frac{C\,b^2x^3}{3}+Bac\,x^2+\frac{B\,b^2x^2}{2}+2Aacx+A\,b^2x+2Bab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x)
```

```
[Out] 1/7*c^2*C*x^7+1/6*B*c^2*x^6+1/5*A*x^5*c^2+2/5*C*x^5*b*c+1/2*b*B*c*x^4+2/3*A
*x^3*b*c+2/3*C*x^3*a*c+1/3*C*x^3*b^2+B*x^2*a*c+1/2*B*x^2*b^2+2*a*A*c*x+A*b^
2*x+2*C*a*b*x+2*a*b*B*ln(x)-2*a/x*A*b-a^2/x*C-1/3*a^2*A/x^3-1/2*a^2*B/x^2
```

maxima [A] time = 0.68, size = 140, normalized size = 0.94

$$\frac{1}{7}\,Cc^2x^7+\frac{1}{6}\,Bc^2x^6+\frac{1}{2}\,Bbcx^4+\frac{1}{5}\left(2\,Cbc+Ac^2\right)x^5+\frac{1}{3}\left(Cb^2+2\,(Ca+Ab)c\right)x^3+2\,Bab\log(x)+\frac{1}{2}\left(Bb^2+2\,Bac\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/7*C*c^2*x^7 + 1/6*B*c^2*x^6 + 1/2*B*b*c*x^4 + 1/5*(2*C*b*c + A*c^2)*x^5 +
 1/3*(C*b^2 + 2*(C*a + A*b)*c)*x^3 + 2*B*a*b*log(x) + 1/2*(B*b^2 + 2*B*a*c)
*x^2 + (2*C*a*b + A*b^2 + 2*A*a*c)*x - 1/6*(3*B*a^2*x + 2*A*a^2 + 6*(C*a^2
+ 2*A*a*b)*x^2)/x^3
```

mupad [B] time = 0.06, size = 137, normalized size = 0.92

$$x^5\left(\frac{A\,c^2}{5}+\frac{2\,C\,b\,c}{5}\right)-\frac{x^2\,\left(C\,a^2+2\,A\,b\,a\right)+\frac{A\,a^2}{3}+\frac{B\,a^2\,x}{2}}{x^3}+x\,\left(A\,b^2+2\,C\,a\,b+2\,A\,a\,c\right)+x^3\,\left(\frac{C\,b^2}{3}+\frac{2\,A\,c\,b}{3}+\frac{2\,C}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^4,x)
```

```
[Out] x^5*((A*c^2)/5 + (2*C*b*c)/5) - (x^2*(C*a^2 + 2*A*a*b) + (A*a^2)/3 + (B*a^2
*x)/2)/x^3 + x*(A*b^2 + 2*A*a*c + 2*C*a*b) + x^3*((C*b^2)/3 + (2*A*b*c)/3 +
 (2*C*a*c)/3) + (B*c^2*x^6)/6 + (C*c^2*x^7)/7 + (B*x^2*(2*a*c + b^2))/2 + (
B*b*c*x^4)/2 + 2*B*a*b*log(x)
```

sympy [A] time = 0.72, size = 160, normalized size = 1.07

$$2Bab\log(x)+\frac{Bbcx^4}{2}+\frac{Bc^2x^6}{6}+\frac{Cc^2x^7}{7}+x^5\left(\frac{Ac^2}{5}+\frac{2Cbc}{5}\right)+x^3\left(\frac{2Abc}{3}+\frac{2Cac}{3}+\frac{Cb^2}{3}\right)+x^2\left(Bac+\frac{Bb^2}{2}\right)+x\left(2Aac+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**4,x)
```

```
[Out] 2*B*a*b*log(x) + B*b*c*x**4/2 + B*c**2*x**6/6 + C*c**2*x**7/7 + x**5*(A*c**
2/5 + 2*C*b*c/5) + x**3*(2*A*b*c/3 + 2*C*a*c/3 + C*b**2/3) + x**2*(B*a*c +
B*b**2/2) + x*(2*A*a*c + A*b**2 + 2*C*a*b) + (-2*A*a**2 - 3*B*a**2*x + x**2
*(-12*A*a*b - 6*C*a**2))/(6*x**3)
```

3.18 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^5} dx$

Optimal. Leaf size=148

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+$$

```
[Out] -1/4*a^2*A/x^4-1/3*a^2*B/x^3-1/2*a*(2*A*b+C*a)/x^2-2*a*b*B/x+B*(2*a*c+b^2)*
x+1/2*(2*A*b*c+(2*a*c+b^2)*C)*x^2+2/3*b*B*c*x^3+1/4*c*(A*c+2*C*b)*x^4+1/5*B
*c^2*x^5+1/6*c^2*C*x^6+(A*(2*a*c+b^2)+2*a*b*C)*ln(x)
```

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}+\frac{1}{2}x^2\left(C\left(2ac+b^2\right)+2Abc\right)+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)-\frac{a(aC+2Ab)}{2x^2}+Bx\left(2ac+b^2\right)-\frac{2abB}{x}+$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -(a^2*A)/(4*x^4) - (a^2*B)/(3*x^3) - (a*(2*A*b + a*C))/(2*x^2) - (2*a*b*B)/
x + B*(b^2 + 2*a*c)*x + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^2)/2 + (2*b*B*c*x^3)
/3 + (c*(A*c + 2*b*C)*x^4)/4 + (B*c^2*x^5)/5 + (c^2*C*x^6)/6 + (A*(b^2 + 2*
a*c) + 2*a*b*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^5}\,dx = \int \left(B\left(b^2+2ac\right)+\frac{a^2A}{x^5}+\frac{a^2B}{x^4}+\frac{a(2Ab+aC)}{x^3}+\frac{2abB}{x^2}+\frac{A\left(b^2+2\right.}{}\right.$$

$$= -\frac{a^2A}{4x^4}-\frac{a^2B}{3x^3}-\frac{a(2Ab+aC)}{2x^2}-\frac{2abB}{x}+B\left(b^2+2ac\right)x+\frac{1}{2}\left(2Abc+\right.$$

Mathematica [A] time = 0.08, size = 130, normalized size = 0.88

$$-\frac{a^2\left(3A+4Bx+6Cx^2\right)}{12x^4}+\log(x)\left(A\left(2ac+b^2\right)+2abC\right)+\frac{a\left(-Ab-2bBx+cx^3(2B+Cx)\right)}{x^2}+\frac{1}{60}x\left(10bcx(6A+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x]
```

```
[Out] -1/12*(a^2*(3*A + 4*B*x + 6*C*x^2))/x^4 + (a*(-(A*b) - 2*b*B*x + c*x^3*(2*B
 + C*x)))/x^2 + (x*(30*b^2*(2*B + C*x) + 10*b*c*x*(6*A + x*(4*B + 3*C*x)) +
 c^2*x^3*(15*A + 2*x*(6*B + 5*C*x))))/60 + (A*(b^2 + 2*a*c) + 2*a*b*C)*Log[
x]
```

fricas [A] time = 0.92, size = 145, normalized size = 0.98

$$\frac{10\,Cc^2x^{10}+12\,Bc^2x^9+40\,Bbcx^7+15\left(2\,Cbc+Ac^2\right)x^8+30\left(Cb^2+2\,(Ca+Ab)c\right)x^6-120\,Babx^3+60\left(Bb^2+\right.}{60\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/60*(10*C*c^2*x^10 + 12*B*c^2*x^9 + 40*B*b*c*x^7 + 15*(2*C*b*c + A*c^2)*x^
8 + 30*(C*b^2 + 2*(C*a + A*b)*c)*x^6 - 120*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)
*x^5 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4*log(x) - 20*B*a^2*x - 15*A*a^2 -
30*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

giac [A] time = 0.38, size = 142, normalized size = 0.96

$$\frac{1}{6}Cc^2x^6+\frac{1}{5}Bc^2x^5+\frac{1}{2}Cbcx^4+\frac{1}{4}Ac^2x^4+\frac{2}{3}Bbcx^3+\frac{1}{2}Cb^2x^2+Cacx^2+Abcx^2+Bb^2x+2Bacx+\left(2Cab+Ab^2+2Aac\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="giac")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 1/2*C*b*c*x^4 + 1/4*A*c^2*x^4 + 2/3*B*b*c*x
^3 + 1/2*C*b^2*x^2 + C*a*c*x^2 + A*b*c*x^2 + B*b^2*x + 2*B*a*c*x + (2*C*a*b
 + A*b^2 + 2*A*a*c)*log(abs(x)) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2
+ 6*(C*a^2 + 2*A*a*b)*x^2)/x^4
```

maple [A] time = 0.01, size = 144, normalized size = 0.97

$$\frac{C\,c^2x^6}{6}+\frac{B\,c^2x^5}{5}+\frac{A\,c^2x^4}{4}+\frac{Cbc\,x^4}{2}+\frac{2Bbc\,x^3}{3}+Abc\,x^2+Cac\,x^2+\frac{C\,b^2x^2}{2}+2Aac\ln(x)+A\,b^2\ln(x)+2Bacx+B\,b^2x+2Ca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x)
```

```
[Out] 1/6*c^2*C*x^6+1/5*B*c^2*x^5+1/4*A*x^4*c^2+1/2*C*x^4*b*c+2/3*b*B*c*x^3+A*x^2
*b*c+C*x^2*a*c+1/2*C*x^2*b^2+2*a*B*c*x+b^2*B*x+2*A*ln(x)*a*c+A*ln(x)*b^2+2*
C*ln(x)*a*b-2*a*b*B/x-1/3*a^2*B/x^3-1/4*a^2*A/x^4-a/x^2*A*b-1/2*a^2/x^2*C
```

maxima [A] time = 0.62, size = 139, normalized size = 0.94

$$\frac{1}{6}Cc^2x^6+\frac{1}{5}Bc^2x^5+\frac{2}{3}Bbcx^3+\frac{1}{4}\left(2Cbc+Ac^2\right)x^4+\frac{1}{2}\left(Cb^2+2(Ca+Ab)c\right)x^2+\left(Bb^2+2Bac\right)x+\left(2Cab+Ab^2+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^5,x, algorithm="maxima")
```

```
[Out] 1/6*C*c^2*x^6 + 1/5*B*c^2*x^5 + 2/3*B*b*c*x^3 + 1/4*(2*C*b*c + A*c^2)*x^4 +
 1/2*(C*b^2 + 2*(C*a + A*b)*c)*x^2 + (B*b^2 + 2*B*a*c)*x + (2*C*a*b + A*b^2
 + 2*A*a*c)*log(x) - 1/12*(24*B*a*b*x^3 + 4*B*a^2*x + 3*A*a^2 + 6*(C*a^2 +
2*A*a*b)*x^2)/x^4
```

mupad [B] time = 0.06, size = 134, normalized size = 0.91

$$x^4\left(\frac{A\,c^2}{4}+\frac{C\,b\,c}{2}\right)-\frac{x^2\left(\frac{C\,a^2}{2}+A\,b\,a\right)+\frac{A\,a^2}{4}+\frac{B\,a^2\,x}{3}+2\,B\,a\,b\,x^3}{x^4}+x^2\left(\frac{C\,b^2}{2}+A\,c\,b+C\,a\,c\right)+\ln(x)\,\left(A\,b^2+2\,C\,a\,b\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^5,x)
```

```
[Out] x^4*((A*c^2)/4 + (C*b*c)/2) - (x^2*((C*a^2)/2 + A*a*b) + (A*a^2)/4 + (B*a^2
*x)/3 + 2*B*a*b*x^3)/x^4 + x^2*((C*b^2)/2 + A*b*c + C*a*c) + log(x)*(A*b^2
+ 2*A*a*c + 2*C*a*b) + (B*c^2*x^5)/5 + (C*c^2*x^6)/6 + B*x*(2*a*c + b^2) +
(2*B*b*c*x^3)/3
```

sympy [A] time = 2.35, size = 153, normalized size = 1.03

$$\frac{2Bbcx^3}{3}+\frac{Bc^2x^5}{5}+\frac{Cc^2x^6}{6}+x^4\left(\frac{Ac^2}{4}+\frac{Cbc}{2}\right)+x^2\left(Abc+Cac+\frac{Cb^2}{2}\right)+x\left(2Bac+Bb^2\right)+\left(2Aac+Ab^2+2Cab\right)\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**5,x)
```

```
[Out] 2*B*b*c*x**3/3 + B*c**2*x**5/5 + C*c**2*x**6/6 + x**4*(A*c**2/4 + C*b*c/2)
+ x**2*(A*b*c + C*a*c + C*b**2/2) + x*(2*B*a*c + B*b**2) + (2*A*a*c + A*b**
2 + 2*C*a*b)*log(x) + (-3*A*a**2 - 4*B*a**2*x - 24*B*a*b*x**3 + x**2*(-12*A
*a*b - 6*C*a**2))/(12*x**4)
```

3.19 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^6}\,dx$

Optimal. Leaf size=143

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac$$

```
[Out] -1/5*a^2*A/x^5-1/4*a^2*B/x^4-1/3*a*(2*A*b+C*a)/x^3-a*b*B/x^2+(-A*(2*a*c+b^2
)-2*a*b*C)/x+(2*A*b*c+(2*a*c+b^2)*C)*x+b*B*c*x^2+1/3*c*(A*c+2*C*b)*x^3+1/4*
B*c^2*x^4+1/5*c^2*C*x^5+B*(2*a*c+b^2)*ln(x)
```

Rubi [A] time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}+x\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{A\left(2ac+b^2\right)+2abC}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -(a^2*A)/(5*x^5) - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*(b^2 + 2*a*c) + 2*a*b*C)/x + (2*A*b*c + (b^2 + 2*a*c)*C)*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^6}\,dx &= \int \left(2Abc\left(1+\frac{b\left(1+\frac{2ac}{b^2}\right)C}{2Ac}\right)+\frac{a^2A}{x^6}+\frac{a^2B}{x^5}+\frac{a(2Ab+aC)}{x^4}+\frac{2abB}{x^3}+\right.\\ &= -\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{a(2Ab+aC)}{3x^3}-\frac{abB}{x^2}-\frac{A\left(b^2+2ac\right)+2abC}{x}+\left(2Abc+\right.\end{aligned}$$

Mathematica [A] time = 0.08, size = 142, normalized size = 0.99

$$-\frac{a^2A}{5x^5}-\frac{a^2B}{4x^4}-\frac{2aAc+2abC+Ab^2}{x}-\frac{a(aC+2Ab)}{3x^3}+B\log(x)\left(2ac+b^2\right)+Cx\left(2ac+b^2\right)-\frac{abB}{x^2}+\frac{1}{3}cx^3(Ac+2bC)+2A$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x]
```

```
[Out] -1/5*(a^2*A)/x^5 - (a^2*B)/(4*x^4) - (a*(2*A*b + a*C))/(3*x^3) - (a*b*B)/x^
2 - (A*b^2 + 2*a*A*c + 2*a*b*C)/x + 2*A*b*c*x + (b^2 + 2*a*c)*C*x + b*B*c*x
^2 + (c*(A*c + 2*b*C)*x^3)/3 + (B*c^2*x^4)/4 + (c^2*C*x^5)/5 + B*(b^2 + 2*a
*c)*Log[x]
```

fricas [A] time = 0.76, size = 145, normalized size = 1.01

$$\frac{12\,Cc^2x^{10} + 15\,Bc^2x^9 + 60\,Bbcx^7 + 20\left(2\,Cbc + Ac^2\right)x^8 + 60\left(Cb^2 + 2\,(Ca + Ab)c\right)x^6 + 60\left(Bb^2 + 2\,Bac\right)x^5 \log}{60\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="fricas")
```

```
[Out] 1/60*(12*C*c^2*x^10 + 15*B*c^2*x^9 + 60*B*b*c*x^7 + 20*(2*C*b*c + A*c^2)*x^
8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6 + 60*(B*b^2 + 2*B*a*c)*x^5*log(x) - 60
*B*a*b*x^3 - 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 15*B*a^2*x - 12*A*a^2 - 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

giac [A] time = 0.29, size = 140, normalized size = 0.98

$$\frac{1}{5}\,Cc^2x^5 + \frac{1}{4}\,Bc^2x^4 + \frac{2}{3}\,Cbcx^3 + \frac{1}{3}\,Ac^2x^3 + Bbcx^2 + Cb^2x + 2\,Cacx + 2\,Abcx + \left(Bb^2 + 2\,Bac\right)\log\left(|x|\right) - \frac{60\,Babx^3 + 60}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="giac")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + 2/3*C*b*c*x^3 + 1/3*A*c^2*x^3 + B*b*c*x^2 +
 C*b^2*x + 2*C*a*c*x + 2*A*b*c*x + (B*b^2 + 2*B*a*c)*log(abs(x)) - 1/60*(60
*B*a*b*x^3 + 60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 2
0*(C*a^2 + 2*A*a*b)*x^2)/x^5
```

maple [A] time = 0.01, size = 144, normalized size = 1.01

$$\frac{C\,c^2x^5}{5} + \frac{B\,c^2x^4}{4} + \frac{A\,c^2x^3}{3} + \frac{2Cbc\,x^3}{3} + Bbc\,x^2 + 2Abcx + 2Bac\ln(x) + B\,b^2\ln(x) + 2Cacx + C\,b^2x - \frac{2Aac}{x} - \frac{A\,b^2}{x} - \frac{2Cab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x)
```

```
[Out] 1/5*c^2*C*x^5+1/4*B*c^2*x^4+1/3*A*x^3*c^2+2/3*C*x^3*b*c+b*B*c*x^2+2*A*b*c*x
+2*a*c*C*x+b^2*C*x+2*B*ln(x)*a*c+B*ln(x)*b^2-2/x*a*A*c-1/x*A*b^2-2/x*C*a*b-
2/3*a/x^3*A*b-1/3*a^2/x^3*C-1/5*a^2*A/x^5-1/4*a^2*B/x^4-a*b*B/x^2
```

maxima [A] time = 0.61, size = 138, normalized size = 0.97

$$\frac{1}{5}\,Cc^2x^5 + \frac{1}{4}\,Bc^2x^4 + Bbcx^2 + \frac{1}{3}\left(2\,Cbc + Ac^2\right)x^3 + \left(Cb^2 + 2\,(Ca + Ab)c\right)x + \left(Bb^2 + 2\,Bac\right)\log(x) - \frac{60\,Babx^3 + 60\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^6,x, algorithm="maxima")
```

```
[Out] 1/5*C*c^2*x^5 + 1/4*B*c^2*x^4 + B*b*c*x^2 + 1/3*(2*C*b*c + A*c^2)*x^3 + (C*
b^2 + 2*(C*a + A*b)*c)*x + (B*b^2 + 2*B*a*c)*log(x) - 1/60*(60*B*a*b*x^3 +
60*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 15*B*a^2*x + 12*A*a^2 + 20*(C*a^2 + 2*
A*a*b)*x^2)/x^5
```

mupad [B] time = 0.05, size = 136, normalized size = 0.95

$$x^3\left(\frac{A\,c^2}{3} + \frac{2\,C\,b\,c}{3}\right) - \frac{x^2\left(\frac{C\,a^2}{3} + \frac{2\,A\,b\,a}{3}\right) + \frac{A\,a^2}{5} + x^4\left(A\,b^2 + 2\,C\,a\,b + 2\,A\,a\,c\right) + \frac{B\,a^2\,x}{4} + B\,a\,b\,x^3}{x^5} + x\left(C\,b^2 + 2\,A\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^6,x)
```

```
[Out] x^3*((A*c^2)/3 + (2*C*b*c)/3) - (x^2*((C*a^2)/3 + (2*A*a*b)/3) + (A*a^2)/5
+ x^4*(A*b^2 + 2*A*a*c + 2*C*a*b) + (B*a^2*x)/4 + B*a*b*x^3)/x^5 + x*(C*b^2
 + 2*A*b*c + 2*C*a*c) + log(x)*(B*b^2 + 2*B*a*c) + (B*c^2*x^4)/4 + (C*c^2*x
^5)/5 + B*b*c*x^2
```

sympy [A] time = 7.81, size = 155, normalized size = 1.08

$$Bbcx^2+\frac{Bc^2x^4}{4}+B\left(2ac+b^2\right)\log\left(x\right)+\frac{Cc^2x^5}{5}+x^3\left(\frac{Ac^2}{3}+\frac{2Cbc}{3}\right)+x\left(2Abc+2Cac+Cb^2\right)+\frac{-12Aa^2-15Ba^2x-60}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**6,x)
```

```
[Out] B*b*c*x**2 + B*c**2*x**4/4 + B*(2*a*c + b**2)*log(x) + C*c**2*x**5/5 + x**3
*(A*c**2/3 + 2*C*b*c/3) + x*(2*A*b*c + 2*C*a*c + C*b**2) + (-12*A*a**2 - 15
*B*a**2*x - 60*B*a*b*x**3 + x**4*(-120*A*a*c - 60*A*b**2 - 120*C*a*b) + x**
2*(-40*A*a*b - 20*C*a**2))/(60*x**5)
```

3.20 $\int \frac{(A+Bx+Cx^2)(a+bx^2+cx^4)^2}{x^7}\,dx$

Optimal. Leaf size=149

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A\left(2ac+b^2\right)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B\left(2ac+b^2\right)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2($$

```
[Out] -1/6*a^2*A/x^6-1/5*a^2*B/x^5-1/4*a*(2*A*b+C*a)/x^4-2/3*a*b*B/x^3+1/2*(-A*(2
*a*c+b^2)-2*a*b*C)/x^2-B*(2*a*c+b^2)/x+2*b*B*c*x+1/2*c*(A*c+2*C*b)*x^2+1/3*
B*c^2*x^3+1/4*c^2*C*x^4+(2*A*b*c+(2*a*c+b^2)*C)*ln(x)
```

Rubi [A] time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {1628}

$$-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{A\left(2ac+b^2\right)+2abC}{2x^2}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a(aC+2Ab)}{4x^4}-\frac{B\left(2ac+b^2\right)}{x}-\frac{2abB}{3x^3}+\frac{1}{2}cx^2($$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -(a^2*A)/(6*x^6) - (a^2*B)/(5*x^5) - (a*(2*A*b + a*C))/(4*x^4) - (2*a*b*B)/
(3*x^3) - (A*(b^2 + 2*a*c) + 2*a*b*C)/(2*x^2) - (B*(b^2 + 2*a*c))/x + 2*b*B
*c*x + (c*(A*c + 2*b*C)*x^2)/2 + (B*c^2*x^3)/3 + (c^2*C*x^4)/4 + (2*A*b*c +
 (b^2 + 2*a*c)*C)*Log[x]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2}{x^7}\,dx=\int \left(2bBc+\frac{a^2A}{x^7}+\frac{a^2B}{x^6}+\frac{a(2Ab+aC)}{x^5}+\frac{2abB}{x^4}+\frac{A\left(b^2+2ac\right)+2}{x^3}\right.$$

$$=-\frac{a^2A}{6x^6}-\frac{a^2B}{5x^5}-\frac{a(2Ab+aC)}{4x^4}-\frac{2abB}{3x^3}-\frac{A\left(b^2+2ac\right)+2abC}{2x^2}-\frac{B(b}{}$$

Mathematica [A] time = 0.09, size = 144, normalized size = 0.97

$$-\frac{a^2(10A+3x(4B+5Cx))}{60x^6}+\log(x)\left(C\left(2ac+b^2\right)+2Abc\right)-\frac{a\left(3A\left(b+2cx^2\right)+2x\left(2bB+3bCx+6Bcx^2\right)\right)}{6x^4}+\frac{A}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x]
```

```
[Out] -((b^2*B)/x) + b*c*x*(2*B + C*x) + (c^2*x^3*(4*B + 3*C*x))/12 + (A*(-b^2 +
c^2*x^4))/(2*x^2) - (a^2*(10*A + 3*x*(4*B + 5*C*x)))/(60*x^6) - (a*(3*A*(b
+ 2*c*x^2) + 2*x*(2*b*B + 3*b*C*x + 6*B*c*x^2)))/(6*x^4) + (2*A*b*c + (b^2
+ 2*a*c)*C)*Log[x]
```

fricas [A] time = 0.59, size = 145, normalized size = 0.97

$$\frac{15\,Cc^2x^{10} + 20\,Bc^2x^9 + 120\,Bbcx^7 + 30\left(2\,Cbc + Ac^2\right)x^8 + 60\left(Cb^2 + 2\,(Ca + Ab)c\right)x^6\log(x) - 40\,Babx^3 - 60\,(B\ldots}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(15*C*c^2*x^10 + 20*B*c^2*x^9 + 120*B*b*c*x^7 + 30*(2*C*b*c + A*c^2)*x
^8 + 60*(C*b^2 + 2*(C*a + A*b)*c)*x^6*log(x) - 40*B*a*b*x^3 - 60*(B*b^2 + 2
*B*a*c)*x^5 - 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 - 12*B*a^2*x - 10*A*a^2 -
15*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

giac [A] time = 0.40, size = 141, normalized size = 0.95

$$\frac{1}{4}\,Cc^2x^4+\frac{1}{3}\,Bc^2x^3+Cbcx^2+\frac{1}{2}\,Ac^2x^2+2\,Bbcx+\left(Cb^2 + 2\,Cac + 2\,Abc\right)\log\left(|x|\right)-\frac{40\,Babx^3 + 60\left(Bb^2 + 2\,Bac\right)x^5 + 3\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="giac")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + C*b*c*x^2 + 1/2*A*c^2*x^2 + 2*B*b*c*x + (C*
b^2 + 2*C*a*c + 2*A*b*c)*log(abs(x)) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B
*a*c)*x^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15
*(C*a^2 + 2*A*a*b)*x^2)/x^6
```

maple [A] time = 0.01, size = 148, normalized size = 0.99

$$\frac{C\,c^2x^4}{4}+\frac{B\,c^2x^3}{3}+\frac{A\,c^2x^2}{2}+Cbc\,x^2+2Abc\ln(x)+2Bbcx+2Cac\ln(x)+C\,b^2\ln(x)-\frac{2Bac}{x}-\frac{B\,b^2}{x}-\frac{Aac}{x^2}-\frac{A\,b^2}{2x^2}-\frac{Cab}{x^2}-\frac{2B\ldots}{3x\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x)
```

```
[Out] 1/4*c^2*C*x^4+1/3*B*c^2*x^3+1/2*A*x^2*c^2+C*x^2*b*c+2*b*B*c*x+2*A*ln(x)*b*c
+2*C*ln(x)*a*c+C*ln(x)*b^2-2*B/x*a*c-B/x*b^2-2/3*a*b*B/x^3-1/5*a^2*B/x^5-1/
2*a/x^4*A*b-1/4*a^2/x^4*C-1/x^2*a*A*c-1/2/x^2*A*b^2-1/x^2*C*a*b-1/6*a^2*A/x
^6
```

maxima [A] time = 0.69, size = 140, normalized size = 0.94

$$\frac{1}{4}\,Cc^2x^4+\frac{1}{3}\,Bc^2x^3+2\,Bbcx+\frac{1}{2}\left(2\,Cbc + Ac^2\right)x^2+\left(Cb^2 + 2\,(Ca + Ab)c\right)\log(x)-\frac{40\,Babx^3 + 60\left(Bb^2 + 2\,Bac\right)x^5 + 3\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2/x^7,x, algorithm="maxima")
```

```
[Out] 1/4*C*c^2*x^4 + 1/3*B*c^2*x^3 + 2*B*b*c*x + 1/2*(2*C*b*c + A*c^2)*x^2 + (C*
b^2 + 2*(C*a + A*b)*c)*log(x) - 1/60*(40*B*a*b*x^3 + 60*(B*b^2 + 2*B*a*c)*x
^5 + 30*(2*C*a*b + A*b^2 + 2*A*a*c)*x^4 + 12*B*a^2*x + 10*A*a^2 + 15*(C*a^2
 + 2*A*a*b)*x^2)/x^6
```

mupad [B] time = 0.06, size = 136, normalized size = 0.91

$$x^2\left(\frac{A\,c^2}{2}+C\,b\,c\right)-\frac{x^2\left(\frac{C\,a^2}{4}+\frac{A\,b\,a}{2}\right)+x^5\left(B\,b^2+2\,B\,a\,c\right)+\frac{A\,a^2}{6}+x^4\left(\frac{A\,b^2}{2}+C\,a\,b+A\,a\,c\right)+\frac{B\,a^2\,x}{5}+\frac{2\,B\,a\,b\,x^3}{3}}{x^6}+\ln(\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2)/x^7,x)
```

```
[Out] x^2*((A*c^2)/2 + C*b*c) - (x^2*((C*a^2)/4 + (A*a*b)/2) + x^5*(B*b^2 + 2*B*a
*c) + (A*a^2)/6 + x^4*((A*b^2)/2 + A*a*c + C*a*b) + (B*a^2*x)/5 + (2*B*a*b*
x^3)/3)/x^6 + log(x)*(C*b^2 + 2*A*b*c + 2*C*a*c) + (B*c^2*x^3)/3 + (C*c^2*x
^4)/4 + 2*B*b*c*x
```

sympy [A] time = 27.40, size = 158, normalized size = 1.06

$$2Bbcx+\frac{Bc^2x^3}{3}+\frac{Cc^2x^4}{4}+x^2\left(\frac{Ac^2}{2}+Cbc\right)+\left(2Abc+2Cac+Cb^2\right)\log\left(x\right)+\frac{-10Aa^2-12Ba^2x-40Babx^3+x^5\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2/x**7,x)
```

```
[Out] 2*B*b*c*x + B*c**2*x**3/3 + C*c**2*x**4/4 + x**2*(A*c**2/2 + C*b*c) + (2*A*
b*c + 2*C*a*c + C*b**2)*log(x) + (-10*A*a**2 - 12*B*a**2*x - 40*B*a*b*x**3
+ x**5*(-120*B*a*c - 60*B*b**2) + x**4*(-60*A*a*c - 30*A*b**2 - 60*C*a*b) +
 x**2*(-30*A*a*b - 15*C*a**2))/(60*x**6)
```

3.21 $\int \frac{x^4(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=339

$$-\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+}}$$

```
[Out] (A*c-C*b)*x/c^2+1/2*B*x^2/c+1/3*C*x^3/c-1/4*b*B*ln(c*x^4+b*x^2+a)/c^2-1/2*B
*(-2*a*c+b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2
)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c-b^2*C+a
*c*C+(-A*c*(-2*a*c+b^2)+b*(-3*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2))*(A*b*c-b^2*C+a*c*C+(A*c*(-2*a*c+b^2)-b*(-3*a*c+b^2)*C)/(-4*
a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.86, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1662, 1279, 1166, 205, 12, 1114, 703, 634, 618, 206, 628}

$$-\frac{\left(-\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{Ac(b^2-2ac)-bC(b^2-3ac)}{\sqrt{b^2-4ac}}+acC+Abc+b^2(-C\right.}{\sqrt{2}c^{5/2}\sqrt{\sqrt{b^2-4ac}+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((A*c - b*C)*x)/c^2 + (B*x^2)/(2*c) + (C*x^3)/(3*c) - ((A*b*c - b^2*C + a*c
*C - (A*c*(b^2 - 2*a*c) - b*(b^2 - 3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - ((A*b*c - b^2*C + a*c*C + (A*c*(b^2 - 2*a*c) - b*(b^2 -
3*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*(b^2 - 2*a*c)
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - (b*B
*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^
(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^5}{a+bx^2+cx^4}\,dx + \int \frac{x^4\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{Cx^3}{3c} + B\int \frac{x^5}{a+bx^2+cx^4}\,dx - \frac{\int \frac{x^2\left(3aC-3(Ac-bC)x^2\right)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Cx^3}{3c} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x^2}{a+bx+cx^2}\,dx, x, x^2\right) + \frac{\int \frac{-3a(Ac-bC)-3\left(Abc-b^2C+\right.}{a+bx^2+cx^4}}{3c^2}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} + \frac{B\,\text{Subst}\left(\int \frac{-a-bx}{a+bx+cx^2}\,dx, x, x^2\right)}{2c} - \frac{\left(Abc-b^2C+acC-\right.}{}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac\left(b^2-2ac\right)-b\left(b^2-3ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b-\sqrt{b^2}}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac\left(b^2-2ac\right)-b\left(b^2-3ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b-\sqrt{b^2}}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

$$= \frac{(Ac-bC)x}{c^2} + \frac{Bx^2}{2c} + \frac{Cx^3}{3c} - \frac{\left(Abc-b^2C+acC-\frac{Ac\left(b^2-2ac\right)-b\left(b^2-3ac\right)C}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{}}{\sqrt{b-\sqrt{b^2}}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.57, size = 460, normalized size = 1.36

$$\frac{\frac{6\sqrt{2}\left(Ac\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)+C\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}+3abc-b^3\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{6\sqrt{2}\left(C\left(b^2\sqrt{b^2-4ac}-ac\sqrt{b^2-4ac}-3abc+b^3\right)-Ac\left(b\right.\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (12*Sqrt[c]*(A*c - b*C)*x + 6*B*c^(3/2)*x^2 + 4*c^(3/2)*C*x^3 + (6*Sqrt[2]*
(A*c*(b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c]) + (-b^3 + 3*a*b*c + b^2*Sqrt[b^2 -
 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqr
t[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (6*Sqrt
[2]*(-(A*c*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])) + (b^3 - 3*a*b*c + b^2*Sqrt
[b^2 - 4*a*c] - a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (
3*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - (3*B*Sqrt[c]*(b^2 - 2*a*c + b*Sqrt[b^2 - 4
*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(12*c^(5/2)
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.75, size = 5304, normalized size = 15.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*B*b*log(abs(c*x^4 + b*x^2 + a))/c^2 - 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 +
 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^
3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2
*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2
 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*C*c^2 + 2*(sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a
*c)*a^2*c^5)*A*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^
2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^
3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5
- 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(c) - (2*b^5*
c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6
 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a
*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4
 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 -
2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*C)*arctan(2*sqrt(1/2)*
x/sqrt((b*c^7 + sqrt(b^2*c^14 - 4*a*c^15))/c^8))/((a*b^4*c^4 - 8*a^2*b^2*c^
5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) +
```

```
1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2
*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*
a*c)*a^2*c^4)*C*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 -
 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(c) + 2*(sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*
b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*
c)*a^2*b*c^4)*C*abs(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c
)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a
*b^2*c^5)*C)*arctan(2*sqrt(1/2)*x/sqrt((b*c^7 - sqrt(b^2*c^14 - 4*a*c^15))/
c^8))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6
+ a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/6*(2*C*c^2*x^3 + 3*B*c^2*x^2 - 6*C*b*c*x
+ 6*A*c^2*x)/c^3 + 1/16*((b^7 - 10*a*b^5*c - 2*b^6*c + 32*a^2*b^3*c^2 + 12*
a*b^4*c^2 + b^5*c^2 - 32*a^3*b*c^3 - 16*a^2*b^2*c^3 - 6*a*b^3*c^3 + 8*a^2*b
*c^4 - (b^6 - 10*a*b^4*c - 2*b^5*c + 32*a^2*b^2*c^2 + 12*a*b^3*c^2 + b^4*c^
2 - 32*a^3*c^3 - 16*a^2*b*c^3 - 6*a*b^2*c^3 + 8*a^2*c^4)*sqrt(b^2 - 4*a*c))
*B*abs(c) + (b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c
^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 -
(b^6*c - 6*a*b^4*c^2 - 2*b^5*c^2 + 8*a^2*b^2*c^3 + 4*a*b^3*c^3 + b^4*c^3 -
2*a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b*c^7 + sqrt(b^2*c^14 - 4
```

```
*a*c^15))/c^8)/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2
*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2*abs(c)) + 1/16*((b^7 - 10*a*b^5*c - 2*b
^6*c + 32*a^2*b^3*c^2 + 12*a*b^4*c^2 + b^5*c^2 - 32*a^3*b*c^3 - 16*a^2*b^2*
c^3 - 6*a*b^3*c^3 + 8*a^2*b*c^4 + (b^6 - 10*a*b^4*c - 2*b^5*c + 32*a^2*b^2*
c^2 + 12*a*b^3*c^2 + b^4*c^2 - 32*a^3*c^3 - 16*a^2*b*c^3 - 6*a*b^2*c^3 + 8*
a^2*c^4)*sqrt(b^2 - 4*a*c))*B*abs(c) + (b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 +
32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6
*a*b^3*c^4 + 8*a^2*b*c^5 + (b^6*c - 6*a*b^4*c^2 - 2*b^5*c^2 + 8*a^2*b^2*c^3
 + 4*a*b^3*c^3 + b^4*c^3 - 2*a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2
*(b*c^7 - sqrt(b^2*c^14 - 4*a*c^15))/c^8)/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b
^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*c^2*abs(c))
```

maple [B] time = 0.06, size = 1622, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*C*x^3/c+1/2*B*x^2/c+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-1/2/c^2/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*C-1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*A*b^3+1/2/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*C+2/(4*a*c-b^2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*A*a*b-1/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a*(-4*a*c+b^2)
^(1/2)-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a*b-1/(4*a*c-b^2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*A*a*(-4*a*c+b^2)^(1/2)-1/c^2*b*C*x-1/2/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*C*(-4*a*c+b^2)^(1/2)*b^3-5/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*
C*a+1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2*(-4*a*c+b^2)^(1/2)+5/2/c/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*C*a-1/2/c/(4*a*c-b^2)*B*ln(2*c*x^2+b+(-4
*a*c+b^2)^(1/2))*a*(-4*a*c+b^2)^(1/2)+A/c*x+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*A*b^2*(-4*a*c+b^2)^(1/2)+3/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-
4*a*c+b^2)^(1/2)*a*b+3/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1
/2)*a*b-1/2/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^3+1/4
/c^2/(4*a*c-b^2)*B*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*b^2*(-4*a*c+b^2)^(1/2)+
1/2/c/(4*a*c-b^2)*B*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*a*(-4*a*c+b^2)^(1/2)-
1/4/c^2/(4*a*c-b^2)*B*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*b^2*(-4*a*c+b^2)^(1
/2)-1/c/(4*a*c-b^2)*B*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*a*b+2/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*C*a^2-2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a^2-1/c/(4*a*
c-b^2)*B*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*a*b+1/4/c^2/(4*a*c-b^2)*B*ln(-2*c
*x^2-b+(-4*a*c+b^2)^(1/2))*b^3+1/4/c^2/(4*a*c-b^2)*B*ln(2*c*x^2+b+(-4*a*c+b
^2)^(1/2))*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/6*(2*C*c*x^3 + 3*B*c*x^2 - 6*(C*b - A*c)*x)/c^2 - integrate((B*b*c*x^3 +
B*a*c*x - C*a*b + A*a*c - (C*b^2 - (C*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a),
 x)/c^2
```

mupad [B] time = 0.96, size = 2588, normalized size = 7.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (C*b)/c^2) + symsum(log((C^3*a^4*c - C^3*a^3*b^2 - A*B^2*a^3*c^2 +
 A*C^2*a^2*b^3 + A^2*C*a^3*c^2 + A^3*a^2*b*c^2 + A*B^2*a^2*b^2*c - 2*A^2*C*
a^2*b^2*c - B^2*C*a^3*b*c)/c^3 - root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 -
256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*
z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80
*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a
*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^
2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A
^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z
 - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^
3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z
 + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^
2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c -
 C^4*a^5, z, k)*(root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4
- 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b
^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z
^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 3
6*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a
^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 -
 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*
b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z +
16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b
*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c -
B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z, k)*
((x*(16*B*a^2*c^5 + 8*B*b^4*c^3 - 36*B*a*b^2*c^4))/c^3 - (16*A*a^2*c^5 - 4*
A*a*b^2*c^4 + 4*C*a*b^3*c^3 - 16*C*a^2*b*c^4)/c^3 + (root(128*a*b^2*c^6*z^4
 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4
*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 +
8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*
b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2 + 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2
- 100*C^2*a^2*b^3*c^2*z^2 - 56*B^2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B
^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z -
 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^
3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z
- 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c
+ 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a
^3*c^2 - B^4*a^4*c - C^4*a^5, z, k)*x*(8*b^3*c^5 - 32*a*b*c^6))/c^3) + (8*B
*C*a^3*c^3 - 4*A*B*a^2*b*c^3)/c^3 + (x*(2*C^2*b^6 + 2*B^2*b^5*c + 4*A^2*a^2
*c^4 + 2*A^2*b^4*c^2 - 4*C^2*a^3*c^3 - 4*A*C*b^5*c + 18*C^2*a^2*b^2*c^2 - 1
2*C^2*a*b^4*c - 8*A^2*a*b^2*c^3 - 10*B^2*a*b^3*c^2 + 6*B^2*a^2*b*c^3 + 20*A
*C*a*b^3*c^2 - 20*A*C*a^2*b*c^3))/c^3) + (x*(B*C^2*a^2*b^3 - B^3*a^3*c^2 +
```

```
B^3*a^2*b^2*c + A^2*B*a^2*b*c^2 + 2*A*B*C*a^3*c^2 - 2*B*C^2*a^3*b*c - 2*A*B
*C*a^2*b^2*c))/c^3)*root(128*a*b^2*c^6*z^4 - 16*b^4*c^5*z^4 - 256*a^2*c^7*z
^4 - 256*B*a^2*b*c^5*z^3 + 128*B*a*b^3*c^4*z^3 - 16*B*b^5*c^3*z^3 - 64*A*C*
a*b^4*c^2*z^2 + 144*A*C*a^2*b^2*c^3*z^2 + 8*A*C*b^6*c*z^2 + 80*C^2*a^3*b*c^
3*z^2 + 32*B^2*a*b^4*c^2*z^2 - 48*A^2*a^2*b*c^4*z^2 + 28*A^2*a*b^3*c^3*z^2
+ 36*C^2*a*b^5*c*z^2 - 64*A*C*a^3*c^4*z^2 - 100*C^2*a^2*b^3*c^2*z^2 - 56*B^
2*a^2*b^2*c^3*z^2 - 4*B^2*b^6*c*z^2 - 32*B^2*a^3*c^4*z^2 - 4*A^2*b^5*c^2*z^
2 - 4*C^2*b^7*z^2 + 32*A*B*C*a^3*b*c^2*z - 8*A*B*C*a^2*b^3*c*z - 20*B*C^2*a
^3*b^2*c*z + 4*A^2*B*a^2*b^2*c^2*z - 16*B^3*a^3*b*c^2*z + 4*B^3*a^2*b^3*c*z
 + 16*B*C^2*a^4*c^2*z + 4*B*C^2*a^2*b^4*z - 16*A^2*B*a^3*c^3*z + 2*A^3*C*a^
3*b*c + 4*A*B^2*C*a^4*c - 2*A^2*C^2*a^4*c + 2*A*C^3*a^4*b - A^2*B^2*a^3*b*c
 - B^2*C^2*a^4*b - A^2*C^2*a^3*b^2 - A^4*a^3*c^2 - B^4*a^4*c - C^4*a^5, z,
k), k, 1, 4) + (B*x^2)/(2*c) + (C*x^3)/(3*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.22 $\int \frac{x^3(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=278

$$\frac{\left(2acC+Abc+b^2(-C)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-$$

```
[Out] B*x/c+1/2*C*x^2/c+1/4*(A*c-C*b)*ln(c*x^4+b*x^2+a)/c^2+1/2*(A*b*c+2*C*a*c-C*
b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)-1/2*B*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*
c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan(x*2
^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(
1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1662, 1251, 773, 634, 618, 206, 628, 12, 1122, 1166, 205}

$$\frac{\left(2acC+Abc+b^2(-C)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(Ac-bC)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{B\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c + (C*x^2)/(2*c) - (B*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b -
Sqrt[b^2 - 4*a*c]]) - (B*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt
[b^2 - 4*a*c]]) + ((A*b*c - b^2*C + 2*a*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((A*c - b*C)*Log[a + b*x^2 + c*x^4]
)/(4*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^3 (A+Bx+Cx^2)}{a+bx^2+cx^4} dx = \int \frac{Bx^4}{a+bx^2+cx^4} dx + \int \frac{x^3 (A+Cx^2)}{a+bx^2+cx^4} dx$$
$$= \frac{1}{2} \text{Subst}\left(\int \frac{x(A+Cx)}{a+bx+cx^2} dx, x, x^2\right) + B \int \frac{x^4}{a+bx^2+cx^4} dx$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} + \frac{\text{Subst}\left(\int \frac{-aC+(Ac-bC)x}{a+bx+cx^2} dx, x, x^2\right)}{2c} - \frac{B \int \frac{a+bx^2}{a+bx^2+cx^4} dx}{c}$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{\left(B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} - \frac{1}{2}\sqrt{b^2-4ac} + cx^2} dx}{2c} - \frac{\left(B\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right) \int \frac{1}{\frac{b}{2} + \frac{1}{2}\sqrt{b^2-4a}}$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$
$$= \frac{Bx}{c} + \frac{Cx^2}{2c} - \frac{B\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\left(b + \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) \tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.42, size = 377, normalized size = 1.36

$$\frac{\frac{\left(Ac\left(\sqrt{b^2-4ac}-b\right)+C\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}} - \frac{\left(C\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-Ac\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} - \frac{2\sqrt{2}}{}}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*B*c*x + 2*c*C*x^2 - (2*Sqrt[2]*B*Sqrt[c]*(-b^2 + 2*a*c + b*Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*B*Sqrt[c]*(b^2 - 2*a*c + b
*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*c*(-b + Sqrt[b^2 -
4*a*c]) + (b^2 - 2*a*c - b*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2])/Sqrt[b^2 - 4*a*c] - ((-(A*c*(b + Sqrt[b^2 - 4*a*c])) + (b^2 -
2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[
b^2 - 4*a*c])/(4*c^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.03, size = 3519, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(C*b - A*c)*log(abs(c*x^4 + b*x^2 + a))/c^2 + 1/2*(C*c*x^2 + 2*B*c*x)/
c^2 - 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*
(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c
^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b
^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2
- 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 + sqrt(b^2*c^10 - 4*a
*c^11))/c^6))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^
2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32
*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
+ 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8
*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^
2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5
+ 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^5*c^
4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 -
 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x
/sqrt((b*c^5 - sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^3 - 8*a^2*b^2*c^4
 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1
/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*
c^3 - 4*a*b^2*c^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b
^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^7 - 10*a*b^5
*c - 2*b^6*c + 32*a^2*b^3*c^2 + 12*a*b^4*c^2 + b^5*c^2 - 32*a^3*b*c^3 - 16*
a^2*b^2*c^3 - 6*a*b^3*c^3 + 8*a^2*b*c^4 - (b^6 - 10*a*b^4*c - 2*b^5*c + 32*
a^2*b^2*c^2 + 12*a*b^3*c^2 + b^4*c^2 - 32*a^3*c^3 - 16*a^2*b*c^3 - 6*a*b^2*
c^3 + 8*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b
^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 + (b^5*c^2 -
```

```
4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A - (b^7*c - 10*a*b^5
*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 -
 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 - (b^6*c - 6*a*b^4*c^2 - 2*b^5*
c^2 + 8*a^2*b^2*c^3 + 4*a*b^3*c^3 + b^4*c^3 - 2*a*b^2*c^4)*sqrt(b^2 - 4*a*c
))*C)*log(x^2 + 1/2*(b*c^5 + sqrt(b^2*c^10 - 4*a*c^11))/c^6)/((a*b^4*c - 8*
a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^
4)*c^2*abs(c)) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 +
8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16
*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c)
 - (b^7 - 10*a*b^5*c - 2*b^6*c + 32*a^2*b^3*c^2 + 12*a*b^4*c^2 + b^5*c^2 -
32*a^3*b*c^3 - 16*a^2*b^2*c^3 - 6*a*b^3*c^3 + 8*a^2*b*c^4 + (b^6 - 10*a*b^4
*c - 2*b^5*c + 32*a^2*b^2*c^2 + 12*a*b^3*c^2 + b^4*c^2 - 32*a^3*c^3 - 16*a^
2*b*c^3 - 6*a*b^2*c^3 + 8*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2 -
 8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2
*c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A -
 (b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^
3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 + (b^6*c - 6*
a*b^4*c^2 - 2*b^5*c^2 + 8*a^2*b^2*c^3 + 4*a*b^3*c^3 + b^4*c^3 - 2*a*b^2*c^4
)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b*c^5 - sqrt(b^2*c^10 - 4*a*c^11))/c
^6)/((a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*
b^2*c^3 - 4*a^2*c^4)*c^2*abs(c))
```

maple [B] time = 0.05, size = 1171, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*C*x^2/c+B/c*x+1/4/c/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4
*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*a*A-1/4/c
/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*b^2+1/2/c/(4*a*c-b^2)*ln(-
2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a-1/4/c^2/(4*a*c-b^2)*ln
(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-1/c/(4*a*c-b^2)*ln
(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*a*b+1/4/c^2/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4
*a*c+b^2)^(1/2))*b^3*C-1/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(
1/2)*a+1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b^2+2/(
4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B-1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^3*B-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)
^(1/2)*b+1/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*a*A-1/4/c/(4*a*c-b^
2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b^2-1/2/c/(4*a*c-b^2)*ln(2*c*x^2+b+(-
4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+b+(
-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b^2-1/c/(4*a*c-b^2)*ln(2*c*x^2+b+(-
4*a*c+b^2)^(1/2))*C*a*b+1/4/c^2/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2)
)*b^3*C-1/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*a+1/2/c/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b^2-2/(4*a*c-b^2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*a*b*B+1/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Cx^2 + 2\,Bx}{2\,c} + \frac{-\int \frac{Bbx^2 + (Cb - Ac)x^3 + Cax + Ba}{cx^4 + bx^2 + a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/2*(C*x^2 + 2*B*x)/c + integrate(-(B*b*x^2 + (C*b - A*c)*x^3 + C*a*x + B*a
)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 1.53, size = 2696, normalized size = 9.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log((B^3*a^2*b*c - B*C^2*a^3*c + A^2*B*a^2*c^2 + B*C^2*a^2*b^2 - 2*A
*B*C*a^2*b*c)/c^2 - root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z
^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C
*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2
 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a
*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c
^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*
a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z +
12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a
*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*
C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A
^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*
a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a
^4, z, k)*(root(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*
C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*
z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C
*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*
z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 -
4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z
^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*
a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z
- 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c
^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*
b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c -
 B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k)
*((x*(16*C*a^2*c^4 - 8*A*b^3*c^3 + 8*C*b^4*c^2 + 32*A*a*b*c^4 - 36*C*a*b^2*
c^3))/c^2 - (16*B*a^2*c^4 - 4*B*a*b^2*c^3)/c^2 + (root(128*a*b^2*c^5*z^4 -
16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*C*a^2*b*c^4*z^3 + 128*C*a*b^3*c^3*z^
3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*z^3 + 16*A*b^4*c^3*z^3 + 256*A*a^2*c
^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C*a*b^3*c^2*z^2 + 8*A*C*b^5*c*z^2 - 4
8*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*z^2 + 40*A^2*a*b^2*c^3*z^2 + 32*C^2*
a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z^2 - 32*C^2*a^3*c^3*z^2
 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^6*z^2 + 4*B^2*C*a^2*b^2
*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*a^2*b^2*c*z + 16*A*B^2*a^2*b*c^2*z +
 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b^4*z - 4*A^3*a*b^2*c^2*
z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c^2*z - 16*C^3*a^3*b*c*z + 4*C^3*a^2*
b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2*C*a^3*c - 2*A^2*C^2*a^
3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*b - A^2*C^2*a^2*b^2 - A
^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k)*x*(8*b^3*c^4 - 32*a*b*c^5))/c^2) +
(8*A*B*a^2*c^3 - 4*B*C*a^2*b*c^2)/c^2 + (x*(2*C^2*b^5 + 2*B^2*b^4*c + 2*A^2
*b^3*c^2 + 4*B^2*a^2*c^3 - 4*A*C*b^4*c - 8*A*C*a^2*c^3 - 10*A^2*a*b*c^3 - 1
0*C^2*a*b^3*c - 8*B^2*a*b^2*c^2 + 6*C^2*a^2*b*c^2 + 20*A*C*a*b^2*c^2))/c^2)
 - (x*(C^3*a^3*c - C^3*a^2*b^2 + A*C^2*a*b^3 + A^3*a*b*c^2 - A*B^2*a^2*c^2
+ A^2*C*a^2*c^2 + A*B^2*a*b^2*c - 2*A^2*C*a*b^2*c - B^2*C*a^2*b*c))/c^2)*ro
ot(128*a*b^2*c^5*z^4 - 16*b^4*c^4*z^4 - 256*a^2*c^6*z^4 - 256*C*a^2*b*c^4*z
^3 + 128*C*a*b^3*c^3*z^3 - 128*A*a*b^2*c^4*z^3 - 16*C*b^5*c^2*z^3 + 16*A*b^
```

```
4*c^3*z^3 + 256*A*a^2*c^5*z^3 + 160*A*C*a^2*b*c^3*z^2 - 72*A*C*a*b^3*c^2*z^
2 + 8*A*C*b^5*c*z^2 - 48*B^2*a^2*b*c^3*z^2 + 28*B^2*a*b^3*c^2*z^2 + 40*A^2*
a*b^2*c^3*z^2 + 32*C^2*a*b^4*c*z^2 - 56*C^2*a^2*b^2*c^2*z^2 - 4*B^2*b^5*c*z
^2 - 32*C^2*a^3*c^3*z^2 - 4*A^2*b^4*c^2*z^2 - 96*A^2*a^2*c^4*z^2 - 4*C^2*b^
6*z^2 + 4*B^2*C*a^2*b^2*c*z - 32*A^2*C*a^2*b*c^2*z + 12*A*C^2*a^2*b^2*c*z +
 16*A*B^2*a^2*b*c^2*z + 8*A^2*C*a*b^3*c*z - 4*A*B^2*a*b^3*c*z - 4*A*C^2*a*b
^4*z - 4*A^3*a*b^2*c^2*z - 16*B^2*C*a^3*c^2*z + 16*A*C^2*a^3*c^2*z - 16*C^3
*a^3*b*c*z + 4*C^3*a^2*b^3*z + 16*A^3*a^2*c^3*z + 2*A^3*C*a^2*b*c + 4*A*B^2
*C*a^3*c - 2*A^2*C^2*a^3*c + 2*A*C^3*a^3*b - A^2*B^2*a^2*b*c - B^2*C^2*a^3*
b - A^2*C^2*a^2*b^2 - A^4*a^2*c^2 - B^4*a^3*c - C^4*a^4, z, k), k, 1, 4) +
(C*x^2)/(2*c) + (B*x)/c
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.23 $\int \frac{x^2(A+Bx+Cx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=270

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB\tanh^{-}}{2c\sqrt{b}}$$

```
[Out] C*x/c+1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1
/2))/c/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(A*c-b*C+(-A*b*c+(-2*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2))*(A*c-b*C+(A*b*c+2*C*a*c-C*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)
*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1662, 1279, 1166, 205, 12, 1114, 634, 618, 206, 628}

$$\frac{\left(-\frac{Abc-C(b^2-2ac)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2acC+Abc+b^2(-C)}{\sqrt{b^2-4ac}}+Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{bB\tanh^{-}}{2c\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (C*x)/c + ((A*c - b*C - (A*b*c - (b^2 - 2*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) + ((A*c - b*C + (A*b*c - b^2*C + 2*a*c*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \int \frac{Bx^3}{a+bx^2+cx^4}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{Cx}{c} + B\int \frac{x^3}{a+bx^2+cx^4}\,dx - \frac{\int \frac{aC+(-Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{Cx}{c} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{x}{a+bx+cx^2}\,dx, x, x^2\right) - \frac{\left(-Ac+bC+\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\int \frac{\cdots}{\frac{b}{2}-\frac{1}{2}\sqrt{\cdots}}}{2c}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-\left(b^2-2ac\right)C}{\sqrt{b^2-4ac}}\right)\cdots}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{\cdots}}}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-\left(b^2-2ac\right)C}{\sqrt{b^2-4ac}}\right)\cdots}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{\cdots}}}$$

$$= \frac{Cx}{c} + \frac{\left(Ac-bC-\frac{Abc-b^2C+2acC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(Ac-bC+\frac{Abc-\left(b^2-2ac\right)C}{\sqrt{b^2-4ac}}\right)\cdots}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{\cdots}}}$$

Mathematica [A] time = 0.37, size = 360, normalized size = 1.33

$$\frac{-\frac{2\sqrt{2}\left(Ac\left(b-\sqrt{b^2-4ac}\right)+C\left(b\sqrt{b^2-4ac}+2ac-b^2\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{2\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-Ac\left(\sqrt{b^2-4ac}+b\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\cdots}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (4*Sqrt[c]*C*x - (2*Sqrt[2]*(A*c*(b - Sqrt[b^2 - 4*a*c]) + (-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*(-(A*c*(
b + Sqrt[b^2 - 4*a*c])) + (b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b +
Sqrt[b^2 - 4*a*c]]) + (B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c]
)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*c^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.57, size = 3843, normalized size = 14.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] C*x/c + 1/4*B*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^3 - 16*a*b^2*c^
4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 -
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3
 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a
*c)*a*b*c^3)*C*c^2 - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 -
 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 -
4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2
*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)
*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*
C)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^
4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5
- 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*
A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*C*c^2 + 2*(sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4
+ 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt
```

```
(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2
- 4*a*c)*a^2*c^4)*C*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 -
 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*
(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*C)*arctan(2*sqrt(1/2)*x/sq
rt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*
a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/16*(
(b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b
^2*c^3 - (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2
- 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(c) + (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2
 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*
c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b*c^3 + sqr
t(b^2*c^6 - 4*a*c^7))/c^4)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 +
 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) - 1/16*((b^6 - 8*a*b^4*c
- 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 + (b^5 - 8
*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt
(b^2 - 4*a*c))*B*abs(c) + (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3
 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 +
 b^3*c^3)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b*c^3 - sqrt(b^2*c^6 - 4*a*c
^7))/c^4)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*
b^2*c^2 - 4*a^2*c^3)*c^2*abs(c))
```

maple [B] time = 0.05, size = 1327, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] C*x/c+1/4/c/(4*a*c-b^2)*B*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1
/2)*b+1/(4*a*c-b^2)*B*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*a-1/4/c/(4*a*c-b^2)
*B*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*b^2-1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a+1/2/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2-1/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*C*(-4*a*c+b^2)*b-1/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2
)*a+1/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^2+1/(4*a
*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*C*a-1/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b^3*C-1/4/c/(4*a*c-b^2)*B*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*(-4*a*c+b^2)^(1
/2)*b+1/(4*a*c-b^2)*B*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*a-1/4/c/(4*a*c-b^2)*
B*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*b^2-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A
*(-4*a*c+b^2)^(1/2)*b+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*a-1/2/(4*a*c-b^2
```

```
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*A*b^2+1/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b
^2)*b-1/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/2/c/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^2-1/(4*a*c-b^2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b*C*a+1/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] C*x/c + integrate((B*c*x^3 - (C*b - A*c)*x^2 - C*a)/(c*x^4 + b*x^2 + a), x)
/c
```

mupad [B] time = 2.00, size = 1890, normalized size = 7.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(- root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 12
8*B*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2
*z^2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C
^2*a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2
- 96*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*
z + 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2
*c*z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c -
 2*A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*
c - A^4*a*c^2 - C^4*a^3, z, k)*((8*B*C*a^2*c^2 - 4*A*B*a*b*c^2)/c - root(12
8*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a*b^2*c^3*z^3 +
16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2 + 8*A*C*b^4*c*z
^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*b^3*c*z^2 + 16*
A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*B^2*a^2*c^3*z^2
 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 16*B*C^2*a^2*b*c
*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z + 16*B^3*a^2*c^
2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^2*C^2*a^2*c + 2
*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A^4*a*c^2 - C^4*
a^3, z, k)*((16*C*a^2*c^3 - 4*C*a*b^2*c^2)/c + (x*(8*B*b^3*c^2 - 32*B*a*b*c
^3))/c - (root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B
*a*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^
2 + 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*
a*b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 9
6*B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z +
 16*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*
z + 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*
A^2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c -
 A^4*a*c^2 - C^4*a^3, z, k)*x*(8*b^3*c^3 - 32*a*b*c^4))/c) + (x*(2*C^2*b^4
- 4*A^2*a*c^3 + 2*B^2*b^3*c + 2*A^2*b^2*c^2 + 4*C^2*a^2*c^2 - 4*A*C*b^3*c -
 10*B^2*a*b*c^2 - 8*C^2*a*b^2*c + 12*A*C*a*b*c^2))/c) - (A^3*a*c^2 - C^3*a^
2*b + A*C^2*a*b^2 + A*C^2*a^2*c - B^2*C*a^2*c + A*B^2*a*b*c - 2*A^2*C*a*b*c
)/c - (x*(B^3*a*b*c + A^2*B*a*c^2 + B*C^2*a*b^2 - B*C^2*a^2*c - 2*A*B*C*a*b
*c))/c)*root(128*a*b^2*c^4*z^4 - 16*b^4*c^3*z^4 - 256*a^2*c^5*z^4 - 128*B*a
```

```
*b^2*c^3*z^3 + 16*B*b^4*c^2*z^3 + 256*B*a^2*c^4*z^3 - 48*A*C*a*b^2*c^2*z^2
+ 8*A*C*b^4*c*z^2 - 48*C^2*a^2*b*c^2*z^2 + 40*B^2*a*b^2*c^2*z^2 + 28*C^2*a*
b^3*c*z^2 + 16*A^2*a*b*c^3*z^2 + 64*A*C*a^2*c^3*z^2 - 4*B^2*b^4*c*z^2 - 96*
B^2*a^2*c^3*z^2 - 4*A^2*b^3*c^2*z^2 - 4*C^2*b^5*z^2 + 8*A*B*C*a*b^2*c*z + 1
6*B*C^2*a^2*b*c*z - 32*A*B*C*a^2*c^2*z - 4*B*C^2*a*b^3*z - 4*B^3*a*b^2*c*z
+ 16*B^3*a^2*c^2*z + 4*A*B^2*C*a^2*c + 2*A^3*C*a*b*c - A^2*B^2*a*b*c - 2*A^
2*C^2*a^2*c + 2*A*C^3*a^2*b - B^2*C^2*a^2*b - A^2*C^2*a*b^2 - B^4*a^2*c - A
^4*a*c^2 - C^4*a^3, z, k), k, 1, 4) + (C*x)/c
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.24 $\int \frac{x(A+Bx+Cx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=223

$$-\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] 1/4*C*ln(c*x^4+b*x^2+a)/c-1/2*(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^
(1/2))/c/(-4*a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/
2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+
b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1662, 1247, 634, 618, 206, 628, 12, 1130, 205}

$$-\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}-\frac{B\sqrt{b-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{\sqrt{b^2-4ac}+b}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((B*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])) + (B*Sqrt[b + Sqrt[b^2
- 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]
*Sqrt[c]*Sqrt[b^2 - 4*a*c]) - ((2*A*c - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*a*c]) + (C*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx &= \int \frac{Bx^2}{a+bx^2+cx^4}\,dx+\int \frac{x\left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{A+Cx}{a+bx+cx^2}\,dx,x,x^2\right)+B\int \frac{x^2}{a+bx^2+cx^4}\,dx \\
&= \frac{1}{2}\left(B\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(B\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}
\\
&= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}} \\
&= -\frac{B\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{B\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}
\end{aligned}
$$

Mathematica [A] time = 0.36, size = 240, normalized size = 1.08

$$
\frac{\left(C\left(\sqrt{b^2-4ac}-b\right)+2Ac\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)-\left(2Ac-C\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}+b+2\right.}{4c\sqrt{b^2-4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*Sqrt[2]*B*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]] + 2*Sqrt[2]*B*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]] + (2*A*c + (-
b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2] - (2*A*c -
(b + Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*c*Sqrt[
b^2 - 4*a*c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.36, size = 2369, normalized size = 10.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*C*log(abs(c*x^4 + b*x^2 + a))/c + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a
^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*
a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/s
qrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a
*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c^4 - 4*a^2*c^5)*c^2) - 1/8*((2
*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2
- 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - (2*b^4*c^4 - 8*a*b^2*c^5
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*
c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a
*b^4*c^2 - 8*a^2*b^2*c^3 - 2*a*b^3*c^3 + 16*a^3*c^4 + 8*a^2*b*c^4 + a*b^2*c
^4 - 4*a^2*c^5)*c^2) + 1/16*(2*(b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*
c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4 - (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2
+ 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) -
 (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*
```

```
b^2*c^3 - (b^5 - 8*a*b^3*c - 2*b^4*c + 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2
 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(c) + 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^
4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b
^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A - (b^6*c - 8*a*b^4*c^2 -
 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c
- 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b
*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a
^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(c)) + 1/16*(2*(b^5*c
- 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^
4 + (b^4*c - 8*a*b^2*c^2 - 2*b^3*c^2 + 16*a^2*c^3 + 8*a*b*c^3 + b^2*c^3 - 4
*a*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - (b^6 - 8*a*b^4*c - 2*b^5*c + 16*a^2*b
^2*c^2 + 8*a*b^3*c^2 + b^4*c^2 - 4*a*b^2*c^3 + (b^5 - 8*a*b^3*c - 2*b^4*c +
 16*a^2*b*c^2 + 8*a*b^2*c^2 + b^3*c^2 - 4*a*b*c^3)*sqrt(b^2 - 4*a*c))*C*abs
(c) + 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b
^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2
 - 4*a*c))*A - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*
c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*s
qrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b*c - sqrt(b^2*c^2 - 4*a*c^3))/c^2)/((a
*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a
^2*c^3)*c^2*abs(c))
```

maple [B] time = 0.04, size = 728, normalized size = 3.26

$$-\frac{2\sqrt{2}\,Bac\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{2\sqrt{2}\,Bac\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,B\,b^2\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)+1/4
/c/(4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b+1/(
4*a*c-b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*a*C-1/4/c/(4*a*c-b^2)*ln(-2*c*
x^2-b+(-4*a*c+b^2)^(1/2))*b^2*C-1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4
*a*c+b^2)^(1/2)*b-2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B+1/2/(4*a*c-b^2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2*B+1/2/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2
))*A*(-4*a*c+b^2)^(1/2)-1/4/c/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*
C*(-4*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*a*C-1
/4/c/(4*a*c-b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*b^2*C-1/2/(4*a*c-b^2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a
*B-1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2+Bx+A\right)x}{cx^4+bx^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)*x/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 1.89, size = 5594, normalized size = 25.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(A^3*c^2*x - B^3*a*c - B*C^2*a*b - 8*root(128*a*b^2*c^3*z^4 - 16*
b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 1
6*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 +
 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z
^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z
 + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z -
 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*
c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a
^2 - A^4*c^2, z, k)^3*b^3*c^2*x - C^3*a*b*x + A*C^2*b^2*x - 2*C^2*root(128*
a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 25
6*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 4
0*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2
 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*
z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z
 + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C
^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c -
A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^3*x + 32*root(128*a*b^2*c^3*z^4 -
16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3
+ 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^
2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^
2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*
c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*
z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C
*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^
4*a^2 - A^4*c^2, z, k)^3*a*b*c^3*x - 4*A*root(128*a*b^2*c^3*z^4 - 16*b^4*c^
2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^
4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^
2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8
*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*
A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3
*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*
A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A
^4*c^2, z, k)^2*b^2*c^2*x - 8*A*B*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 -
 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3
 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c
^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^
2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a
*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*
z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a
*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2,
 z, k)*a*c^2 + A*B^2*b*c*x + A*C^2*a*c*x - 2*A^2*C*b*c*x - B^2*C*a*c*x + 2*
A*B*C*a*c + 16*A*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4
- 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2
*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^
3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2
*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*
b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z
 + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b
- A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)^2*a*c^3*x
+ 2*A^2*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a
*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*
A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 -
```

```
 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2
- 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z -
 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^
2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2
*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b*c^2*x + 4*B^2*roo
t(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^
3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z
^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c
^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*
a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a
*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2
*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*
a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2*x - 2*B^2*root(128*a*b^2
*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a
^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2
*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96
*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4
*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16
*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*
c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C
^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*b^2*c*x + 8*C*root(128*a*b^2*c^3*z^4 - 16
*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 +
16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2
+ 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*
z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*
z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z
- 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b
*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*
a^2 - A^4*c^2, z, k)^2*b^3*c*x - 32*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z
^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c
*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a
*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^
2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2
*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*
b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C
^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*
c^2, z, k)^2*a*b*c^2*x + 4*B*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 25
6*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 +
32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*
z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c
^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^
2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z +
 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b
- B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z,
 k)*a*b*c - 8*A*C*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4
 - 128*C*a*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^
2*z^2 - 8*A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b
^3*c*z^2 - 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^
2*b^4*z^2 - 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2
*b^2*c*z - 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*
z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b
 - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*c^2*x +
 10*C^2*root(128*a*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a
*b^2*c^2*z^3 + 256*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*
A*C*b^3*c*z^2 + 40*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 -
 32*A^2*a*c^3*z^2 - 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2
- 16*A*C^2*a*b*c*z - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z -
 16*A*B^2*a*c^2*z + 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^
2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2
```

```
*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*a*b*c*x)*root(128*a
*b^2*c^3*z^4 - 16*b^4*c^2*z^4 - 256*a^2*c^4*z^4 - 128*C*a*b^2*c^2*z^3 + 256
*C*a^2*c^3*z^3 + 16*C*b^4*c*z^3 + 32*A*C*a*b*c^2*z^2 - 8*A*C*b^3*c*z^2 + 40
*C^2*a*b^2*c*z^2 + 16*B^2*a*b*c^2*z^2 - 4*B^2*b^3*c*z^2 - 32*A^2*a*c^3*z^2
- 96*C^2*a^2*c^2*z^2 + 8*A^2*b^2*c^2*z^2 - 4*C^2*b^4*z^2 - 16*A*C^2*a*b*c*z
 - 4*A^2*C*b^2*c*z + 16*A^2*C*a*c^2*z + 4*A*B^2*b^2*c*z - 16*A*B^2*a*c^2*z
+ 16*C^3*a^2*c*z - 4*C^3*a*b^2*z + 4*A*C^2*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^
2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A
^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.25 $\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4} dx$

Optimal. Leaf size=211

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

```
[Out] -B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(
1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(
1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(C+(-2*A*c+C*b)/(-4*a*c+b^2)^(1/2))*2^(1
/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1673, 1166, 205, 12, 1107, 618, 206}

$$\frac{\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((C -
(2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (B*ArcTanh[(
b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{a+bx^2+cx^4}\,dx = \int \frac{Bx}{a+bx^2+cx^4}\,dx + \int \frac{A+Cx^2}{a+bx^2+cx^4}\,dx$$

$$= B\int \frac{x}{a+bx^2+cx^4}\,dx + \frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(C+\frac{2Ac}{\sqrt{b^2-}}\right.$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{1}{2}B\,\mathrm{Subst}\left(\int\right.$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - B\,\mathrm{Subst}\left(\int \frac{}{b^2}\right.$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}} - \frac{B\tanh^{-1}\left(\frac{b+2}{\sqrt{b^2}}\right.}{\sqrt{b^2-4ac}}$$

Mathematica [A] time = 0.21, size = 234, normalized size = 1.11

$$\frac{\frac{\sqrt{2}\left(C\left(\sqrt{b^2-4ac}-b\right)+2Ac\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\left(C\left(\sqrt{b^2-4ac}+b\right)-2Ac\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}} + B\log\left(\sqrt{b^2-4ac}-b-2cx^2\right) - B}{2\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((Sqrt[2]*(2*A*c + (-b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[
2]*(-2*A*c + (b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + B*Log[-b + Sq
```

```
rt[b^2 - 4*a*c] - 2*c*x^2] - B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(2*Sqr
t[b^2 - 4*a*c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 4.37, size = 1616, normalized size = 7.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c)*B*log(x^2 + 1/2*
(b + sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b
*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/2*(b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sq
rt(b^2 - 4*a*c)*B*log(x^2 + 1/2*(b - sqrt(b^2 - 4*a*c))/c)/((b^4 - 8*a*b^2*
c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2) + 1/4*((sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*
c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^2*c^2 + 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c^3 - 32*a^2*c^3 + 8*a*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2
+ 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A
+ 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2
*(b^2 - 4*a*c)*a*c^2)*C)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2 - 4*a*c))/
c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^
2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*c^2 - 4*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*c^3 + sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c
)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*C)*arctan(2*sqrt(1
/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c +
16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

maple [B] time = 0.02, size = 616, normalized size = 2.92

$$-\frac{2\sqrt{2}\,Cac\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{2\sqrt{2}\,Cac\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\,Cb^2\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\left(4ac-b^2\right)\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*B*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))+c*(
-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-2*c/(4*a*c-b^2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*C*a+1/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*b^2-1/2*(-4*a*
c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*C+1/2*(-4*a*c+b^2)^(1/2)/(
4*a*c-b^2)*B*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))+c*(-4*a*c+b^2)^(1/2)/(4*a*c-b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*A+2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a-1/2/(4*a*c-
b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*C*b^2-1/2*(-4*a*c+b^2)^(1/2)/(4*a*c-b^2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b*C
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{Cx^2 + Bx + A}{cx^4 + bx^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((C*x^2 + B*x + A)/(c*x^4 + b*x^2 + a), x)
```

mupad [B] time = 2.31, size = 3942, normalized size = 18.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] symsum(log(A*B^2*c^2 - A^2*C*c^2 + B^3*c^2*x - C^3*a*c + A*C^2*b*c - 8*root
(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^
2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^
2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C
^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2
*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*
b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*b^3*c^2*x - 16*A*r
oot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c
*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*
B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*
B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B
^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*a*c^3 - 4*A^2*r
```

```
oot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c
*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C
*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*
B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*
B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B
^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*c^3*x + 4*A*root(
16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2
 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2
*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^
2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*
C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b
*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*b^2*c^2 + 32*root(1
6*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2
- 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*
c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2
*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C
*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*b*
c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^3*a*b*c^3*x - 4*B*root
(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^
2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*a^
2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B*C
^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B^2
*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^2*
b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)^2*b^2*c^2*x + 4*A*B*
root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*
c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*
C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16
*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A
*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*
B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*b*c^2 - 8*B*C*ro
ot(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3*z^4 - 16*A*C*a*b^2*c*
z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2*a*b*c^2*z^2 + 64*A*C*
a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*B^2*a^2*c^2*z^2 + 16*B
*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16*A^2*B*a*c^2*z - 4*A*B
^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b + B^2*C^2*a*b + A^2*B^
2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z, k)*a*c^2 - 2*A*B*C*c^
2*x + B*C^2*b*c*x + 16*B*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^
3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 1
6*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2
+ 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z
 - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3
*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^
2, z, k)^2*a*c^3*x + 2*B^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*b*c^2*x + 4*C^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*a*c^2*x - 2*C^2*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
```

```
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*b^2*c*x + 4*A*C*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*
a^3*c^3*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 -
 16*A^2*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^
2 + 32*B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2
*z - 16*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C
^3*a*b + B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*
c^2, z, k)*b*c^2*x)*root(16*a*b^4*c*z^4 - 128*a^2*b^2*c^2*z^4 + 256*a^3*c^3
*z^4 - 16*A*C*a*b^2*c*z^2 - 16*C^2*a^2*b*c*z^2 - 8*B^2*a*b^2*c*z^2 - 16*A^2
*a*b*c^2*z^2 + 64*A*C*a^2*c^2*z^2 + 4*C^2*a*b^3*z^2 + 4*A^2*b^3*c*z^2 + 32*
B^2*a^2*c^2*z^2 + 16*B*C^2*a^2*c*z + 4*A^2*B*b^2*c*z - 4*B*C^2*a*b^2*z - 16
*A^2*B*a*c^2*z - 4*A*B^2*C*a*c + 2*A^2*C^2*a*c - 2*A^3*C*b*c - 2*A*C^3*a*b
+ B^2*C^2*a*b + A^2*B^2*b*c + B^4*a*c + A^2*C^2*b^2 + C^4*a^2 + A^4*c^2, z,
 k), k, 1, 4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.26 $\int \frac{A+Bx+Cx^2}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=229

$$\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}+\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\cdots}{\sqrt{b^2-4ac}\sqrt{\cdots}}$$

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*C*a)*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)+B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*2^(1/2)*c^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2)-B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*2^(1/2)*c
^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1662, 1251, 800, 634, 618, 206, 628, 12, 1093, 205}

$$\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}+\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}B\sqrt{c}\tan^{-1}\cdots}{\sqrt{b^2-4ac}\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4]
)/(4*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)}dx = \int \frac{B}{a+bx^2+cx^4}dx + \int \frac{A+Cx^2}{x\left(a+bx^2+cx^4\right)}dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x\left(a+bx+cx^2\right)}dx, x, x^2\right) + B\int \frac{1}{a+bx^2+cx^4}dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aC-Acx}{a\left(a+bx+cx^2\right)}\right)dx, x, x^2\right) + \frac{(Bc)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}dx}{\sqrt{b^2-4ac}} - \frac{(Bc)}{}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} + \frac{\text{Subst}\left(\right.}{}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{A\,\text{Subs}}{}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{A\log(x)}{a} - \frac{A\log\left(a\right.}{}$$
$$= \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2}B\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}} + \frac{(Ab-2aC)\tanh^{-1}}{2a\sqrt{b^2-4a}}$$

Mathematica [A] time = 0.44, size = 285, normalized size = 1.24

$$-\frac{\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{4a\sqrt{b^2-4ac}} - \frac{\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\log\left(\sqrt{b^2-4ac}+b+\right.}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[2]*B*Sqrt[c]*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])
/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b + Sqrt[b^2 - 4*a*c]]) + (A*Log[x])/a - ((A*(b + Sqrt[b^2 - 4*a*c]) - 2*a
*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c]) - ((A*(-
b + Sqrt[b^2 - 4*a*c]) + 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*
Sqrt[b^2 - 4*a*c])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.08, size = 2336, normalized size = 10.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*A*log(abs(c*x^4 + b*x^2 + a))/a + A*log(abs(x))/a + 1/4*((sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 -
 16*a*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^
3 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) + (2*b^3*c^3 -
8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*
b*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b^2*c^2 - 4*a^5*c^3
))/(a^2*c^2)))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
 + a*b^2*c^2 - 4*a^2*c^3)*c^2) + 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2
+ sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 - 4*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 + 2*(b^2 - 4*a*c)*b^2*
c - 8*(b^2 - 4*a*c)*a*c^2)*B*abs(c) - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*B)*arctan(2*sqrt(
1/2)*x/sqrt((a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a*b^4 -
 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3
)*c^2) - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*
c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c
^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - 2*(a*
b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*
c^3 - 4*a^2*b*c^4 - (a*b^4*c - 8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8
*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2
- 8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^
2*c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A
- 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4
 + a*b^3*c^4 - 4*a^2*b*c^5 - (a*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b
^2*c^4)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^2*b*c + sqrt(a^4*b^2*c^2 - 4
*a^5*c^3))/(a^2*c^2))/((a^2*b^4 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 +
8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*c^2*abs(c)) - 1/16*((b^6*c - 8*a*b^4
*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (
b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^3 - 4*
a*b*c^4)*sqrt(b^2 - 4*a*c))*A*abs(c) - 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4
*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 + (a*b^4*c -
8*a^2*b^2*c^2 - 2*a*b^3*c^2 + 16*a^3*c^3 + 8*a^2*b*c^3 + a*b^2*c^3 - 4*a^2*
c^4)*sqrt(b^2 - 4*a*c))*C*abs(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b^5*c^3 + 16*
a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 + (b^5*c^2 - 4*a*b^3*c^3
- 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*A - 2*(a*b^5*c^2 - 8*a^2*b^3*c^3
- 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 + (a
*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*C)*l
og(x^2 + 1/2*(a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2))/((a^2*b^4
 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a
^3*c^3)*c^2*abs(c))
```

maple [B] time = 0.04, size = 488, normalized size = 2.13

$$\frac{A\,b^2\ln\left(-2c\,x^2-b+\sqrt{-4ac+b^2}\right)}{\left(16ac-4b^2\right)a}+\frac{A\,b^2\ln\left(2c\,x^2+b+\sqrt{-4ac+b^2}\right)}{\left(16ac-4b^2\right)a}-\frac{4Ac\ln\left(-2c\,x^2-b+\sqrt{-4ac+b^2}\right)}{16ac-4b^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] A/a*ln(x)-4*c/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A+1/a/(16*a*
c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*b^2+1/a*(-4*a*c+b^2)^(1/2)/(16
*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*b-2*(-4*a*c+b^2)^(1/2)/(16*
a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C+4*c*(-4*a*c+b^2)^(1/2)/(16*a
*c-4*b^2)*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-4*c/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b
^2)^(1/2))*A+1/a/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b^2-1/a*
(-4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b+2*(-
4*a*c+b^2)^(1/2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C+4*c*(-4*
a*c+b^2)^(1/2)/(16*a*c-4*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] A*log(x)/a - integrate((A*c*x^3 - B*a - (C*a - A*b)*x)/(c*x^4 + b*x^2 + a),
 x)/a
```

mupad [B] time = 1.49, size = 2258, normalized size = 9.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(x*(B^4*c^3 + C^4*a*c^2 + A^2*C^2*c^3 - 3*A*B^2*C*c^3 - A*C^3*b*c
^2 + B^2*C^2*b*c^2) - root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4
*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^
2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*
C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 -
4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*
B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*
b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2
*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(A
*B^2*b*c^3 - 5*A^3*c^4 - 13*A*C^2*a*c^3 + 6*A^2*C*b*c^3 + 17*B^2*C*a*c^3 +
C^3*a*b*c^2 + A*C^2*b^2*c^2 - 4*B^2*C*b^2*c^2) - root(128*a^3*b^2*c*z^4 - 2
56*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 -
 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2
 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*
z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*
c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z
- 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*
b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4
```

```
*a^2 - A^4*c^2, z, k)*(x*(60*A^2*a*c^4 - 16*A^2*b^2*c^3 + 4*B^2*b^3*c^2 + 3
6*C^2*a^2*c^3 + 8*A*C*b^3*c^2 - 14*B^2*a*b*c^3 - 10*C^2*a*b^2*c^2 - 28*A*C*
a*b*c^3) + root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*
A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 -
 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z
^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z
^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2
*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A
*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*
B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*(x*(240*A*a^2*c^
4 + 12*A*b^4*c^2 - 108*A*a*b^2*c^3 + 4*C*a*b^3*c^2 - 16*C*a^2*b*c^3) + root
(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^4 + 128*A*a^2*b^2*c*z^3
 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b*c*z^2 - 8*A*C*a*b^3*z^
2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2*a^3*c*z^2 - 4*B^2*a*b^
3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A^2*b^4*z^2 + 16*A^2*C*a
*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2*C*a*b^2*z + 4*A*C^2*a*
b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3*z + 4*A*B^2*C*a*c - 2*
A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*b - A^2*B^2*b*c - B^4*a
*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k)*x*(320*a^3*c^4 + 24*a*b^4*c^2 -
 176*a^2*b^2*c^3) - 4*B*a*b^3*c^2 + 16*B*a^2*b*c^3) + 4*A*B*b^3*c^2 + 8*B*C
*a^2*c^3 - 12*A*B*a*b*c^3 - 4*B*C*a*b^2*c^2) + B^3*a*c^3 + 4*A^2*B*b*c^3 +
6*A*B*C*a*c^3 - 4*A*B*C*b^2*c^2 + B*C^2*a*b*c^2) + A*B^3*c^3 - 2*A^2*B*C*c^
3 + A*B*C^2*b*c^2)*root(128*a^3*b^2*c*z^4 - 256*a^4*c^2*z^4 - 16*a^2*b^4*z^
4 + 128*A*a^2*b^2*c*z^3 - 256*A*a^3*c^2*z^3 - 16*A*a*b^4*z^3 + 32*A*C*a^2*b
*c*z^2 - 8*A*C*a*b^3*z^2 + 16*B^2*a^2*b*c*z^2 + 40*A^2*a*b^2*c*z^2 - 32*C^2
*a^3*c*z^2 - 4*B^2*a*b^3*z^2 + 8*C^2*a^2*b^2*z^2 - 96*A^2*a^2*c^2*z^2 - 4*A
^2*b^4*z^2 + 16*A^2*C*a*b*c*z + 16*B^2*C*a^2*c*z - 16*A*C^2*a^2*c*z - 4*B^2
*C*a*b^2*z + 4*A*C^2*a*b^2*z + 4*A^3*b^2*c*z - 16*A^3*a*c^2*z - 4*A^2*C*b^3
*z + 4*A*B^2*C*a*c - 2*A^2*C^2*a*c + 2*A^3*C*b*c + 2*A*C^3*a*b - B^2*C^2*a*
b - A^2*B^2*b*c - B^4*a*c - A^2*C^2*b^2 - C^4*a^2 - A^4*c^2, z, k), k, 1, 4
) + (A*log(x))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.27 $\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=260

$$-\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\cdots$$

```
[Out] -A/a/x+B*ln(x)/a-1/4*B*ln(c*x^4+b*x^2+a)/a+1/2*b*B*arctanh((2*c*x^2+b)/(-4*
a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(A*b-2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(A+(-A*b+2*C*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*
c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1662, 1281, 1166, 205, 12, 1114, 705, 29, 634, 618, 206, 628}

$$-\frac{\sqrt{c}\left(\frac{Ab-2aC}{\sqrt{b^2-4ac}}+A\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}+\frac{bB\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
]) + (b*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*a*c])
 + (B*Log[x])/a - (B*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 705

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)}dx = \int \frac{B}{x\left(a+bx^2+cx^4\right)}dx + \int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)}dx$$

$$= -\frac{A}{ax} - \frac{\int \frac{Ab-aC+Acx^2}{a+bx^2+cx^4}dx}{a} + B\int \frac{1}{x\left(a+bx^2+cx^4\right)}dx$$

$$= -\frac{A}{ax} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{x\left(a+bx+cx^2\right)}dx, x, x^2\right) - \frac{\left(c\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac+}}}{2a}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{b^2-}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{b^2-}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{b^2-}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

$$= -\frac{A}{ax} - \frac{\sqrt{c}\left(A+\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{c}\left(A-\frac{Ab-2aC}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+\sqrt{b^2-}}}\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 1.08, size = 315, normalized size = 1.21

$$-\frac{\frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}+b\right)-2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{2\sqrt{2}\sqrt{c}\left(A\left(\sqrt{b^2-4ac}-b\right)+2aC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}} + \frac{B\left(\sqrt{b^2-4ac}+b\right)\log\left(\sqrt{b^2-4a}\right.}{\sqrt{b^2-4ac}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*((4*A)/x + (2*Sqrt[2]*Sqrt[c]*(A*(b + Sqrt[b^2 - 4*a*c]) - 2*a*C)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) + (2*Sqrt[2]*Sqrt[c]*(A*(-b + Sqrt[b^2 - 4*a*c])
+ 2*a*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - 4*B*Log[x] + (B*(b + Sqrt[b^2 - 4*
a*c])*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (B*(-b + S
qrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/
a
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.39, size = 3507, normalized size = 13.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*B*log(abs(c*x^4 + b*x^2 + a))/a + B*log(abs(x))/a - A/(a*x) - 1/8*((2*
b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 -
 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^
2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3
- 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2
*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(c) - 2*(s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 +
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 -
 4*a*c)*a^2*c^3)*C*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*A - 2*(2*a*b^3*c^
4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 -
2*(b^2 - 4*a*c)*a*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c + sqrt(a^4*b
^2*c^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^
2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) + 1/8*((2*b^4*
c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a
*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*c^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16
*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c
^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(c) + 2*(sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
```

```
- 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a
*c)*a^2*c^3)*C*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*A - 2*(2*a*b^3*c^4 -
8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b
^2 - 4*a*c)*a*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b*c - sqrt(a^4*b^2*c
^2 - 4*a^5*c^3))/(a^2*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 - 2*a^2*b^3*c^2 +
16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) - 1/16*((b^6*c - 8
*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c
^4 - (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8*a*b^2*c^3 + b^3*c^
3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*B*abs(c) + (b^6*c^2 - 8*a*b^4*c^3 - 2*b^5
*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*c^5 - (b^5*c^2 - 4*
a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(a^2*b
*c + sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2))/((a^2*b^4 - 8*a^3*b^2*c - 2*
a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a^3*c^3)*c^2*abs(c))
 - 1/16*((b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 +
b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 8*a*b^3*c^2 - 2*b^4*c^2 + 16*a^2*b*c^3 + 8
*a*b^2*c^3 + b^3*c^3 - 4*a*b*c^4)*sqrt(b^2 - 4*a*c))*B*abs(c) + (b^6*c^2 -
8*a*b^4*c^3 - 2*b^5*c^3 + 16*a^2*b^2*c^4 + 8*a*b^3*c^4 + b^4*c^4 - 4*a*b^2*
c^5 + (b^5*c^2 - 4*a*b^3*c^3 - 2*b^4*c^3 + b^3*c^4)*sqrt(b^2 - 4*a*c))*B)*l
og(x^2 + 1/2*(a^2*b*c - sqrt(a^4*b^2*c^2 - 4*a^5*c^3))/(a^2*c^2))/((a^2*b^4
 - 8*a^3*b^2*c - 2*a^2*b^3*c + 16*a^4*c^2 + 8*a^3*b*c^2 + a^2*b^2*c^2 - 4*a
^3*c^3)*c^2*abs(c))
```

maple [B] time = 0.04, size = 811, normalized size = 3.12

$$-\frac{2\sqrt{2}\,A\,b^{2}c\,\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(16ac-4b^{2}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\;a}+\frac{2\sqrt{2}\,A\,b^{2}c\,\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(16ac-4b^{2}\right)\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\;a}+\frac{8\sqrt{2}\,A\,c^{2}\operatorname{arctanh}\left(\cdots\right)}{\left(16ac-4b^{2}\right)\sqrt{\left(-b+\sqrt{-4a\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -A/a/x+B/a*ln(x)+1/a/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*B*(-4
*a*c+b^2)^(1/2)*b-4*c/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*B+1/
a/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*b^2*B-2/a*c/(16*a*c-4*b^
2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b+8*c^2/(16*a*c-4*b^2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*A-2/a*c/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+4*c/(16*
a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)-1/a/(16*a*c-4*b^2)*ln
(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b-4*c/(16*a*c-4*b^2)*ln
(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*B+1/a/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^
2)^(1/2))*b^2*B-2/a*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2
)*b-8*c^2/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A+2/a*c/(16*a*c-4*b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*b^2+4*c/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*log(x)/a - integrate((B*c*x^3 + A*c*x^2 + B*b*x - C*a + A*b)/(c*x^4 + b*x
^2 + a), x)/a - A/(a*x)
```

mupad [B] time = 1.02, size = 2588, normalized size = 9.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*
B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z
^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2
*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 -
96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z
- 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c
*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2
*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c
- B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16
*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 -
 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2
*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 -
 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 -
 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*
c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c
- B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2
*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*(root(128*a^4*b^2*c*z^4 - 2
56*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 -
 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2
*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 6
4*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^
2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c
^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c
^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*
C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k)*((16*A*a
^3*c^4 + 4*A*a*b^4*c^2 + 16*C*a^3*b*c^3 - 20*A*a^2*b^2*c^3 - 4*C*a^2*b^3*c^
2)/a + (x*(240*B*a^4*c^4 + 12*B*a^2*b^4*c^2 - 108*B*a^3*b^2*c^3))/a^2 + (ro
ot(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 - 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z
^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*
b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2
+ 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^
2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*
b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a
^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^
2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 -
 A^4*c^3, z, k)*x*(320*a^5*c^4 + 24*a^3*b^4*c^2 - 176*a^4*b^2*c^3))/a^2) -
(8*A*B*a^2*c^4 + 4*A*B*b^4*c^2 - 16*A*B*a*b^2*c^3 - 4*B*C*a*b^3*c^2 + 12*B*
C*a^2*b*c^3)/a + (x*(4*A^2*b^5*c^2 + 60*B^2*a^3*c^4 - 16*B^2*a^2*b^2*c^3 +
4*C^2*a^2*b^3*c^2 - 72*A*C*a^3*c^4 - 28*A^2*a*b^3*c^3 + 50*A^2*a^2*b*c^4 -
14*C^2*a^3*b*c^3 + 48*A*C*a^2*b^2*c^3 - 8*A*C*a*b^4*c^2))/a^2) - (C^3*a^2*c
^3 + 7*A*B^2*a*c^4 + A^2*C*a*c^4 - 4*A*B^2*b^2*c^3 - A*C^2*a*b*c^3 + 4*B^2*
C*a*b*c^3)/a + (x*(5*B^3*a^2*c^4 - 4*A^2*B*b^3*c^3 - B*C^2*a^2*b*c^3 - 26*A
```

```
*B*C*a^2*c^4 + 14*A^2*B*a*b*c^4 + 8*A*B*C*a*b^2*c^3))/a^2) - (A*B^3*c^4 - A
^2*B*C*c^4 - B*C^3*a*c^3 + A*B*C^2*b*c^3)/a + (x*(A^4*c^5 + C^4*a^2*c^3 + A
^2*C^2*b^2*c^3 - 2*A^3*C*b*c^4 + A^2*B^2*b*c^4 + 2*A^2*C^2*a*c^4 - 2*A*B^2*
C*a*c^4 - 2*A*C^3*a*b*c^3))/a^2)*root(128*a^4*b^2*c*z^4 - 256*a^5*c^2*z^4 -
 16*a^3*b^4*z^4 + 128*B*a^3*b^2*c*z^3 - 256*B*a^4*c^2*z^3 - 16*B*a^2*b^4*z^
3 - 48*A*C*a^2*b^2*c*z^2 + 8*A*C*a*b^4*z^2 + 40*B^2*a^2*b^2*c*z^2 - 48*A^2*
a^2*b*c^2*z^2 + 16*C^2*a^3*b*c*z^2 + 28*A^2*a*b^3*c*z^2 + 64*A*C*a^3*c^2*z^
2 - 4*B^2*a*b^4*z^2 - 96*B^2*a^3*c^2*z^2 - 4*C^2*a^2*b^3*z^2 - 4*A^2*b^5*z^
2 - 8*A*B*C*a*b^2*c*z - 16*A^2*B*a*b*c^2*z + 32*A*B*C*a^2*c^2*z + 4*A^2*B*b
^3*c*z + 4*B^3*a*b^2*c*z - 16*B^3*a^2*c^2*z + 4*A*B^2*C*a*c^2 + 2*A*C^3*a*b
*c - B^2*C^2*a*b*c - 2*A^2*C^2*a*c^2 + 2*A^3*C*b*c^2 - A^2*C^2*b^2*c - A^2*
B^2*b*c^2 - C^4*a^2*c - B^4*a*c^2 - A^4*c^3, z, k), k, 1, 4) - A/(a*x) + (B
*log(x))/a
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.28 $\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(A\left(b^2-2ac\right)-abC\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4a}}\right.}{\sqrt{2}\,a}$$

```
[Out] -1/2*A/a/x^2-B/a/x-(A*b-C*a)*ln(x)/a^2+1/4*(A*b-C*a)*ln(c*x^4+b*x^2+a)/a^2-
1/2*(A*(-2*a*c+b^2)-a*b*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*
a*c+b^2)^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*c^(1/2)*(1+b/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/
2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(1-b/(-4
*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.393, Rules used = {1662, 1251, 800, 634, 618, 206, 628, 12, 1123, 1166, 205}

$$-\frac{\left(A\left(b^2-2ac\right)-abC\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aC)}{a^2}-\frac{A}{2ax^2}-\frac{B\sqrt{c}\left(\frac{b}{\sqrt{b^2-4a}}\right.}{\sqrt{2}\,a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -A/(2*a*x^2) - B/(a*x) - (B*Sqrt[c]*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - (B*Sqrt[c]*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((A
*(b^2 - 2*a*c) - a*b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sq
rt[b^2 - 4*a*c]) - ((A*b - a*C)*Log[x])/a^2 + ((A*b - a*C)*Log[a + b*x^2 +
c*x^4])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx = \int \frac{B}{x^2\left(a+bx^2+cx^4\right)}\,dx + \int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right) + B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

$$= -\frac{B}{ax} + \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2} + \frac{-Ab+aC}{a^2x} + \frac{A\left(b^2-ac\right)-abC+c(Ab-aC)x}{a^2\left(a+bx+cx^2\right)}\right) dx, x, x^2$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{(Ab-aC)\log(x)}{a^2} + \frac{\text{Subst}\left(\int \frac{A\left(b^2-ac\right)-abC+c(Ab-aC)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2} - \frac{\left(Bc\left(1\right.\right.}{}$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4a}}}$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4a}}}$$

$$= -\frac{A}{2ax^2} - \frac{B}{ax} - \frac{B\sqrt{c}\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}} - \frac{B\sqrt{c}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4a}}}$$

Mathematica [A] time = 0.89, size = 377, normalized size = 1.31

$$\frac{\frac{\left(A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)-aC\left(\sqrt{b^2-4ac}+b\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\sqrt{b^2-4ac}} + \frac{\left(A\left(b\sqrt{b^2-4ac}+2ac-b^2\right)+aC\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)}{\sqrt{b^2-4ac}} + 4\log}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*Sqrt[2]*a*B*Sqrt[c]*(b + Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]
*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (2*Sqrt[2]*a*B*Sqrt[c]*(-b + Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*(-(A*b) + a*C)*Log[x] + ((A*(b^2 - 2
*a*c + b*Sqrt[b^2 - 4*a*c]) - a*(b + Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^
2 - 4*a*c] - 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((A*(-b^2 + 2*a*c + b*Sqrt[b^2 -
 4*a*c]) + a*(b - Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.87, size = 3353, normalized size = 11.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(C*a - A*b)*log(abs(c*x^4 + b*x^2 + a))/a^2 + (C*a - A*b)*log(abs(x))/
a^2 - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 + 2*(sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5
*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B
*abs(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b
*c + sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2
 - 2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2)
 + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3
 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*B*c^2 - 2*(sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^
2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c
^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*B*ab
s(c) + (2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b*c
- sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2)))/((a^2*b^4*c - 8*a^3*b^2*c^2 -
2*a^2*b^3*c^2 + 16*a^4*c^3 + 8*a^3*b*c^3 + a^2*b^2*c^3 - 4*a^3*c^4)*c^2) +
1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 12*a*b^4*c^3 + b
^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2*b*c^5 - (b^6*c
 - 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 32*
a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 - 4*a*c))*A*abs(
c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3 + 8*a^2*b^3*c^
3 + a*b^4*c^3 - 4*a^2*b^2*c^4 - (a*b^5*c - 8*a^2*b^3*c^2 - 2*a*b^4*c^2 + 16
*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*
abs(c) + (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^4 + 12*a*b^4*c^
4 + b^5*c^4 - 32*a^3*b*c^5 - 16*a^2*b^2*c^5 - 6*a*b^3*c^5 + 8*a^2*b*c^6 + (
```

```
b^6*c^2 - 6*a*b^4*c^3 - 2*b^5*c^3 + 8*a^2*b^2*c^4 + 4*a*b^3*c^4 + b^4*c^4 -
 2*a*b^2*c^5)*sqrt(b^2 - 4*a*c))*A - (a*b^6*c^2 - 8*a^2*b^4*c^3 - 2*a*b^5*c
^3 + 16*a^3*b^2*c^4 + 8*a^2*b^3*c^4 + a*b^4*c^4 - 4*a^2*b^2*c^5 - (a*b^5*c^
2 - 4*a^2*b^3*c^3 - 2*a*b^4*c^3 + a*b^3*c^4)*sqrt(b^2 - 4*a*c))*C)*log(x^2
+ 1/2*(a^4*b*c + sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2))/((a^3*b^4 - 8*a^
4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)
*c^2*abs(c)) + 1/16*((b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 32*a^2*b^3*c^3 + 1
2*a*b^4*c^3 + b^5*c^3 - 32*a^3*b*c^4 - 16*a^2*b^2*c^4 - 6*a*b^3*c^4 + 8*a^2
*b*c^5 + (b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 32*a^2*b^2*c^3 + 12*a*b^3*c^3
+ b^4*c^3 - 32*a^3*c^4 - 16*a^2*b*c^4 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt(b^2 -
 4*a*c))*A*abs(c) - (a*b^6*c - 8*a^2*b^4*c^2 - 2*a*b^5*c^2 + 16*a^3*b^2*c^3
 + 8*a^2*b^3*c^3 + a*b^4*c^3 - 4*a^2*b^2*c^4 + (a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4)*sqrt(b
^2 - 4*a*c))*C*abs(c) - (b^7*c^2 - 10*a*b^5*c^3 - 2*b^6*c^3 + 32*a^2*b^3*c^
4 + 12*a*b^4*c^4 + b^5*c^4 - 32*a^3*b*c^5 - 16*a^2*b^2*c^5 - 6*a*b^3*c^5 +
8*a^2*b*c^6 + (b^6*c^2 - 6*a*b^4*c^3 - 2*b^5*c^3 + 8*a^2*b^2*c^4 + 4*a*b^3*
c^4 + b^4*c^4 - 2*a*b^2*c^5)*sqrt(b^2 - 4*a*c))*A + (a*b^6*c^2 - 8*a^2*b^4*
c^3 - 2*a*b^5*c^3 + 16*a^3*b^2*c^4 + 8*a^2*b^3*c^4 + a*b^4*c^4 - 4*a^2*b^2*
c^5 + (a*b^5*c^2 - 4*a^2*b^3*c^3 - 2*a*b^4*c^3 + a*b^3*c^4)*sqrt(b^2 - 4*a*
c))*C)*log(x^2 + 1/2*(a^4*b*c - sqrt(a^8*b^2*c^2 - 4*a^9*c^3))/(a^4*c^2))/(
(a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c
^2 - 4*a^4*c^3)*c^2*abs(c)) - 1/2*(2*B*a*x + A*a)/(a^2*x^2)
```

maple [B] time = 0.06, size = 1054, normalized size = 3.66

$$-\frac{4\sqrt{2}\,B\,b^{2}c\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(32ac-8b^{2}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}\;a}+\frac{4\sqrt{2}\,B\,b^{2}c\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(32ac-8b^{2}\right)\sqrt{\left(b+\sqrt{-4ac+b^{2}}\right)c}\;a}+\frac{16\sqrt{2}\,B\,c^{2}\,\mathrm{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}\right)}{\left(32ac-8b^{2}\right)\sqrt{\left(-b+\sqrt{-4ac+b^{2}}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*A/a/x^2-B/a/x-A/a^2*b*ln(x)+1/a*ln(x)*C+8/a*c/(32*a*c-8*b^2)*ln(-2*c*x
^2-b+(-4*a*c+b^2)^(1/2))*A*b-2/a^2/(32*a*c-8*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2
)^(1/2))*A*b^3+4/a*c/(32*a*c-8*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4
*a*c+b^2)^(1/2)-2/a^2/(32*a*c-8*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-
4*a*c+b^2)^(1/2)*b^2+2/a/(32*a*c-8*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C
*(-4*a*c+b^2)^(1/2)*b-8*c/(32*a*c-8*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*
C+2/a/(32*a*c-8*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*b^2-4/a*c/(32*a*c-
8*b^2)*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*(-4*a*c+b^2)^(1/2)*b+16*c^2/(32*a*c-8*b^2)*B
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)-4/a/c/(32*a*c-8*b^2)*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2+8/a
*c/(32*a*c-8*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b-2/a^2/(32*a*c-8*b^2)
*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b^3-4/a*c/(32*a*c-8*b^2)*ln(2*c*x^2+b+(
-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)+2/a^2/(32*a*c-8*b^2)*ln(2*c*x^2+b+(
-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2-2/a/(32*a*c-8*b^2)*ln(2*c*x^2+b
+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b-8*c/(32*a*c-8*b^2)*ln(2*c*x^2+b
+(-4*a*c+b^2)^(1/2))*C+2/a/(32*a*c-8*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*
C*b^2-4/a*c/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*(-4*a*c+b^2)^(1/2)*b-16*c^2
/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+4/a*c/(32*a*c-8*b^2)*B*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{(Ca-Ab)\log(x)}{a^2}+\frac{-\int\frac{Bacx^2+(Ca-Ab)cx^3+Bab+(Cab-Ab^2+Aac)x}{cx^4+bx^2+a}\,dx}{a^2}-\frac{2\,Bx+A}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] (C*a - A*b)*log(x)/a^2 + integrate(-(B*a*c*x^2 + (C*a - A*b)*c*x^3 + B*a*b
+ (C*a*b - A*b^2 + A*a*c)*x)/(c*x^4 + b*x^2 + a), x)/a^2 - 1/2*(2*B*x + A)/
(a*x^2)
```

mupad [B] time = 1.17, size = 3563, normalized size = 12.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*
C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2
*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C
*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*
z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 -
4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z
^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C
*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z
+ 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3
*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*
c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 -
 A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)
*(root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2
*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16
*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*
c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*
B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b
^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^
2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^
2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*
a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A
*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A
*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*
b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)*(root(12
8*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 +
256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^
4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8
*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b
^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 -
96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2
 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A
*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z
 + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*
c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*
c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A
^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k)*((16*B*a^5*c^4 +
4*B*a^3*b^4*c^2 - 20*B*a^4*b^2*c^3)/a^3 + (x*(240*C*a^5*c^4 - 224*A*a^4*b*c
^4 - 12*A*a^2*b^5*c^2 + 104*A*a^3*b^3*c^3 + 12*C*a^3*b^4*c^2 - 108*C*a^4*b^
2*c^3))/a^3 + (root(128*a^5*b^2*c*z^4 - 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 +
```

```
128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5
*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72
*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 + 40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*
c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^
2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c
^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A
^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z + 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*
c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2
*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*
a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c
^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 - C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z
, k)*x*(320*a^6*c^4 + 24*a^4*b^4*c^2 - 176*a^5*b^2*c^3))/a^3) - (8*B*C*a^4*
c^4 + 20*A*B*a^2*b^3*c^3 + 4*B*C*a^2*b^4*c^2 - 16*B*C*a^3*b^2*c^3 - 4*A*B*a
*b^5*c^2 - 20*A*B*a^3*b*c^4)/a^3 + (x*(36*A^2*a^3*c^5 + 60*C^2*a^4*c^4 + 22
*A^2*a^2*b^2*c^4 - 28*B^2*a^2*b^3*c^3 - 16*C^2*a^3*b^2*c^3 - 8*A^2*a*b^4*c^
3 + 4*B^2*a*b^5*c^2 + 50*B^2*a^3*b*c^4 + 24*A*C*a^2*b^3*c^3 - 92*A*C*a^3*b*
c^4))/a^3) - (A^2*B*a^2*c^5 + 7*B*C^2*a^3*c^4 - 4*A^2*B*a*b^2*c^4 - 4*B*C^2
*a^2*b^2*c^3 + 4*A*B*C*a*b^3*c^3 - 4*A*B*C*a^2*b*c^4)/a^3 + (x*(2*A^3*b^3*c
^4 + 5*C^3*a^3*c^4 - 12*A^3*a*b*c^5 - 17*A*B^2*a^2*c^5 + 13*A^2*C*a^2*c^5 +
 6*A*B^2*a*b^2*c^4 - 9*A*C^2*a^2*b*c^4 + 2*A^2*C*a*b^2*c^4 - 4*B^2*C*a*b^3*
c^3 + 14*B^2*C*a^2*b*c^4))/a^3) - (A^3*B*b*c^5 + B*C^3*a^2*c^4 - A^2*B*C*a*
c^5 - A*B*C^2*a*b*c^4)/a^3 + (x*(A^4*c^6 + B^4*a*c^5 - A^3*C*b*c^5 + A^2*C^
2*a*c^5 + B^2*C^2*a*b*c^4 - 3*A*B^2*C*a*c^5))/a^3)*root(128*a^5*b^2*c*z^4 -
 256*a^6*c^2*z^4 - 16*a^4*b^4*z^4 + 128*C*a^4*b^2*c*z^3 + 256*A*a^4*b*c^2*z
^3 - 128*A*a^3*b^3*c*z^3 - 256*C*a^5*c^2*z^3 - 16*C*a^3*b^4*z^3 + 16*A*a^2*
b^5*z^3 + 160*A*C*a^3*b*c^2*z^2 - 72*A*C*a^2*b^3*c*z^2 + 8*A*C*a*b^5*z^2 +
40*C^2*a^3*b^2*c*z^2 - 48*B^2*a^3*b*c^2*z^2 + 28*B^2*a^2*b^3*c*z^2 + 32*A^2
*a*b^4*c*z^2 - 56*A^2*a^2*b^2*c^2*z^2 - 4*B^2*a*b^5*z^2 - 96*C^2*a^4*c^2*z^
2 - 4*C^2*a^2*b^4*z^2 - 32*A^2*a^3*c^3*z^2 - 4*A^2*b^6*z^2 - 16*B^2*C*a^2*b
*c^2*z + 32*A*C^2*a^2*b*c^2*z - 12*A^2*C*a*b^2*c^2*z - 4*A*B^2*a*b^2*c^2*z
+ 4*B^2*C*a*b^3*c*z - 8*A*C^2*a*b^3*c*z + 16*A^3*a*b*c^3*z + 4*A^2*C*b^4*c*
z + 4*C^3*a^2*b^2*c*z - 16*A^2*C*a^2*c^3*z + 16*A*B^2*a^2*c^3*z - 16*C^3*a^
3*c^2*z - 4*A^3*b^3*c^2*z + 2*A*C^3*a*b*c^2 + 4*A*B^2*C*a*c^3 - 2*A^2*C^2*a
*c^3 + 2*A^3*C*b*c^3 - B^2*C^2*a*b*c^2 - A^2*B^2*b*c^3 - A^2*C^2*b^2*c^2 -
C^4*a^2*c^2 - B^4*a*c^3 - A^4*c^4, z, k), k, 1, 4) - (A/(2*a) + (B*x)/a)/x^
2 - (log(x)*(A*b - C*a))/a^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.29 $\int \frac{x^4(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=412

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\ldots\right.}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-\ldots}}}$$

```
[Out] 1/2*(2*A*c-C*b)*x/c/(-4*a*c+b^2)+1/2*B*x^2*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)-1/2*x^3*(A*b-2*a*C+(2*A*c-C*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2
*a*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(
x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(-A*c
*(4*a*c+b^2)-b*(-8*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2))*(A*b*c+(-6*a*c+b^2)*C+(A*c*(4*a*c+b^2)+b*(-8*a*c+b^2)*C)/
(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] time = 1.33, antiderivative size = 412, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {1662, 1275, 1279, 1166, 205, 12, 1114, 722, 618, 206}

$$\frac{\left(-\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{Ac(4ac+b^2)+bC(b^2-8ac)}{\sqrt{b^2-4ac}}+C\left(b^2-6ac\right)+Abc\ldots\right.}{2\sqrt{2}\,c^{3/2}\left(b^2-4ac\right)\sqrt{\sqrt{b^2-\ldots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*A*c - b*C)*x)/(2*c*(b^2 - 4*a*c)) + (B*x^2*(2*a + b*x^2))/(2*(b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) - (x^3*(A*b - 2*a*C + (2*A*c - b*C)*x^2))/(2*(b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + ((A*b*c + (b^2 - 6*a*c)*C - (A*c*(b^2 + 4*a
*c) + b*(b^2 - 8*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^
2 - 4*a*c]]) + ((A*b*c + (b^2 - 6*a*c)*C + (A*c*(b^2 + 4*a*c) + b*(b^2 - 8*
a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4
*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2
*a*B*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 722

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*(a + b*x
+ c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*(2*p + 3)*(c*d^2 -
b*d*e + a*e^2))/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^4\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^5}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^4\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^5}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{x^2\left(3(Ab-2aC)+(2Ac\right.}{a+bx^2+cx^4}}{2\left(b^2-4ac\right.}$$

$$= \frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)} - \frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x^2}{\left(a+bx+cx^2\right)^2}\,d\right.$$

$$= \frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)} + \frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} -$$

$$= \frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)} + \frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} +$$

$$= \frac{(2Ac-bC)x}{2c\left(b^2-4ac\right)} + \frac{Bx^2\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x^3\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} +$$

Mathematica [A] time = 1.38, size = 444, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2\left(a(b(B+Cx)-2cx(A+x(B+Cx)))+bx^2(b(B+Cx)-Acx)\right)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b^2\sqrt{b^2-4ac}-6ac\sqrt{b^2-4ac}+\right.\right.}{c^{3/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(b*x^2*(-(A*c*x) + b*(B + C*x)) + a*(b*(B + C*x) - 2*c*x*(A + x*(B + C*
x)))))/(c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(A*c*(b^2 + 4*a*
c - b*Sqrt[b^2 - 4*a*c])) + (-b^3 + 8*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 6*a*c
*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(
A*c*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c]) + (b^3 - 8*a*b*c + b^2*Sqrt[b^2 - 4
*a*c] - 6*a*c*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(c^(3/2)*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 - (4*a*B*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*a
*B*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 8.47, size = 5219, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(C*b^2*x^3 - 2*C*a*c*x^3 - A*b*c*x^3 + B*b^2*x^2 - 2*B*a*c*x^2 + C*a*b
*x - 2*A*a*c*x + B*a*b)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*
b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2
 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4
*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 +
16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3
*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4
*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a
*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(b^2*c - 4*a*c^2) - (2*b^7*
c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b
*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 +
 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*C)*arctan(2*sqr
t(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c
- 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a
^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64
```

```
*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*
abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20
*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*
c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 -
12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^
2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*C + 4*(sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*
b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c
^5)*A*abs(b^2*c - 4*a*c^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a
^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b
*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*C*abs(b^2*c -
 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*
(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 -
256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^
3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2
 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c
^6)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c
^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))
/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c
^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*ab
s(b^2*c - 4*a*c^2)*abs(c)) - 1/4*((b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3 +
(b^2*c - 4*a*c^2 - 2*b*c^2 + c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2*c - 4*a*c^2)
 + (b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^
4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*
a*c))*B)*log(x^2 + 1/2*(b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*
(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))/((b^4 - 8*
a*b^2*c - 2*b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2*abs(b^2
```

```
*c - 4*a*c^2)) - 1/4*((b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3 - (b^2*c - 4*a
*c^2 - 2*b*c^2 + c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2*c - 4*a*c^2) - (b^5*c^2
- 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^
5 - (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log
(x^2 + 1/2*(b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4
*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3))/((b^4 - 8*a*b^2*c - 2*
b^3*c + 16*a^2*c^2 + 8*a*b*c^2 + b^2*c^2 - 4*a*c^3)*c^2*abs(b^2*c - 4*a*c^2
))
```

maple [B] time = 0.06, size = 1429, normalized size = 3.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(A*b*c+2*C*a*c-C*b^2)/(4*a*c-b^2)/c*x^3-1/2*B*(2*a*c-b^2)/(4*a*c-b^2)
/c*x^2-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c*x+1/2*a*b*B/c/(4*a*c-b^2))/(c*x^4+b*
x^2+a)-1/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*a*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1
/2))+1/(4*a*c-b^2)^2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*a+1/4/(4*
a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b^2+1/(4*a*c-b^2)^2*c
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a*A*b-1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-
2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a*b+1/4/(4*a*c-b
^2)^2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^3-6/(4*a*c-b^2)^2*c*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*C*a^2+5/2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b^2-1
/4/(4*a*c-b^2)^2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*b^4+1/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*a*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))+1/(4*a*c-b^2)^2*c*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*A*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c
+b^2)^(1/2)*b^2-1/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A*b+1/4/(4*a*c-b^2)^
2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*A*b^3-2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2
)^(1/2)*a*b+1/4/(4*a*c-b^2)^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^3
+6/(4*a*c-b^2)^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a^2-5/2/(4*a*c-b^2)^2*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*C*a*b^2+1/4/(4*a*c-b^2)^2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*b^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(Cb^2-(2\,Ca+Ab)c\right)x^3+Bab+\left(Bb^2-2\,Bac\right)x^2+(Cab-2\,Aac)x}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}+\frac{-\int\frac{4\,Bacx-Cab+2\,Aac-\left(Cb^2-(6\,Ca-Ab)c\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((C*b^2 - (2*C*a + A*b)*c)*x^3 + B*a*b + (B*b^2 - 2*B*a*c)*x^2 + (C*a*
b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4
*a*b*c^2)*x^2) + 1/2*integrate(-(4*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (6*
C*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

mupad [B] time = 1.77, size = 4754, normalized size = 11.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(- root(1572864*a^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680
*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c
^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^
5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c
*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*
z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5
*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*
a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536
*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*
b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*
a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c
^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^
3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*
c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4
*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c -
960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144
*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C
^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*
c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c
^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*
a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*c^4, z, k)*(root(1572864*a^5*b^2*c^8*
z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2*b^8*c^5*z
^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^4 + 576*A*C
*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3*z^2 + 204
8*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*z^2 + 12288
*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2 - 61440*C^
2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*c^2*z^2 +
24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^2*b^6*c^3*
z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768*B^2*a^5*c
^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^3*c^3*z +
768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7*c*z + 672
*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c^3*z - 499
2*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*b^4*c^3*z
- 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z + 192*A*B
^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2 - 16*A^2*
B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*B^2*C^2*a^
3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2*B^2*a^3*b
*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3*b*c^3 + 7
68*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C^3*a^2*b^5
 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 16*B^2*C^2*
a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 16*A^4*a^3*
c^4, z, k)*((x*(1024*B*a^4*c^6 - 16*B*a*b^6*c^3 + 192*B*a^2*b^4*c^4 - 768*B
*a^3*b^2*c^5))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (
2048*A*a*a^4*c^6 - 32*A*a*b^6*c^3 + 16*C*a*b^7*c^2 - 1024*C*a^4*b*c^5 + 384*A
*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*C*a^2*b^5*c^3 + 768*C*a^3*b^3*c^4)/
(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (root(1572864*a^
```

```
5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^2
*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^4
 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^3
*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*z
^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2
 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7*
c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a^
2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 32768
*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b^
3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^7
*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*c
^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2*
b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*z
 + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^2
 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240*
B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^2
*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^3
*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*C
^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 - 1
6*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 - 1
6*A^4*a^3*c^4, z, k)*x*(16*b^9*c^3 - 256*a*b^7*c^4 + 4096*a^4*b*c^7 + 1536*
a^2*b^5*c^5 - 4096*a^3*b^3*c^6))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48
*a^2*b^2*c^3))) - (1536*B*C*a^4*c^4 + 128*A*B*a^2*b^3*c^3 + 32*B*C*a^2*b^4*
c^2 - 512*B*C*a^3*b^2*c^3 - 512*A*B*a^3*b*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*
a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(C^2*b^8 - 32*A^2*a^3*c^5 + A^2*b^6*c^2 +
 288*C^2*a^4*c^4 + 2*A*C*b^7*c - 16*B^2*a^2*b^3*c^3 + 138*C^2*a^2*b^4*c^2 -
 368*C^2*a^3*b^2*c^3 - 20*C^2*a*b^6*c - 2*A^2*a*b^4*c^3 + 64*B^2*a^3*b*c^4
 + 48*A*C*a^2*b^3*c^3 - 22*A*C*a*b^5*c^2 + 32*A*C*a^3*b*c^4))/(2*(b^6*c - 64
*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))) - (3*A*C^2*a*b^5 - 216*C^3*a^4*
c^2 - 5*C^3*a^2*b^4 + 32*A*B^2*a^3*c^3 - 24*A^2*C*a^3*c^3 + 3*A^3*a*b^3*c^2
 + 4*A^3*a^2*b*c^3 + 66*C^3*a^3*b^2*c - 51*A*C^2*a^2*b^3*c + 204*A*C^2*a^3*
b*c^2 - 16*B^2*C*a^3*b*c^2 - 42*A^2*C*a^2*b^2*c^2 + 6*A^2*C*a*b^4*c)/(8*(b^
6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(16*B^3*a^3*c^3 + B
*C^2*a*b^5 + A^2*B*a*b^3*c^2 + 4*A^2*B*a^2*b*c^3 - 14*B*C^2*a^2*b^3*c + 48*
B*C^2*a^3*b*c^2 - 24*A*B*C*a^3*c^3 - 10*A*B*C*a^2*b^2*c^2 + 2*A*B*C*a*b^4*c
))/(2*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)))*root(1572864*a
^5*b^2*c^8*z^4 - 983040*a^4*b^4*c^7*z^4 + 327680*a^3*b^6*c^6*z^4 - 61440*a^
2*b^8*c^5*z^4 + 6144*a*b^10*c^4*z^4 - 256*b^12*c^3*z^4 - 1048576*a^6*c^9*z^
4 + 576*A*C*a*b^8*c^2*z^2 + 24576*A*C*a^4*b^2*c^5*z^2 - 3072*A*C*a^2*b^6*c^
3*z^2 + 2048*A*C*a^3*b^4*c^4*z^2 - 32*A*C*b^10*c*z^2 + 61440*C^2*a^5*b*c^5*
z^2 + 12288*A^2*a^4*b*c^6*z^2 + 432*C^2*a*b^9*c*z^2 - 49152*A*C*a^5*c^6*z^2
 - 61440*C^2*a^4*b^3*c^4*z^2 + 24064*C^2*a^3*b^5*c^3*z^2 - 4608*C^2*a^2*b^7
*c^2*z^2 + 24576*B^2*a^4*b^2*c^5*z^2 - 6144*B^2*a^3*b^4*c^4*z^2 + 512*B^2*a
^2*b^6*c^3*z^2 - 8192*A^2*a^3*b^3*c^5*z^2 + 1536*A^2*a^2*b^5*c^4*z^2 - 3276
8*B^2*a^5*c^6*z^2 - 16*A^2*b^9*c^2*z^2 - 16*C^2*b^11*z^2 - 3072*A*B*C*a^3*b
^3*c^3*z + 768*A*B*C*a^2*b^5*c^2*z + 4096*A*B*C*a^4*b*c^4*z - 64*A*B*C*a*b^
7*c*z + 672*B*C^2*a^2*b^6*c*z - 32*A^2*B*a*b^6*c^2*z + 15872*B*C^2*a^4*b^2*
c^3*z - 4992*B*C^2*a^3*b^4*c^2*z - 1536*A^2*B*a^3*b^2*c^4*z + 384*A^2*B*a^2
*b^4*c^3*z - 32*B*C^2*a*b^8*z - 18432*B*C^2*a^5*c^4*z + 2048*A^2*B*a^4*c^5*
z + 192*A*B^2*C*a^3*b^2*c^2 - 32*A*B^2*C*a^2*b^4*c - 960*A^2*C^2*a^3*b^2*c^
2 - 16*A^2*B^2*a^2*b^3*c^2 - 18*A^3*C*a*b^5*c - 960*B^2*C^2*a^4*b*c^2 + 240
*B^2*C^2*a^3*b^3*c + 198*A^2*C^2*a^2*b^4*c + 144*A^3*C*a^2*b^3*c^2 - 192*A^
2*B^2*a^3*b*c^3 + 2016*A*C^3*a^4*b*c^2 - 496*A*C^3*a^3*b^3*c + 224*A^3*C*a^
3*b*c^3 + 768*A*B^2*C*a^4*c^3 + 360*C^4*a^4*b^2*c - 9*A^4*a*b^4*c^2 + 30*A*
C^3*a^2*b^5 - 9*A^2*C^2*a*b^6 - 24*A^4*a^2*b^2*c^3 - 288*A^2*C^2*a^4*c^3 -
16*B^2*C^2*a^2*b^5 - 1296*C^4*a^5*c^2 - 256*B^4*a^4*c^3 - 25*C^4*a^3*b^4 -
16*A^4*a^3*c^4, z, k), k, 1, 4) - ((x^3*(A*b*c - C*b^2 + 2*C*a*c))/(2*c*(4*
a*c - b^2)) + (x*(2*A*a*c - C*a*b))/(2*c*(4*a*c - b^2)) - (B*a*b)/(2*c*(4*a
*c - b^2)) + (B*x^2*(2*a*c - b^2))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.30 $\int \frac{x^3(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=347

$$-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(b-\right.}{2\sqrt{2}\,\sqrt{}}$$

```
[Out] 1/2*B*x*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(a*(2*A*c-C*b)+(A*b*c+
2*C*a*c-C*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(A*b-2*C*a)*arctanh((2*c
*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*B*arctan(x*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c-b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*
c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*arctan(x*2^(1/2)*
c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2+4*a*c+b*(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(3/2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1662, 1251, 777, 618, 206, 12, 1120, 1166, 205}

$$\frac{x^2\left(2acC+Abc+b^2(-C)\right)+a(2Ac-bC)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{(Ab-2aC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}+\frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{B\left(b-\right.}{2\sqrt{2}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*x*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (a*(2*A*c - b*C
) + (A*b*c - b^2*C + 2*a*c*C)*x^2)/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
+ (B*(b - (b^2 + 4*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) + (B*(b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^(3/2)*S
qrt[b + Sqrt[b^2 - 4*a*c]]) - ((A*b - 2*a*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 777

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> -Simp[((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x)*(a + b*x + c*x^2)^(p +
1))/(c*(p + 1)*(b^2 - 4*a*c)), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(
2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^3\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^4}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^3\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Cx)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + B\int \frac{x^4}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{a(2Ac-bC)+\left(Abc-b^2C+2acC\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{B\int \frac{2a-bx^2}{a+bx^2+cx}}{2\left(b^2-4a\right.}$$

$$= \frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{a(2Ac-bC)+\left(Abc-b^2C+2acC\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(B\left(b^2+4a\right.\right.}{}$$

$$= \frac{Bx\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{a(2Ac-bC)+\left(Abc-b^2C+2acC\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{B\left(b^2+4ac\right.}{2\sqrt{2}\sqrt{c}}$$

Mathematica [A] time = 0.89, size = 358, normalized size = 1.03

$$\frac{1}{4}\left(-\frac{2\left(a(2Ac-bC+2cx(B+Cx))+bx^2(Ac-bC+Bcx)\right)}{c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{2(Ab-2aC)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{2(Ab-}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(b*x^2*(A*c - b*C + B*c*x) + a*(2*A*c - b*C + 2*c*x*(B + C*x))))/(c*(-
b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*B*(-b^2 - 4*a*c + b*Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*B*(b^2 + 4*a*c
+ b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*(A*b -
2*a*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*(A*b
 - 2*a*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.37, size = 3228, normalized size = 9.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*c*x^3 - C*b^2*x^2 + 2*C*a*c*x^2 + A*b*c*x^2 + 2*B*a*c*x - C*a*b +
2*A*a*c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^2 - 8*a*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2
 - 4*a*c)^2*B - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*
a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2*b^7*c^2 - 8*a*
b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^4)*B)*arctan(2*
sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*
a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a
^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(c)) - 1/16*((2*
b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*
c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a
*c)*b*c^2)*(b^2 - 4*a*c)^2*B + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b
^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 - 2
*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c) - (2
*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*b*c^
4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4
*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c - 12*a
^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3 - 64
*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*abs(
c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c - 4*a*b*c^2
- 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - 2*(a*b^3*c - 4
*a^2*b*c^2 - 2*a*b^2*c^2 + a*b*c^3 + (a*b^2*c - 4*a^2*c^2 - 2*a*b*c^2 + a*c
^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^
2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3
*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*A + 2*(a*b^5*c - 8*a^2*b^3*c
^2 - 2*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 +
 (a*b^4*c - 4*a^2*b^2*c^2 - 2*a*b^3*c^2 + a*b^2*c^3)*sqrt(b^2 - 4*a*c))*C)*
log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)
```

```
*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c +
 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c)) +
1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c - 4*a*b*c^2 - 2*b^
2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - 2*(a*b^3*c - 4*a^2*b
*c^2 - 2*a*b^2*c^2 + a*b*c^3 - (a*b^2*c - 4*a^2*c^2 - 2*a*b*c^2 + a*c^3)*sq
rt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16
*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 -
 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*A + 2*(a*b^5*c - 8*a^2*b^3*c^2 - 2
*a*b^4*c^2 + 16*a^3*b*c^3 + 8*a^2*b^2*c^3 + a*b^3*c^3 - 4*a^2*b*c^4 - (a*b^
4*c - 4*a^2*b^2*c^2 - 2*a*b^3*c^2 + a*b^2*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^
2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*
c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^
3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.04, size = 831, normalized size = 2.39

$$\frac{\sqrt{2}\,Babc\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)^2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,Babc\arctan\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\left(4ac-b^2\right)^2\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\,B\,b^3\operatorname{arctanh}\left(\frac{\sqrt{2}\,cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{4\left(4ac-b^2\right)^2\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*b*B/(4*a*c-b^2)*x^3-1/2*(A*b*c+2*C*a*c-C*b^2)/(4*a*c-b^2)/c*x^2-a*B/(
4*a*c-b^2)*x-1/2*a*(2*A*c-C*b)/(4*a*c-b^2)/c)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^
2)^2*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b-1/(4*a*c-b^2)
^2*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+c/(4*a*c-b^2)^2
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a
*b*B-1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*B-1/2/(4*a*c-b^2)^2*ln(2*c
*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b+1/(4*a*c-b^2)^2*ln(2*c*x^
2+b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*a+c/(4*a*c-b^2)^2*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*B*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*
c+b^2)^(1/2)*b^2-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*B+1/4/(4*a*c-b^2)^2
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^3*B
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c*x^3 + 2*B*a*c*x - C*a*b + 2*A*a*c - (C*b^2 - (2*C*a + A*b)*c)*x^
2)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2
) + 1/2*integrate((B*b*x^2 - 2*B*a - 2*(2*C*a - A*b)*x)/(c*x^4 + b*x^2 + a)
, x)/(b^2 - 4*a*c)
```

mupad [B] time = 1.61, size = 3278, normalized size = 9.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log(root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a
^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*
c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 -
24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^
2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^
3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*
b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2
*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z
 - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z
+ 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*
B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c +
 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^
2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*
B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*((256*A*B*a^2*b
^2*c^3 + 128*B*C*a^2*b^3*c^2 - 64*A*B*a*b^4*c^2 - 512*B*C*a^3*b*c^3)/(8*(b^
6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(1572864*a^5*b^2*c^6*z
^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^
4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4 + 32768*A*C*
a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*
a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 - 1536*A^
2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 81
92*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^
2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^8*c*z^2 - 32768*C^2*a^5*c^4*z^2 -
16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z - 192*A*B^2
*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*
a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a^2*b^2*c + 5
12*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b
^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2
*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*
c - 9*B^4*a*b^4, z, k)*((x*(16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*A*a*b^5*c^3
 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C*a^2*b^4*c^
3 - 1536*C*a^3*b^2*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c
)) - (2048*B*a^4*c^5 - 32*B*a*b^6*c^2 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*
c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(1572864*a
^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*b^6*c^4*z^4 - 61440*a^
2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7*z^4 - 256*b^12*c*z^4
+ 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2
 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z
^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z^2 - 6144*C^2*a^3*b^4*
c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 - 8192*A^2*a^
3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^8*c*z^2 - 32768*C^2*a^
5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z - 1024*A*B^2*a^3*b*c^3*z
 - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 768*A*B^2*a^2*b^3*c^2*z
 - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2*b^7*z + 192*A*B^2*C*a
^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16*A*B^2*C*a*b^4 - 384*A
^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a*b^3*c - 24*B^4*a^2*b^
2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2*b^5 - 256*C^4*a^4*c -
 16*A^4*b^4*c - 9*B^4*a*b^4, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4
*b*c^6 + 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^
2*b^2*c^2 - 12*a*b^4*c))) + (x*(8*A^2*b^5*c^2 - 2*B^2*b^6*c + 64*B^2*a^3*c^
4 + 32*C^2*a^2*b^3*c^2 - 32*A^2*a*b^3*c^3 + 4*B^2*a*b^4*c^2 - 128*C^2*a^3*b
*c^3 + 128*A*C*a^2*b^2*c^3 - 32*A*C*a*b^4*c^2))/(4*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c))) - (3*B^3*a*b^3*c + 32*B*C^2*a^3*c^2 + 4*B^3*a^2*
b*c^2 + 8*A^2*B*a*b^2*c^2 - 32*A*B*C*a^2*b*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a
^2*b^2*c^2 - 12*a*b^4*c)) + (x*(4*A^3*b^3*c^2 - 32*C^3*a^3*c^2 + A*B^2*b^4*
c + 4*A*B^2*a*b^2*c^2 + 48*A*C^2*a^2*b*c^2 - 24*A^2*C*a*b^2*c^2 - 8*B^2*C*a
```

```
^2*b*c^2 - 2*B^2*C*a*b^3*c))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b
^4*c)))*root(1572864*a^5*b^2*c^6*z^4 - 983040*a^4*b^4*c^5*z^4 + 327680*a^3*
b^6*c^4*z^4 - 61440*a^2*b^8*c^3*z^4 + 6144*a*b^10*c^2*z^4 - 1048576*a^6*c^7
*z^4 - 256*b^12*c*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 245
76*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 + 512*C^2*a^2*b^6*c*z^2 +
 12288*B^2*a^4*b*c^4*z^2 - 1536*A^2*a*b^6*c^2*z^2 + 24576*C^2*a^4*b^2*c^3*z
^2 - 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5
*c^2*z^2 - 8192*A^2*a^3*b^2*c^4*z^2 + 6144*A^2*a^2*b^4*c^3*z^2 + 128*A^2*b^
8*c*z^2 - 32768*C^2*a^5*c^4*z^2 - 16*B^2*b^9*z^2 + 384*B^2*C*a^2*b^4*c*z -
1024*A*B^2*a^3*b*c^3*z - 192*A*B^2*a*b^5*c*z - 1536*B^2*C*a^3*b^2*c^2*z + 7
68*A*B^2*a^2*b^3*c^2*z - 32*B^2*C*a*b^6*z + 2048*B^2*C*a^4*c^3*z + 16*A*B^2
*b^7*z + 192*A*B^2*C*a^2*b^2*c + 512*A*C^3*a^3*b*c + 128*A^3*C*a*b^3*c + 16
*A*B^2*C*a*b^4 - 384*A^2*C^2*a^2*b^2*c - 192*B^2*C^2*a^3*b*c - 48*A^2*B^2*a
*b^3*c - 24*B^4*a^2*b^2*c - 16*B^2*C^2*a^2*b^3 - 16*B^4*a^3*c^2 - 4*A^2*B^2
*b^5 - 256*C^4*a^4*c - 16*A^4*b^4*c - 9*B^4*a*b^4, z, k), k, 1, 4) - ((B*a*
x)/(4*a*c - b^2) + (x^2*(A*b*c - C*b^2 + 2*C*a*c))/(2*c*(4*a*c - b^2)) + (B
*b*x^3)/(2*(4*a*c - b^2)) + (a*(2*A*c - C*b))/(2*c*(4*a*c - b^2)))/(a + b*x
^2 + c*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.31 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4a\right.}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^2 \left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2 \left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc}{\sqrt{}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc}{\sqrt{}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc}{\sqrt{}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2\right.}$$

Mathematica [A] time = 1.04, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 7.05, size = 4440, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
```

```
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.06, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/2*b*B/(4*a*c-b^2)*x^2+1/2*(A*b-2*C*a)/(4
*a*c-b^2)*x-a*B/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^
2)^(1/2)*b*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((-b+(
```

```
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*
A+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*C*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b-1/4/(
4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(
1/2)*b*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
A*(-4*a*c+b^2)^(1/2)*b+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*A-1/2*c/(4*
a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-
4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2
)*b^2-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b+1/4/(4*a*c-b^2)^2*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b^3*C
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 1.67, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
```

*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2

```
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.32 $\int \frac{x(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=317

$$\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-4\right.}{\sqrt{2}(b^2-4ac)}$$

```
[Out] -1/2*B*x*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(-A*b+2*a*C-(2*A*c-C*
b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+(2*A*c-C*b)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)*2^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*B*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(2*b+(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)*2^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {1662, 1247, 638, 618, 206, 12, 1119, 1166, 205}

$$-\frac{-2aC+x^2(2Ac-bC)+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{(2Ac-bC)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{Bx(b+2cx^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{B\sqrt{c}\left(2b-\sqrt{b^2-\right.}{\sqrt{2}(b^2-4a}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(B*x*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (A*b - 2*a*C +
 (2*A*c - b*C)*x^2)/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (B*Sqrt[c]*(2*b
 - Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*
(2*b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/(Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((2*A*c
 - b*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1119

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d*(d*x)^(m - 1)*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p
+ 1)*(b^2 - 4*a*c)), x] - Dist[d^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m
- 2)*(b*(m - 1) + 2*c*(m + 4*p + 5)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x
] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m,
1] && LeQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + B\int \frac{x^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{Bx\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\int \frac{b-2cx^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)} -$$

$$= -\frac{Bx\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(Bc\left(2b-\sqrt{b^2-4a\right.\right.}{2\left(b^2\right.}$$

$$= -\frac{Bx\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{Ab-2aC+(2Ac-bC)x^2}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(2b-\sqrt{b^2-4\right.}{\sqrt{2}\left(b^2-4ac\right)}$$

Mathematica [A] time = 1.25, size = 335, normalized size = 1.06

$$\frac{1}{2}\left(\frac{2aC-A\left(b+2cx^2\right)+x\left(-bB+bCx-2Bcx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(bC-2Ac)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} + \frac{(2Ac-bC)\log}{(b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*C - A*(b + 2*c*x^2) + x*(-(b*B) + b*C*x - 2*B*c*x^2))/((b^2 - 4*a*c)*
(a + b*x^2 + c*x^4)) - (Sqrt[2]*B*Sqrt[c]*(-2*b + Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqr
t[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*B*Sqrt[c]*(2*b + Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((-2*A*c + b*C)*Log[-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*c - b*C)*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 5.17, size = 3013, normalized size = 9.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*c*x^3 - C*b*x^2 + 2*A*c*x^2 + B*b*x - 2*C*a + A*b)/((c*x^4 + b*x^
2 + a)*(b^2 - 4*a*c)) - 1/8*((2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 - 4*a*c)^2*B - (sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
- 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c + 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a
*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3
 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 - 4*a*c)*a*b*c^2)*B*abs(b^2 - 4*a*c) - 2*
(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^2 - 4*a*c)*a*b^2*c^3)*B)*arctan(2*s
qrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*
c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5
*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^
3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((2*b^2*c^2
- 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*(b^2 -
4*a*c)^2*B + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
+ 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*a*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*
c)*a*b*c^2)*B*abs(b^2 - 4*a*c) - 2*(2*b^6*c^2 - 16*a*b^4*c^3 + 32*a^2*b^2*c
^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 8*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 8*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 + 8*(b^
2 - 4*a*c)*a*b^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - sqrt((b
^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)
))/((a*b^6 - 12*a^2*b^4*c - 2*a*b^5*c + 48*a^3*b^2*c^2 + 16*a^2*b^3*c^2 + a
*b^4*c^2 - 64*a^4*c^3 - 32*a^3*b*c^3 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*abs(b^2
- 4*a*c)*abs(c)) - 1/8*(2*(b^3*c^2 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 + (b^2*c
^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - (b^4*
c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*
c^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2 - 4*a*c) - 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b
^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^4 + b^3*c^4 - 4*a*b*c^5 + (b^4*c^2 - 4*a*
b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqrt(b^2 - 4*a*c))*A + (b^6*c - 8*a*b^4*c^2
- 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 + (b^5*c
 - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(
b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c
^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8
*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c)) - 1/8*(2*(b^3*c^2
 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 - (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt
(b^2 - 4*a*c))*A*abs(b^2 - 4*a*c) - (b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
```

```
c^3 - (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*C*abs(b^2
- 4*a*c) - 2*(b^5*c^2 - 8*a*b^3*c^3 - 2*b^4*c^3 + 16*a^2*b*c^4 + 8*a*b^2*c^
4 + b^3*c^4 - 4*a*b*c^5 - (b^4*c^2 - 4*a*b^2*c^3 - 2*b^3*c^3 + b^2*c^4)*sqr
t(b^2 - 4*a*c))*A + (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a
*b^3*c^3 + b^4*c^3 - 4*a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c
^3)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*
c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 -
 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3
)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.18, size = 1344, normalized size = 4.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B+1/2*c/(4*a*c-b^2)^2*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*b^2*B+2*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*B-1/2*c/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*B-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c
+b^2)^(1/2)/c)*A*b^2-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/
c)*A*b^2-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*(-4*a*c+b^2)^(1/2)*b*B-c/(4*a*c
-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*(-4*a*c+b^2)^(1/2)*b*B+1/4/c/(4*a*c-b^2)^2/(x
^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*b^3*C+c/(4*a*c-b^2)^2*ln(2*c*x^2+b+(-4
*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-
4*a*c+b^2)^(1/2)/c)*A*a+1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(
1/2)/c)*b^3*C-c/(4*a*c-b^2)^2*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b
^2)^(1/2)+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*A*a-1/2/
(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*B*x*b^2+1/(4*a*c-b^2)^
2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*C*(-4*a*c+b^2)^(1/2)*a-1/(4*a*c-b^
2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*C*a*b+1/2/(4*a*c-b^2)^2*ln(-2*c
*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b-1/2/(4*a*c-b^2)^2/(x^2+1/
2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*B*x*b^2-1/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-
4*a*c+b^2)^(1/2)/c)*C*(-4*a*c+b^2)^(1/2)*a-1/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2
*(-4*a*c+b^2)^(1/2)/c)*C*a*b-1/2/(4*a*c-b^2)^2*ln(2*c*x^2+b+(-4*a*c+b^2)^(1
/2))*C*(-4*a*c+b^2)^(1/2)*b+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)
^(1/2)/c)*B*a*x-1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*
C*(-4*a*c+b^2)^(1/2)*b^2+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1
/2)/c)*B*a*x+1/4/c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*C*(
-4*a*c+b^2)^(1/2)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*c*x^3 + B*b*x - (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/((b^2*c - 4*a*c^
2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate((2*B*c*x^2
- B*b - 2*(C*b - 2*A*c)*x)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 1.60, size = 3198, normalized size = 10.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((4*B^3*a*c^4 + 3*B^3*b^2*c^3 + 8*A^2*B*b*c^4 + 2*B*C^2*b^3*c^2 -
 8*A*B*C*b^2*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - ro
ot(1572864*a^6*b^2*c^5*z^4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^
4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256
*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3
*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^
2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C
^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 +
24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 -
32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2
*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a
^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 1
92*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b
^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 2
4*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B
^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^4, z, k)*(root(1572864*a^6*b^2*c^5*z^
4 - 983040*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4
 + 6144*a^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a
^4*b*c^4*z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a
^2*b^5*c^2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2
*a*b^6*c^2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192
*B^2*a^3*b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2
 - 6144*A^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 1
6*B^2*b^9*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*
a*b^5*c*z - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b
^6*c*z - 2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 51
2*A^3*C*a*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*
c^2 - 48*B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*
B^2*b^3*c^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 -
 256*A^4*a*c^4, z, k)*((x*(512*A*a^3*c^6 - 8*A*b^6*c^3 + 4*C*b^7*c^2 + 96*A
*a*b^4*c^4 - 48*C*a*b^5*c^3 - 256*C*a^3*b*c^5 - 384*A*a^2*b^2*c^5 + 192*C*a
^2*b^3*c^4))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c) - (8*B*b^7*c^
2 - 96*B*a*b^5*c^3 - 512*B*a^3*b*c^5 + 384*B*a^2*b^3*c^4)/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (root(1572864*a^6*b^2*c^5*z^4 - 98304
0*a^5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a
^2*b^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*
z^2 - 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^
2*z^2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^
2*z^2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*
b^3*c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A
^2*a^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9
*z^2 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z
 - 768*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z -
2048*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a
*b*c^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*
B^2*C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c
^2 - 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*
a*c^4, z, k)*x*(8*b^9*c^2 - 128*a*b^7*c^3 + 2048*a^4*b*c^6 + 768*a^2*b^5*c^
4 - 2048*a^3*b^3*c^5))/(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) -
(256*A*B*a^2*c^5 - 16*A*B*b^4*c^3 + 8*B*C*b^5*c^2 - 128*B*C*a^2*b*c^4)/(4*(
b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(16*B^2*a^2*c^5 - 8*A
^2*b^3*c^4 + 5*B^2*b^4*c^3 - 2*C^2*b^5*c^2 + 8*A*C*b^4*c^3 + 32*A^2*a*b*c^5
 - 24*B^2*a*b^2*c^4 + 8*C^2*a*b^3*c^3 - 32*A*C*a*b^2*c^4))/(b^6 - 64*a^3*c^
3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(8*A^3*c^5 - C^3*b^3*c^2 + 4*A*B^2*b
*c^4 - 12*A^2*C*b*c^4 + 6*A*C^2*b^2*c^3 - 2*B^2*C*b^2*c^3))/(b^6 - 64*a^3*c
^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))*root(1572864*a^6*b^2*c^5*z^4 - 983040*a^
```

```
5*b^4*c^4*z^4 + 327680*a^4*b^6*c^3*z^4 - 61440*a^3*b^8*c^2*z^4 + 6144*a^2*b
^10*c*z^4 - 1048576*a^7*c^6*z^4 - 256*a*b^12*z^4 + 32768*A*C*a^4*b*c^4*z^2
- 512*A*C*a*b^7*c*z^2 - 24576*A*C*a^3*b^3*c^3*z^2 + 6144*A*C*a^2*b^5*c^2*z^
2 - 1536*C^2*a^2*b^6*c*z^2 + 12288*B^2*a^4*b*c^4*z^2 + 512*A^2*a*b^6*c^2*z^
2 - 8192*C^2*a^4*b^2*c^3*z^2 + 6144*C^2*a^3*b^4*c^2*z^2 - 8192*B^2*a^3*b^3*
c^3*z^2 + 1536*B^2*a^2*b^5*c^2*z^2 + 24576*A^2*a^3*b^2*c^4*z^2 - 6144*A^2*a
^2*b^4*c^3*z^2 + 128*C^2*a*b^8*z^2 - 32768*A^2*a^4*c^5*z^2 - 16*B^2*b^9*z^2
 + 1024*B^2*C*a^3*b*c^3*z - 384*A*B^2*a*b^4*c^2*z + 192*B^2*C*a*b^5*c*z - 7
68*B^2*C*a^2*b^3*c^2*z + 1536*A*B^2*a^2*b^2*c^3*z + 32*A*B^2*b^6*c*z - 2048
*A*B^2*a^3*c^4*z - 16*B^2*C*b^7*z + 192*A*B^2*C*a*b^2*c^2 + 512*A^3*C*a*b*c
^3 + 128*A*C^3*a*b^3*c + 16*A*B^2*C*b^4*c - 384*A^2*C^2*a*b^2*c^2 - 48*B^2*
C^2*a*b^3*c - 192*A^2*B^2*a*b*c^3 - 24*B^4*a*b^2*c^2 - 16*A^2*B^2*b^3*c^2 -
 16*B^4*a^2*c^3 - 4*B^2*C^2*b^5 - 9*B^4*b^4*c - 16*C^4*a*b^4 - 256*A^4*a*c^
4, z, k), k, 1, 4) + ((A*b - 2*C*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - C*b))
/(2*(4*a*c - b^2)) + (B*b*x)/(2*(4*a*c - b^2)) + (B*c*x^3)/(4*a*c - b^2))/(
a + b*x^2 + c*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.33 $\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=368

$$\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4a}{\sqrt{b^2-}}\right.}{2\sqrt{2}}$$

```
[Out] -1/2*B*(2*c*x^2+b)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*x*(A*b^2-2*a*A*c-a*b*C+
c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+2*B*c*arctanh((2*c*x^2+b)
/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*a*C+(A*(-12*a*c+b^2)+4*a*b*C)/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arcta
n(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2*a*C+(12*A*
a*c-A*b^2-4*C*a*b)/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {1673, 1178, 1166, 205, 12, 1107, 614, 618, 206}

$$\frac{x\left(cx^2(Ab-2aC)-2aAc-abC+Ab^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(\frac{A\left(b^2-12ac\right)+4abC}{\sqrt{b^2-4ac}}-2aC+Ab\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4a}{\sqrt{b^2-}}\right.}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(B*(b + 2*c*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (x*(A*b^2 - 2*a*
A*c - a*b*C + c*(A*b - 2*a*C)*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 + (Sqrt[c]*(A*b - 2*a*C + (A*(b^2 - 12*a*c) + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a*C - (A*b^2 - 12
*a*A*c + 4*a*b*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
+ (2*B*c*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1107

```
Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2,
 Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1673

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Module[{q
 = Expon[Pq, x], k}, Int[Sum[Coeff[Pq, x, 2*k]*x^(2*k), {k, 0, q/2}]*(a + b
*x^2 + c*x^4)^p, x] + Int[x*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q -
1)/2}]*(a + b*x^2 + c*x^4)^p, x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x]
 &&  !PolyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{Bx}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x}{\left(a+bx^2+cx^4\right)^2}\,dx - \frac{\int \frac{-Ab^2+6aAc-abC\cdots}{a+bx^2+\cdots}}{2a\left(b^2-\cdots\right.}$$

$$= \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{1}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + \cdots$$

$$= -\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(A\left(b^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(A\left(b^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{B\left(b+2cx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{x\left(Ab^2-2aAc-abC+c(Ab-2aC)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(A\left(b^2\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 1.22, size = 393, normalized size = 1.07

$$\frac{1}{4}\left(\frac{4acx(A+x(B+Cx))+2ab(B+Cx)-2Abx\left(b+cx^2\right)}{a\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(A\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-2aC\left(\sqrt{b^2-\cdots}\right.\right.}{a\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-\cdots}}}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*a*b*(B + C*x) - 2*A*b*x*(b + c*x^2) + 4*a*c*x*(A + x*(B + C*x)))/(a*(-b
^2 + 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c + b*Sq
rt[b^2 - 4*a*c]) - 2*a*(-2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2
 - 4*a*c]]) - (Sqrt[2]*Sqrt[c]*(A*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*c]) + 2*
a*(2*b + Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (4*B*c*L
og[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + (4*B*c*Log[b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 7.85, size = 5158, normalized size = 14.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*C*a*c*x^3 - A*b*c*x^3 + 2*B*a*c*x^2 + C*a*b*x - A*b^2*x + 2*A*a*c*x
 + B*a*b)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*
b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*
b^2 - 4*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C + 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*
a*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c^2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 -
 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 +
 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2
- 4*a*c)*a*b^4*c - 20*(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)
*A*abs(a*b^2 - 4*a^2*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^
3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3
+ 2*(b^2 - 4*a*c)*a^2*b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*C*abs(a*b^2 - 4*a^
2*c) + (2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a
^2*b^5*c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A +
 4*(2*a^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 -
4*a*c)*a^4*b^2*c^3)*C)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt(
(a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^
2*c - 4*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2
+ 16*a^4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3
+ 16*a^5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/16*((2*b^3*c^2 - 8*a*b*c^3 -
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4
*a^2*c)^2*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*C - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c
 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c^2 - 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 128*a^3*b^2*c^3 + 24*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 192*a^4*c^4 - 2*(b^2 - 4*a*c
)*a*b^4*c + 20*(b^2 - 4*a*c)*a^2*b^2*c^2 - 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(
a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 16*a^3*b^3*c^2 -
 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 32*a^4*b*c^3 - 2*(b^
2 - 4*a*c)*a^2*b^3*c + 8*(b^2 - 4*a*c)*a^3*b*c^2)*C*abs(a*b^2 - 4*a^2*c) +
(2*a^2*b^7*c^2 - 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*
c^2 + 32*(b^2 - 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a
^3*b^6*c^2 - 16*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*
a^4*b^2*c^3)*C)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^3
- 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4
*a^2*c^2)))/((a^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^
4*b^3*c^2 + a^3*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^
5*c^4)*abs(a*b^2 - 4*a^2*c)*abs(c)) - 1/4*((b^3*c^2 - 4*a*b*c^3 - 2*b^2*c^3
 + b*c^4 + (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sqrt(b^2 - 4*a*c))*B*abs(a*b
^2 - 4*a^2*c) - (a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*b^4*c^3 + 16*a^3*b*c^4 + 8
*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 + (a*b^4*c^2 - 4*a^2*b^2*c^3 - 2*a*b
^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2 + 1/2*(a*b^3 - 4*a^2*b*c
+ sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2))
)/(a*b^2*c - 4*a^2*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8
*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(a*b^2 - 4*a^2*c)) - 1/4*((b^3*c
^2 - 4*a*b*c^3 - 2*b^2*c^3 + b*c^4 - (b^2*c^2 - 4*a*c^3 - 2*b*c^3 + c^4)*sq
rt(b^2 - 4*a*c))*B*abs(a*b^2 - 4*a^2*c) - (a*b^5*c^2 - 8*a^2*b^3*c^3 - 2*a*
```

```
b^4*c^3 + 16*a^3*b*c^4 + 8*a^2*b^2*c^4 + a*b^3*c^4 - 4*a^2*b*c^5 - (a*b^4*c
^2 - 4*a^2*b^2*c^3 - 2*a*b^3*c^3 + a*b^2*c^4)*sqrt(b^2 - 4*a*c))*B)*log(x^2
 + 1/2*(a*b^3 - 4*a^2*b*c - sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3
*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2))/((a*b^4 - 8*a^2*b^2*c -
2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(a*b^2
 - 4*a^2*c))
```

maple [B] time = 0.15, size = 1813, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*B*b^2-1/2/(4*a*c-
b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*B*b^2-1/2/(4*a*c-b^2)^2/(x^2+
1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*x*C*b^2+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/
2*(-4*a*c+b^2)^(1/2)/c)*B*a-c/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*ln(-2*c*x^
2-b+(-4*a*c+b^2)^(1/2))+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/
2)/c)*B*a+c/(4*a*c-b^2)^2*B*(-4*a*c+b^2)^(1/2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1
/2))+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)-c^2/(4*a*c
-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*A*b+2*c^2/(4*a*c-b^2)^2*a*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C-1
/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^3-c/(4*a*c-b^2)^2*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b+1/4*c/(4*a*c-b^2)^2/a*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*A*b^3-c/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b+1/4/(4*a*c
-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)/a*x*A*b^3+1/4/(4*a*c-b^2)^2/
(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)/a*x*A*b^3-c/(4*a*c-b^2)^2/(x^2+1/2*b
/c-1/2*(-4*a*c+b^2)^(1/2)/c)*x*A*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/
2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*A*b*x+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-
4*a*c+b^2)^(1/2)/c)*a*C*x+c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/
2)/c)*x*A*(-4*a*c+b^2)^(1/2)-c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^
(1/2)/c)*A*b*x+2*c/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)*a*C
*x-1/2/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)*x*C*b^2-1/4*c/(
4*a*c-b^2)^2/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b^2-1/4*c/(4*a*c-
b^2)^2/a*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b^2-1/2*c/(4*a*c-b^2)^2*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*C*b^2+3*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+
b^2)^(1/2)+c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
anh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b-2*c^2/(4*a*c-b^2)^2*
a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*C+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*b^2+1
/4/(4*a*c-b^2)^2/(x^2+1/2*b/c-1/2*(-4*a*c+b^2)^(1/2)/c)/a*x*A*(-4*a*c+b^2)^
(1/2)*b^2-1/4/(4*a*c-b^2)^2/(x^2+1/2*b/c+1/2*(-4*a*c+b^2)^(1/2)/c)/a*x*A*(-
4*a*c+b^2)^(1/2)*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\,Bacx^2 + (2\,Ca - Ab)cx^3 + Bab + \left(Cab - Ab^2 + 2\,Aac\right)x}{2\left(\left(ab^2c - 4\,a^2c^2\right)x^4 + a^2b^2 - 4\,a^3c + \left(ab^3 - 4\,a^2bc\right)x^2\right)} + \frac{-\int \frac{4\,Bacx + (2\,Ca - Ab)cx^2 - Cab - Ab^2 + 6\,Aac}{cx^4 + bx^2 + a}\,dx}{2\left(ab^2 - 4\,a^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*B*a*c*x^2 + (2*C*a - A*b)*c*x^3 + B*a*b + (C*a*b - A*b^2 + 2*A*a*c)
*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^
2) + 1/2*integrate(-(4*B*a*c*x + (2*C*a - A*b)*c*x^2 - C*a*b - A*b^2 + 6*A*
a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

mupad [B] time = 1.67, size = 4707, normalized size = 12.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((B*b)/(2*(4*a*c - b^2)) + (x*(2*A*a*c - A*b^2 + C*a*b))/(2*a*(4*a*c - b^2)
) + (B*c*x^2)/(4*a*c - b^2) - (c*x^3*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) + symsum(log((5*A^3*b^3*c^4 + 8*C^3*a^3*c^4 + 6*C^3*a^2*b
^2*c^3 - 36*A^3*a*b*c^5 - 96*A*B^2*a^2*c^5 + 72*A^2*C*a^2*c^5 - 3*A^2*C*b^4
*c^3 + 16*A*B^2*a*b^2*c^4 + 3*A*C^2*a*b^3*c^3 - 60*A*C^2*a^2*b*c^4 + 18*A^2
*C*a*b^2*c^4 + 16*B^2*C*a^2*b*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) - root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
+ 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 10
48576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^
5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*
C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*
a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a
^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512
*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^
2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 -
 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 409
6*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B
*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A
^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C
^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C
*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*
b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c
^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 -
 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^
2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a
^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 25
6*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k)*(root(1572864*a^8*
b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b
^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 +
 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z
^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2
 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 -
8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4
*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 512*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b
^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B
^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2 - 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*
c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 4096*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c
*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*
z - 384*B*C^2*a^3*b^4*c^2*z - 15872*A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^
4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z +
 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 -
960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^
2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B
^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4 - 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b
```

```
*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C
*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*
A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 256*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*
A^4*a^2*c^5, z, k)*((x*(1024*B*a^5*c^6 - 16*B*a^2*b^6*c^3 + 192*B*a^3*b^4*c
^4 - 768*B*a^4*b^2*c^5))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b
^2*c^2)) - (6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*C*a^5*b*c^5 - 288*A*a^2*
b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*C*a^2*b^7*c^2 - 192*
C*a^3*b^5*c^3 + 768*C*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c
+ 48*a^4*b^2*c^2)) + (root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1
048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a
^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A
*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2
*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*
a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 51
2*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z
^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2
- 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 40
96*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*
B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*
A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*
C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*
C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a
*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*
c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4
- 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a
^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*
a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 2
56*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k)*x*(4096*a^6*b*c^6
 + 16*a^2*b^9*c^2 - 256*a^3*b^7*c^3 + 1536*a^4*b^5*c^4 - 4096*a^5*b^3*c^5))
/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) + (32*B*C*a^2*
b^4*c^3 - 384*A*B*a^2*b^3*c^4 - 512*B*C*a^4*c^5 + 32*A*B*a*b^5*c^3 + 1024*A
*B*a^3*b*c^5)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) +
(x*(A^2*b^6*c^3 - 288*A^2*a^3*c^6 + 32*C^2*a^4*c^5 + 128*A^2*a^2*b^2*c^5 -
16*B^2*a^2*b^3*c^4 + 10*C^2*a^2*b^4*c^3 - 48*C^2*a^3*b^2*c^4 - 18*A^2*a*b^4
*c^4 + 64*B^2*a^3*b*c^5 - 48*A*C*a^2*b^3*c^4 + 2*A*C*a*b^5*c^3 + 160*A*C*a^
3*b*c^5))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2))) - (x*
(16*B^3*a^2*c^5 - A^2*B*b^3*c^4 + 8*B*C^2*a^2*b*c^4 - 24*A*B*C*a^2*c^5 + 12
*A^2*B*a*b*c^5 - 2*A*B*C*a*b^2*c^4))/(2*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*
c + 48*a^4*b^2*c^2)))*root(1572864*a^8*b^2*c^5*z^4 - 983040*a^7*b^4*c^4*z^4
 + 327680*a^6*b^6*c^3*z^4 - 61440*a^5*b^8*c^2*z^4 + 6144*a^4*b^10*c*z^4 - 1
048576*a^9*c^6*z^4 - 256*a^3*b^12*z^4 + 576*A*C*a^2*b^8*c*z^2 + 24576*A*C*a
^5*b^2*c^4*z^2 - 3072*A*C*a^3*b^6*c^2*z^2 + 2048*A*C*a^4*b^4*c^3*z^2 - 32*A
*C*a*b^10*z^2 + 12288*C^2*a^6*b*c^4*z^2 + 61440*A^2*a^5*b*c^5*z^2 + 432*A^2
*a*b^9*c*z^2 - 49152*A*C*a^6*c^5*z^2 - 8192*C^2*a^5*b^3*c^3*z^2 + 1536*C^2*
a^4*b^5*c^2*z^2 + 24576*B^2*a^5*b^2*c^4*z^2 - 6144*B^2*a^4*b^4*c^3*z^2 + 51
2*B^2*a^3*b^6*c^2*z^2 - 61440*A^2*a^4*b^3*c^4*z^2 + 24064*A^2*a^3*b^5*c^3*z
^2 - 4608*A^2*a^2*b^7*c^2*z^2 - 32768*B^2*a^6*c^5*z^2 - 16*C^2*a^2*b^9*z^2
- 16*A^2*b^11*z^2 + 3072*A*B*C*a^3*b^3*c^3*z - 768*A*B*C*a^2*b^5*c^2*z - 40
96*A*B*C*a^4*b*c^4*z + 64*A*B*C*a*b^7*c*z + 32*B*C^2*a^2*b^6*c*z - 672*A^2*
B*a*b^6*c^2*z + 1536*B*C^2*a^4*b^2*c^3*z - 384*B*C^2*a^3*b^4*c^2*z - 15872*
A^2*B*a^3*b^2*c^4*z + 4992*A^2*B*a^2*b^4*c^3*z + 32*A^2*B*b^8*c*z - 2048*B*
C^2*a^5*c^4*z + 18432*A^2*B*a^4*c^5*z + 192*A*B^2*C*a^2*b^2*c^3 - 32*A*B^2*
C*a*b^4*c^2 - 16*B^2*C^2*a^2*b^3*c^2 - 960*A^2*C^2*a^2*b^2*c^3 - 18*A*C^3*a
*b^5*c - 192*B^2*C^2*a^3*b*c^3 + 198*A^2*C^2*a*b^4*c^2 + 144*A*C^3*a^2*b^3*
c^2 - 960*A^2*B^2*a^2*b*c^4 + 240*A^2*B^2*a*b^3*c^3 + 2016*A^3*C*a^2*b*c^4
- 496*A^3*C*a*b^3*c^3 + 224*A*C^3*a^3*b*c^3 + 768*A*B^2*C*a^3*c^4 - 9*C^4*a
^2*b^4*c + 360*A^4*a*b^2*c^4 + 30*A^3*C*b^5*c^2 - 9*A^2*C^2*b^6*c - 24*C^4*
a^3*b^2*c^2 - 288*A^2*C^2*a^3*c^4 - 16*A^2*B^2*b^5*c^2 - 16*C^4*a^4*c^3 - 2
```

```
56*B^4*a^3*c^4 - 25*A^4*b^4*c^3 - 1296*A^4*a^2*c^5, z, k), k, 1, 4)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.34 $\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=403

$$\frac{\left(4a^2cC+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx\right.}$$

```
[Out] 1/2*B*x*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+b
^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-6*a*b*
c+b^3)+4*a^2*c*C)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^
(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2+1/4*B*arctan(x*2^(1/2)*c^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))/a
/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*B*arctan(x*2^(
1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2-12*a*c-b*(-4*a*c+b^
2)^(1/2))/a/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1662, 1251, 822, 800, 634, 618, 206, 628, 12, 1092, 1166, 205}

$$\frac{\left(4a^2cC+A\left(b^3-6abc\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}+\frac{A\left(b^2-2ac\right)+cx^2(Ab-2aC)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*x*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*
(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*(a + b*x^2
+ c*x^4)) + (B*Sqrt[c]*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)^(3/2)*
Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (B*Sqrt[c]*(b^2 - 12*a*c - b*Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + ((A*(b^3 - 6*a*b*c) + 4*a
^2*c*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2
)) + (A*Log[x])/a^2 - (A*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{B}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{1}{2}\text{Subst}\left(\int \frac{A+Cx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) + B\int \frac{1}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-\right.}{2\sqrt{}}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-\right.}{2\sqrt{}}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-\right.}{2\sqrt{}}$$

$$= \frac{Bx\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{B\sqrt{c}\left(b^2-\right.}{2\sqrt{}}$$

Mathematica [A] time = 1.47, size = 458, normalized size = 1.14

$$-\frac{\frac{\left(4a^2cC+A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}-6abc+b^3\right)\right)\log\left(\sqrt{b^2-4ac}-b-2cx^2\right)}{\left(b^2-4ac\right)^{3/2}} - \frac{\left(A\left(b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}+6abc-b^3\right)-4a^2cC\right)\log\left(\sqrt{b^2-4ac}+b\right.}{\left(b^2-4ac\right)^{3/2}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*b*C + 2*a*c*x*(B + C*x) - b*B*x*(b + c*x^2) - A*(b^2 - 2*a*c + b*
c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(b^2 -
12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*
B*Sqrt[c]*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*
c]]) + 4*A*Log[x] - ((A*(b^3 - 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt
[b^2 - 4*a*c]) + 4*a^2*c*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2])/(b^2 - 4
*a*c)^(3/2) - ((A*(-b^3 + 6*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2
- 4*a*c]) - 4*a^2*c*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^
(3/2))/(4*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 6.55, size = 6022, normalized size = 14.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*A*log(abs(c*x^4 + b*x^2 + a))/a^2 + A*log(abs(x))/a^2 + 1/16*((a^4*b^4
*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*B + 2*(sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 - 2*a^
4*b^8*c^2 + 120*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 28*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^6*b^3*c^4 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 240*
a^6*b^4*c^4 + 384*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 192*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 704*a^7*b^2*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^7*c^6 - 768*a^8*c^6 + 2*(b^2 - 4*a*c)*a^4*b^6*c^2 - 28*(
b^2 - 4*a*c)*a^5*b^4*c^3 + 128*(b^2 - 4*a*c)*a^6*b^2*c^4 - 192*(b^2 - 4*a*c
)*a^7*c^5)*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (2*a^8*b^11*c^4
- 56*a^9*b^9*c^5 + 576*a^10*b^7*c^6 - 2816*a^11*b^5*c^7 + 6656*a^12*b^3*c^8
 - 6144*a^13*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^11*c^2 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^9*b^9*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^10*c^3 - 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^10*b^7*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^9*b^8*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^8*b^9*c^4 + 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^11*b^5*c^5 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^10*b^6*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^9*b^7*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
```

```
4*a*c)*c)*a^12*b^3*c^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^11*b^4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^10*b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^13*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^12*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^11*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^12*b*c^8 - 2*(b^2 - 4*a*c)*a^8*b^9*c^4 + 48*(b^2
- 4*a*c)*a^9*b^7*c^5 - 384*(b^2 - 4*a*c)*a^10*b^5*c^6 + 1280*(b^2 - 4*a*c)*
a^11*b^3*c^7 - 1536*(b^2 - 4*a*c)*a^12*b*c^8)*B)*arctan(2*sqrt(1/2)*x/sqrt(
(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^2
 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^
2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4
)))/((a^6*b^8*c - 16*a^7*b^6*c^2 - 2*a^6*b^7*c^2 + 96*a^8*b^4*c^3 + 24*a^7*
b^5*c^3 + a^6*b^6*c^3 - 256*a^9*b^2*c^4 - 96*a^8*b^3*c^4 - 12*a^7*b^4*c^4 +
 256*a^10*c^5 + 128*a^9*b*c^5 + 48*a^8*b^2*c^5 - 64*a^9*c^6)*abs(a^4*b^4*c
- 8*a^5*b^2*c^2 + 16*a^6*c^3)*abs(c)) - 1/16*((a^4*b^4*c - 8*a^5*b^2*c^2 +
16*a^6*c^3)^2*(2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c
^2 - 2*(b^2 - 4*a*c)*b*c^2)*B - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^8*c - 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 - 2*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 + 2*a^4*b^8*c^2 + 120*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 28*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^6*c^3 - 36*a^5*b^6*c^3 - 352*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*
b^2*c^4 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 - 14*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 + 240*a^6*b^4*c^4 + 384*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^5 + 192*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^7*b*c^5 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*
b^2*c^5 - 704*a^7*b^2*c^5 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*
c^6 + 768*a^8*c^6 - 2*(b^2 - 4*a*c)*a^4*b^6*c^2 + 28*(b^2 - 4*a*c)*a^5*b^4*
c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4 + 192*(b^2 - 4*a*c)*a^7*c^5)*B*abs(a^4*
b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (2*a^8*b^11*c^4 - 56*a^9*b^9*c^5 + 57
6*a^10*b^7*c^6 - 2816*a^11*b^5*c^7 + 6656*a^12*b^3*c^8 - 6144*a^13*b*c^9 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^11*c^2 + 28
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^9*c^3 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^3 - 28
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^7*c^4 -
48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^9*c^4 + 140
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^5*c^5 +
384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^6*c^5
+ 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^7*c^5
- 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^3*c
^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^
4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*
b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
13*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^12*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^11*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^12*b*c^8 - 2*(b^2 - 4*a*c)*a^8*b^9*c^4 + 48*(b^2 - 4*a*c)*a^9*b^7*c^5
- 384*(b^2 - 4*a*c)*a^10*b^5*c^6 + 1280*(b^2 - 4*a*c)*a^11*b^3*c^7 - 1536*(
b^2 - 4*a*c)*a^12*b*c^8)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b^5*c - 8*a^5*b^
3*c^2 + 16*a^6*b*c^3 - sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 -
4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 1
6*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/((a^6*b^8*c - 16*
a^7*b^6*c^2 - 2*a^6*b^7*c^2 + 96*a^8*b^4*c^3 + 24*a^7*b^5*c^3 + a^6*b^6*c^3
 - 256*a^9*b^2*c^4 - 96*a^8*b^3*c^4 - 12*a^7*b^4*c^4 + 256*a^10*c^5 + 128*a
```

```
^9*b*c^5 + 48*a^8*b^2*c^5 - 64*a^9*c^6)*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*
a^6*c^3)*abs(c)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24*a^2*b^2*c^3
 + 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 + (b^5*c - 10*a*b^3*c^2 - 2*b^4*c^2
 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2 - 4*a*c))*A*
abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + 4*(a^2*b^3*c^2 - 4*a^3*b*c^3
- 2*a^2*b^2*c^3 + a^2*b*c^4 + (a^2*b^2*c^2 - 4*a^3*c^3 - 2*a^2*b*c^3 + a^2*
c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (a^
4*b^10*c^2 - 18*a^5*b^8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 + 28*a^5*b^7*
c^4 + a^4*b^8*c^4 - 352*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*b^6*c^5 + 38
4*a^8*b^2*c^6 + 192*a^7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c^7 + (a^4*b^
9*c^2 - 14*a^5*b^7*c^3 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^5*b^6*c^4 +
a^4*b^7*c^4 - 96*a^7*b^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5 + 24*a^6*b^3
*c^6)*sqrt(b^2 - 4*a*c))*A + 4*(a^6*b^7*c^3 - 12*a^7*b^5*c^4 - 2*a^6*b^6*c^
4 + 48*a^8*b^3*c^5 + 16*a^7*b^4*c^5 + a^6*b^5*c^5 - 64*a^9*b*c^6 - 32*a^8*b
^2*c^6 - 8*a^7*b^3*c^6 + 16*a^8*b*c^7 + (a^6*b^6*c^3 - 8*a^7*b^4*c^4 - 2*a^
6*b^5*c^4 + 16*a^8*b^2*c^5 + 8*a^7*b^3*c^5 + a^6*b^4*c^5 - 4*a^7*b^2*c^6)*s
qrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^
3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^
6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b
^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c
 + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(a^4*b^4*c -
8*a^5*b^2*c^2 + 16*a^6*c^3)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24
*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 - (b^5*c - 10*a*b^3*c^2
 - 2*b^4*c^2 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2
- 4*a*c))*A*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + 4*(a^2*b^3*c^2 -
4*a^3*b*c^3 - 2*a^2*b^2*c^3 + a^2*b*c^4 - (a^2*b^2*c^2 - 4*a^3*c^3 - 2*a^2*
b*c^3 + a^2*c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^
6*c^3) - (a^4*b^10*c^2 - 18*a^5*b^8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 +
 28*a^5*b^7*c^4 + a^4*b^8*c^4 - 352*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*
b^6*c^5 + 384*a^8*b^2*c^6 + 192*a^7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c
^7 - (a^4*b^9*c^2 - 14*a^5*b^7*c^3 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^
5*b^6*c^4 + a^4*b^7*c^4 - 96*a^7*b^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5
+ 24*a^6*b^3*c^6)*sqrt(b^2 - 4*a*c))*A - 4*(a^6*b^7*c^3 - 12*a^7*b^5*c^4 -
2*a^6*b^6*c^4 + 48*a^8*b^3*c^5 + 16*a^7*b^4*c^5 + a^6*b^5*c^5 - 64*a^9*b*c^
6 - 32*a^8*b^2*c^6 - 8*a^7*b^3*c^6 + 16*a^8*b*c^7 - (a^6*b^6*c^3 - 8*a^7*b^
4*c^4 - 2*a^6*b^5*c^4 + 16*a^8*b^2*c^5 + 8*a^7*b^3*c^5 + a^6*b^4*c^5 - 4*a^
7*b^2*c^6)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 +
 16*a^6*b*c^3 - sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*
b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c
^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c -
 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(
a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)) + 1/2*(B*a*b*c*x^3 - C*a^2*b + A*a
*b^2 - 2*A*a^2*c - (2*C*a^2*c - A*a*b*c)*x^2 + (B*a*b^2 - 2*B*a^2*c)*x)/((c
*x^4 + b*x^2 + a)*(b^2 - 4*a*c)*a^2)
```

maple [B] time = 0.06, size = 1603, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b
^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1
/2)*b^2-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a*b^2*x-1/2/(c*x^4+b*x^2+a)/(4*a*
c-b^2)*B/a*b*c*x^3+A/a^2*ln(x)+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*C*b+1/(c*x^4
+b*x^2+a)/(4*a*c-b^2)*A*c-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a*b*c*x^2-1/a^2
/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*b^4-1/a^2/(
```

```
4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b^4-4*c/(4*a*c
-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)
+4*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+
b^2)^(1/2)-6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2)
)*A*(-4*a*c+b^2)^(1/2)*b+12*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*B*(-4*a*c+b^2)^(1/2)+4*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b*B-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/
2))*A-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A-
1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*
c+b^2)^(1/2)*b^3+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2
)^(1/2))*A*b^2+1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(
1/2))*A*(-4*a*c+b^2)^(1/2)*b^3+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+
b+(-4*a*c+b^2)^(1/2))*A*b^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*B*b^3+1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^3-1/2
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a*b^2+1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*C+
12*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)-4
*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*B+6/a*c/(4*a*c-b^2)/(1
6*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b+1/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*B*c*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*c*x^3 - (2*C*a - A*b)*c*x^2 - C*a*b + A*b^2 - 2*A*a*c + (B*b^2 - 2
*B*a*c)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*
b*c)*x^2) + 1/2*integrate((B*a*b*c*x^2 + B*a*b^2 - 6*B*a^2*c - 2*(A*b^2*c -
 4*A*a*c^2)*x^3 - 2*(A*b^3 + (2*C*a^2 - 5*A*a*b)*c)*x)/(c*x^4 + b*x^2 + a),
 x)/(a^2*b^2 - 4*a^3*c) + A*log(x)/a^2
```

mupad [B] time = 1.84, size = 8129, normalized size = 20.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*A*a*c - A*b^2 + C*a*b)/(2*a*(4*a*c - b^2)) + (B*x*(2*a*c - b^2))/(2*a*(
4*a*c - b^2)) - (c*x^2*(A*b - 2*C*a))/(2*a*(4*a*c - b^2)) - (B*b*c*x^3)/(2*
a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + symsum(log(root(1572864*a^9*b^2*c^5
*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*
z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 - 256*a^4*b^12*z^4 + 15728
64*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3
- 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 -
 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 901
12*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z
^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^
2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*
c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*
a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 +
 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2
```

- 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*
A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*
B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z - 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2
*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15
360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^
3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^
5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c
^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z
 - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*
A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 96
0*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*
C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^
3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 + 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*
a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B
^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 - 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^
5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2
c^6, z, k)(root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680
*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^
10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^
4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3
*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*
c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4
*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2
*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C
^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 +
24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c
^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2
*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a
^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c
^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z -
 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c
^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*
a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A
*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^
3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^
2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c
^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^
3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5
*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c
^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 +
 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*
a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 -
2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3
*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*((1032*A*B*a^3*b^5*c^4 - 1
52*A*B*a^2*b^7*c^3 - 768*B*C*a^6*c^6 - 2944*A*B*a^4*b^3*c^5 + 16*B*C*a^3*b^
6*c^3 - 208*B*C*a^4*b^4*c^4 + 768*B*C*a^5*b^2*c^5 + 8*A*B*a*b^9*c^2 + 2944*
A*B*a^5*b*c^6)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) +
 root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 327680*a^7*b^6*c^3
*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a^10*c^6*z^4 -
 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b^4*c^4*z^3 +
327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^3*b^10*c*z^3
 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b*c^5*z^2 + 25
6*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^4*b^5*c^3*z^2
 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^2*a^2*b^9*c*z
^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*C^2*a^5*b^4*c
^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 + 24064*B^2*a^
4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*c^5*z^2 - 288
768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^2*a^2*b^8*c^2
*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*a^6*c^6*z^2 -

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 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*c^2*z + 3072*
A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z - 15872*B^2*C
*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*c^2*z - 45056
*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2*a^4*b^2*c^4*
z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*A*B^2*a^3*b^3
*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A^3*a^3*b^2*c^
5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B^2*C*a^5*c^5*
z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*c^6*z - 1088*
A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b^3*c^3 - 1920
*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^5*c^2 + 768*A
^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*c^5 + 1104*A^
2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4 + 1536*A*C^3*
a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4*a*b^2*c^5 +
192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 - 2048*A^2*C^2
*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^3*c^5 - 144*A
^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*((x*(983040*A*a^8*c^8 - 32768*C*a^8*b*
c^7 + 192*A*a^2*b^12*c^2 - 4736*A*a^3*b^10*c^3 + 48896*A*a^4*b^8*c^4 - 2703
36*A*a^5*b^6*c^5 + 843776*A*a^6*b^4*c^6 - 1409024*A*a^7*b^2*c^7 - 128*C*a^4
*b^9*c^3 + 2048*C*a^5*b^7*c^4 - 12288*C*a^6*b^5*c^5 + 32768*C*a^7*b^3*c^6))
/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c
^3)) - (3584*B*a^7*b*c^6 + 8*B*a^3*b^9*c^2 - 152*B*a^4*b^7*c^3 + 1056*B*a^5
*b^5*c^4 - 3200*B*a^6*b^3*c^5)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)) + (root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 3
27680*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 10485
76*a^10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a
^6*b^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*
A*a^3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a
^6*b*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*
C*a^4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 43
2*B^2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6
144*C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z
^2 + 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*
b^2*c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 1574
4*A^2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*
A^2*a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*
b^7*c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*
c*z - 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*
b^6*c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A
*C^2*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2
304*A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 614
40*A^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 184
32*B^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*
a^4*c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a
^2*b^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*
a*b^5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^
2*b*c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*
c^4 + 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536
*A^4*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c
^2 - 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^
4*a^3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k)*x*(1310720*a^10*c^8 +
 384*a^4*b^12*c^2 - 8960*a^5*b^10*c^3 + 87040*a^6*b^8*c^4 - 450560*a^7*b^6*
c^5 + 1310720*a^8*b^4*c^6 - 2031616*a^9*b^2*c^7))/(16*(a^3*b^8 + 256*a^7*c^
4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) - (x*(26560*A^2*a^3*
b^6*c^5 - 36864*C^2*a^7*c^7 - 2912*A^2*a^2*b^8*c^4 - 245760*A^2*a^6*c^8 - 1
20832*A^2*a^4*b^4*c^6 + 273408*A^2*a^5*b^2*c^7 + 432*B^2*a^2*b^9*c^3 - 4616
*B^2*a^3*b^7*c^4 + 24032*B^2*a^4*b^5*c^5 - 60800*B^2*a^5*b^3*c^6 + 640*C^2*
a^4*b^6*c^4 - 7424*C^2*a^5*b^4*c^5 + 28672*C^2*a^6*b^2*c^6 + 128*A^2*a*b^10
*c^3 - 16*B^2*a*b^11*c^2 + 59904*B^2*a^6*b*c^7 + 256*A*C*a^2*b^9*c^3 - 4608
*A*C*a^3*b^7*c^4 + 30464*A*C*a^4*b^5*c^5 - 88064*A*C*a^5*b^3*c^6 + 94208*A*
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C*a^6*b*c^7))/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 -
256*a^6*b^2*c^3))) + (108*B^3*a^4*c^6 - 15*B^3*a^3*b^2*c^5 + 24*A^2*B*a*b^5
*c^4 + 704*A^2*B*a^3*b*c^6 + 56*B*C^2*a^4*b*c^5 - 266*A^2*B*a^2*b^3*c^5 - 8
*B*C^2*a^3*b^3*c^4 + 576*A*B*C*a^4*c^6 - 16*A*B*C*a*b^6*c^3 + 208*A*B*C*a^2
*b^4*c^4 - 744*A*B*C*a^3*b^2*c^5)/(4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
 48*a^5*b^2*c^2)) + (x*(20480*A^3*a^4*c^8 - 32*A^3*b^8*c^4 + 1216*A^3*a^2*b
^4*c^6 - 11008*A^3*a^3*b^2*c^7 + 128*C^3*a^4*b^3*c^5 + 13312*A*C^2*a^5*c^7
- 19584*B^2*C*a^5*c^7 + 192*A^3*a*b^6*c^5 - 512*C^3*a^5*b*c^6 + 40*A*B^2*a*
b^7*c^4 - 2496*A*B^2*a^4*b*c^7 + 256*A^2*C*a*b^7*c^4 - 25600*A^2*C*a^4*b*c^
7 - 32*B^2*C*a*b^8*c^3 - 508*A*B^2*a^2*b^5*c^5 + 2016*A*B^2*a^3*b^3*c^6 - 6
4*A*C^2*a^2*b^6*c^4 + 1152*A*C^2*a^3*b^4*c^5 - 6912*A*C^2*a^4*b^2*c^6 - 355
2*A^2*C*a^2*b^5*c^5 + 16512*A^2*C*a^3*b^3*c^6 + 672*B^2*C*a^2*b^6*c^4 - 500
0*B^2*C*a^3*b^4*c^5 + 16192*B^2*C*a^4*b^2*c^6))/(16*(a^3*b^8 + 256*a^7*c^4
- 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a^6*b^2*c^3))) - (108*A*B^3*a^2*c^6 -
 10*A^3*B*b^3*c^5 - 192*A^2*B*C*a^2*c^6 - 15*A*B^3*a*b^2*c^5 + 64*A^3*B*a*b
*c^6 - 8*A*B*C^2*a*b^3*c^4 + 56*A*B*C^2*a^2*b*c^5 + 24*A^2*B*C*a*b^2*c^5)/(
4*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)) + (x*(1296*B^4*a^
3*c^7 - 48*A^4*b^4*c^6 + 256*C^4*a^4*c^6 + 1024*A^2*C^2*a^3*c^7 - 360*B^4*a
^2*b^2*c^6 + 32*A^3*C*b^5*c^5 + 256*A^4*a*b^2*c^7 + 25*B^4*a*b^4*c^5 - 3456
*A*B^2*C*a^3*c^7 - 1024*A*C^3*a^3*b*c^6 - 1024*A^3*C*a^2*b*c^7 - 176*A^2*B^
2*a*b^3*c^6 + 960*A^2*B^2*a^2*b*c^7 + 128*A*C^3*a^2*b^3*c^5 - 128*A^2*C^2*a
*b^4*c^5 + 16*B^2*C^2*a*b^5*c^4 + 960*B^2*C^2*a^3*b*c^6 + 640*A^2*C^2*a^2*b
^2*c^6 - 240*B^2*C^2*a^2*b^3*c^5 - 40*A*B^2*C*a*b^4*c^5 + 768*A*B^2*C*a^2*b
^2*c^6))/(16*(a^3*b^8 + 256*a^7*c^4 - 16*a^4*b^6*c + 96*a^5*b^4*c^2 - 256*a
^6*b^2*c^3)))*root(1572864*a^9*b^2*c^5*z^4 - 983040*a^8*b^4*c^4*z^4 + 32768
0*a^7*b^6*c^3*z^4 - 61440*a^6*b^8*c^2*z^4 + 6144*a^5*b^10*c*z^4 - 1048576*a
^10*c^6*z^4 - 256*a^4*b^12*z^4 + 1572864*A*a^7*b^2*c^5*z^3 - 983040*A*a^6*b
^4*c^4*z^3 + 327680*A*a^5*b^6*c^3*z^3 - 61440*A*a^4*b^8*c^2*z^3 + 6144*A*a^
3*b^10*c*z^3 - 1048576*A*a^8*c^6*z^3 - 256*A*a^2*b^12*z^3 + 98304*A*C*a^6*b
*c^5*z^2 + 256*A*C*a^2*b^9*c*z^2 - 90112*A*C*a^5*b^3*c^4*z^2 + 30720*A*C*a^
4*b^5*c^3*z^2 - 4608*A*C*a^3*b^7*c^2*z^2 + 61440*B^2*a^6*b*c^5*z^2 + 432*B^
2*a^2*b^9*c*z^2 + 1536*A^2*a*b^10*c*z^2 + 24576*C^2*a^6*b^2*c^4*z^2 - 6144*
C^2*a^5*b^4*c^3*z^2 + 512*C^2*a^4*b^6*c^2*z^2 - 61440*B^2*a^5*b^3*c^4*z^2 +
 24064*B^2*a^4*b^5*c^3*z^2 - 4608*B^2*a^3*b^7*c^2*z^2 + 516096*A^2*a^5*b^2*
c^5*z^2 - 288768*A^2*a^4*b^4*c^4*z^2 + 88576*A^2*a^3*b^6*c^3*z^2 - 15744*A^
2*a^2*b^8*c^2*z^2 - 16*B^2*a*b^11*z^2 - 32768*C^2*a^7*c^5*z^2 - 393216*A^2*
a^6*c^6*z^2 - 64*A^2*b^12*z^2 + 49152*A^2*C*a^4*b*c^5*z - 2304*A^2*C*a*b^7*
c^2*z + 3072*A*B^2*a^4*b*c^5*z - 48*A*B^2*a*b^7*c^2*z + 32*B^2*C*a*b^8*c*z
- 15872*B^2*C*a^4*b^2*c^4*z + 4992*B^2*C*a^3*b^4*c^3*z - 672*B^2*C*a^2*b^6*
c^2*z - 45056*A^2*C*a^3*b^3*c^4*z + 15360*A^2*C*a^2*b^5*c^3*z + 12288*A*C^2
*a^4*b^2*c^4*z - 3072*A*C^2*a^3*b^4*c^3*z + 256*A*C^2*a^2*b^6*c^2*z - 2304*
A*B^2*a^3*b^3*c^4*z + 576*A*B^2*a^2*b^5*c^3*z + 128*A^2*C*b^9*c*z + 61440*A
^3*a^3*b^2*c^5*z - 21504*A^3*a^2*b^4*c^4*z + 3328*A^3*a*b^6*c^3*z + 18432*B
^2*C*a^5*c^5*z - 16384*A*C^2*a^5*c^5*z - 192*A^3*b^8*c^2*z - 65536*A^3*a^4*
c^6*z - 1088*A*B^2*C*a^2*b^2*c^4 + 48*A*B^2*C*a*b^4*c^3 + 240*B^2*C^2*a^2*b
^3*c^3 - 1920*A^2*C^2*a^2*b^2*c^4 - 960*B^2*C^2*a^3*b*c^4 - 16*B^2*C^2*a*b^
5*c^2 + 768*A^2*C^2*a*b^4*c^3 - 256*A*C^3*a^2*b^3*c^3 - 3072*A^2*B^2*a^2*b*
c^5 + 1104*A^2*B^2*a*b^3*c^4 + 6144*A^3*C*a^2*b*c^5 - 2176*A^3*C*a*b^3*c^4
+ 1536*A*C^3*a^3*b*c^4 + 4608*A*B^2*C*a^3*c^5 - 25*B^4*a*b^4*c^3 + 1536*A^4
*a*b^2*c^5 + 192*A^3*C*b^5*c^3 + 360*B^4*a^2*b^2*c^4 - 64*A^2*C^2*b^6*c^2 -
 2048*A^2*C^2*a^3*c^5 - 100*A^2*B^2*b^5*c^3 - 256*C^4*a^4*c^4 - 1296*B^4*a^
3*c^5 - 144*A^4*b^4*c^4 - 4096*A^4*a^2*c^6, z, k), k, 1, 4) + (A*log(x))/a^
2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.35 $\int \frac{A+Bx+Cx^2}{x^2(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=514

$$-\frac{-10aAc-abC+3Ab^2}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right.\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*(10*A*a*c-3*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a
/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/
a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/2*b*B*(-6*a*c+b^2)*arctanh((2*c*x^2+b)/(
-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+B*ln(x)/a^2-1/4*B*ln(c*x^4+b*x^2+
a)/a^2-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
-a*C*(b^2-12*a*c+b*(-4*a*c+b^2)^(1/2))+A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)
^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*
c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(3*A*b^2-10*a*A*c-a*b*C+(-A*(-16*a*b*c+3*b^3)+a*(-12*a*c+b^2)*C
)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.49, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1662, 1277, 1281, 1166, 205, 12, 1114, 740, 800, 634, 618, 206, 628}

$$-\frac{-10aAc-abC+3Ab^2}{2a^2x(b^2-4ac)}-\frac{\sqrt{c}\left(A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-aC\left(b\sqrt{b^2-4ac}-12ac+b^2\right.\right.}{2\sqrt{2}a^2(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(3*A*b^2 - 10*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b
*c*x^2))/(2*a*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C
 + c*(A*b - 2*a*C)*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) - (Sqrt[c
]*(A*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]
) - a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqr
t[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - 10*a*A*c - a*b*C - (A*(3*b^3 - 16*a*
b*c) - a*(b^2 - 12*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2
- 4*a*c]]) + (b*B*(b^2 - 6*a*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
2*a^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[x])/a^2 - (B*Log[a + b*x^2 + c*x^4])/(4
*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 740

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[((d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e
)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
```

```
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{B}{x\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{A+Cx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + B\int \frac{1}{x\left(a+bx^2+cx^4\right)^2}\,dx - \frac{\int \frac{-3Ab^2+10a}{x^2}}{2a}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int -\right.$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c}{2a\left(b^2-4ac\right)x\left(a+\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c}{2a\left(b^2-4ac\right)x\left(a+\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c}{2a\left(b^2-4ac\right)x\left(a+\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c}{2a\left(b^2-4ac\right)x\left(a+\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c}{2a\left(b^2-4ac\right)x\left(a+\right.}$$

$$= -\frac{3Ab^2-10aAc-abC}{2a^2\left(b^2-4ac\right)x} + \frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\left(b^2-2ac\right)-abC+c}{2a\left(b^2-4ac\right)x\left(a+\right.}$$

Mathematica [A] time = 2.03, size = 559, normalized size = 1.09

$$\frac{\frac{-4a^2c(B+Cx)+2a\left(bcx(3A+x(B+Cx))+2Ac^2x^3+b^2(B+Cx)\right)-2Ab^2x\left(b+cx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\left(A\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}+16abc-3b^3\right)+aC\left(b\sqrt{b^2-4}\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (-4*a^2*c*(B + C*x) - 2*A*b^2*x*(b + c*x^2) + 2*a*(2*A*c^2*x^3
+ b^2*(B + C*x) + b*c*x*(3*A + x*(B + C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c
*x^4)) + (Sqrt[2]*Sqrt[c]*(A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] +
 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcT
an[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(A*(3*b^3 - 16*a*b*c - 3*b^2
*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + a*(-b^2 + 12*a*c + b*Sqrt[
```

```
b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + 4*B*Log[x] - (B*(b^3 - 6
*a*b*c + b^2*Sqrt[b^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (B*(-b^3 + 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c])*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2))/(4*a^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 11.55, size = 9015, normalized size = 17.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*B*log(abs(c*x^4 + b*x^2 + a))/a^2 + B*log(abs(x))/a^2 + 1/2*(C*a*b*c*x
^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b*c*x^3 + C*a*b^2*x^2 - 3*A*b^3*x
^2 - 2*C*a^2*c*x^2 + 11*A*a*b*c*x^2 + B*a*b^2*x - 2*B*a^2*c*x - 2*A*a*b^2 +
 8*A*a^2*c)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((a^4*b^4*c
- 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c
)*a*c^3)*A - (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b^3*c^2 - 8*a^
2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*
a*b*c^2)*C + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 49*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 6*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^8*c^2 - 6*a^4*b^9*c^2 + 300*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^6*b^5*c^3 + 74*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 98*a^
5*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 304*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 37*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 600*a^6*b^5*c^4 + 832*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5 + 416*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^7*b^2*c^5 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 +
1632*a^7*b^3*c^5 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 -
1664*a^8*b*c^6 + 6*(b^2 - 4*a*c)*a^4*b^7*c^2 - 74*(b^2 - 4*a*c)*a^5*b^5*c^3
 + 304*(b^2 - 4*a*c)*a^6*b^3*c^4 - 416*(b^2 - 4*a*c)*a^7*b*c^5)*A*abs(a^4*b
^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^8*c - 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^2 - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 2*a^5*b^8*c^2 + 120*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^3 + 28*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^5*b^6*c^3 + 36*a^6*b^6*c^3 - 352*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*b^2*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 14
```

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*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 240*a^7*b^4*c^4 + 38
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*c^5 + 192*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^8*b*c^5 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^7*b^2*c^5 + 704*a^8*b^2*c^5 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^8*c^6 - 768*a^9*c^6 + 2*(b^2 - 4*a*c)*a^5*b^6*c^2 - 28*(b^2 - 4*a*c)*a^6
*b^4*c^3 + 128*(b^2 - 4*a*c)*a^7*b^2*c^4 - 192*(b^2 - 4*a*c)*a^8*c^5)*C*abs
(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) + (6*a^8*b^12*c^4 - 128*a^9*b^10*c
^5 + 1088*a^10*b^8*c^6 - 4608*a^11*b^6*c^7 + 9728*a^12*b^4*c^8 - 8192*a^13*
b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b
^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*
b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*
b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
10*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^9*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^8*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^11*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^10*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^9*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^12*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^11*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^10*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^13*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^12*b^3*c^7 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^11*b^4*c^7 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^12*b^2*c^8 - 6*(b^2 - 4*a*c)*a^8*b^10*c^4 + 104*(b
^2 - 4*a*c)*a^9*b^8*c^5 - 672*(b^2 - 4*a*c)*a^10*b^6*c^6 + 1920*(b^2 - 4*a*
c)*a^11*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^12*b^2*c^8)*A - (2*a^9*b^11*c^4 - 56
*a^10*b^9*c^5 + 576*a^11*b^7*c^6 - 2816*a^12*b^5*c^7 + 6656*a^13*b^3*c^8 -
6144*a^14*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^9*b^11*c^2 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^10*b^9*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^9*b^10*c^3 - 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^11*b^7*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^10*b^8*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^9*b^9*c^4 + 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^12*b^5*c^5 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^11*b^6*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^10*b^7*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^13*b^3*c^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^12*b^4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^11*b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^14*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^13*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^12*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^13*b*c^8 - 2*(b^2 - 4*a*c)*a^9*b^9*c^4 + 48*(b^2
- 4*a*c)*a^10*b^7*c^5 - 384*(b^2 - 4*a*c)*a^11*b^5*c^6 + 1280*(b^2 - 4*a*c)
*a^12*b^3*c^7 - 1536*(b^2 - 4*a*c)*a^13*b*c^8)*C)*arctan(2*sqrt(1/2)*x/sqrt
((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + sqrt((a^4*b^5*c - 8*a^5*b^3*c^
2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c
^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^
4)))/((a^7*b^8*c - 16*a^8*b^6*c^2 - 2*a^7*b^7*c^2 + 96*a^9*b^4*c^3 + 24*a^8
*b^5*c^3 + a^7*b^6*c^3 - 256*a^10*b^2*c^4 - 96*a^9*b^3*c^4 - 12*a^8*b^4*c^4
 + 256*a^11*c^5 + 128*a^10*b*c^5 + 48*a^9*b^2*c^5 - 64*a^10*c^6)*abs(a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*abs(c)) + 1/16*((a^4*b^4*c - 8*a^5*b^2*c^
2 + 16*a^6*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
```

```
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*A - (
a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)^2*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*C - 2
*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^9*c - 49*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^4*b^8*c^2 + 6*a^4*b^9*c^2 + 300*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^6*b^5*c^3 + 74*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 +
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^3 - 98*a^5*b^7*c^3 - 81
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 304*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^5*b^5*c^4 + 600*a^6*b^5*c^4 + 832*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^8*b*c^5 + 416*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5
 + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 - 1632*a^7*b^3*c
^5 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^6 + 1664*a^8*b*c^6
 - 6*(b^2 - 4*a*c)*a^4*b^7*c^2 + 74*(b^2 - 4*a*c)*a^5*b^5*c^3 - 304*(b^2 -
4*a*c)*a^6*b^3*c^4 + 416*(b^2 - 4*a*c)*a^7*b*c^5)*A*abs(a^4*b^4*c - 8*a^5*b
^2*c^2 + 16*a^6*c^3) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^8*c
 - 18*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c^2 + 2*a^5*b^8*c^2 + 120*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^3 + 28*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^6*b^5*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 -
 36*a^6*b^6*c^3 - 352*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^4 -
 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^4 - 14*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 + 240*a^7*b^4*c^4 + 384*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*c^5 + 192*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b*c^5 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 -
 704*a^8*b^2*c^5 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^6 + 768
*a^9*c^6 - 2*(b^2 - 4*a*c)*a^5*b^6*c^2 + 28*(b^2 - 4*a*c)*a^6*b^4*c^3 - 128
*(b^2 - 4*a*c)*a^7*b^2*c^4 + 192*(b^2 - 4*a*c)*a^8*c^5)*C*abs(a^4*b^4*c - 8
*a^5*b^2*c^2 + 16*a^6*c^3) + (6*a^8*b^12*c^4 - 128*a^9*b^10*c^5 + 1088*a^10
*b^8*c^6 - 4608*a^11*b^6*c^7 + 9728*a^12*b^4*c^8 - 8192*a^13*b^2*c^9 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^12*c^2 + 64*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^10*c^3 + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^11*c^3 - 544
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^8*c^4 - 1
04*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^9*c^4 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^10*c^4 +
2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^6*c^5
 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^7*c
^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^8*c
^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^
4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11
*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
10*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^13*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^12*b^3*c^7 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^11*b^4*c^7 - 1024*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^12*b^2*c^8 - 6*(b^2 - 4*a*c)*a^8*b^10*c^4 + 104*(b^2 - 4*a*c)*a^
9*b^8*c^5 - 672*(b^2 - 4*a*c)*a^10*b^6*c^6 + 1920*(b^2 - 4*a*c)*a^11*b^4*c^
7 - 2048*(b^2 - 4*a*c)*a^12*b^2*c^8)*A - (2*a^9*b^11*c^4 - 56*a^10*b^9*c^5
+ 576*a^11*b^7*c^6 - 2816*a^12*b^5*c^7 + 6656*a^13*b^3*c^8 - 6144*a^14*b*c^
9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^11*c^2
+ 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^9*c^3
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^10*c^3
```

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 - 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^7*c
^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^8*
c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^9*c^4
 + 1408*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^12*b^5*
c^5 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b^
6*c^5 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b
^7*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^1
3*b^3*c^6 - 1280*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^12*b^4*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^11*b^5*c^6 + 3072*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^14*b*c^7 + 1536*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^13*b^2*c^7 + 640*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^12*b^3*c^7 - 768*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^13*b*c^8 - 2*(b^2 - 4*a*c)*a^9*b^9*c^4 + 48*(b^2 - 4*a*c)*a^10*
b^7*c^5 - 384*(b^2 - 4*a*c)*a^11*b^5*c^6 + 1280*(b^2 - 4*a*c)*a^12*b^3*c^7
- 1536*(b^2 - 4*a*c)*a^13*b*c^8)*C)*arctan(2*sqrt(1/2)*x/sqrt((a^4*b^5*c -
8*a^5*b^3*c^2 + 16*a^6*b*c^3 - sqrt((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c
^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2
*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/((a^7*b^8
*c - 16*a^8*b^6*c^2 - 2*a^7*b^7*c^2 + 96*a^9*b^4*c^3 + 24*a^8*b^5*c^3 + a^7
*b^6*c^3 - 256*a^10*b^2*c^4 - 96*a^9*b^3*c^4 - 12*a^8*b^4*c^4 + 256*a^11*c^
5 + 128*a^10*b*c^5 + 48*a^9*b^2*c^5 - 64*a^10*c^6)*abs(a^4*b^4*c - 8*a^5*b^
2*c^2 + 16*a^6*c^3)*abs(c)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24*
a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 + (b^5*c - 10*a*b^3*c^2
- 2*b^4*c^2 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2 -
 4*a*c))*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - (a^4*b^10*c^2 - 18
*a^5*b^8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 + 28*a^5*b^7*c^4 + a^4*b^8*c
^4 - 352*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*b^6*c^5 + 384*a^8*b^2*c^6 +
 192*a^7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c^7 + (a^4*b^9*c^2 - 14*a^5*
b^7*c^3 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^5*b^6*c^4 + a^4*b^7*c^4 - 9
6*a^7*b^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5 + 24*a^6*b^3*c^6)*sqrt(b^2
- 4*a*c))*B)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 + sqrt
((a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^
2 + 16*a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 -
 8*a^5*b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^
5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(a^4*b^4*c - 8*a^5*b^
2*c^2 + 16*a^6*c^3)) - 1/16*((b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 24*a^2*b^2
*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 6*a*b^2*c^4 - (b^5*c - 10*a*b^3*c^2 - 2*b^4
*c^2 + 24*a^2*b*c^3 + 12*a*b^2*c^3 + b^3*c^3 - 6*a*b*c^4)*sqrt(b^2 - 4*a*c)
)*B*abs(a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3) - (a^4*b^10*c^2 - 18*a^5*b^
8*c^3 - 2*a^4*b^9*c^3 + 120*a^6*b^6*c^4 + 28*a^5*b^7*c^4 + a^4*b^8*c^4 - 35
2*a^7*b^4*c^5 - 128*a^6*b^5*c^5 - 14*a^5*b^6*c^5 + 384*a^8*b^2*c^6 + 192*a^
7*b^3*c^6 + 64*a^6*b^4*c^6 - 96*a^7*b^2*c^7 - (a^4*b^9*c^2 - 14*a^5*b^7*c^3
 - 2*a^4*b^8*c^3 + 64*a^6*b^5*c^4 + 20*a^5*b^6*c^4 + a^4*b^7*c^4 - 96*a^7*b
^3*c^5 - 48*a^6*b^4*c^5 - 10*a^5*b^5*c^5 + 24*a^6*b^3*c^6)*sqrt(b^2 - 4*a*c
))*B)*log(x^2 + 1/2*(a^4*b^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3 - sqrt((a^4*b
^5*c - 8*a^5*b^3*c^2 + 16*a^6*b*c^3)^2 - 4*(a^5*b^4*c - 8*a^6*b^2*c^2 + 16*
a^7*c^3)*(a^4*b^4*c^2 - 8*a^5*b^2*c^3 + 16*a^6*c^4)))/(a^4*b^4*c^2 - 8*a^5*
b^2*c^3 + 16*a^6*c^4))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 +
 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*c^2*abs(a^4*b^4*c - 8*a^5*b^2*c^2 +
 16*a^6*c^3))
```

maple [B] time = 0.08, size = 2398, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x)
[Out] -1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a*b^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2
```

```
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b
^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*b^2-16/a*c^2/(4*a*c-b^2)/(16*a*
c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b+3/a^2*c/(4*a*c-b^2)/(
16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b^3+3/a^2*c/(4*a*c-b^
2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b^3-16/a*c^2/(
4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b+B/a^2*l
n(x)-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a*c^2*x^3+1/2/(c*x^4+b*x^2+a)/(4*a*c-b
^2)*A/a^2*b^3*x+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a^2*b^2*c*x^3-3/2/(c*x^4+
b*x^2+a)/(4*a*c-b^2)*A/a*b*c*x+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B*c-1/2/(c*x^4
+b*x^2+a)/(4*a*c-b^2)*B/a*b*c*x^2-1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*C*b^3+22/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
A*b^2-3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4+3/a^2*c/(
4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^4-22/a*c^2/(4*a*c-b^2)/
(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+1/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*C*b^3+6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-
4*a*c+b^2)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b-6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*l
n(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b+12*c^2/(4*a*c-b^2)/(
16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)+12*c^2/(4*a*c-b^2)
/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)-4*c^2/(4*a*c-b^2)/(
16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*b+4*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*C*b+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*C-1/2/a/(c*x^4+b*x^2
+a)*c/(4*a*c-b^2)*x^3*C*b-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-
4*a*c+b^2)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b^3+1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)
*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*B*(-4*a*c+b^2)^(1/2)*b^3+8/a*c/(4*a*c-b^2
)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*b^2*B+8/a*c/(4*a*c-b^2)/(
16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*b^2*B+40*c^3/(4*a*c-b^2)/(1
6*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A-40*c^3/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*A-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2
)^(1/2))*B-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/
2))*B-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*C*b^2-1/a^2/(4*a*c-b^2)/(16*a*c-4
*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*b^4*B-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^
2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*b^4*B-A/a^2/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*a*b*c*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A
*a^2*c + (C*a*b^2 - 3*A*b^3 - (2*C*a^2 - 11*A*a*b)*c)*x^2 + (B*a*b^2 - 2*B*
```

```
a^2*c)*x)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b
^2 - 4*a^4*c)*x) + 1/2*integrate((C*a*b^2 - 3*A*b^3 - 2*(B*b^2*c - 4*B*a*c^
2)*x^3 + (10*A*a*c^2 + (C*a*b - 3*A*b^2)*c)*x^2 - (6*C*a^2 - 13*A*a*b)*c -
2*(B*b^3 - 5*B*a*b*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c) + B*lo
g(x)/a^2
```

mupad [B] time = 2.47, size = 8684, normalized size = 16.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*
a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a^1
1*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4
*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*
b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10
*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A
*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*
C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 43008
0*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 6144
0*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^
2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4
*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483
840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^
2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 -
144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z -
 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z
- 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*
z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c
^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a
^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a
*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b
^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^
2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a
*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a
*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*
c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 420
0*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*
c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 3
24*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5
- 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4 - 983040*a^9*b^4*
c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*
z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8*b^2*c^5*z^3 -
 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*
z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a^3*b^12*z^3 -
2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400*A*C*a^5*b^4*c
^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*
b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b
^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2 + 245760*A*C*a
^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^3*z^2 - 4608*C
^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*a^5*b^4*c^4*z^
2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 716800*A^2*a^5*
b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7*c^3*z^2 - 332
32*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c^6*z^2 - 16*C^
2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*z + 36864*A*B*
C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*c^2*z + 3072*B
*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^7*c^3*z + 1228
80*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3*b^5*c^3*z - 4
```

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8*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^2*B*a^2*b^5*c^
4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*
c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3*a^5*c^6*z - 5
568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4
 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3
 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6
 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*C*a*b^3*c^5 +
5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^3*c^6 - 144*B^
4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4
 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6*c^3 - 7200*A^
2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 -
 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k)*(root(1572864*a^10*b^2*c^5*z^4
 - 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 +
 6144*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*
a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 614
40*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*
B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358
400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^
2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2
 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z
^2 + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5
*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B
^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2
+ 716800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*
b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^
7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c
^5*z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b
^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a
*b^7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*
a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656
*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z +
3328*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*
B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2
*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*
B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*
A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A
^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C
*a^3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360
*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*
b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4
096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k)*((x*(983040*B*
a^9*c^8 + 192*B*a^3*b^12*c^2 - 4736*B*a^4*b^10*c^3 + 48896*B*a^5*b^8*c^4 -
270336*B*a^6*b^6*c^5 + 843776*B*a^7*b^4*c^6 - 1409024*B*a^8*b^2*c^7))/(16*(
a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)) -
 (10240*A*a^8*c^7 + 7168*C*a^8*b*c^6 - 48*A*a^3*b^10*c^2 + 832*A*a^4*b^8*c^
3 - 5536*A*a^5*b^6*c^4 + 17280*A*a^6*b^4*c^5 - 24064*A*a^7*b^2*c^6 + 16*C*a
^4*b^9*c^2 - 304*C*a^5*b^7*c^3 + 2112*C*a^6*b^5*c^4 - 6400*C*a^7*b^3*c^5)/(
8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (root(1572864*a
^10*b^2*c^5*z^4 - 983040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a
^7*b^8*c^2*z^4 + 6144*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*
z^4 + 1572864*B*a^8*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b
^6*c^3*z^3 - 61440*B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^
9*c^6*z^3 - 256*B*a^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b
^2*c^5*z^2 + 358400*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 2476
8*A*C*a^3*b^8*c^2*z^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C
^2*a^3*b^9*c*z^2 + 1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 340
8*A^2*a*b^11*c*z^2 + 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 2
4064*C^2*a^5*b^5*c^3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^
5*z^2 - 288768*B^2*a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*
```

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a^3*b^8*c^2*z^2 + 716800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 +
 170496*A^2*a^3*b^7*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2
 - 393216*B^2*a^7*c^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592
*A*B*C*a^4*b^2*c^5*z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z
 + 288*A*B*C*a*b^8*c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*
z + 7344*A^2*B*a*b^7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^
4*z + 576*B*C^2*a^3*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b
^3*c^5*z - 46656*A^2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*
a^3*b^4*c^4*z + 3328*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^
9*c^2*z - 65536*B^3*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^
4*c^4 + 1104*B^2*C^2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*
a^3*b*c^5 - 100*B^2*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2
*b^3*c^4 - 7680*A^2*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^
2*b*c^6 - 6192*A^3*C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3
 + 15360*A*B^2*C*a^3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3
*C*b^5*c^4 + 360*C^4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^
5 - 225*A^2*C^2*b^6*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296
*C^4*a^4*c^5 - 4096*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k
)*x*(1310720*a^11*c^8 + 384*a^5*b^12*c^2 - 8960*a^6*b^10*c^3 + 87040*a^7*b^
8*c^4 - 450560*a^8*b^6*c^5 + 1310720*a^9*b^4*c^6 - 2031616*a^10*b^2*c^7))/(
16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
))) + (5120*A*B*a^6*c^7 + 832*A*B*a^2*b^8*c^3 - 5392*A*B*a^3*b^6*c^4 + 1574
4*A*B*a^4*b^4*c^5 - 18944*A*B*a^5*b^2*c^6 + 16*B*C*a^2*b^9*c^2 - 304*B*C*a^
3*b^7*c^3 + 2064*B*C*a^4*b^5*c^4 - 5888*B*C*a^5*b^3*c^5 - 48*A*B*a*b^10*c^2
 + 5888*B*C*a^6*b*c^6)/(8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2
*c^2)) + (x*(144*A^2*b^13*c^2 + 245760*B^2*a^7*c^8 + 33304*A^2*a^2*b^9*c^4
 - 171768*A^2*a^3*b^7*c^5 + 492320*A^2*a^4*b^5*c^6 - 742016*A^2*a^5*b^3*c^7
 - 128*B^2*a^2*b^10*c^3 + 2912*B^2*a^3*b^8*c^4 - 26560*B^2*a^4*b^6*c^5 + 120
832*B^2*a^5*b^4*c^6 - 273408*B^2*a^6*b^2*c^7 + 16*C^2*a^2*b^11*c^2 - 432*C^
2*a^3*b^9*c^3 + 4616*C^2*a^4*b^7*c^4 - 24032*C^2*a^5*b^5*c^5 + 60800*C^2*a^
6*b^3*c^6 - 276480*A*C*a^7*c^8 - 3408*A^2*a*b^11*c^3 + 458240*A^2*a^6*b*c^8
 - 59904*C^2*a^7*b*c^7 + 2432*A*C*a^2*b^10*c^3 - 24816*A*C*a^3*b^8*c^4 + 12
9952*A*C*a^4*b^6*c^5 - 365440*A*C*a^5*b^4*c^6 + 515584*A*C*a^6*b^2*c^7 - 96
*A*C*a*b^12*c^2))/(16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^
2 - 256*a^7*b^2*c^3))) + (216*C^3*a^5*c^6 + 63*A^3*a^2*b^3*c^6 - 30*C^3*a^4
*b^2*c^5 + 4480*A*B^2*a^4*c^7 + 600*A^2*C*a^4*c^7 - 300*A^3*a^3*b*c^7 - 144
*A*B^2*a*b^6*c^4 - 564*A*C^2*a^4*b*c^6 + 1408*B^2*C*a^4*b*c^6 + 1536*A*B^2*
a^2*b^4*c^5 - 4984*A*B^2*a^3*b^2*c^6 + 105*A*C^2*a^3*b^3*c^5 - 45*A^2*C*a^2
*b^4*c^5 + 102*A^2*C*a^3*b^2*c^6 + 48*B^2*C*a^2*b^5*c^4 - 532*B^2*C*a^3*b^3
*c^5)/(8*(a^4*b^6 - 64*a^7*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (x*(2048
0*B^3*a^5*c^8 + 192*B^3*a^2*b^6*c^5 + 1216*B^3*a^3*b^4*c^6 - 11008*B^3*a^4*
b^2*c^7 + 360*A^2*B*b^9*c^4 - 32*B^3*a*b^8*c^4 - 6072*A^2*B*a*b^7*c^5 + 112
320*A^2*B*a^4*b*c^8 - 2496*B*C^2*a^5*b*c^7 + 38284*A^2*B*a^2*b^5*c^6 - 1071
04*A^2*B*a^3*b^3*c^7 + 40*B*C^2*a^2*b^7*c^4 - 508*B*C^2*a^3*b^5*c^5 + 2016*
B*C^2*a^4*b^3*c^6 - 99840*A*B*C*a^5*c^8 - 240*A*B*C*a*b^8*c^4 + 4448*A*B*C*
a^2*b^6*c^5 - 30176*A*B*C*a^3*b^4*c^6 + 89856*A*B*C*a^4*b^2*c^7))/(16*(a^4*
b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3))) - (6
3*A^3*B*b^3*c^6 - 640*A*B^3*a^2*c^7 + 216*B*C^3*a^3*c^6 + 600*A^2*B*C*a^2*c
^7 - 45*A^2*B*C*b^4*c^5 + 136*A*B^3*a*b^2*c^6 - 20*B^3*C*a*b^3*c^5 + 128*B^
3*C*a^2*b*c^6 - 30*B*C^3*a^2*b^2*c^5 - 300*A^3*B*a*b*c^7 + 105*A*B*C^2*a*b^
3*c^5 - 564*A*B*C^2*a^2*b*c^6 + 102*A^2*B*C*a*b^2*c^6)/(8*(a^4*b^6 - 64*a^7
*c^3 - 12*a^5*b^4*c + 48*a^6*b^2*c^2)) + (x*(10000*A^4*a^2*c^9 + 441*A^4*b^
4*c^7 + 1296*C^4*a^4*c^7 + 216*A^2*B^2*b^5*c^6 + 7200*A^2*C^2*a^3*c^8 + 225
*A^2*C^2*b^6*c^5 + 256*B^4*a^2*b^2*c^7 + 25*C^4*a^2*b^4*c^5 - 360*C^4*a^3*b
^2*c^6 - 630*A^3*C*b^5*c^6 - 4200*A^4*a*b^2*c^8 - 48*B^4*a*b^4*c^6 - 7680*A
*B^2*C*a^3*c^8 - 150*A*C^3*a*b^5*c^5 - 5472*A*C^3*a^3*b*c^7 + 6192*A^3*C*a*
b^3*c^7 - 15200*A^3*C*a^2*b*c^8 - 2160*A^2*B^2*a*b^3*c^7 + 5440*A^2*B^2*a^2
*b*c^8 + 1840*A*C^3*a^2*b^3*c^6 - 2070*A^2*C^2*a*b^4*c^6 + 960*B^2*C^2*a^3*
b*c^7 + 3264*A^2*C^2*a^2*b^2*c^7 - 176*B^2*C^2*a^2*b^3*c^6 - 144*A*B^2*C*a*
```

```
b^4*c^6 + 2240*A*B^2*C*a^2*b^2*c^7))/(16*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^
6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3)))*root(1572864*a^10*b^2*c^5*z^4 - 9
83040*a^9*b^4*c^4*z^4 + 327680*a^8*b^6*c^3*z^4 - 61440*a^7*b^8*c^2*z^4 + 61
44*a^6*b^10*c*z^4 - 1048576*a^11*c^6*z^4 - 256*a^5*b^12*z^4 + 1572864*B*a^8
*b^2*c^5*z^3 - 983040*B*a^7*b^4*c^4*z^3 + 327680*B*a^6*b^6*c^3*z^3 - 61440*
B*a^5*b^8*c^2*z^3 + 6144*B*a^4*b^10*c*z^3 - 1048576*B*a^9*c^6*z^3 - 256*B*a
^3*b^12*z^3 - 2432*A*C*a^2*b^10*c*z^2 - 491520*A*C*a^6*b^2*c^5*z^2 + 358400
*A*C*a^5*b^4*c^4*z^2 - 129024*A*C*a^4*b^6*c^3*z^2 + 24768*A*C*a^3*b^8*c^2*z
^2 + 96*A*C*a*b^12*z^2 + 61440*C^2*a^7*b*c^5*z^2 + 432*C^2*a^3*b^9*c*z^2 +
1536*B^2*a^2*b^10*c*z^2 - 430080*A^2*a^6*b*c^6*z^2 + 3408*A^2*a*b^11*c*z^2
+ 245760*A*C*a^7*c^6*z^2 - 61440*C^2*a^6*b^3*c^4*z^2 + 24064*C^2*a^5*b^5*c^
3*z^2 - 4608*C^2*a^4*b^7*c^2*z^2 + 516096*B^2*a^6*b^2*c^5*z^2 - 288768*B^2*
a^5*b^4*c^4*z^2 + 88576*B^2*a^4*b^6*c^3*z^2 - 15744*B^2*a^3*b^8*c^2*z^2 + 7
16800*A^2*a^5*b^3*c^5*z^2 - 483840*A^2*a^4*b^5*c^4*z^2 + 170496*A^2*a^3*b^7
*c^3*z^2 - 33232*A^2*a^2*b^9*c^2*z^2 - 64*B^2*a*b^12*z^2 - 393216*B^2*a^7*c
^6*z^2 - 16*C^2*a^2*b^11*z^2 - 144*A^2*b^13*z^2 - 110592*A*B*C*a^4*b^2*c^5*
z + 36864*A*B*C*a^3*b^4*c^4*z - 5376*A*B*C*a^2*b^6*c^3*z + 288*A*B*C*a*b^8*
c^2*z + 3072*B*C^2*a^5*b*c^5*z - 138240*A^2*B*a^4*b*c^6*z + 7344*A^2*B*a*b^
7*c^3*z + 122880*A*B*C*a^5*c^6*z - 2304*B*C^2*a^4*b^3*c^4*z + 576*B*C^2*a^3
*b^5*c^3*z - 48*B*C^2*a^2*b^7*c^2*z + 131328*A^2*B*a^3*b^3*c^5*z - 46656*A^
2*B*a^2*b^5*c^4*z + 61440*B^3*a^4*b^2*c^5*z - 21504*B^3*a^3*b^4*c^4*z + 332
8*B^3*a^2*b^6*c^3*z - 192*B^3*a*b^8*c^2*z - 432*A^2*B*b^9*c^2*z - 65536*B^3
*a^5*c^6*z - 5568*A*B^2*C*a^2*b^2*c^5 + 496*A*B^2*C*a*b^4*c^4 + 1104*B^2*C^
2*a^2*b^3*c^4 - 3264*A^2*C^2*a^2*b^2*c^5 - 3072*B^2*C^2*a^3*b*c^5 - 100*B^2
*C^2*a*b^5*c^3 + 2070*A^2*C^2*a*b^4*c^4 - 1840*A*C^3*a^2*b^3*c^4 - 7680*A^2
*B^2*a^2*b*c^6 + 3152*A^2*B^2*a*b^3*c^5 + 15200*A^3*C*a^2*b*c^6 - 6192*A^3*
C*a*b^3*c^5 + 5472*A*C^3*a^3*b*c^5 + 150*A*C^3*a*b^5*c^3 + 15360*A*B^2*C*a^
3*c^6 - 144*B^4*a*b^4*c^4 + 4200*A^4*a*b^2*c^6 + 630*A^3*C*b^5*c^4 + 360*C^
4*a^3*b^2*c^4 - 25*C^4*a^2*b^4*c^3 + 1536*B^4*a^2*b^2*c^5 - 225*A^2*C^2*b^6
*c^3 - 7200*A^2*C^2*a^3*c^6 - 324*A^2*B^2*b^5*c^4 - 1296*C^4*a^4*c^5 - 4096
*B^4*a^3*c^6 - 441*A^4*b^4*c^5 - 10000*A^4*a^2*c^7, z, k), k, 1, 4) - (A/a
- (x^2*(3*A*b^3 - C*a*b^2 + 2*C*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c - b^2))
+ (x^4*(10*A*a*c^2 - 3*A*b^2*c + C*a*b*c))/(2*a^2*(4*a*c - b^2)) - (B*x*(2*
a*c - b^2))/(2*a*(4*a*c - b^2)) + (B*b*c*x^3)/(2*a*(4*a*c - b^2)))/(a*x + b
*x^3 + c*x^5) + (B*log(x))/a^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.36 $\int \frac{A+Bx+Cx^2}{x^3(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=534

$$\frac{(2Ab-aC)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aC)}{a^3}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}-\frac{B\left(3b^2-10ac\right)}{2a^2x\left(b^2-4ac\right)}-\frac{B\sqrt{c}\left(\left(3b^2-10\right.\right.}{2}$$

```
[Out] 1/2*(6*A*a*c-2*A*b^2+C*a*b)/a^2/(-4*a*c+b^2)/x^2-1/2*B*(-10*a*c+3*b^2)/a^2/
(-4*a*c+b^2)/x+1/2*B*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1
/2*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2
+a)-1/2*(2*A*(6*a^2*c^2-6*a*b^2*c+b^4)-a*b*(-6*a*c+b^2)*C)*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-C*a)*ln(x)/a^3+1/4*(2
*A*b-C*a)*ln(c*x^4+b*x^2+a)/a^3-1/4*B*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b
^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c+(-10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2
))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*B*arctan
(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^3-16*a*b*c-(-
10*a*c+3*b^2)*(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.99, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {1662, 1251, 822, 800, 634, 618, 206, 628, 12, 1121, 1281, 1166, 205}

$$-\frac{\left(2A\left(6a^2c^2-6ab^2c+b^4\right)-abC\left(b^2-6ac\right)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-6aAc-abC+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(2Ab-aC)\log\left(a+b\right.}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(2*A*b^2 - 6*a*A*c - a*b*C)/(2*a^2*(b^2 - 4*a*c)*x^2) - (B*(3*b^2 - 10*a*c
))/(2*a^2*(b^2 - 4*a*c)*x) + (B*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)
*x*(a + b*x^2 + c*x^4)) + (A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)/(
2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4)) - (B*Sqrt[c]*(3*b^3 - 16*a*b*c +
 (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4
*a*c]]) + (B*Sqrt[c]*(3*b^3 - 16*a*b*c - (3*b^2 - 10*a*c)*Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(
b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - ((2*A*(b^4 - 6*a*b^2*c +
6*a^2*c^2) - a*b*(b^2 - 6*a*c)*C)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])
/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*C)*Log[x])/a^3 + ((2*A*b - a*C)*
Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 822

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(f*(b*c*d - b^2*e +
2*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*(a
+ b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{A+Bx+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{B}{x^2\left(a+bx^2+cx^4\right)^2}\,dx+\int \frac{A+Cx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\frac{1}{2}\,\text{Subst}\left(\int \frac{A+Cx}{x^2\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)+B\int \frac{1}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A\left(b^2-2ac\right)-abC+c(Ab-2aC)x^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}-\frac{\text{Subst}\left(\int \right.}{}$$

$$=-\frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x}+\frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A\left(b^2-2ac\right)-abC+c(Ab-}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+\right.}$$

$$=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x}+\frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A\left(b^2-\right.}{2a\left(\right.}$$

$$=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x}+\frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A\left(b^2-\right.}{2a\left(\right.}$$

$$=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x}+\frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A\left(b^2-\right.}{2a\left(\right.}$$

$$=-\frac{2Ab^2-6aAc-abC}{2a^2\left(b^2-4ac\right)x^2}-\frac{B\left(3b^2-10ac\right)}{2a^2\left(b^2-4ac\right)x}+\frac{B\left(b^2-2ac+bcx^2\right)}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{A\left(b^2-\right.}{2a\left(\right.}$$

Mathematica [A] time = 2.47, size = 655, normalized size = 1.23

$$-\frac{2a\left(2a^2cC+A\left(-3abc-2ac^2x^2+b^3+b^2cx^2\right)-a\left(b^2C+bcx(3B+Cx)+2Bc^2x^3\right)+b^2Bx\left(b+cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(2A\left(6a^2c^2-6ab^2c-4abc\sqrt{b^2-4ac}+b^3\sqrt{b^2-4ac}+b^4\right)+aC\right.}{\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (4*a*B)/x - (2*a*(2*a^2*c*C + b^2*B*x*(b + c*x^2) + A*(b^3
- 3*a*b*c + b^2*c*x^2 - 2*a*c^2*x^2) - a*(b^2*C + 2*B*c^2*x^3 + b*c*x*(3*B
+ C*x))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*a*B*Sqrt[c]*(-3*b^
3 + 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*a*B*Sqrt[c]*(3*b^3 - 16*a*b*c - 3*b^2*Sq
rt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
 + 4*(-2*A*b + a*C)*Log[x] + ((2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[
b^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(-b^3 + 6*a*b*c - b^2*Sqrt[b^
2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x
^2])/(b^2 - 4*a*c)^(3/2) + ((2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b
^2 - 4*a*c] - 4*a*b*c*Sqrt[b^2 - 4*a*c]) + a*(b^3 - 6*a*b*c - b^2*Sqrt[b^2
```

```
- 4*a*c] + 4*a*c*Sqrt[b^2 - 4*a*c])*C)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2]
)/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 7.56, size = 6938, normalized size = 12.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c^3
 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2
 + 20*(b^2 - 4*a*c)*a*c^3)*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^2 - 6*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 - 6*a^6*b^9*c^2 + 300*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^7*c^3 + 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^4 - 37*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 - 600*a^8*b^5*c^4 + 832*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^8*b^3*c^5 + 1632*a^9*b^3*c^5 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^9*b*c^6 - 1664*a^10*b*c^6 + 6*(b^2 - 4*a*c)*a^6*b^7*c^2 - 74*(b^2
- 4*a*c)*a^7*b^5*c^3 + 304*(b^2 - 4*a*c)*a^8*b^3*c^4 - 416*(b^2 - 4*a*c)*a^
9*b*c^5)*B*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) + (6*a^12*b^12*c^4 -
 128*a^13*b^10*c^5 + 1088*a^14*b^8*c^6 - 4608*a^15*b^6*c^7 + 9728*a^16*b^4*
c^8 - 8192*a^17*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^12*b^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^13*b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^12*b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^14*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^13*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^12*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^15*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^14*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^13*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^15*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^14*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^17*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^3*c^7 + 960*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^15*b^4*c^7 - 1024*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^16*b^2*c^8 - 6*(b^2 - 4*a*c)
*a^12*b^10*c^4 + 104*(b^2 - 4*a*c)*a^13*b^8*c^5 - 672*(b^2 - 4*a*c)*a^14*b^
```

```
6*c^6 + 1920*(b^2 - 4*a*c)*a^15*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^16*b^2*c^8)*
B)*arctan(2*sqrt(1/2)*x/sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3 + sq
rt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3)^2 - 4*(a^7*b^4*c - 8*a^8*b^2*
c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/(a^6*b^4*c^2
 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/((a^9*b^8*c - 16*a^10*b^6*c^2 - 2*a^9*b^7*
c^2 + 96*a^11*b^4*c^3 + 24*a^10*b^5*c^3 + a^9*b^6*c^3 - 256*a^12*b^2*c^4 -
96*a^11*b^3*c^4 - 12*a^10*b^4*c^4 + 256*a^13*c^5 + 128*a^12*b*c^5 + 48*a^11
*b^2*c^5 - 64*a^12*c^6)*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*abs(c))
 + 1/16*((a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)^2*(6*b^4*c^2 - 44*a*b^2*c
^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c
^2 + 20*(b^2 - 4*a*c)*a*c^3)*B - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^6*b^9*c - 49*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c^2 - 6*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c^2 + 6*a^6*b^9*c^2 + 300*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^3 + 74*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^7*b^6*c^3 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^6*b^7*c^3 - 98*a^7*b^7*c^3 - 816*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^9*b^3*c^4 - 304*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^4 - 37
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^4 + 600*a^8*b^5*c^4 + 83
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^5 + 416*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^5 + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^8*b^3*c^5 - 1632*a^9*b^3*c^5 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^9*b*c^6 + 1664*a^10*b*c^6 - 6*(b^2 - 4*a*c)*a^6*b^7*c^2 + 74*(b^
2 - 4*a*c)*a^7*b^5*c^3 - 304*(b^2 - 4*a*c)*a^8*b^3*c^4 + 416*(b^2 - 4*a*c)*
a^9*b*c^5)*B*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) + (6*a^12*b^12*c^4
 - 128*a^13*b^10*c^5 + 1088*a^14*b^8*c^6 - 4608*a^15*b^6*c^7 + 9728*a^16*b^
4*c^8 - 8192*a^17*b^2*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^12*b^12*c^2 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^13*b^10*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^12*b^11*c^3 - 544*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^14*b^8*c^4 - 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^13*b^9*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^12*b^10*c^4 + 2304*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^15*b^6*c^5 + 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^14*b^7*c^5 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^13*b^8*c^5 - 4864*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^16*b^4*c^6 - 1920*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^15*b^5*c^6 - 336*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^14*b^6*c^6 + 4096*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^17*b^2*c^7 + 2048*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^16*b^3*c^7 + 960*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^15*b^4*c^7 - 1024*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^16*b^2*c^8 - 6*(b^2 - 4*a*
c)*a^12*b^10*c^4 + 104*(b^2 - 4*a*c)*a^13*b^8*c^5 - 672*(b^2 - 4*a*c)*a^14*
b^6*c^6 + 1920*(b^2 - 4*a*c)*a^15*b^4*c^7 - 2048*(b^2 - 4*a*c)*a^16*b^2*c^8
)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3 -
sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3)^2 - 4*(a^7*b^4*c - 8*a^8*b^
2*c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/(a^6*b^4*c
^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4)))/((a^9*b^8*c - 16*a^10*b^6*c^2 - 2*a^9*b^
7*c^2 + 96*a^11*b^4*c^3 + 24*a^10*b^5*c^3 + a^9*b^6*c^3 - 256*a^12*b^2*c^4
 - 96*a^11*b^3*c^4 - 12*a^10*b^4*c^4 + 256*a^13*c^5 + 128*a^12*b*c^5 + 48*a^
11*b^2*c^5 - 64*a^12*c^6)*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)*abs(c
)) - 1/4*(C*a - 2*A*b)*log(abs(c*x^4 + b*x^2 + a))/a^3 + (C*a - 2*A*b)*log(
abs(x))/a^3 + 1/16*(2*(b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 30*a^2*b^3*c^3 +
```

```
12*a*b^4*c^3 + b^5*c^3 - 24*a^3*b*c^4 - 12*a^2*b^2*c^4 - 6*a*b^3*c^4 + 6*a^
2*b*c^5 + (b^6*c - 10*a*b^4*c^2 - 2*b^5*c^2 + 30*a^2*b^2*c^3 + 12*a*b^3*c^3
 + b^4*c^3 - 24*a^3*c^4 - 12*a^2*b*c^4 - 6*a*b^2*c^4 + 6*a^2*c^5)*sqrt(b^2
- 4*a*c))*A*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) - (a*b^6*c - 10*a^2
*b^4*c^2 - 2*a*b^5*c^2 + 24*a^3*b^2*c^3 + 12*a^2*b^3*c^3 + a*b^4*c^3 - 6*a^
2*b^2*c^4 + (a*b^5*c - 10*a^2*b^3*c^2 - 2*a*b^4*c^2 + 24*a^3*b*c^3 + 12*a^2
*b^2*c^3 + a*b^3*c^3 - 6*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^6*b^4*c - 8*
a^7*b^2*c^2 + 16*a^8*c^3) - 2*(a^6*b^11*c^2 - 18*a^7*b^9*c^3 - 2*a^6*b^10*c
^3 + 126*a^8*b^7*c^4 + 28*a^7*b^8*c^4 + a^6*b^9*c^4 - 424*a^9*b^5*c^5 - 140
*a^8*b^6*c^5 - 14*a^7*b^7*c^5 + 672*a^10*b^3*c^6 + 288*a^9*b^4*c^6 + 70*a^8
*b^5*c^6 - 384*a^11*b*c^7 - 192*a^10*b^2*c^7 - 144*a^9*b^3*c^7 + 96*a^10*b*
c^8 + (a^6*b^10*c^2 - 14*a^7*b^8*c^3 - 2*a^6*b^9*c^3 + 70*a^8*b^6*c^4 + 20*
a^7*b^7*c^4 + a^6*b^8*c^4 - 144*a^9*b^4*c^5 - 60*a^8*b^5*c^5 - 10*a^7*b^6*c
^5 + 96*a^10*b^2*c^6 + 48*a^9*b^3*c^6 + 30*a^8*b^4*c^6 - 24*a^9*b^2*c^7)*sq
rt(b^2 - 4*a*c))*A + (a^7*b^10*c^2 - 18*a^8*b^8*c^3 - 2*a^7*b^9*c^3 + 120*a
^9*b^6*c^4 + 28*a^8*b^7*c^4 + a^7*b^8*c^4 - 352*a^10*b^4*c^5 - 128*a^9*b^5*
c^5 - 14*a^8*b^6*c^5 + 384*a^11*b^2*c^6 + 192*a^10*b^3*c^6 + 64*a^9*b^4*c^6
 - 96*a^10*b^2*c^7 + (a^7*b^9*c^2 - 14*a^8*b^7*c^3 - 2*a^7*b^8*c^3 + 64*a^9
*b^5*c^4 + 20*a^8*b^6*c^4 + a^7*b^7*c^4 - 96*a^10*b^3*c^5 - 48*a^9*b^4*c^5
- 10*a^8*b^5*c^5 + 24*a^9*b^3*c^6)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^6
*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3 + sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 1
6*a^8*b*c^3)^2 - 4*(a^7*b^4*c - 8*a^8*b^2*c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 -
8*a^7*b^2*c^3 + 16*a^8*c^4)))/(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4))/(
(a^4*b^4 - 8*a^5*b^2*c - 2*a^4*b^3*c + 16*a^6*c^2 + 8*a^5*b*c^2 + a^4*b^2*c
^2 - 4*a^5*c^3)*c^2*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)) + 1/16*(2*
(b^7*c - 10*a*b^5*c^2 - 2*b^6*c^2 + 30*a^2*b^3*c^3 + 12*a*b^4*c^3 + b^5*c^3
 - 24*a^3*b*c^4 - 12*a^2*b^2*c^4 - 6*a*b^3*c^4 + 6*a^2*b*c^5 - (b^6*c - 10*
a*b^4*c^2 - 2*b^5*c^2 + 30*a^2*b^2*c^3 + 12*a*b^3*c^3 + b^4*c^3 - 24*a^3*c^
4 - 12*a^2*b*c^4 - 6*a*b^2*c^4 + 6*a^2*c^5)*sqrt(b^2 - 4*a*c))*A*abs(a^6*b^
4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3) - (a*b^6*c - 10*a^2*b^4*c^2 - 2*a*b^5*c^2
 + 24*a^3*b^2*c^3 + 12*a^2*b^3*c^3 + a*b^4*c^3 - 6*a^2*b^2*c^4 - (a*b^5*c -
 10*a^2*b^3*c^2 - 2*a*b^4*c^2 + 24*a^3*b*c^3 + 12*a^2*b^2*c^3 + a*b^3*c^3 -
 6*a^2*b*c^4)*sqrt(b^2 - 4*a*c))*C*abs(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c
^3) - 2*(a^6*b^11*c^2 - 18*a^7*b^9*c^3 - 2*a^6*b^10*c^3 + 126*a^8*b^7*c^4 +
 28*a^7*b^8*c^4 + a^6*b^9*c^4 - 424*a^9*b^5*c^5 - 140*a^8*b^6*c^5 - 14*a^7*
b^7*c^5 + 672*a^10*b^3*c^6 + 288*a^9*b^4*c^6 + 70*a^8*b^5*c^6 - 384*a^11*b*
c^7 - 192*a^10*b^2*c^7 - 144*a^9*b^3*c^7 + 96*a^10*b*c^8 - (a^6*b^10*c^2 -
14*a^7*b^8*c^3 - 2*a^6*b^9*c^3 + 70*a^8*b^6*c^4 + 20*a^7*b^7*c^4 + a^6*b^8*
c^4 - 144*a^9*b^4*c^5 - 60*a^8*b^5*c^5 - 10*a^7*b^6*c^5 + 96*a^10*b^2*c^6 +
 48*a^9*b^3*c^6 + 30*a^8*b^4*c^6 - 24*a^9*b^2*c^7)*sqrt(b^2 - 4*a*c))*A + (
a^7*b^10*c^2 - 18*a^8*b^8*c^3 - 2*a^7*b^9*c^3 + 120*a^9*b^6*c^4 + 28*a^8*b^
7*c^4 + a^7*b^8*c^4 - 352*a^10*b^4*c^5 - 128*a^9*b^5*c^5 - 14*a^8*b^6*c^5 +
 384*a^11*b^2*c^6 + 192*a^10*b^3*c^6 + 64*a^9*b^4*c^6 - 96*a^10*b^2*c^7 - (
a^7*b^9*c^2 - 14*a^8*b^7*c^3 - 2*a^7*b^8*c^3 + 64*a^9*b^5*c^4 + 20*a^8*b^6*
c^4 + a^7*b^7*c^4 - 96*a^10*b^3*c^5 - 48*a^9*b^4*c^5 - 10*a^8*b^5*c^5 + 24*
a^9*b^3*c^6)*sqrt(b^2 - 4*a*c))*C)*log(x^2 + 1/2*(a^6*b^5*c - 8*a^7*b^3*c^2
 + 16*a^8*b*c^3 - sqrt((a^6*b^5*c - 8*a^7*b^3*c^2 + 16*a^8*b*c^3)^2 - 4*(a^
7*b^4*c - 8*a^8*b^2*c^2 + 16*a^9*c^3)*(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8
*c^4)))/(a^6*b^4*c^2 - 8*a^7*b^2*c^3 + 16*a^8*c^4))/((a^4*b^4 - 8*a^5*b^2*c
 - 2*a^4*b^3*c + 16*a^6*c^2 + 8*a^5*b*c^2 + a^4*b^2*c^2 - 4*a^5*c^3)*c^2*ab
s(a^6*b^4*c - 8*a^7*b^2*c^2 + 16*a^8*c^3)) - 1/2*((3*B*a*b^2*c - 10*B*a^2*c
^2)*x^5 + A*a^2*b^2 - 4*A*a^3*c - (C*a^2*b*c - 2*A*a*b^2*c + 6*A*a^2*c^2)*x
^4 + (3*B*a*b^3 - 11*B*a^2*b*c)*x^3 - (C*a^2*b^2 - 2*A*a*b^3 - 2*C*a^3*c +
7*A*a^2*b*c)*x^2 + 2*(B*a^2*b^2 - 4*B*a^3*c)*x)/((c*x^4 + b*x^2 + a)*(b^2 -
 4*a*c)*a^3*x^2)
```

maple [B] time = 0.10, size = 2512, normalized size = 4.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a^2*b^3-1/2*A/a^2/x^2-1/(c*x^4+b*x^2+a)/(
4*a*c-b^2)*A/a*c^2*x^2-3/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*A/a*b*c+1/2/(c*x^4+b
*x^2+a)/(4*a*c-b^2)*B/a^2*b^2*c*x^3-3/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a*b*c
*x+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*C-2*A/a^3*b*ln(x)+1/2/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*A/a^2*b^2*c*x^2+6/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-
4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b-12/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2
)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^2-6/a*c/(4*a*c-b
^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*b+
12/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*
a*c+b^2)^(1/2)*b^2+2/a^3/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^
2)^(1/2))*A*b^5+2/a^3/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)
^(1/2))*A*b^5-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(
1/2))*C*b^4-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2
))*C*b^4+1/a^2*ln(x)*C-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a
*c+b^2)^(1/2))*C-16*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^
2)^(1/2))*C-B/a^2/x-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
B*(-4*a*c+b^2)^(1/2)*b+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*B*(-4*a*c+b^2)^(1/2)*b^3+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b^3-16/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*B*(-4*a*c+b^2)^(1/2)*b-22/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*B*b^2-3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*B*b^4+3/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^
4+22/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*B*b^2-1/(c*x^4+b*x
^2+a)/(4*a*c-b^2)*B/a*c^2*x^3+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*B/a^2*b^3*x-1
/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*C*b-40*c^3/(4*a*c-b^2)/(16*a*c-4*b^2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*B+40*c^3/(4*a*c-b^2)/(16*a*c-4*b^2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*B+12/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))
*A*(-4*a*c+b^2)^(1/2)+2/a^3/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*
c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^4+32/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*
ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*b-16/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*l
n(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b^3+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2
*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*b^2-16/a^2*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-
2*c*x^2-b+(-4*a*c+b^2)^(1/2))*A*b^3+8/a*c/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*
c*x^2-b+(-4*a*c+b^2)^(1/2))*C*b^2-12/a*c^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*
c*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)-2/a^3/(4*a*c-b^2)/(16*a*c-
4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*(-4*a*c+b^2)^(1/2)*b^4+32/a*c^2/(
4*a*c-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*A*b+1/a^2/(4*a*c
-b^2)/(16*a*c-4*b^2)*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))*C*(-4*a*c+b^2)^(1/2)*
b^3-1/a^2/(4*a*c-b^2)/(16*a*c-4*b^2)*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))*C*(-
4*a*c+b^2)^(1/2)*b^3-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*C*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^2+B*x+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((3*B*b^2*c - 10*B*a*c^2)*x^5 - (6*A*a*c^2 + (C*a*b - 2*A*b^2)*c)*x^4
+ A*a*b^2 - 4*A*a^2*c + (3*B*b^3 - 11*B*a*b*c)*x^3 - (C*a*b^2 - 2*A*b^3 - (
2*C*a^2 - 7*A*a*b)*c)*x^2 + 2*(B*a*b^2 - 4*B*a^2*c)*x)/((a^2*b^2*c - 4*a^3*
c^2)*x^6 + (a^2*b^3 - 4*a^3*b*c)*x^4 + (a^3*b^2 - 4*a^4*c)*x^2) - 1/2*integ
rate((3*B*a*b^3 - 13*B*a^2*b*c - 2*(4*(C*a^2 - 2*A*a*b)*c^2 - (C*a*b^2 - 2*
A*b^3)*c)*x^3 + (3*B*a*b^2*c - 10*B*a^2*c^2)*x^2 + 2*(C*a*b^3 - 2*A*b^4 - 6
*A*a^2*c^2 - 5*(C*a^2*b - 2*A*a*b^2)*c)*x)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2
 - 4*a^4*c) + (C*a - 2*A*b)*log(x)/a^3
```

mupad [B] time = 2.77, size = 10595, normalized size = 19.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x + C*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 327680
*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*a^
12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*b^
4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*A*
a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 + 12
2880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3 -
12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 512*
A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 1794
048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^7*
c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b^1
0*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a*b
^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 88576
*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5*z
^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*a^
3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 -
761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b^1
0*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12*z
^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z -
 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3*z
 + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5
*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^
4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 8294
4*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3
*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^
2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*
a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A
^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2
*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3
*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C*
a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2*
C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C
^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3
*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a*
b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 +
1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*
a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5
*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4*
a^2*c^8, z, k)*(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 3
27680*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 10485
76*a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a
^8*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 31457
28*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3
 + 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z
^3 - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 +
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512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 -
1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4
*b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^
3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^
2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 +
88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*
c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B
^2*a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z
^2 - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^
2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b
^12*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^
6*z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*
c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^
3*c^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C
^2*a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z +
82944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^
7*c^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B
^2*a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328
*C^3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104
448*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 76
8*A^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 6553
6*C^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B
^2*C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680
*B^2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*
A^2*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 3225
6*A^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C
^3*a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c
^7 + 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200
*B^4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^
2*b^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736
*A^4*a^2*c^8, z, k)*(root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^
4 + 327680*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 -
1048576*a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 98304
0*C*a^8*b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 -
3145728*A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^
3*z^3 + 122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*
c^6*z^3 - 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*
z^3 + 512*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*
z^2 - 1794048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*
C*a^4*b^7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C
^2*a^3*b^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 61
44*A^2*a*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z
^2 + 88576*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6
*b^3*c^5*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33
232*B^2*a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*
c^4*z^2 - 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A
^2*a^2*b^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*
a^2*b^12*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5
*b*c^6*z - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a
*b^8*c^3*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a
^4*b^3*c^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 23347
2*A*C^2*a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^
6*z + 82944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a
^2*b^7*c^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 931
2*A*B^2*a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z +
3328*C^3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z
+ 104448*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z
- 768*A^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z -
65536*C^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 164
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8*A*B^2*C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 -
 7680*B^2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 +
4608*A^2*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 +
 32256*A^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 57
6*A*C^3*a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*
b^2*c^7 + 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 +
 4200*B^4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A
^2*B^2*b^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 -
20736*A^4*a^2*c^8, z, k)*((x*(983040*C*a^11*c^8 - 1867776*A*a^10*b*c^8 - 38
4*A*a^4*b^13*c^2 + 9472*A*a^5*b^11*c^3 - 97408*A*a^6*b^9*c^4 + 534528*A*a^7
*b^7*c^5 - 1650688*A*a^8*b^5*c^6 + 2719744*A*a^9*b^3*c^7 + 192*C*a^5*b^12*c
^2 - 4736*C*a^6*b^10*c^3 + 48896*C*a^7*b^8*c^4 - 270336*C*a^8*b^6*c^5 + 843
776*C*a^9*b^4*c^6 - 1409024*C*a^10*b^2*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 -
16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3)) - (10240*B*a^10*c^7 - 48*
B*a^5*b^10*c^2 + 832*B*a^6*b^8*c^3 - 5536*B*a^7*b^6*c^4 + 17280*B*a^8*b^4*c
^5 - 24064*B*a^9*b^2*c^6)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*
b^2*c^2)) + (root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 3276
80*a^9*b^6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*
a^12*c^6*z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*
b^4*c^4*z^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*
A*a^8*b^3*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 +
122880*A*a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3
- 12288*A*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 51
2*A*a^3*b^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 17
94048*A*C*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^
7*c^3*z^2 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b
^10*c*z^2 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a
*b^12*c*z^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 885
76*C^2*a^5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5
*z^2 - 483840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*
a^3*b^9*c^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2
- 761856*A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b
^10*c^2*z^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12
*z^2 - 294912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z
 - 432*B^2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3
*z + 768*A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c
^5*z - 46656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*
a^4*b^3*c^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 82
944*A*C^2*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c
^3*z - 152064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*
a^2*b^6*c^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^
3*a^3*b^6*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448
*A^3*a^2*b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A
^2*C*b^10*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C
^3*a^6*c^6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*
C*a*b^4*c^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^
2*C^2*a^3*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2
*C^2*a*b^4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A
^3*C*a^2*b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*
a*b^5*c^4 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7
+ 1536*A^3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^
4*a^2*b^2*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b
^5*c^5 - 4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^
4*a^2*c^8, z, k)*x*(1310720*a^13*c^8 + 384*a^7*b^12*c^2 - 8960*a^8*b^10*c^3
 + 87040*a^9*b^8*c^4 - 450560*a^10*b^6*c^5 + 1310720*a^11*b^4*c^6 - 2031616
*a^12*b^2*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2
 - 256*a^9*b^2*c^3))) + (5120*B*C*a^8*c^7 + 96*A*B*a^2*b^11*c^2 - 1664*A*B*
a^3*b^9*c^3 + 11072*A*B*a^4*b^7*c^4 - 34752*A*B*a^5*b^5*c^5 + 49792*A*B*a^6
*b^3*c^6 - 48*B*C*a^3*b^10*c^2 + 832*B*C*a^4*b^8*c^3 - 5392*B*C*a^5*b^6*c^4
```

```
 + 15744*B*C*a^6*b^4*c^5 - 18944*B*C*a^7*b^2*c^6 - 24064*A*B*a^7*b*c^7)/(8*
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)) + (x*(331776*A^2*a^
8*c^9 + 245760*C^2*a^9*c^8 - 512*A^2*a^2*b^12*c^3 + 10112*A^2*a^3*b^10*c^4
- 78592*A^2*a^4*b^8*c^5 + 294784*A^2*a^5*b^6*c^6 - 498432*A^2*a^6*b^4*c^7 +
 159744*A^2*a^7*b^2*c^8 + 144*B^2*a^2*b^13*c^2 - 3408*B^2*a^3*b^11*c^3 + 33
304*B^2*a^4*b^9*c^4 - 171768*B^2*a^5*b^7*c^5 + 492320*B^2*a^6*b^5*c^6 - 742
016*B^2*a^7*b^3*c^7 - 128*C^2*a^4*b^10*c^3 + 2912*C^2*a^5*b^8*c^4 - 26560*C
^2*a^6*b^6*c^5 + 120832*C^2*a^7*b^4*c^6 - 273408*C^2*a^8*b^2*c^7 + 458240*B
^2*a^8*b*c^8 + 512*A*C*a^3*b^11*c^3 - 10880*A*C*a^4*b^9*c^4 + 92416*A*C*a^5
*b^7*c^5 - 391936*A*C*a^6*b^5*c^6 + 829440*A*C*a^7*b^3*c^7 - 700416*A*C*a^8
*b*c^8))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*
a^9*b^2*c^3))) + (63*B^3*a^4*b^3*c^6 + 1440*A^2*B*a^5*c^8 + 4480*B*C^2*a^6*
c^7 - 300*B^3*a^5*b*c^7 - 384*A^2*B*a^2*b^6*c^5 + 3440*A^2*B*a^3*b^4*c^6 -
8000*A^2*B*a^4*b^2*c^7 - 144*B*C^2*a^3*b^6*c^4 + 1536*B*C^2*a^4*b^4*c^5 - 4
984*B*C^2*a^5*b^2*c^6 - 6112*A*B*C*a^5*b*c^7 + 288*A*B*C*a^2*b^7*c^4 - 2880
*A*B*C*a^3*b^5*c^5 + 8464*A*B*C*a^4*b^3*c^6)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*
a^7*b^4*c + 48*a^8*b^2*c^2)) + (x*(256*A^3*b^11*c^4 + 20480*C^3*a^7*c^8 + 3
4048*A^3*a^2*b^7*c^6 - 130816*A^3*a^3*b^5*c^7 + 264320*A^3*a^4*b^3*c^8 - 32
*C^3*a^3*b^8*c^4 + 192*C^3*a^4*b^6*c^5 + 1216*C^3*a^5*b^4*c^6 - 11008*C^3*a
^6*b^2*c^7 - 163200*A*B^2*a^6*c^9 + 119808*A^2*C*a^6*c^9 - 4608*A^3*a*b^9*c
^5 - 225792*A^3*a^5*b*c^9 + 144*A*B^2*a*b^10*c^4 - 46080*A*C^2*a^6*b*c^8 -
384*A^2*C*a*b^10*c^4 + 112320*B^2*C*a^6*b*c^8 - 3120*A*B^2*a^2*b^8*c^5 + 26
272*A*B^2*a^3*b^6*c^6 - 107416*A*B^2*a^4*b^4*c^7 + 212928*A*B^2*a^5*b^2*c^8
 + 192*A*C^2*a^2*b^9*c^4 - 1920*A*C^2*a^3*b^7*c^5 + 3360*A*C^2*a^4*b^5*c^6
+ 16512*A*C^2*a^5*b^3*c^7 + 5376*A^2*C*a^2*b^8*c^5 - 28608*A^2*C*a^3*b^6*c^
6 + 76416*A^2*C*a^4*b^4*c^7 - 123648*A^2*C*a^5*b^2*c^8 + 360*B^2*C*a^2*b^9*
c^4 - 6072*B^2*C*a^3*b^7*c^5 + 38284*B^2*C*a^4*b^5*c^6 - 107104*B^2*C*a^5*b
^3*c^7))/(16*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*
a^9*b^2*c^3))) + (224*A^3*B*b^5*c^6 + 640*B*C^3*a^4*c^7 - 1440*A^2*B*C*a^3*
c^8 + 126*A*B^3*a*b^4*c^6 - 1664*A^3*B*a*b^3*c^7 + 2880*A^3*B*a^2*b*c^8 + 3
00*B^3*C*a^3*b*c^7 - 600*A*B^3*a^2*b^2*c^7 - 136*B*C^3*a^3*b^2*c^6 - 63*B^3
*C*a^2*b^3*c^6 - 1824*A*B*C^2*a^3*b*c^7 - 336*A^2*B*C*a*b^4*c^6 + 384*A*B*C
^2*a^2*b^3*c^6 + 1920*A^2*B*C*a^2*b^2*c^7)/(8*(a^6*b^6 - 64*a^9*c^3 - 12*a^
7*b^4*c + 48*a^8*b^2*c^2)) + (x*(20736*A^4*a^3*c^10 - 512*A^4*b^6*c^7 + 100
00*B^4*a^4*c^9 + 9216*A^2*C^2*a^4*c^9 - 18432*A^4*a^2*b^2*c^9 + 441*B^4*a^2
*b^4*c^7 - 4200*B^4*a^3*b^2*c^8 - 48*C^4*a^3*b^4*c^6 + 256*C^4*a^4*b^2*c^7
+ 384*A^3*C*b^7*c^6 + 5376*A^4*a*b^4*c^8 - 28800*A*B^2*C*a^4*c^9 + 3072*A*C
^3*a^4*b*c^8 - 3584*A^3*C*a*b^5*c^7 - 9216*A^3*C*a^3*b*c^9 - 288*A^2*B^2*a*
b^5*c^7 - 2880*A^2*B^2*a^3*b*c^9 + 288*A*C^3*a^2*b^5*c^6 - 2048*A*C^3*a^3*b
^3*c^7 - 576*A^2*C^2*a*b^6*c^6 + 10368*A^3*C*a^2*b^3*c^8 + 5440*B^2*C^2*a^4
*b*c^8 + 1936*A^2*B^2*a^2*b^3*c^8 + 4992*A^2*C^2*a^2*b^4*c^7 - 12672*A^2*C^
2*a^3*b^2*c^8 + 216*B^2*C^2*a^2*b^5*c^6 - 2160*B^2*C^2*a^3*b^3*c^7 + 216*A*
B^2*C*a*b^6*c^6 - 3096*A*B^2*C*a^2*b^4*c^7 + 15872*A*B^2*C*a^3*b^2*c^8))/(1
6*(a^6*b^8 + 256*a^10*c^4 - 16*a^7*b^6*c + 96*a^8*b^4*c^2 - 256*a^9*b^2*c^3
)))*root(1572864*a^11*b^2*c^5*z^4 - 983040*a^10*b^4*c^4*z^4 + 327680*a^9*b^
6*c^3*z^4 - 61440*a^8*b^8*c^2*z^4 + 6144*a^7*b^10*c*z^4 - 1048576*a^12*c^6*
z^4 - 256*a^6*b^12*z^4 + 1572864*C*a^9*b^2*c^5*z^3 - 983040*C*a^8*b^4*c^4*z
^3 + 327680*C*a^7*b^6*c^3*z^3 - 61440*C*a^6*b^8*c^2*z^3 - 3145728*A*a^8*b^3
*c^5*z^3 + 1966080*A*a^7*b^5*c^4*z^3 - 655360*A*a^6*b^7*c^3*z^3 + 122880*A*
a^5*b^9*c^2*z^3 + 6144*C*a^5*b^10*c*z^3 + 2097152*A*a^9*b*c^6*z^3 - 12288*A
*a^4*b^11*c*z^3 - 1048576*C*a^10*c^6*z^3 - 256*C*a^4*b^12*z^3 + 512*A*a^3*b
^13*z^3 + 1277952*A*C*a^7*b*c^6*z^2 - 6144*A*C*a^2*b^11*c*z^2 - 1794048*A*C
*a^6*b^3*c^5*z^2 + 1062912*A*C*a^5*b^5*c^4*z^2 - 340480*A*C*a^4*b^7*c^3*z^2
 + 62208*A*C*a^3*b^9*c^2*z^2 + 256*A*C*a*b^13*z^2 + 1536*C^2*a^3*b^10*c*z^2
 - 430080*B^2*a^7*b*c^6*z^2 + 3408*B^2*a^2*b^11*c*z^2 + 6144*A^2*a*b^12*c*z
^2 + 516096*C^2*a^7*b^2*c^5*z^2 - 288768*C^2*a^6*b^4*c^4*z^2 + 88576*C^2*a^
5*b^6*c^3*z^2 - 15744*C^2*a^4*b^8*c^2*z^2 + 716800*B^2*a^6*b^3*c^5*z^2 - 48
3840*B^2*a^5*b^5*c^4*z^2 + 170496*B^2*a^4*b^7*c^3*z^2 - 33232*B^2*a^3*b^9*c
^2*z^2 + 1468416*A^2*a^5*b^4*c^5*z^2 - 966144*A^2*a^4*b^6*c^4*z^2 - 761856*
```

```
A^2*a^6*b^2*c^6*z^2 + 326656*A^2*a^3*b^8*c^3*z^2 - 61440*A^2*a^2*b^10*c^2*z
^2 - 144*B^2*a*b^13*z^2 - 393216*C^2*a^8*c^6*z^2 - 64*C^2*a^2*b^12*z^2 - 29
4912*A^2*a^7*c^7*z^2 - 256*A^2*b^14*z^2 - 138240*B^2*C*a^5*b*c^6*z - 432*B^
2*C*a*b^9*c^2*z + 245760*A*C^2*a^5*b*c^6*z + 12288*A^2*C*a*b^8*c^3*z + 768*
A*C^2*a*b^9*c^2*z + 576*A*B^2*a*b^8*c^3*z + 131328*B^2*C*a^4*b^3*c^5*z - 46
656*B^2*C*a^3*b^5*c^4*z + 7344*B^2*C*a^2*b^7*c^3*z - 233472*A*C^2*a^4*b^3*c
^5*z + 168960*A^2*C*a^3*b^4*c^5*z - 86016*A^2*C*a^4*b^2*c^6*z + 82944*A*C^2
*a^3*b^5*c^4*z - 71424*A^2*C*a^2*b^6*c^4*z - 13056*A*C^2*a^2*b^7*c^3*z - 15
2064*A*B^2*a^4*b^2*c^6*z + 56448*A*B^2*a^3*b^4*c^5*z - 9312*A*B^2*a^2*b^6*c
^4*z + 61440*C^3*a^5*b^2*c^5*z - 21504*C^3*a^4*b^4*c^4*z + 3328*C^3*a^3*b^6
*c^3*z - 192*C^3*a^2*b^8*c^2*z - 286720*A^3*a^3*b^3*c^6*z + 104448*A^3*a^2*
b^5*c^5*z + 294912*A^3*a^4*b*c^7*z - 16896*A^3*a*b^7*c^4*z - 768*A^2*C*b^10
*c^2*z - 147456*A^2*C*a^5*c^7*z + 153600*A*B^2*a^5*c^7*z - 65536*C^3*a^6*c^
6*z + 1024*A^3*b^9*c^3*z - 15936*A*B^2*C*a^2*b^2*c^6 + 1648*A*B^2*C*a*b^4*c
^5 + 3152*B^2*C^2*a^2*b^3*c^5 - 4992*A^2*C^2*a^2*b^2*c^6 - 7680*B^2*C^2*a^3
*b*c^6 - 324*B^2*C^2*a*b^5*c^4 - 5760*A*C^3*a^2*b^3*c^5 + 4608*A^2*C^2*a*b^
4*c^5 - 16320*A^2*B^2*a^2*b*c^7 + 7152*A^2*B^2*a*b^3*c^6 + 32256*A^3*C*a^2*
b*c^7 + 14336*A*C^3*a^3*b*c^6 - 14080*A^3*C*a*b^3*c^6 + 576*A*C^3*a*b^5*c^4
 + 38400*A*B^2*C*a^3*c^7 - 441*B^4*a*b^4*c^5 + 9216*A^4*a*b^2*c^7 + 1536*A^
3*C*b^5*c^5 + 1536*C^4*a^3*b^2*c^5 - 144*C^4*a^2*b^4*c^4 + 4200*B^4*a^2*b^2
*c^6 - 576*A^2*C^2*b^6*c^4 - 18432*A^2*C^2*a^3*c^7 - 784*A^2*B^2*b^5*c^5 -
4096*C^4*a^4*c^6 - 10000*B^4*a^3*c^7 - 1024*A^4*b^4*c^6 - 20736*A^4*a^2*c^8
, z, k), k, 1, 4) - (A/(2*a) + (B*x)/a - (x^2*(2*A*b^3 - C*a*b^2 + 2*C*a^2*
c - 7*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (B*x^5*(10*a*c^2 - 3*b^2*c))/(2*a^2
*(4*a*c - b^2)) + (c*x^4*(6*A*a*c - 2*A*b^2 + C*a*b))/(2*a^2*(4*a*c - b^2))
 + (B*b*x^3*(11*a*c - 3*b^2))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6
) - (log(x)*(2*A*b - C*a))/a^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**2+B*x+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.37 $\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=399

$$\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)}+\frac{3c(dx)^{m+11}\left(C\left(ac+b^2\right)+Abc\right)}{d^{11}(m+11)}+\frac{(dx)^{m+9}\left(3A\right.}{}$$

```
[Out] a^3*A*(d*x)^(1+m)/d/(1+m)+a^3*B*(d*x)^(2+m)/d^2/(2+m)+a^2*(3*A*b+C*a)*(d*x)
^(3+m)/d^3/(3+m)+3*a^2*b*B*(d*x)^(4+m)/d^4/(4+m)+3*a*(A*(a*c+b^2)+a*b*C)*(d
*x)^(5+m)/d^5/(5+m)+3*a*B*(a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(A*(6*a*b*c+b^3)+
3*a*(a*c+b^2)*C)*(d*x)^(7+m)/d^7/(7+m)+b*B*(6*a*c+b^2)*(d*x)^(8+m)/d^8/(8+m
)+(3*A*c*(a*c+b^2)+b*(6*a*c+b^2)*C)*(d*x)^(9+m)/d^9/(9+m)+3*B*c*(a*c+b^2)*(
d*x)^(10+m)/d^10/(10+m)+3*c*(A*b*c+(a*c+b^2)*C)*(d*x)^(11+m)/d^11/(11+m)+3*
b*B*c^2*(d*x)^(12+m)/d^12/(12+m)+c^2*(A*c+3*C*b)*(d*x)^(13+m)/d^13/(13+m)+B
*c^3*(d*x)^(14+m)/d^14/(14+m)+c^3*C*(d*x)^(15+m)/d^15/(15+m)
```

Rubi [A] time = 0.42, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1628}

$$\frac{a^2(dx)^{m+3}(aC+3Ab)}{d^3(m+3)}+\frac{a^3A(dx)^{m+1}}{d(m+1)}+\frac{3a^2bB(dx)^{m+4}}{d^4(m+4)}+\frac{a^3B(dx)^{m+2}}{d^2(m+2)}+\frac{3a(dx)^{m+5}\left(A\left(ac+b^2\right)+abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(A\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^3*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a^2*(3*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (3*a^2*b*B*(d*x)^(4 + m))/
(d^4*(4 + m)) + (3*a*(A*(b^2 + a*c) + a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) +
 (3*a*B*(b^2 + a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((A*(b^3 + 6*a*b*c) + 3*
a*(b^2 + a*c)*C)*(d*x)^(7 + m))/(d^7*(7 + m)) + (b*B*(b^2 + 6*a*c)*(d*x)^(8
 + m))/(d^8*(8 + m)) + ((3*A*c*(b^2 + a*c) + b*(b^2 + 6*a*c)*C)*(d*x)^(9 +
m))/(d^9*(9 + m)) + (3*B*c*(b^2 + a*c)*(d*x)^(10 + m))/(d^10*(10 + m)) + (3
*c*(A*b*c + (b^2 + a*c)*C)*(d*x)^(11 + m))/(d^11*(11 + m)) + (3*b*B*c^2*(d*
x)^(12 + m))/(d^12*(12 + m)) + (c^2*(A*c + 3*b*C)*(d*x)^(13 + m))/(d^13*(13
 + m)) + (B*c^3*(d*x)^(14 + m))/(d^14*(14 + m)) + (c^3*C*(d*x)^(15 + m))/(d
^15*(15 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^3\,dx = \int \left(a^3A(dx)^m+\frac{a^3B(dx)^{1+m}}{d}+\frac{a^2(3Ab+aC)(dx)^{2+m}}{d^2}+\frac{3a^2bB(d}{d^3}\right.$$

$$=\frac{a^3A(dx)^{1+m}}{d(1+m)}+\frac{a^3B(dx)^{2+m}}{d^2(2+m)}+\frac{a^2(3Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{3a^2bB(dx)}{d^4(4+}$$

Mathematica [A] time = 0.92, size = 296, normalized size = 0.74

$$x(dx)^m\left(\frac{a^3A}{m+1}+\frac{a^3Bx}{m+2}+\frac{a^2x^2(aC+3Ab)}{m+3}+\frac{3a^2bBx^3}{m+4}+\frac{3cx^{10}\left(C\left(ac+b^2\right)+Abc\right)}{m+11}+\frac{x^8\left(3Ac\left(ac+b^2\right)+bC\left(6\right.\right.}{m+9}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(d*x)^m*((a^3*A)/(1 + m) + (a^3*B*x)/(2 + m) + (a^2*(3*A*b + a*C)*x^2)/(3
 + m) + (3*a^2*b*B*x^3)/(4 + m) + (3*a*(A*(b^2 + a*c) + a*b*C)*x^4)/(5 + m)
 + (3*a*B*(b^2 + a*c)*x^5)/(6 + m) + ((A*(b^3 + 6*a*b*c) + 3*a*(b^2 + a*c)*
C)*x^6)/(7 + m) + (b*B*(b^2 + 6*a*c)*x^7)/(8 + m) + ((3*A*c*(b^2 + a*c) + b
*(b^2 + 6*a*c)*C)*x^8)/(9 + m) + (3*B*c*(b^2 + a*c)*x^9)/(10 + m) + (3*c*(A
*b*c + (b^2 + a*c)*C)*x^10)/(11 + m) + (3*b*B*c^2*x^11)/(12 + m) + (c^2*(A*
c + 3*b*C)*x^12)/(13 + m) + (B*c^3*x^13)/(14 + m) + (c^3*C*x^14)/(15 + m))
```

fricas [B] time = 1.91, size = 3898, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((C*c^3*m^14 + 105*C*c^3*m^13 + 5005*C*c^3*m^12 + 143325*C*c^3*m^11 + 27497
47*C*c^3*m^10 + 37312275*C*c^3*m^9 + 368411615*C*c^3*m^8 + 2681453775*C*c^3
*m^7 + 14409322928*C*c^3*m^6 + 56663366760*C*c^3*m^5 + 159721605680*C*c^3*m
^4 + 310989260400*C*c^3*m^3 + 392156797824*C*c^3*m^2 + 283465647360*C*c^3*m
 + 87178291200*C*c^3)*x^15 + (B*c^3*m^14 + 106*B*c^3*m^13 + 5096*B*c^3*m^12
 + 147056*B*c^3*m^11 + 2840838*B*c^3*m^10 + 38786748*B*c^3*m^9 + 385081268*
B*c^3*m^8 + 2816490248*B*c^3*m^7 + 15200266081*B*c^3*m^6 + 59999485546*B*c^
3*m^5 + 169679309436*B*c^3*m^4 + 331303013496*B*c^3*m^3 + 418753514880*B*c^
3*m^2 + 303268406400*B*c^3*m + 93405312000*B*c^3)*x^14 + ((3*C*b*c^2 + A*c^
3)*m^14 + 107*(3*C*b*c^2 + A*c^3)*m^13 + 5189*(3*C*b*c^2 + A*c^3)*m^12 + 15
0943*(3*C*b*c^2 + A*c^3)*m^11 + 2937363*(3*C*b*c^2 + A*c^3)*m^10 + 40372761
*(3*C*b*c^2 + A*c^3)*m^9 + 403249847*(3*C*b*c^2 + A*c^3)*m^8 + 2965379989*(
3*C*b*c^2 + A*c^3)*m^7 + 16081189696*(3*C*b*c^2 + A*c^3)*m^6 + 63747744632*
(3*C*b*c^2 + A*c^3)*m^5 + 180951426864*(3*C*b*c^2 + A*c^3)*m^4 + 3017710080
00*C*b*c^2 + 100590336000*A*c^3 + 354444796368*(3*C*b*c^2 + A*c^3)*m^3 + 44
9213351040*(3*C*b*c^2 + A*c^3)*m^2 + 326044051200*(3*C*b*c^2 + A*c^3)*m)*x^
13 + 3*(B*b*c^2*m^14 + 108*B*b*c^2*m^13 + 5284*B*b*c^2*m^12 + 154992*B*b*c^
2*m^11 + 3039718*B*b*c^2*m^10 + 42081864*B*b*c^2*m^9 + 423113372*B*b*c^2*m^
8 + 3130267536*B*b*c^2*m^7 + 17067919121*B*b*c^2*m^6 + 67988181228*B*b*c^2*
m^5 + 193813932344*B*b*c^2*m^4 + 381046157472*B*b*c^2*m^3 + 484441814160*B*
b*c^2*m^2 + 352515844800*B*b*c^2*m + 108972864000*B*b*c^2)*x^12 + 3*((C*b^2
*c + (C*a + A*b)*c^2)*m^14 + 109*(C*b^2*c + (C*a + A*b)*c^2)*m^13 + 5381*(C
*b^2*c + (C*a + A*b)*c^2)*m^12 + 159209*(C*b^2*c + (C*a + A*b)*c^2)*m^11 +
3148323*(C*b^2*c + (C*a + A*b)*c^2)*m^10 + 43926927*(C*b^2*c + (C*a + A*b)*
c^2)*m^9 + 444899543*(C*b^2*c + (C*a + A*b)*c^2)*m^8 + 3313733027*(C*b^2*c
+ (C*a + A*b)*c^2)*m^7 + 18180066256*(C*b^2*c + (C*a + A*b)*c^2)*m^6 + 7282
2481864*(C*b^2*c + (C*a + A*b)*c^2)*m^5 + 208624806576*(C*b^2*c + (C*a + A*
b)*c^2)*m^4 + 118879488000*C*b^2*c + 411940473264*(C*b^2*c + (C*a + A*b)*c^
2)*m^3 + 118879488000*(C*a + A*b)*c^2 + 525650497920*(C*b^2*c + (C*a + A*b)
*c^2)*m^2 + 383662137600*(C*b^2*c + (C*a + A*b)*c^2)*m)*x^11 + 3*((B*b^2*c
+ B*a*c^2)*m^14 + 110*(B*b^2*c + B*a*c^2)*m^13 + 5480*(B*b^2*c + B*a*c^2)*m
^12 + 163600*(B*b^2*c + B*a*c^2)*m^11 + 3263622*(B*b^2*c + B*a*c^2)*m^10 +
45922260*(B*b^2*c + B*a*c^2)*m^9 + 468873140*(B*b^2*c + B*a*c^2)*m^8 + 3518
896600*(B*b^2*c + B*a*c^2)*m^7 + 19442163553*(B*b^2*c + B*a*c^2)*m^6 + 7838
1575150*(B*b^2*c + B*a*c^2)*m^5 + 225856355580*(B*b^2*c + B*a*c^2)*m^4 + 13
0767436800*B*b^2*c + 130767436800*B*a*c^2 + 448249789800*(B*b^2*c + B*a*c^2
)*m^3 + 574497805824*(B*b^2*c + B*a*c^2)*m^2 + 420839556480*(B*b^2*c + B*a*
c^2)*m)*x^10 + ((C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^14 + 111*(C*b
^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^13 + 5581*(C*b^3 + 3*A*a*c^2 + 3*
(2*C*a*b + A*b^2)*c)*m^12 + 168171*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)
*c)*m^11 + 3386083*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^10 + 48083
```

```
733*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^9 + 495342143*(C*b^3 + 3*
A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^8 + 3749548713*(C*b^3 + 3*A*a*c^2 + 3*(2
*C*a*b + A*b^2)*c)*m^7 + 20885191136*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^
2)*c)*m^6 + 84836490456*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^5 + 2
46143692976*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)*m^4 + 145297152000*
C*b^3 + 435891456000*A*a*c^2 + 491520108816*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b
 + A*b^2)*c)*m^3 + 633314724480*(C*b^3 + 3*A*a*c^2 + 3*(2*C*a*b + A*b^2)*c)
*m^2 + 435891456000*(2*C*a*b + A*b^2)*c + 465985094400*(C*b^3 + 3*A*a*c^2 +
 3*(2*C*a*b + A*b^2)*c)*m)*x^9 + ((B*b^3 + 6*B*a*b*c)*m^14 + 112*(B*b^3 + 6
*B*a*b*c)*m^13 + 5684*(B*b^3 + 6*B*a*b*c)*m^12 + 172928*(B*b^3 + 6*B*a*b*c)
*m^11 + 3516198*(B*b^3 + 6*B*a*b*c)*m^10 + 50428896*(B*b^3 + 6*B*a*b*c)*m^9
 + 524664572*(B*b^3 + 6*B*a*b*c)*m^8 + 4010311424*(B*b^3 + 6*B*a*b*c)*m^7 +
 22548638161*(B*b^3 + 6*B*a*b*c)*m^6 + 92414105392*(B*b^3 + 6*B*a*b*c)*m^5
+ 270359263944*(B*b^3 + 6*B*a*b*c)*m^4 + 163459296000*B*b^3 + 980755776000*
B*a*b*c + 543939234048*(B*b^3 + 6*B*a*b*c)*m^3 + 705481831440*(B*b^3 + 6*B*
a*b*c)*m^2 + 521962963200*(B*b^3 + 6*B*a*b*c)*m)*x^8 + ((3*C*a*b^2 + A*b^3
+ 3*(C*a^2 + 2*A*a*b)*c)*m^14 + 113*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b
)*c)*m^13 + 5789*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^12 + 177877*
(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^11 + 3654483*(3*C*a*b^2 + A*b
^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^10 + 52977099*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 +
 2*A*a*b)*c)*m^9 + 557256047*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^
8 + 4306835671*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^7 + 2448327985
6*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^6 + 101420251688*(3*C*a*b^2
 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^5 + 299730345264*(3*C*a*b^2 + A*b^3 + 3
*(C*a^2 + 2*A*a*b)*c)*m^4 + 560431872000*C*a*b^2 + 186810624000*A*b^3 + 608
700928752*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^3 + 796089202560*(3
*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m^2 + 560431872000*(C*a^2 + 2*A*a
*b)*c + 593193196800*(3*C*a*b^2 + A*b^3 + 3*(C*a^2 + 2*A*a*b)*c)*m)*x^7 + 3
*((B*a*b^2 + B*a^2*c)*m^14 + 114*(B*a*b^2 + B*a^2*c)*m^13 + 5896*(B*a*b^2 +
 B*a^2*c)*m^12 + 183024*(B*a*b^2 + B*a^2*c)*m^11 + 3801478*(B*a*b^2 + B*a^2
*c)*m^10 + 55749612*(B*a*b^2 + B*a^2*c)*m^9 + 593598068*(B*a*b^2 + B*a^2*c)
*m^8 + 4646039592*(B*a*b^2 + B*a^2*c)*m^7 + 26754892001*(B*a*b^2 + B*a^2*c)
*m^6 + 112273858674*(B*a*b^2 + B*a^2*c)*m^5 + 336028955036*(B*a*b^2 + B*a^2
*c)*m^4 + 217945728000*B*a*b^2 + 217945728000*B*a^2*c + 690639615384*(B*a*b
^2 + B*a^2*c)*m^3 + 913158011520*(B*a*b^2 + B*a^2*c)*m^2 + 686869545600*(B*
a*b^2 + B*a^2*c)*m)*x^6 + 3*((C*a^2*b + A*a*b^2 + A*a^2*c)*m^14 + 115*(C*a^
2*b + A*a*b^2 + A*a^2*c)*m^13 + 6005*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^12 + 1
88375*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^11 + 3957747*(C*a^2*b + A*a*b^2 + A*a
^2*c)*m^10 + 58769745*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^9 + 634247015*(C*a^2*
b + A*a*b^2 + A*a^2*c)*m^8 + 5036392925*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^7 +
 29449164928*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^6 + 125557386040*(C*a^2*b + A*
a*b^2 + A*a^2*c)*m^5 + 381885176880*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^4 + 261
534873600*C*a^2*b + 261534873600*A*a*b^2 + 261534873600*A*a^2*c + 797387461
200*(C*a^2*b + A*a*b^2 + A*a^2*c)*m^3 + 1070058397824*(C*a^2*b + A*a*b^2 +
A*a^2*c)*m^2 + 815525625600*(C*a^2*b + A*a*b^2 + A*a^2*c)*m)*x^5 + 3*(B*a^2
*b*m^14 + 116*B*a^2*b*m^13 + 6116*B*a^2*b*m^12 + 193936*B*a^2*b*m^11 + 4123
878*B*a^2*b*m^10 + 62062968*B*a^2*b*m^9 + 679843868*B*a^2*b*m^8 + 548825252
8*B*a^2*b*m^7 + 32678119441*B*a^2*b*m^6 + 142090732916*B*a^2*b*m^5 + 441309
175416*B*a^2*b*m^4 + 941576643936*B*a^2*b*m^3 + 1290689128080*B*a^2*b*m^2 +
 1003061102400*B*a^2*b*m + 326918592000*B*a^2*b)*x^4 + ((C*a^3 + 3*A*a^2*b)
*m^14 + 117*(C*a^3 + 3*A*a^2*b)*m^13 + 6229*(C*a^3 + 3*A*a^2*b)*m^12 + 1997
13*(C*a^3 + 3*A*a^2*b)*m^11 + 4300483*(C*a^3 + 3*A*a^2*b)*m^10 + 65657031*(
C*a^3 + 3*A*a^2*b)*m^9 + 731124647*(C*a^3 + 3*A*a^2*b)*m^8 + 6014254059*(C*
a^3 + 3*A*a^2*b)*m^7 + 36588367376*(C*a^3 + 3*A*a^2*b)*m^6 + 163038108552*(
C*a^3 + 3*A*a^2*b)*m^5 + 520557781424*(C*a^3 + 3*A*a^2*b)*m^4 + 43589145600
0*C*a^3 + 1307674368000*A*a^2*b + 1145140001328*(C*a^3 + 3*A*a^2*b)*m^3 + 1
621575699840*(C*a^3 + 3*A*a^2*b)*m^2 + 1301090515200*(C*a^3 + 3*A*a^2*b)*m)
*x^3 + (B*a^3*m^14 + 118*B*a^3*m^13 + 6344*B*a^3*m^12 + 205712*B*a^3*m^11 +
 4488198*B*a^3*m^10 + 69582084*B*a^3*m^9 + 788931572*B*a^3*m^8 + 6629764856
```

```
*B*a^3*m^7 + 41371599841*B*a^3*m^6 + 190060010998*B*a^3*m^5 + 629552085084*
B*a^3*m^4 + 1447709175432*B*a^3*m^3 + 2161577352960*B*a^3*m^2 + 18426629088
00*B*a^3*m + 653837184000*B*a^3)*x^2 + (A*a^3*m^14 + 119*A*a^3*m^13 + 6461*
A*a^3*m^12 + 211939*A*a^3*m^11 + 4687683*A*a^3*m^10 + 73870797*A*a^3*m^9 +
854224943*A*a^3*m^8 + 7353403057*A*a^3*m^7 + 47277726496*A*a^3*m^6 + 225525
484184*A*a^3*m^5 + 784146622896*A*a^3*m^4 + 1922666722704*A*a^3*m^3 + 31343
28981120*A*a^3*m^2 + 3031488633600*A*a^3*m + 1307674368000*A*a^3)*x)*(d*x)^
m/(m^15 + 120*m^14 + 6580*m^13 + 218400*m^12 + 4899622*m^11 + 78558480*m^10
 + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m^6 + 10
09672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614720*m^2
 + 4339163001600*m + 1307674368000)
```

giac [B] time = 1.13, size = 7808, normalized size = 19.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c^3*m^14*x^15 + (d*x)^m*B*c^3*m^14*x^14 + 105*(d*x)^m*C*c^3*m^13
*x^15 + 3*(d*x)^m*C*b*c^2*m^14*x^13 + (d*x)^m*A*c^3*m^14*x^13 + 106*(d*x)^m
*B*c^3*m^13*x^14 + 5005*(d*x)^m*C*c^3*m^12*x^15 + 3*(d*x)^m*B*b*c^2*m^14*x^
12 + 321*(d*x)^m*C*b*c^2*m^13*x^13 + 107*(d*x)^m*A*c^3*m^13*x^13 + 5096*(d*
x)^m*B*c^3*m^12*x^14 + 143325*(d*x)^m*C*c^3*m^11*x^15 + 3*(d*x)^m*C*b^2*c*m
^14*x^11 + 3*(d*x)^m*C*a*c^2*m^14*x^11 + 3*(d*x)^m*A*b*c^2*m^14*x^11 + 324*
(d*x)^m*B*b*c^2*m^13*x^12 + 15567*(d*x)^m*C*b*c^2*m^12*x^13 + 5189*(d*x)^m*
A*c^3*m^12*x^13 + 147056*(d*x)^m*B*c^3*m^11*x^14 + 2749747*(d*x)^m*C*c^3*m^
10*x^15 + 3*(d*x)^m*B*b^2*c*m^14*x^10 + 3*(d*x)^m*B*a*c^2*m^14*x^10 + 327*(
d*x)^m*C*b^2*c*m^13*x^11 + 327*(d*x)^m*C*a*c^2*m^13*x^11 + 327*(d*x)^m*A*b*
c^2*m^13*x^11 + 15852*(d*x)^m*B*b*c^2*m^12*x^12 + 452829*(d*x)^m*C*b*c^2*m^
11*x^13 + 150943*(d*x)^m*A*c^3*m^11*x^13 + 2840838*(d*x)^m*B*c^3*m^10*x^14
+ 37312275*(d*x)^m*C*c^3*m^9*x^15 + (d*x)^m*C*b^3*m^14*x^9 + 6*(d*x)^m*C*a*
b*c*m^14*x^9 + 3*(d*x)^m*A*b^2*c*m^14*x^9 + 3*(d*x)^m*A*a*c^2*m^14*x^9 + 33
0*(d*x)^m*B*b^2*c*m^13*x^10 + 330*(d*x)^m*B*a*c^2*m^13*x^10 + 16143*(d*x)^m
*C*b^2*c*m^12*x^11 + 16143*(d*x)^m*C*a*c^2*m^12*x^11 + 16143*(d*x)^m*A*b*c^
2*m^12*x^11 + 464976*(d*x)^m*B*b*c^2*m^11*x^12 + 8812089*(d*x)^m*C*b*c^2*m^
10*x^13 + 2937363*(d*x)^m*A*c^3*m^10*x^13 + 38786748*(d*x)^m*B*c^3*m^9*x^14
 + 368411615*(d*x)^m*C*c^3*m^8*x^15 + (d*x)^m*B*b^3*m^14*x^8 + 6*(d*x)^m*B*
a*b*c*m^14*x^8 + 111*(d*x)^m*C*b^3*m^13*x^9 + 666*(d*x)^m*C*a*b*c*m^13*x^9
+ 333*(d*x)^m*A*b^2*c*m^13*x^9 + 333*(d*x)^m*A*a*c^2*m^13*x^9 + 16440*(d*x)
^m*B*b^2*c*m^12*x^10 + 16440*(d*x)^m*B*a*c^2*m^12*x^10 + 477627*(d*x)^m*C*b
^2*c*m^11*x^11 + 477627*(d*x)^m*C*a*c^2*m^11*x^11 + 477627*(d*x)^m*A*b*c^2*
m^11*x^11 + 9119154*(d*x)^m*B*b*c^2*m^10*x^12 + 121118283*(d*x)^m*C*b*c^2*m
^9*x^13 + 40372761*(d*x)^m*A*c^3*m^9*x^13 + 385081268*(d*x)^m*B*c^3*m^8*x^1
4 + 2681453775*(d*x)^m*C*c^3*m^7*x^15 + 3*(d*x)^m*C*a*b^2*m^14*x^7 + (d*x)^
m*A*b^3*m^14*x^7 + 3*(d*x)^m*C*a^2*c*m^14*x^7 + 6*(d*x)^m*A*a*b*c*m^14*x^7
+ 112*(d*x)^m*B*b^3*m^13*x^8 + 672*(d*x)^m*B*a*b*c*m^13*x^8 + 5581*(d*x)^m*
C*b^3*m^12*x^9 + 33486*(d*x)^m*C*a*b*c*m^12*x^9 + 16743*(d*x)^m*A*b^2*c*m^1
2*x^9 + 16743*(d*x)^m*A*a*c^2*m^12*x^9 + 490800*(d*x)^m*B*b^2*c*m^11*x^10 +
 490800*(d*x)^m*B*a*c^2*m^11*x^10 + 9444969*(d*x)^m*C*b^2*c*m^10*x^11 + 944
4969*(d*x)^m*C*a*c^2*m^10*x^11 + 9444969*(d*x)^m*A*b*c^2*m^10*x^11 + 126245
592*(d*x)^m*B*b*c^2*m^9*x^12 + 1209749541*(d*x)^m*C*b*c^2*m^8*x^13 + 403249
847*(d*x)^m*A*c^3*m^8*x^13 + 2816490248*(d*x)^m*B*c^3*m^7*x^14 + 1440932292
8*(d*x)^m*C*c^3*m^6*x^15 + 3*(d*x)^m*B*a*b^2*m^14*x^6 + 3*(d*x)^m*B*a^2*c*m
^14*x^6 + 339*(d*x)^m*C*a*b^2*m^13*x^7 + 113*(d*x)^m*A*b^3*m^13*x^7 + 339*(
d*x)^m*C*a^2*c*m^13*x^7 + 678*(d*x)^m*A*a*b*c*m^13*x^7 + 5684*(d*x)^m*B*b^3
*m^12*x^8 + 34104*(d*x)^m*B*a*b*c*m^12*x^8 + 168171*(d*x)^m*C*b^3*m^11*x^9
+ 1009026*(d*x)^m*C*a*b*c*m^11*x^9 + 504513*(d*x)^m*A*b^2*c*m^11*x^9 + 5045
13*(d*x)^m*A*a*c^2*m^11*x^9 + 9790866*(d*x)^m*B*b^2*c*m^10*x^10 + 9790866*(
d*x)^m*B*a*c^2*m^10*x^10 + 131780781*(d*x)^m*C*b^2*c*m^9*x^11 + 131780781*(
```

d*x)^m*C*a*c^2*m^9*x^11 + 131780781*(d*x)^m*A*b*c^2*m^9*x^11 + 1269340116*(
d*x)^m*B*b*c^2*m^8*x^12 + 8896139967*(d*x)^m*C*b*c^2*m^7*x^13 + 2965379989*
(d*x)^m*A*c^3*m^7*x^13 + 15200266081*(d*x)^m*B*c^3*m^6*x^14 + 56663366760*(
d*x)^m*C*c^3*m^5*x^15 + 3*(d*x)^m*C*a^2*b*m^14*x^5 + 3*(d*x)^m*A*a*b^2*m^14
x^5 + 3(d*x)^m*A*a^2*c*m^14*x^5 + 342*(d*x)^m*B*a*b^2*m^13*x^6 + 342*(d*x
)^m*B*a^2*c*m^13*x^6 + 17367*(d*x)^m*C*a*b^2*m^12*x^7 + 5789*(d*x)^m*A*b^3*
m^12*x^7 + 17367*(d*x)^m*C*a^2*c*m^12*x^7 + 34734*(d*x)^m*A*a*b*c*m^12*x^7
+ 172928*(d*x)^m*B*b^3*m^11*x^8 + 1037568*(d*x)^m*B*a*b*c*m^11*x^8 + 338608
3*(d*x)^m*C*b^3*m^10*x^9 + 20316498*(d*x)^m*C*a*b*c*m^10*x^9 + 10158249*(d*
x)^m*A*b^2*c*m^10*x^9 + 10158249*(d*x)^m*A*a*c^2*m^10*x^9 + 137766780*(d*x)
^m*B*b^2*c*m^9*x^10 + 137766780*(d*x)^m*B*a*c^2*m^9*x^10 + 1334698629*(d*x)
^m*C*b^2*c*m^8*x^11 + 1334698629*(d*x)^m*C*a*c^2*m^8*x^11 + 1334698629*(d*x
)^m*A*b*c^2*m^8*x^11 + 9390802608*(d*x)^m*B*b*c^2*m^7*x^12 + 48243569088*(d
*x)^m*C*b*c^2*m^6*x^13 + 16081189696*(d*x)^m*A*c^3*m^6*x^13 + 59999485546*(
d*x)^m*B*c^3*m^5*x^14 + 159721605680*(d*x)^m*C*c^3*m^4*x^15 + 3*(d*x)^m*B*a
^2*b*m^14*x^4 + 345*(d*x)^m*C*a^2*b*m^13*x^5 + 345*(d*x)^m*A*a*b^2*m^13*x^5
 + 345*(d*x)^m*A*a^2*c*m^13*x^5 + 17688*(d*x)^m*B*a*b^2*m^12*x^6 + 17688*(d
*x)^m*B*a^2*c*m^12*x^6 + 533631*(d*x)^m*C*a*b^2*m^11*x^7 + 177877*(d*x)^m*A
*b^3*m^11*x^7 + 533631*(d*x)^m*C*a^2*c*m^11*x^7 + 1067262*(d*x)^m*A*a*b*c*m
^11*x^7 + 3516198*(d*x)^m*B*b^3*m^10*x^8 + 21097188*(d*x)^m*B*a*b*c*m^10*x^
8 + 48083733*(d*x)^m*C*b^3*m^9*x^9 + 288502398*(d*x)^m*C*a*b*c*m^9*x^9 + 14
4251199*(d*x)^m*A*b^2*c*m^9*x^9 + 144251199*(d*x)^m*A*a*c^2*m^9*x^9 + 14066
19420*(d*x)^m*B*b^2*c*m^8*x^10 + 1406619420*(d*x)^m*B*a*c^2*m^8*x^10 + 9941
199081*(d*x)^m*C*b^2*c*m^7*x^11 + 9941199081*(d*x)^m*C*a*c^2*m^7*x^11 + 994
1199081*(d*x)^m*A*b*c^2*m^7*x^11 + 51203757363*(d*x)^m*B*b*c^2*m^6*x^12 + 1
91243233896*(d*x)^m*C*b*c^2*m^5*x^13 + 63747744632*(d*x)^m*A*c^3*m^5*x^13 +
 169679309436*(d*x)^m*B*c^3*m^4*x^14 + 310989260400*(d*x)^m*C*c^3*m^3*x^15
+ (d*x)^m*C*a^3*m^14*x^3 + 3*(d*x)^m*A*a^2*b*m^14*x^3 + 348*(d*x)^m*B*a^2*b
*m^13*x^4 + 18015*(d*x)^m*C*a^2*b*m^12*x^5 + 18015*(d*x)^m*A*a*b^2*m^12*x^5
 + 18015*(d*x)^m*A*a^2*c*m^12*x^5 + 549072*(d*x)^m*B*a*b^2*m^11*x^6 + 54907
2*(d*x)^m*B*a^2*c*m^11*x^6 + 10963449*(d*x)^m*C*a*b^2*m^10*x^7 + 3654483*(d
*x)^m*A*b^3*m^10*x^7 + 10963449*(d*x)^m*C*a^2*c*m^10*x^7 + 21926898*(d*x)^m
*A*a*b*c*m^10*x^7 + 50428896*(d*x)^m*B*b^3*m^9*x^8 + 302573376*(d*x)^m*B*a*
b*c*m^9*x^8 + 495342143*(d*x)^m*C*b^3*m^8*x^9 + 2972052858*(d*x)^m*C*a*b*c*
m^8*x^9 + 1486026429*(d*x)^m*A*b^2*c*m^8*x^9 + 1486026429*(d*x)^m*A*a*c^2*m
^8*x^9 + 10556689800*(d*x)^m*B*b^2*c*m^7*x^10 + 10556689800*(d*x)^m*B*a*c^2
*m^7*x^10 + 54540198768*(d*x)^m*C*b^2*c*m^6*x^11 + 54540198768*(d*x)^m*C*a*
c^2*m^6*x^11 + 54540198768*(d*x)^m*A*b*c^2*m^6*x^11 + 203964543684*(d*x)^m*
B*b*c^2*m^5*x^12 + 542854280592*(d*x)^m*C*b*c^2*m^4*x^13 + 180951426864*(d*
x)^m*A*c^3*m^4*x^13 + 331303013496*(d*x)^m*B*c^3*m^3*x^14 + 392156797824*(d
*x)^m*C*c^3*m^2*x^15 + (d*x)^m*B*a^3*m^14*x^2 + 117*(d*x)^m*C*a^3*m^13*x^3
+ 351*(d*x)^m*A*a^2*b*m^13*x^3 + 18348*(d*x)^m*B*a^2*b*m^12*x^4 + 565125*(d
*x)^m*C*a^2*b*m^11*x^5 + 565125*(d*x)^m*A*a*b^2*m^11*x^5 + 565125*(d*x)^m*A
*a^2*c*m^11*x^5 + 11404434*(d*x)^m*B*a*b^2*m^10*x^6 + 11404434*(d*x)^m*B*a^
2*c*m^10*x^6 + 158931297*(d*x)^m*C*a*b^2*m^9*x^7 + 52977099*(d*x)^m*A*b^3*m
^9*x^7 + 158931297*(d*x)^m*C*a^2*c*m^9*x^7 + 317862594*(d*x)^m*A*a*b*c*m^9*
x^7 + 524664572*(d*x)^m*B*b^3*m^8*x^8 + 3147987432*(d*x)^m*B*a*b*c*m^8*x^8
+ 3749548713*(d*x)^m*C*b^3*m^7*x^9 + 22497292278*(d*x)^m*C*a*b*c*m^7*x^9 +
11248646139*(d*x)^m*A*b^2*c*m^7*x^9 + 11248646139*(d*x)^m*A*a*c^2*m^7*x^9 +
 58326490659*(d*x)^m*B*b^2*c*m^6*x^10 + 58326490659*(d*x)^m*B*a*c^2*m^6*x^1
0 + 218467445592*(d*x)^m*C*b^2*c*m^5*x^11 + 218467445592*(d*x)^m*C*a*c^2*m^
5*x^11 + 218467445592*(d*x)^m*A*b*c^2*m^5*x^11 + 581441797032*(d*x)^m*B*b*c
^2*m^4*x^12 + 1063334389104*(d*x)^m*C*b*c^2*m^3*x^13 + 354444796368*(d*x)^m
*A*c^3*m^3*x^13 + 418753514880*(d*x)^m*B*c^3*m^2*x^14 + 283465647360*(d*x)^
m*C*c^3*m*x^15 + (d*x)^m*A*a^3*m^14*x + 118*(d*x)^m*B*a^3*m^13*x^2 + 6229*(
d*x)^m*C*a^3*m^12*x^3 + 18687*(d*x)^m*A*a^2*b*m^12*x^3 + 581808*(d*x)^m*B*a
^2*b*m^11*x^4 + 11873241*(d*x)^m*C*a^2*b*m^10*x^5 + 11873241*(d*x)^m*A*a*b^
2*m^10*x^5 + 11873241*(d*x)^m*A*a^2*c*m^10*x^5 + 167248836*(d*x)^m*B*a*b^2*
m^9*x^6 + 167248836*(d*x)^m*B*a^2*c*m^9*x^6 + 1671768141*(d*x)^m*C*a*b^2*m^

8*x^7 + 557256047*(d*x)^m*A*b^3*m^8*x^7 + 1671768141*(d*x)^m*C*a^2*c*m^8*x^
7 + 3343536282*(d*x)^m*A*a*b*c*m^8*x^7 + 4010311424*(d*x)^m*B*b^3*m^7*x^8 +
 24061868544*(d*x)^m*B*a*b*c*m^7*x^8 + 20885191136*(d*x)^m*C*b^3*m^6*x^9 +
125311146816*(d*x)^m*C*a*b*c*m^6*x^9 + 62655573408*(d*x)^m*A*b^2*c*m^6*x^9
+ 62655573408*(d*x)^m*A*a*c^2*m^6*x^9 + 235144725450*(d*x)^m*B*b^2*c*m^5*x^
10 + 235144725450*(d*x)^m*B*a*c^2*m^5*x^10 + 625874419728*(d*x)^m*C*b^2*c*m
^4*x^11 + 625874419728*(d*x)^m*C*a*c^2*m^4*x^11 + 625874419728*(d*x)^m*A*b*
c^2*m^4*x^11 + 1143138472416*(d*x)^m*B*b*c^2*m^3*x^12 + 1347640053120*(d*x)
^m*C*b*c^2*m^2*x^13 + 449213351040*(d*x)^m*A*c^3*m^2*x^13 + 303268406400*(d
*x)^m*B*c^3*m*x^14 + 87178291200*(d*x)^m*C*c^3*x^15 + 119*(d*x)^m*A*a^3*m^1
3*x + 6344*(d*x)^m*B*a^3*m^12*x^2 + 199713*(d*x)^m*C*a^3*m^11*x^3 + 599139*
(d*x)^m*A*a^2*b*m^11*x^3 + 12371634*(d*x)^m*B*a^2*b*m^10*x^4 + 176309235*(d
*x)^m*C*a^2*b*m^9*x^5 + 176309235*(d*x)^m*A*a*b^2*m^9*x^5 + 176309235*(d*x)
^m*A*a^2*c*m^9*x^5 + 1780794204*(d*x)^m*B*a*b^2*m^8*x^6 + 1780794204*(d*x)^
m*B*a^2*c*m^8*x^6 + 12920507013*(d*x)^m*C*a*b^2*m^7*x^7 + 4306835671*(d*x)^
m*A*b^3*m^7*x^7 + 12920507013*(d*x)^m*C*a^2*c*m^7*x^7 + 25841014026*(d*x)^m
*A*a*b*c*m^7*x^7 + 22548638161*(d*x)^m*B*b^3*m^6*x^8 + 135291828966*(d*x)^m
*B*a*b*c*m^6*x^8 + 84836490456*(d*x)^m*C*b^3*m^5*x^9 + 509018942736*(d*x)^m
*C*a*b*c*m^5*x^9 + 254509471368*(d*x)^m*A*b^2*c*m^5*x^9 + 254509471368*(d*x
)^m*A*a*c^2*m^5*x^9 + 677569066740*(d*x)^m*B*b^2*c*m^4*x^10 + 677569066740*
(d*x)^m*B*a*c^2*m^4*x^10 + 1235821419792*(d*x)^m*C*b^2*c*m^3*x^11 + 1235821
419792*(d*x)^m*C*a*c^2*m^3*x^11 + 1235821419792*(d*x)^m*A*b*c^2*m^3*x^11 +
1453325442480*(d*x)^m*B*b*c^2*m^2*x^12 + 978132153600*(d*x)^m*C*b*c^2*m*x^1
3 + 326044051200*(d*x)^m*A*c^3*m*x^13 + 93405312000*(d*x)^m*B*c^3*x^14 + 64
61*(d*x)^m*A*a^3*m^12*x + 205712*(d*x)^m*B*a^3*m^11*x^2 + 4300483*(d*x)^m*C
*a^3*m^10*x^3 + 12901449*(d*x)^m*A*a^2*b*m^10*x^3 + 186188904*(d*x)^m*B*a^2
*b*m^9*x^4 + 1902741045*(d*x)^m*C*a^2*b*m^8*x^5 + 1902741045*(d*x)^m*A*a*b^
2*m^8*x^5 + 1902741045*(d*x)^m*A*a^2*c*m^8*x^5 + 13938118776*(d*x)^m*B*a*b^
2*m^7*x^6 + 13938118776*(d*x)^m*B*a^2*c*m^7*x^6 + 73449839568*(d*x)^m*C*a*b
^2*m^6*x^7 + 24483279856*(d*x)^m*A*b^3*m^6*x^7 + 73449839568*(d*x)^m*C*a^2*
c*m^6*x^7 + 146899679136*(d*x)^m*A*a*b*c*m^6*x^7 + 92414105392*(d*x)^m*B*b^
3*m^5*x^8 + 554484632352*(d*x)^m*B*a*b*c*m^5*x^8 + 246143692976*(d*x)^m*C*b
^3*m^4*x^9 + 1476862157856*(d*x)^m*C*a*b*c*m^4*x^9 + 738431078928*(d*x)^m*A
*b^2*c*m^4*x^9 + 738431078928*(d*x)^m*A*a*c^2*m^4*x^9 + 1344749369400*(d*x)
^m*B*b^2*c*m^3*x^10 + 1344749369400*(d*x)^m*B*a*c^2*m^3*x^10 + 157695149376
0*(d*x)^m*C*b^2*c*m^2*x^11 + 1576951493760*(d*x)^m*C*a*c^2*m^2*x^11 + 15769
51493760*(d*x)^m*A*b*c^2*m^2*x^11 + 1057547534400*(d*x)^m*B*b*c^2*m*x^12 +
301771008000*(d*x)^m*C*b*c^2*x^13 + 100590336000*(d*x)^m*A*c^3*x^13 + 21193
9*(d*x)^m*A*a^3*m^11*x + 4488198*(d*x)^m*B*a^3*m^10*x^2 + 65657031*(d*x)^m*
C*a^3*m^9*x^3 + 196971093*(d*x)^m*A*a^2*b*m^9*x^3 + 2039531604*(d*x)^m*B*a^
2*b*m^8*x^4 + 15109178775*(d*x)^m*C*a^2*b*m^7*x^5 + 15109178775*(d*x)^m*A*a
*b^2*m^7*x^5 + 15109178775*(d*x)^m*A*a^2*c*m^7*x^5 + 80264676003*(d*x)^m*B*
a*b^2*m^6*x^6 + 80264676003*(d*x)^m*B*a^2*c*m^6*x^6 + 304260755064*(d*x)^m*
C*a*b^2*m^5*x^7 + 101420251688*(d*x)^m*A*b^3*m^5*x^7 + 304260755064*(d*x)^m
*C*a^2*c*m^5*x^7 + 608521510128*(d*x)^m*A*a*b*c*m^5*x^7 + 270359263944*(d*x
)^m*B*b^3*m^4*x^8 + 1622155583664*(d*x)^m*B*a*b*c*m^4*x^8 + 491520108816*(d
*x)^m*C*b^3*m^3*x^9 + 2949120652896*(d*x)^m*C*a*b*c*m^3*x^9 + 1474560326448
*(d*x)^m*A*b^2*c*m^3*x^9 + 1474560326448*(d*x)^m*A*a*c^2*m^3*x^9 + 17234934
17472*(d*x)^m*B*b^2*c*m^2*x^10 + 1723493417472*(d*x)^m*B*a*c^2*m^2*x^10 + 1
150986412800*(d*x)^m*C*b^2*c*m*x^11 + 1150986412800*(d*x)^m*C*a*c^2*m*x^11
+ 1150986412800*(d*x)^m*A*b*c^2*m*x^11 + 326918592000*(d*x)^m*B*b*c^2*x^12
+ 4687683*(d*x)^m*A*a^3*m^10*x + 69582084*(d*x)^m*B*a^3*m^9*x^2 + 731124647
*(d*x)^m*C*a^3*m^8*x^3 + 2193373941*(d*x)^m*A*a^2*b*m^8*x^3 + 16464757584*(
d*x)^m*B*a^2*b*m^7*x^4 + 88347494784*(d*x)^m*C*a^2*b*m^6*x^5 + 88347494784*
(d*x)^m*A*a*b^2*m^6*x^5 + 88347494784*(d*x)^m*A*a^2*c*m^6*x^5 + 33682157602
2*(d*x)^m*B*a*b^2*m^5*x^6 + 336821576022*(d*x)^m*B*a^2*c*m^5*x^6 + 89919103
5792*(d*x)^m*C*a*b^2*m^4*x^7 + 299730345264*(d*x)^m*A*b^3*m^4*x^7 + 8991910
35792*(d*x)^m*C*a^2*c*m^4*x^7 + 1798382071584*(d*x)^m*A*a*b*c*m^4*x^7 + 543
939234048*(d*x)^m*B*b^3*m^3*x^8 + 3263635404288*(d*x)^m*B*a*b*c*m^3*x^8 + 6

33314724480*(d*x)^m*C*b^3*m^2*x^9 + 3799888346880*(d*x)^m*C*a*b*c*m^2*x^9 + 1899944173440*(d*x)^m*A*b^2*c*m^2*x^9 + 1899944173440*(d*x)^m*A*a*c^2*m^2*x^9 + 1262518669440*(d*x)^m*B*b^2*c*m*x^10 + 1262518669440*(d*x)^m*B*a*c^2*m*x^10 + 356638464000*(d*x)^m*C*b^2*c*x^11 + 356638464000*(d*x)^m*C*a*c^2*x^11 + 356638464000*(d*x)^m*A*b*c^2*x^11 + 73870797*(d*x)^m*A*a^3*m^9*x + 788931572*(d*x)^m*B*a^3*m^8*x^2 + 6014254059*(d*x)^m*C*a^3*m^7*x^3 + 18042762177*(d*x)^m*A*a^2*b*m^7*x^3 + 98034358323*(d*x)^m*B*a^2*b*m^6*x^4 + 376672158120*(d*x)^m*C*a^2*b*m^5*x^5 + 376672158120*(d*x)^m*A*a*b^2*m^5*x^5 + 376672158120*(d*x)^m*A*a^2*c*m^5*x^5 + 1008086865108*(d*x)^m*B*a*b^2*m^4*x^6 + 1008086865108*(d*x)^m*B*a^2*c*m^4*x^6 + 1826102786256*(d*x)^m*C*a*b^2*m^3*x^7 + 608700928752*(d*x)^m*A*b^3*m^3*x^7 + 1826102786256*(d*x)^m*C*a^2*c*m^3*x^7 + 3652205572512*(d*x)^m*A*a*b*c*m^3*x^7 + 705481831440*(d*x)^m*B*b^3*m^2*x^8 + 4232890988640*(d*x)^m*B*a*b*c*m^2*x^8 + 465985094400*(d*x)^m*C*b^3*m*x^9 + 2795910566400*(d*x)^m*C*a*b*c*m*x^9 + 1397955283200*(d*x)^m*A*b^2*c*m*x^9 + 1397955283200*(d*x)^m*A*a*c^2*m*x^9 + 392302310400*(d*x)^m*B*b^2*c*x^10 + 392302310400*(d*x)^m*B*a*c^2*x^10 + 854224943*(d*x)^m*A*a^3*m^8*x + 6629764856*(d*x)^m*B*a^3*m^7*x^2 + 36588367376*(d*x)^m*C*a^3*m^6*x^3 + 109765102128*(d*x)^m*A*a^2*b*m^6*x^3 + 426272198748*(d*x)^m*B*a^2*b*m^5*x^4 + 1145655530640*(d*x)^m*C*a^2*b*m^4*x^5 + 1145655530640*(d*x)^m*A*a*b^2*m^4*x^5 + 1145655530640*(d*x)^m*A*a^2*c*m^4*x^5 + 2071918846152*(d*x)^m*B*a*b^2*m^3*x^6 + 2071918846152*(d*x)^m*B*a^2*c*m^3*x^6 + 2388267607680*(d*x)^m*C*a*b^2*m^2*x^7 + 796089202560*(d*x)^m*A*b^3*m^2*x^7 + 2388267607680*(d*x)^m*C*a^2*c*m^2*x^7 + 4776535215360*(d*x)^m*A*a*b*c*m^2*x^7 + 521962963200*(d*x)^m*B*b^3*m*x^8 + 3131777779200*(d*x)^m*B*a*b*c*m*x^8 + 145297152000*(d*x)^m*C*b^3*x^9 + 871782912000*(d*x)^m*C*a*b*c*x^9 + 435891456000*(d*x)^m*A*b^2*c*x^9 + 435891456000*(d*x)^m*A*a*c^2*x^9 + 7353403057*(d*x)^m*A*a^3*m^7*x + 41371599841*(d*x)^m*B*a^3*m^6*x^2 + 163038108552*(d*x)^m*C*a^3*m^5*x^3 + 489114325656*(d*x)^m*A*a^2*b*m^5*x^3 + 1323927526248*(d*x)^m*B*a^2*b*m^4*x^4 + 2392162383600*(d*x)^m*C*a^2*b*m^3*x^5 + 2392162383600*(d*x)^m*A*a*b^2*m^3*x^5 + 2392162383600*(d*x)^m*A*a^2*c*m^3*x^5 + 2739474034560*(d*x)^m*B*a*b^2*m^2*x^6 + 2739474034560*(d*x)^m*B*a^2*c*m^2*x^6 + 1779579590400*(d*x)^m*C*a*b^2*m*x^7 + 593193196800*(d*x)^m*A*b^3*m*x^7 + 1779579590400*(d*x)^m*C*a^2*c*m*x^7 + 3559159180800*(d*x)^m*A*a*b*c*m*x^7 + 163459296000*(d*x)^m*B*b^3*x^8 + 980755776000*(d*x)^m*B*a*b*c*x^8 + 47277726496*(d*x)^m*A*a^3*m^6*x + 190060010998*(d*x)^m*B*a^3*m^5*x^2 + 520557781424*(d*x)^m*C*a^3*m^4*x^3 + 1561673344272*(d*x)^m*A*a^2*b*m^4*x^3 + 2824729931808*(d*x)^m*B*a^2*b*m^3*x^4 + 3210175193472*(d*x)^m*C*a^2*b*m^2*x^5 + 3210175193472*(d*x)^m*A*a*b^2*m^2*x^5 + 3210175193472*(d*x)^m*A*a^2*c*m^2*x^5 + 2060608636800*(d*x)^m*B*a*b^2*m*x^6 + 2060608636800*(d*x)^m*B*a^2*c*m*x^6 + 560431872000*(d*x)^m*C*a*b^2*x^7 + 186810624000*(d*x)^m*A*b^3*x^7 + 560431872000*(d*x)^m*C*a^2*c*x^7 + 1120863744000*(d*x)^m*A*a*b*c*x^7 + 225525484184*(d*x)^m*A*a^3*m^5*x + 629552085084*(d*x)^m*B*a^3*m^4*x^2 + 1145140001328*(d*x)^m*C*a^3*m^3*x^3 + 3435420003984*(d*x)^m*A*a^2*b*m^3*x^3 + 3872067384240*(d*x)^m*B*a^2*b*m^2*x^4 + 2446576876800*(d*x)^m*C*a^2*b*m*x^5 + 2446576876800*(d*x)^m*A*a*b^2*m*x^5 + 2446576876800*(d*x)^m*A*a^2*c*m*x^5 + 653837184000*(d*x)^m*B*a*b^2*x^6 + 653837184000*(d*x)^m*B*a^2*c*x^6 + 784146622896*(d*x)^m*A*a^3*m^4*x + 1447709175432*(d*x)^m*B*a^3*m^3*x^2 + 1621575699840*(d*x)^m*C*a^3*m^2*x^3 + 4864727099520*(d*x)^m*A*a^2*b*m^2*x^3 + 3009183307200*(d*x)^m*B*a^2*b*m*x^4 + 784604620800*(d*x)^m*C*a^2*b*x^5 + 784604620800*(d*x)^m*A*a*b^2*x^5 + 784604620800*(d*x)^m*A*a^2*c*x^5 + 1922666722704*(d*x)^m*A*a^3*m^3*x + 2161577352960*(d*x)^m*B*a^3*m^2*x^2 + 1301090515200*(d*x)^m*C*a^3*m*x^3 + 3903271545600*(d*x)^m*A*a^2*b*m*x^3 + 980755776000*(d*x)^m*B*a^2*b*x^4 + 3134328981120*(d*x)^m*A*a^3*m^2*x + 1842662908800*(d*x)^m*B*a^3*m*x^2 + 435891456000*(d*x)^m*C*a^3*x^3 + 1307674368000*(d*x)^m*A*a^2*b*x^3 + 3031488633600*(d*x)^m*A*a^3*m*x + 653837184000*(d*x)^m*B*a^3*x^2 + 1307674368000*(d*x)^m*A*a^3*x)/(m^15 + 120*m^14 + 6580*m^13 + 218400*m^12 + 4899622*m^11 + 78558480*m^10 + 928095740*m^9 + 8207628000*m^8 + 54631129553*m^7 + 272803210680*m^6 + 1009672107080*m^5 + 2706813345600*m^4 + 5056995703824*m^3 + 6165817614720*m^2 + 4339163001600*m + 1307674368000)

maple [B] time = 0.01, size = 5520, normalized size = 13.83

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x)
```

```
[Out] result too large to display
```

maxima [A] time = 1.71, size = 611, normalized size = 1.53

$$\frac{Cc^3d^mx^{15}x^m}{m+15}+\frac{Bc^3d^mx^{14}x^m}{m+14}+\frac{3\,Cbc^2d^mx^{13}x^m}{m+13}+\frac{Ac^3d^mx^{13}x^m}{m+13}+\frac{3\,Bbc^2d^mx^{12}x^m}{m+12}+\frac{3\,Cb^2cd^mx^{11}x^m}{m+11}+\frac{3\,Cac^2d^mx^{11}x}{m+11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] C*c^3*d^m*x^15*x^m/(m + 15) + B*c^3*d^m*x^14*x^m/(m + 14) + 3*C*b*c^2*d^m*x
^13*x^m/(m + 13) + A*c^3*d^m*x^13*x^m/(m + 13) + 3*B*b*c^2*d^m*x^12*x^m/(m
+ 12) + 3*C*b^2*c*d^m*x^11*x^m/(m + 11) + 3*C*a*c^2*d^m*x^11*x^m/(m + 11) +
 3*A*b*c^2*d^m*x^11*x^m/(m + 11) + 3*B*b^2*c*d^m*x^10*x^m/(m + 10) + 3*B*a*
c^2*d^m*x^10*x^m/(m + 10) + C*b^3*d^m*x^9*x^m/(m + 9) + 6*C*a*b*c*d^m*x^9*x
^m/(m + 9) + 3*A*b^2*c*d^m*x^9*x^m/(m + 9) + 3*A*a*c^2*d^m*x^9*x^m/(m + 9)
+ B*b^3*d^m*x^8*x^m/(m + 8) + 6*B*a*b*c*d^m*x^8*x^m/(m + 8) + 3*C*a*b^2*d^m
*x^7*x^m/(m + 7) + A*b^3*d^m*x^7*x^m/(m + 7) + 3*C*a^2*c*d^m*x^7*x^m/(m + 7
) + 6*A*a*b*c*d^m*x^7*x^m/(m + 7) + 3*B*a*b^2*d^m*x^6*x^m/(m + 6) + 3*B*a^2
*c*d^m*x^6*x^m/(m + 6) + 3*C*a^2*b*d^m*x^5*x^m/(m + 5) + 3*A*a*b^2*d^m*x^5*
x^m/(m + 5) + 3*A*a^2*c*d^m*x^5*x^m/(m + 5) + 3*B*a^2*b*d^m*x^4*x^m/(m + 4)
 + C*a^3*d^m*x^3*x^m/(m + 3) + 3*A*a^2*b*d^m*x^3*x^m/(m + 3) + B*a^3*d^m*x^
2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^3/(d*(m + 1))
```

mupad [B] time = 3.28, size = 2443, normalized size = 6.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (x^7*(d*x)^m*(A*b^3 + 3*C*a*b^2 + 3*C*a^2*c + 6*A*a*b*c)*(593193196800*m +
796089202560*m^2 + 608700928752*m^3 + 299730345264*m^4 + 101420251688*m^5 +
 24483279856*m^6 + 4306835671*m^7 + 557256047*m^8 + 52977099*m^9 + 3654483*
m^10 + 177877*m^11 + 5789*m^12 + 113*m^13 + m^14 + 186810624000))/(43391630
01600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 10096
72107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928095
740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m^14
 + m^15 + 1307674368000) + (x^9*(d*x)^m*(C*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*
C*a*b*c)*(465985094400*m + 633314724480*m^2 + 491520108816*m^3 + 2461436929
76*m^4 + 84836490456*m^5 + 20885191136*m^6 + 3749548713*m^7 + 495342143*m^8
 + 48083733*m^9 + 3386083*m^10 + 168171*m^11 + 5581*m^12 + 111*m^13 + m^14
+ 145297152000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 +
 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7
 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m
^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (B*c^3*x^14*(d*x)^m*(3
03268406400*m + 418753514880*m^2 + 331303013496*m^3 + 169679309436*m^4 + 59
999485546*m^5 + 15200266081*m^6 + 2816490248*m^7 + 385081268*m^8 + 38786748
*m^9 + 2840838*m^10 + 147056*m^11 + 5096*m^12 + 106*m^13 + m^14 + 934053120
00))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 27068133456
00*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 820762800
0*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m
```

```
^13 + 120*m^14 + m^15 + 1307674368000) + (B*a^3*x^2*(d*x)^m*(1842662908800*
m + 2161577352960*m^2 + 1447709175432*m^3 + 629552085084*m^4 + 190060010998
*m^5 + 41371599841*m^6 + 6629764856*m^7 + 788931572*m^8 + 69582084*m^9 + 44
88198*m^10 + 205712*m^11 + 6344*m^12 + 118*m^13 + m^14 + 653837184000))/(43
39163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 +
 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 +
928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 12
0*m^14 + m^15 + 1307674368000) + (3*a*x^5*(d*x)^m*(A*b^2 + A*a*c + C*a*b)*(
815525625600*m + 1070058397824*m^2 + 797387461200*m^3 + 381885176880*m^4 +
125557386040*m^5 + 29449164928*m^6 + 5036392925*m^7 + 634247015*m^8 + 58769
745*m^9 + 3957747*m^10 + 188375*m^11 + 6005*m^12 + 115*m^13 + m^14 + 261534
873600))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813
345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 82076
28000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 65
80*m^13 + 120*m^14 + m^15 + 1307674368000) + (3*c*x^11*(d*x)^m*(C*b^2 + A*b
*c + C*a*c)*(383662137600*m + 525650497920*m^2 + 411940473264*m^3 + 2086248
06576*m^4 + 72822481864*m^5 + 18180066256*m^6 + 3313733027*m^7 + 444899543*
m^8 + 43926927*m^9 + 3148323*m^10 + 159209*m^11 + 5381*m^12 + 109*m^13 + m^
14 + 118879488000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^
3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*
m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 21840
0*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (a^2*x^3*(d*x)^m*(3
*A*b + C*a)*(1301090515200*m + 1621575699840*m^2 + 1145140001328*m^3 + 5205
57781424*m^4 + 163038108552*m^5 + 36588367376*m^6 + 6014254059*m^7 + 731124
647*m^8 + 65657031*m^9 + 4300483*m^10 + 199713*m^11 + 6229*m^12 + 117*m^13
 + m^14 + 435891456000))/(4339163001600*m + 6165817614720*m^2 + 505699570382
4*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129
553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 2
18400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (c^2*x^13*(d*x)
^m*(A*c + 3*C*b)*(326044051200*m + 449213351040*m^2 + 354444796368*m^3 + 18
0951426864*m^4 + 63747744632*m^5 + 16081189696*m^6 + 2965379989*m^7 + 40324
9847*m^8 + 40372761*m^9 + 2937363*m^10 + 150943*m^11 + 5189*m^12 + 107*m^13
 + m^14 + 100590336000))/(4339163001600*m + 6165817614720*m^2 + 50569957038
24*m^3 + 2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 5463112
9553*m^7 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 +
218400*m^12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (A*a^3*x*(d*x)
^m*(3031488633600*m + 3134328981120*m^2 + 1922666722704*m^3 + 784146622896*
m^4 + 225525484184*m^5 + 47277726496*m^6 + 7353403057*m^7 + 854224943*m^8 +
 73870797*m^9 + 4687683*m^10 + 211939*m^11 + 6461*m^12 + 119*m^13 + m^14 +
1307674368000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 +
2706813345600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7
 + 8207628000*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^
12 + 6580*m^13 + 120*m^14 + m^15 + 1307674368000) + (C*c^3*x^15*(d*x)^m*(28
3465647360*m + 392156797824*m^2 + 310989260400*m^3 + 159721605680*m^4 + 566
63366760*m^5 + 14409322928*m^6 + 2681453775*m^7 + 368411615*m^8 + 37312275*
m^9 + 2749747*m^10 + 143325*m^11 + 5005*m^12 + 105*m^13 + m^14 + 8717829120
0))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 270681334560
0*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000
*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^
13 + 120*m^14 + m^15 + 1307674368000) + (3*B*c*x^10*(d*x)^m*(a*c + b^2)*(42
0839556480*m + 574497805824*m^2 + 448249789800*m^3 + 225856355580*m^4 + 783
81575150*m^5 + 19442163553*m^6 + 3518896600*m^7 + 468873140*m^8 + 45922260*
m^9 + 3263622*m^10 + 163600*m^11 + 5480*m^12 + 110*m^13 + m^14 + 1307674368
00))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 27068133456
00*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 820762800
0*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m
^13 + 120*m^14 + m^15 + 1307674368000) + (3*B*a*x^6*(d*x)^m*(a*c + b^2)*(68
6869545600*m + 913158011520*m^2 + 690639615384*m^3 + 336028955036*m^4 + 112
273858674*m^5 + 26754892001*m^6 + 4646039592*m^7 + 593598068*m^8 + 55749612
```

```
*m^9 + 3801478*m^10 + 183024*m^11 + 5896*m^12 + 114*m^13 + m^14 + 217945728
000))/(4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345
600*m^4 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 82076280
00*m^8 + 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*
m^13 + 120*m^14 + m^15 + 1307674368000) + (3*B*b*c^2*x^12*(d*x)^m*(35251584
4800*m + 484441814160*m^2 + 381046157472*m^3 + 193813932344*m^4 + 679881812
28*m^5 + 17067919121*m^6 + 3130267536*m^7 + 423113372*m^8 + 42081864*m^9 +
3039718*m^10 + 154992*m^11 + 5284*m^12 + 108*m^13 + m^14 + 108972864000))/(
4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4
 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8
+ 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 +
120*m^14 + m^15 + 1307674368000) + (B*b*x^8*(d*x)^m*(6*a*c + b^2)*(52196296
3200*m + 705481831440*m^2 + 543939234048*m^3 + 270359263944*m^4 + 924141053
92*m^5 + 22548638161*m^6 + 4010311424*m^7 + 524664572*m^8 + 50428896*m^9 +
3516198*m^10 + 172928*m^11 + 5684*m^12 + 112*m^13 + m^14 + 163459296000))/(
4339163001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4
 + 1009672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8
+ 928095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 +
120*m^14 + m^15 + 1307674368000) + (3*B*a^2*b*x^4*(d*x)^m*(1003061102400*m
+ 1290689128080*m^2 + 941576643936*m^3 + 441309175416*m^4 + 142090732916*m^
5 + 32678119441*m^6 + 5488252528*m^7 + 679843868*m^8 + 62062968*m^9 + 41238
78*m^10 + 193936*m^11 + 6116*m^12 + 116*m^13 + m^14 + 326918592000))/(43391
63001600*m + 6165817614720*m^2 + 5056995703824*m^3 + 2706813345600*m^4 + 10
09672107080*m^5 + 272803210680*m^6 + 54631129553*m^7 + 8207628000*m^8 + 928
095740*m^9 + 78558480*m^10 + 4899622*m^11 + 218400*m^12 + 6580*m^13 + 120*m
^14 + m^15 + 1307674368000)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

3.38 $\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx$

Optimal. Leaf size=260

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)}+\frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)}+\frac{a(dx)^{m+3}(aC+2A}{d^3(m+3)}$$

```
[Out] a^2*A*(d*x)^(1+m)/d/(1+m)+a^2*B*(d*x)^(2+m)/d^2/(2+m)+a*(2*A*b+C*a)*(d*x)^(
3+m)/d^3/(3+m)+2*a*b*B*(d*x)^(4+m)/d^4/(4+m)+(A*(2*a*c+b^2)+2*a*b*C)*(d*x)^
(5+m)/d^5/(5+m)+B*(2*a*c+b^2)*(d*x)^(6+m)/d^6/(6+m)+(2*A*b*c+(2*a*c+b^2)*C)
*(d*x)^(7+m)/d^7/(7+m)+2*b*B*c*(d*x)^(8+m)/d^8/(8+m)+c*(A*c+2*C*b)*(d*x)^(9
+m)/d^9/(9+m)+B*c^2*(d*x)^(10+m)/d^10/(10+m)+c^2*C*(d*x)^(11+m)/d^11/(11+m)
```

Rubi [A] time = 0.22, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.033, Rules used = {1628}

$$\frac{a^2A(dx)^{m+1}}{d(m+1)}+\frac{a^2B(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}\left(A\left(2ac+b^2\right)+2abC\right)}{d^5(m+5)}+\frac{(dx)^{m+7}\left(C\left(2ac+b^2\right)+2Abc\right)}{d^7(m+7)}+\frac{a(dx)^{m+3}(aC+2A}{d^3(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*A*(d*x)^(1 + m))/(d*(1 + m)) + (a^2*B*(d*x)^(2 + m))/(d^2*(2 + m)) + (
a*(2*A*b + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (2*a*b*B*(d*x)^(4 + m))/(d^4
*(4 + m)) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*
(b^2 + 2*a*c)*(d*x)^(6 + m))/(d^6*(6 + m)) + ((2*A*b*c + (b^2 + 2*a*c)*C)*(
d*x)^(7 + m))/(d^7*(7 + m)) + (2*b*B*c*(d*x)^(8 + m))/(d^8*(8 + m)) + (c*(A
*c + 2*b*C)*(d*x)^(9 + m))/(d^9*(9 + m)) + (B*c^2*(d*x)^(10 + m))/(d^10*(10
 + m)) + (c^2*C*(d*x)^(11 + m))/(d^11*(11 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)^2\,dx=\int \left(a^2A(dx)^m+\frac{a^2B(dx)^{1+m}}{d}+\frac{a(2Ab+aC)(dx)^{2+m}}{d^2}+\frac{2abB(dx)}{d^3}\right.$$

$$=\frac{a^2A(dx)^{1+m}}{d(1+m)}+\frac{a^2B(dx)^{2+m}}{d^2(2+m)}+\frac{a(2Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{2abB(dx)^{4}}{d^4(4+m}$$

Mathematica [A] time = 0.28, size = 185, normalized size = 0.71

$$x(dx)^m\left(\frac{a^2A}{m+1}+\frac{a^2Bx}{m+2}+\frac{x^6\left(C\left(2ac+b^2\right)+2Abc\right)}{m+7}+\frac{x^4\left(A\left(2ac+b^2\right)+2abC\right)}{m+5}+\frac{ax^2(aC+2Ab)}{m+3}+\frac{Bx^5\left(2ac+\right.}{m+6}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(d*x)^m*((a^2*A)/(1 + m) + (a^2*B*x)/(2 + m) + (a*(2*A*b + a*C)*x^2)/(3 +
 m) + (2*a*b*B*x^3)/(4 + m) + ((A*(b^2 + 2*a*c) + 2*a*b*C)*x^4)/(5 + m) + (
```

```
B*(b^2 + 2*a*c)*x^5)/(6 + m) + ((2*A*b*c + (b^2 + 2*a*c)*C)*x^6)/(7 + m) +
(2*b*B*c*x^7)/(8 + m) + (c*(A*c + 2*b*C)*x^8)/(9 + m) + (B*c^2*x^9)/(10 + m
) + (c^2*C*x^10)/(11 + m))
```

fricas [B] time = 1.04, size = 1603, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((C*c^2*m^10 + 55*C*c^2*m^9 + 1320*C*c^2*m^8 + 18150*C*c^2*m^7 + 157773*C*c
^2*m^6 + 902055*C*c^2*m^5 + 3416930*C*c^2*m^4 + 8409500*C*c^2*m^3 + 1275357
6*C*c^2*m^2 + 10628640*C*c^2*m + 3628800*C*c^2)*x^11 + (B*c^2*m^10 + 56*B*c
^2*m^9 + 1365*B*c^2*m^8 + 19020*B*c^2*m^7 + 167223*B*c^2*m^6 + 965328*B*c^2
*m^5 + 3686255*B*c^2*m^4 + 9133180*B*c^2*m^3 + 13926276*B*c^2*m^2 + 1165521
6*B*c^2*m + 3991680*B*c^2)*x^10 + ((2*C*b*c + A*c^2)*m^10 + 57*(2*C*b*c + A
*c^2)*m^9 + 1412*(2*C*b*c + A*c^2)*m^8 + 19962*(2*C*b*c + A*c^2)*m^7 + 1777
65*(2*C*b*c + A*c^2)*m^6 + 1037673*(2*C*b*c + A*c^2)*m^5 + 4000478*(2*C*b*c
 + A*c^2)*m^4 + 9991428*(2*C*b*c + A*c^2)*m^3 + 8870400*C*b*c + 4435200*A*c
^2 + 15335224*(2*C*b*c + A*c^2)*m^2 + 12900960*(2*C*b*c + A*c^2)*m)*x^9 + 2
*(B*b*c*m^10 + 58*B*b*c*m^9 + 1461*B*b*c*m^8 + 20982*B*b*c*m^7 + 189567*B*b
*c*m^6 + 1121022*B*b*c*m^5 + 4371359*B*b*c*m^4 + 11024858*B*b*c*m^3 + 17059
212*B*b*c*m^2 + 14444280*B*b*c*m + 4989600*B*b*c)*x^8 + ((C*b^2 + 2*(C*a +
A*b)*c)*m^10 + 59*(C*b^2 + 2*(C*a + A*b)*c)*m^9 + 1512*(C*b^2 + 2*(C*a + A*
b)*c)*m^8 + 22086*(C*b^2 + 2*(C*a + A*b)*c)*m^7 + 202821*(C*b^2 + 2*(C*a +
A*b)*c)*m^6 + 1217811*(C*b^2 + 2*(C*a + A*b)*c)*m^5 + 4814858*(C*b^2 + 2*(C
*a + A*b)*c)*m^4 + 12291724*(C*b^2 + 2*(C*a + A*b)*c)*m^3 + 5702400*C*b^2 +
 19216008*(C*b^2 + 2*(C*a + A*b)*c)*m^2 + 11404800*(C*a + A*b)*c + 16405920
*(C*b^2 + 2*(C*a + A*b)*c)*m)*x^7 + ((B*b^2 + 2*B*a*c)*m^10 + 60*(B*b^2 + 2
*B*a*c)*m^9 + 1565*(B*b^2 + 2*B*a*c)*m^8 + 23280*(B*b^2 + 2*B*a*c)*m^7 + 21
7743*(B*b^2 + 2*B*a*c)*m^6 + 1331100*(B*b^2 + 2*B*a*c)*m^5 + 5352935*(B*b^2
 + 2*B*a*c)*m^4 + 13878120*(B*b^2 + 2*B*a*c)*m^3 + 6652800*B*b^2 + 13305600
*B*a*c + 21989356*(B*b^2 + 2*B*a*c)*m^2 + 18981840*(B*b^2 + 2*B*a*c)*m)*x^6
 + ((2*C*a*b + A*b^2 + 2*A*a*c)*m^10 + 61*(2*C*a*b + A*b^2 + 2*A*a*c)*m^9 +
 1620*(2*C*a*b + A*b^2 + 2*A*a*c)*m^8 + 24570*(2*C*a*b + A*b^2 + 2*A*a*c)*m
^7 + 234573*(2*C*a*b + A*b^2 + 2*A*a*c)*m^6 + 1464693*(2*C*a*b + A*b^2 + 2*
A*a*c)*m^5 + 6016070*(2*C*a*b + A*b^2 + 2*A*a*c)*m^4 + 15915380*(2*C*a*b +
A*b^2 + 2*A*a*c)*m^3 + 15966720*C*a*b + 7983360*A*b^2 + 15966720*A*a*c + 25
681176*(2*C*a*b + A*b^2 + 2*A*a*c)*m^2 + 22512096*(2*C*a*b + A*b^2 + 2*A*a*
c)*m)*x^5 + 2*(B*a*b*m^10 + 62*B*a*b*m^9 + 1677*B*a*b*m^8 + 25962*B*a*b*m^7
 + 253575*B*a*b*m^6 + 1623258*B*a*b*m^5 + 6846503*B*a*b*m^4 + 18609718*B*a*
b*m^3 + 30819204*B*a*b*m^2 + 27641160*B*a*b*m + 9979200*B*a*b)*x^4 + ((C*a^
2 + 2*A*a*b)*m^10 + 63*(C*a^2 + 2*A*a*b)*m^9 + 1736*(C*a^2 + 2*A*a*b)*m^8 +
 27462*(C*a^2 + 2*A*a*b)*m^7 + 275037*(C*a^2 + 2*A*a*b)*m^6 + 1812447*(C*a^
2 + 2*A*a*b)*m^5 + 7902194*(C*a^2 + 2*A*a*b)*m^4 + 22289148*(C*a^2 + 2*A*a*
b)*m^3 + 13305600*C*a^2 + 26611200*A*a*b + 38390632*(C*a^2 + 2*A*a*b)*m^2 +
 35746080*(C*a^2 + 2*A*a*b)*m)*x^3 + (B*a^2*m^10 + 64*B*a^2*m^9 + 1797*B*a^
2*m^8 + 29076*B*a^2*m^7 + 299271*B*a^2*m^6 + 2039016*B*a^2*m^5 + 9261503*B*
a^2*m^4 + 27472724*B*a^2*m^3 + 50312628*B*a^2*m^2 + 50292720*B*a^2*m + 1995
8400*B*a^2)*x^2 + (A*a^2*m^10 + 65*A*a^2*m^9 + 1860*A*a^2*m^8 + 30810*A*a^2
*m^7 + 326613*A*a^2*m^6 + 2310945*A*a^2*m^5 + 11028590*A*a^2*m^4 + 34967140
*A*a^2*m^3 + 70290936*A*a^2*m^2 + 80627040*A*a^2*m + 39916800*A*a^2)*x)*(d*
x)^m/(m^11 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13
339535*m^5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 3
9916800)
```

giac [B] time = 0.73, size = 3203, normalized size = 12.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c^2*m^10*x^11 + (d*x)^m*B*c^2*m^10*x^10 + 55*(d*x)^m*C*c^2*m^9*x
^11 + 2*(d*x)^m*C*b*c*m^10*x^9 + (d*x)^m*A*c^2*m^10*x^9 + 56*(d*x)^m*B*c^2*
m^9*x^10 + 1320*(d*x)^m*C*c^2*m^8*x^11 + 2*(d*x)^m*B*b*c*m^10*x^8 + 114*(d*
x)^m*C*b*c*m^9*x^9 + 57*(d*x)^m*A*c^2*m^9*x^9 + 1365*(d*x)^m*B*c^2*m^8*x^10
 + 18150*(d*x)^m*C*c^2*m^7*x^11 + (d*x)^m*C*b^2*m^10*x^7 + 2*(d*x)^m*C*a*c*
m^10*x^7 + 2*(d*x)^m*A*b*c*m^10*x^7 + 116*(d*x)^m*B*b*c*m^9*x^8 + 2824*(d*x
)^m*C*b*c*m^8*x^9 + 1412*(d*x)^m*A*c^2*m^8*x^9 + 19020*(d*x)^m*B*c^2*m^7*x^
10 + 157773*(d*x)^m*C*c^2*m^6*x^11 + (d*x)^m*B*b^2*m^10*x^6 + 2*(d*x)^m*B*a
*c*m^10*x^6 + 59*(d*x)^m*C*b^2*m^9*x^7 + 118*(d*x)^m*C*a*c*m^9*x^7 + 118*(d
*x)^m*A*b*c*m^9*x^7 + 2922*(d*x)^m*B*b*c*m^8*x^8 + 39924*(d*x)^m*C*b*c*m^7*
x^9 + 19962*(d*x)^m*A*c^2*m^7*x^9 + 167223*(d*x)^m*B*c^2*m^6*x^10 + 902055*
(d*x)^m*C*c^2*m^5*x^11 + 2*(d*x)^m*C*a*b*m^10*x^5 + (d*x)^m*A*b^2*m^10*x^5
+ 2*(d*x)^m*A*a*c*m^10*x^5 + 60*(d*x)^m*B*b^2*m^9*x^6 + 120*(d*x)^m*B*a*c*m
^9*x^6 + 1512*(d*x)^m*C*b^2*m^8*x^7 + 3024*(d*x)^m*C*a*c*m^8*x^7 + 3024*(d*
x)^m*A*b*c*m^8*x^7 + 41964*(d*x)^m*B*b*c*m^7*x^8 + 355530*(d*x)^m*C*b*c*m^6
*x^9 + 177765*(d*x)^m*A*c^2*m^6*x^9 + 965328*(d*x)^m*B*c^2*m^5*x^10 + 34169
30*(d*x)^m*C*c^2*m^4*x^11 + 2*(d*x)^m*B*a*b*m^10*x^4 + 122*(d*x)^m*C*a*b*m^
9*x^5 + 61*(d*x)^m*A*b^2*m^9*x^5 + 122*(d*x)^m*A*a*c*m^9*x^5 + 1565*(d*x)^m
*B*b^2*m^8*x^6 + 3130*(d*x)^m*B*a*c*m^8*x^6 + 22086*(d*x)^m*C*b^2*m^7*x^7 +
 44172*(d*x)^m*C*a*c*m^7*x^7 + 44172*(d*x)^m*A*b*c*m^7*x^7 + 379134*(d*x)^m
*B*b*c*m^6*x^8 + 2075346*(d*x)^m*C*b*c*m^5*x^9 + 1037673*(d*x)^m*A*c^2*m^5*
x^9 + 3686255*(d*x)^m*B*c^2*m^4*x^10 + 8409500*(d*x)^m*C*c^2*m^3*x^11 + (d*
x)^m*C*a^2*m^10*x^3 + 2*(d*x)^m*A*a*b*m^10*x^3 + 124*(d*x)^m*B*a*b*m^9*x^4
+ 3240*(d*x)^m*C*a*b*m^8*x^5 + 1620*(d*x)^m*A*b^2*m^8*x^5 + 3240*(d*x)^m*A*
a*c*m^8*x^5 + 23280*(d*x)^m*B*b^2*m^7*x^6 + 46560*(d*x)^m*B*a*c*m^7*x^6 + 2
02821*(d*x)^m*C*b^2*m^6*x^7 + 405642*(d*x)^m*C*a*c*m^6*x^7 + 405642*(d*x)^m
*A*b*c*m^6*x^7 + 2242044*(d*x)^m*B*b*c*m^5*x^8 + 8000956*(d*x)^m*C*b*c*m^4*
x^9 + 4000478*(d*x)^m*A*c^2*m^4*x^9 + 9133180*(d*x)^m*B*c^2*m^3*x^10 + 1275
3576*(d*x)^m*C*c^2*m^2*x^11 + (d*x)^m*B*a^2*m^10*x^2 + 63*(d*x)^m*C*a^2*m^9
*x^3 + 126*(d*x)^m*A*a*b*m^9*x^3 + 3354*(d*x)^m*B*a*b*m^8*x^4 + 49140*(d*x)
^m*C*a*b*m^7*x^5 + 24570*(d*x)^m*A*b^2*m^7*x^5 + 49140*(d*x)^m*A*a*c*m^7*x^
5 + 217743*(d*x)^m*B*b^2*m^6*x^6 + 435486*(d*x)^m*B*a*c*m^6*x^6 + 1217811*(
d*x)^m*C*b^2*m^5*x^7 + 2435622*(d*x)^m*C*a*c*m^5*x^7 + 2435622*(d*x)^m*A*b*
c*m^5*x^7 + 8742718*(d*x)^m*B*b*c*m^4*x^8 + 19982856*(d*x)^m*C*b*c*m^3*x^9
+ 9991428*(d*x)^m*A*c^2*m^3*x^9 + 13926276*(d*x)^m*B*c^2*m^2*x^10 + 1062864
0*(d*x)^m*C*c^2*m*x^11 + (d*x)^m*A*a^2*m^10*x + 64*(d*x)^m*B*a^2*m^9*x^2 +
1736*(d*x)^m*C*a^2*m^8*x^3 + 3472*(d*x)^m*A*a*b*m^8*x^3 + 51924*(d*x)^m*B*a
*b*m^7*x^4 + 469146*(d*x)^m*C*a*b*m^6*x^5 + 234573*(d*x)^m*A*b^2*m^6*x^5 +
469146*(d*x)^m*A*a*c*m^6*x^5 + 1331100*(d*x)^m*B*b^2*m^5*x^6 + 2662200*(d*x
)^m*B*a*c*m^5*x^6 + 4814858*(d*x)^m*C*b^2*m^4*x^7 + 9629716*(d*x)^m*C*a*c*m
^4*x^7 + 9629716*(d*x)^m*A*b*c*m^4*x^7 + 22049716*(d*x)^m*B*b*c*m^3*x^8 + 3
0670448*(d*x)^m*C*b*c*m^2*x^9 + 15335224*(d*x)^m*A*c^2*m^2*x^9 + 11655216*(
d*x)^m*B*c^2*m*x^10 + 3628800*(d*x)^m*C*c^2*x^11 + 65*(d*x)^m*A*a^2*m^9*x +
 1797*(d*x)^m*B*a^2*m^8*x^2 + 27462*(d*x)^m*C*a^2*m^7*x^3 + 54924*(d*x)^m*A
*a*b*m^7*x^3 + 507150*(d*x)^m*B*a*b*m^6*x^4 + 2929386*(d*x)^m*C*a*b*m^5*x^5
 + 1464693*(d*x)^m*A*b^2*m^5*x^5 + 2929386*(d*x)^m*A*a*c*m^5*x^5 + 5352935*
(d*x)^m*B*b^2*m^4*x^6 + 10705870*(d*x)^m*B*a*c*m^4*x^6 + 12291724*(d*x)^m*C
*b^2*m^3*x^7 + 24583448*(d*x)^m*C*a*c*m^3*x^7 + 24583448*(d*x)^m*A*b*c*m^3*
x^7 + 34118424*(d*x)^m*B*b*c*m^2*x^8 + 25801920*(d*x)^m*C*b*c*m*x^9 + 12900
960*(d*x)^m*A*c^2*m*x^9 + 3991680*(d*x)^m*B*c^2*x^10 + 1860*(d*x)^m*A*a^2*m
^8*x + 29076*(d*x)^m*B*a^2*m^7*x^2 + 275037*(d*x)^m*C*a^2*m^6*x^3 + 550074*
(d*x)^m*A*a*b*m^6*x^3 + 3246516*(d*x)^m*B*a*b*m^5*x^4 + 12032140*(d*x)^m*C*
a*b*m^4*x^5 + 6016070*(d*x)^m*A*b^2*m^4*x^5 + 12032140*(d*x)^m*A*a*c*m^4*x^
5 + 13878120*(d*x)^m*B*b^2*m^3*x^6 + 27756240*(d*x)^m*B*a*c*m^3*x^6 + 19216
008*(d*x)^m*C*b^2*m^2*x^7 + 38432016*(d*x)^m*C*a*c*m^2*x^7 + 38432016*(d*x)
^m*A*b*c*m^2*x^7 + 28888560*(d*x)^m*B*b*c*m*x^8 + 8870400*(d*x)^m*C*b*c*x^9
```

```
 + 4435200*(d*x)^m*A*c^2*x^9 + 30810*(d*x)^m*A*a^2*m^7*x + 299271*(d*x)^m*B
*a^2*m^6*x^2 + 1812447*(d*x)^m*C*a^2*m^5*x^3 + 3624894*(d*x)^m*A*a*b*m^5*x^
3 + 13693006*(d*x)^m*B*a*b*m^4*x^4 + 31830760*(d*x)^m*C*a*b*m^3*x^5 + 15915
380*(d*x)^m*A*b^2*m^3*x^5 + 31830760*(d*x)^m*A*a*c*m^3*x^5 + 21989356*(d*x)
^m*B*b^2*m^2*x^6 + 43978712*(d*x)^m*B*a*c*m^2*x^6 + 16405920*(d*x)^m*C*b^2*
m*x^7 + 32811840*(d*x)^m*C*a*c*m*x^7 + 32811840*(d*x)^m*A*b*c*m*x^7 + 99792
00*(d*x)^m*B*b*c*x^8 + 326613*(d*x)^m*A*a^2*m^6*x + 2039016*(d*x)^m*B*a^2*m
^5*x^2 + 7902194*(d*x)^m*C*a^2*m^4*x^3 + 15804388*(d*x)^m*A*a*b*m^4*x^3 + 3
7219436*(d*x)^m*B*a*b*m^3*x^4 + 51362352*(d*x)^m*C*a*b*m^2*x^5 + 25681176*(
d*x)^m*A*b^2*m^2*x^5 + 51362352*(d*x)^m*A*a*c*m^2*x^5 + 18981840*(d*x)^m*B*
b^2*m*x^6 + 37963680*(d*x)^m*B*a*c*m*x^6 + 5702400*(d*x)^m*C*b^2*x^7 + 1140
4800*(d*x)^m*C*a*c*x^7 + 11404800*(d*x)^m*A*b*c*x^7 + 2310945*(d*x)^m*A*a^2
*m^5*x + 9261503*(d*x)^m*B*a^2*m^4*x^2 + 22289148*(d*x)^m*C*a^2*m^3*x^3 + 4
4578296*(d*x)^m*A*a*b*m^3*x^3 + 61638408*(d*x)^m*B*a*b*m^2*x^4 + 45024192*(
d*x)^m*C*a*b*m*x^5 + 22512096*(d*x)^m*A*b^2*m*x^5 + 45024192*(d*x)^m*A*a*c*
m*x^5 + 6652800*(d*x)^m*B*b^2*x^6 + 13305600*(d*x)^m*B*a*c*x^6 + 11028590*(
d*x)^m*A*a^2*m^4*x + 27472724*(d*x)^m*B*a^2*m^3*x^2 + 38390632*(d*x)^m*C*a^
2*m^2*x^3 + 76781264*(d*x)^m*A*a*b*m^2*x^3 + 55282320*(d*x)^m*B*a*b*m*x^4 +
 15966720*(d*x)^m*C*a*b*x^5 + 7983360*(d*x)^m*A*b^2*x^5 + 15966720*(d*x)^m*
A*a*c*x^5 + 34967140*(d*x)^m*A*a^2*m^3*x + 50312628*(d*x)^m*B*a^2*m^2*x^2 +
 35746080*(d*x)^m*C*a^2*m*x^3 + 71492160*(d*x)^m*A*a*b*m*x^3 + 19958400*(d*
x)^m*B*a*b*x^4 + 70290936*(d*x)^m*A*a^2*m^2*x + 50292720*(d*x)^m*B*a^2*m*x^
2 + 13305600*(d*x)^m*C*a^2*x^3 + 26611200*(d*x)^m*A*a*b*x^3 + 80627040*(d*x
)^m*A*a^2*m*x + 19958400*(d*x)^m*B*a^2*x^2 + 39916800*(d*x)^m*A*a^2*x)/(m^1
1 + 66*m^10 + 1925*m^9 + 32670*m^8 + 357423*m^7 + 2637558*m^6 + 13339535*m^
5 + 45995730*m^4 + 105258076*m^3 + 150917976*m^2 + 120543840*m + 39916800)
```

maple [B] time = 0.01, size = 2187, normalized size = 8.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x)
```

```
[Out] x*(C*c^2*m^10*x^10+B*c^2*m^10*x^9+55*C*c^2*m^9*x^10+A*c^2*m^10*x^8+56*B*c^2
*m^9*x^9+2*C*b*c*m^10*x^8+1320*C*c^2*m^8*x^10+57*A*c^2*m^9*x^8+2*B*b*c*m^10
*x^7+1365*B*c^2*m^8*x^9+114*C*b*c*m^9*x^8+18150*C*c^2*m^7*x^10+2*A*b*c*m^10
*x^6+1412*A*c^2*m^8*x^8+116*B*b*c*m^9*x^7+19020*B*c^2*m^7*x^9+2*C*a*c*m^10*
x^6+C*b^2*m^10*x^6+2824*C*b*c*m^8*x^8+157773*C*c^2*m^6*x^10+118*A*b*c*m^9*x
^6+19962*A*c^2*m^7*x^8+2*B*a*c*m^10*x^5+B*b^2*m^10*x^5+2922*B*b*c*m^8*x^7+1
67223*B*c^2*m^6*x^9+118*C*a*c*m^9*x^6+59*C*b^2*m^9*x^6+39924*C*b*c*m^7*x^8+
902055*C*c^2*m^5*x^10+2*A*a*c*m^10*x^4+A*b^2*m^10*x^4+3024*A*b*c*m^8*x^6+17
7765*A*c^2*m^6*x^8+120*B*a*c*m^9*x^5+60*B*b^2*m^9*x^5+41964*B*b*c*m^7*x^7+9
65328*B*c^2*m^5*x^9+2*C*a*b*m^10*x^4+3024*C*a*c*m^8*x^6+1512*C*b^2*m^8*x^6+
355530*C*b*c*m^6*x^8+3416930*C*c^2*m^4*x^10+122*A*a*c*m^9*x^4+61*A*b^2*m^9*
x^4+44172*A*b*c*m^7*x^6+1037673*A*c^2*m^5*x^8+2*B*a*b*m^10*x^3+3130*B*a*c*m
^8*x^5+1565*B*b^2*m^8*x^5+379134*B*b*c*m^6*x^7+3686255*B*c^2*m^4*x^9+122*C*
a*b*m^9*x^4+44172*C*a*c*m^7*x^6+22086*C*b^2*m^7*x^6+2075346*C*b*c*m^5*x^8+8
409500*C*c^2*m^3*x^10+2*A*a*b*m^10*x^2+3240*A*a*c*m^8*x^4+1620*A*b^2*m^8*x^
4+405642*A*b*c*m^6*x^6+4000478*A*c^2*m^4*x^8+124*B*a*b*m^9*x^3+46560*B*a*c*
m^7*x^5+23280*B*b^2*m^7*x^5+2242044*B*b*c*m^5*x^7+9133180*B*c^2*m^3*x^9+C*a
^2*m^10*x^2+3240*C*a*b*m^8*x^4+405642*C*a*c*m^6*x^6+202821*C*b^2*m^6*x^6+80
00956*C*b*c*m^4*x^8+12753576*C*c^2*m^2*x^10+126*A*a*b*m^9*x^2+49140*A*a*c*m
^7*x^4+24570*A*b^2*m^7*x^4+2435622*A*b*c*m^5*x^6+9991428*A*c^2*m^3*x^8+B*a^
2*m^10*x+3354*B*a*b*m^8*x^3+435486*B*a*c*m^6*x^5+217743*B*b^2*m^6*x^5+87427
18*B*b*c*m^4*x^7+13926276*B*c^2*m^2*x^9+63*C*a^2*m^9*x^2+49140*C*a*b*m^7*x^
4+2435622*C*a*c*m^5*x^6+1217811*C*b^2*m^5*x^6+19982856*C*b*c*m^3*x^8+106286
40*C*c^2*m*x^10+A*a^2*m^10+3472*A*a*b*m^8*x^2+469146*A*a*c*m^6*x^4+234573*A
*b^2*m^6*x^4+9629716*A*b*c*m^4*x^6+15335224*A*c^2*m^2*x^8+64*B*a^2*m^9*x+51
924*B*a*b*m^7*x^3+2662200*B*a*c*m^5*x^5+1331100*B*b^2*m^5*x^5+22049716*B*b*
```

```
c*m^3*x^7+11655216*B*c^2*m*x^9+1736*C*a^2*m^8*x^2+469146*C*a*b*m^6*x^4+9629
716*C*a*c*m^4*x^6+4814858*C*b^2*m^4*x^6+30670448*C*b*c*m^2*x^8+3628800*C*c^
2*x^10+65*A*a^2*m^9+54924*A*a*b*m^7*x^2+2929386*A*a*c*m^5*x^4+1464693*A*b^2
*m^5*x^4+24583448*A*b*c*m^3*x^6+12900960*A*c^2*m*x^8+1797*B*a^2*m^8*x+50715
0*B*a*b*m^6*x^3+10705870*B*a*c*m^4*x^5+5352935*B*b^2*m^4*x^5+34118424*B*b*c
*m^2*x^7+3991680*B*c^2*x^9+27462*C*a^2*m^7*x^2+2929386*C*a*b*m^5*x^4+245834
48*C*a*c*m^3*x^6+12291724*C*b^2*m^3*x^6+25801920*C*b*c*m*x^8+1860*A*a^2*m^8
+550074*A*a*b*m^6*x^2+12032140*A*a*c*m^4*x^4+6016070*A*b^2*m^4*x^4+38432016
*A*b*c*m^2*x^6+4435200*A*c^2*x^8+29076*B*a^2*m^7*x+3246516*B*a*b*m^5*x^3+27
756240*B*a*c*m^3*x^5+13878120*B*b^2*m^3*x^5+28888560*B*b*c*m*x^7+275037*C*a
^2*m^6*x^2+12032140*C*a*b*m^4*x^4+38432016*C*a*c*m^2*x^6+19216008*C*b^2*m^2
*x^6+8870400*C*b*c*x^8+30810*A*a^2*m^7+3624894*A*a*b*m^5*x^2+31830760*A*a*c
*m^3*x^4+15915380*A*b^2*m^3*x^4+32811840*A*b*c*m*x^6+299271*B*a^2*m^6*x+136
93006*B*a*b*m^4*x^3+43978712*B*a*c*m^2*x^5+21989356*B*b^2*m^2*x^5+9979200*B
*b*c*x^7+1812447*C*a^2*m^5*x^2+31830760*C*a*b*m^3*x^4+32811840*C*a*c*m*x^6+
16405920*C*b^2*m*x^6+326613*A*a^2*m^6+15804388*A*a*b*m^4*x^2+51362352*A*a*c
*m^2*x^4+25681176*A*b^2*m^2*x^4+11404800*A*b*c*x^6+2039016*B*a^2*m^5*x+3721
9436*B*a*b*m^3*x^3+37963680*B*a*c*m*x^5+18981840*B*b^2*m*x^5+7902194*C*a^2*
m^4*x^2+51362352*C*a*b*m^2*x^4+11404800*C*a*c*x^6+5702400*C*b^2*x^6+2310945
*A*a^2*m^5+44578296*A*a*b*m^3*x^2+45024192*A*a*c*m*x^4+22512096*A*b^2*m*x^4
+9261503*B*a^2*m^4*x+61638408*B*a*b*m^2*x^3+13305600*B*a*c*x^5+6652800*B*b^
2*x^5+22289148*C*a^2*m^3*x^2+45024192*C*a*b*m*x^4+11028590*A*a^2*m^4+767812
64*A*a*b*m^2*x^2+15966720*A*a*c*x^4+7983360*A*b^2*x^4+27472724*B*a^2*m^3*x+
55282320*B*a*b*m*x^3+38390632*C*a^2*m^2*x^2+15966720*C*a*b*x^4+34967140*A*a
^2*m^3+71492160*A*a*b*m*x^2+50312628*B*a^2*m^2*x+19958400*B*a*b*x^3+3574608
0*C*a^2*m*x^2+70290936*A*a^2*m^2+26611200*A*a*b*x^2+50292720*B*a^2*m*x+1330
5600*C*a^2*x^2+80627040*A*a^2*m+19958400*B*a^2*x+39916800*A*a^2)*(d*x)^m/(m
+11)/(10+m)/(m+9)/(8+m)/(m+7)/(6+m)/(m+5)/(4+m)/(m+3)/(2+m)/(m+1)
```

maxima [A] time = 1.73, size = 344, normalized size = 1.32

$$\frac{Cc^2d^mx^{11}x^m}{m+11}+\frac{Bc^2d^mx^{10}x^m}{m+10}+\frac{2\,Cbcd^mx^9x^m}{m+9}+\frac{Ac^2d^mx^9x^m}{m+9}+\frac{2\,Bbcd^mx^8x^m}{m+8}+\frac{Cb^2d^mx^7x^m}{m+7}+\frac{2\,Cacd^mx^7x^m}{m+7}+\frac{2\,Abcd^mx}{m+7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] C*c^2*d^m*x^11*x^m/(m + 11) + B*c^2*d^m*x^10*x^m/(m + 10) + 2*C*b*c*d^m*x^9
*x^m/(m + 9) + A*c^2*d^m*x^9*x^m/(m + 9) + 2*B*b*c*d^m*x^8*x^m/(m + 8) + C*
b^2*d^m*x^7*x^m/(m + 7) + 2*C*a*c*d^m*x^7*x^m/(m + 7) + 2*A*b*c*d^m*x^7*x^m
/(m + 7) + B*b^2*d^m*x^6*x^m/(m + 6) + 2*B*a*c*d^m*x^6*x^m/(m + 6) + 2*C*a*
b*d^m*x^5*x^m/(m + 5) + A*b^2*d^m*x^5*x^m/(m + 5) + 2*A*a*c*d^m*x^5*x^m/(m
+ 5) + 2*B*a*b*d^m*x^4*x^m/(m + 4) + C*a^2*d^m*x^3*x^m/(m + 3) + 2*A*a*b*d^
m*x^3*x^m/(m + 3) + B*a^2*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a^2/(d*(m +
 1))
```

mupad [B] time = 1.81, size = 1314, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (x^5*(d*x)^m*(A*b^2 + 2*A*a*c + 2*C*a*b)*(22512096*m + 25681176*m^2 + 15915
380*m^3 + 6016070*m^4 + 1464693*m^5 + 234573*m^6 + 24570*m^7 + 1620*m^8 + 6
1*m^9 + m^10 + 7983360))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 459
95730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9
+ 66*m^10 + m^11 + 39916800) + (x^7*(d*x)^m*(C*b^2 + 2*A*b*c + 2*C*a*c)*(16
405920*m + 19216008*m^2 + 12291724*m^3 + 4814858*m^4 + 1217811*m^5 + 202821
```

```
*m^6 + 22086*m^7 + 1512*m^8 + 59*m^9 + m^10 + 5702400))/(120543840*m + 1509
17976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357
423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*x^6*(d*x)^
m*(2*a*c + b^2)*(18981840*m + 21989356*m^2 + 13878120*m^3 + 5352935*m^4 + 1
331100*m^5 + 217743*m^6 + 23280*m^7 + 1565*m^8 + 60*m^9 + m^10 + 6652800))/
(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5
+ 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 399168
00) + (A*a^2*x*(d*x)^m*(80627040*m + 70290936*m^2 + 34967140*m^3 + 11028590
*m^4 + 2310945*m^5 + 326613*m^6 + 30810*m^7 + 1860*m^8 + 65*m^9 + m^10 + 39
916800))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1333
9535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
 + 39916800) + (c*x^9*(d*x)^m*(A*c + 2*C*b)*(12900960*m + 15335224*m^2 + 99
91428*m^3 + 4000478*m^4 + 1037673*m^5 + 177765*m^6 + 19962*m^7 + 1412*m^8 +
 57*m^9 + m^10 + 4435200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 4
5995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^
9 + 66*m^10 + m^11 + 39916800) + (a*x^3*(d*x)^m*(2*A*b + C*a)*(35746080*m +
 38390632*m^2 + 22289148*m^3 + 7902194*m^4 + 1812447*m^5 + 275037*m^6 + 274
62*m^7 + 1736*m^8 + 63*m^9 + m^10 + 13305600))/(120543840*m + 150917976*m^2
 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 +
 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*c^2*x^10*(d*x)^m*(1
1655216*m + 13926276*m^2 + 9133180*m^3 + 3686255*m^4 + 965328*m^5 + 167223*
m^6 + 19020*m^7 + 1365*m^8 + 56*m^9 + m^10 + 3991680))/(120543840*m + 15091
7976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^6 + 3574
23*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (C*c^2*x^11*(d
*x)^m*(10628640*m + 12753576*m^2 + 8409500*m^3 + 3416930*m^4 + 902055*m^5 +
 157773*m^6 + 18150*m^7 + 1320*m^8 + 55*m^9 + m^10 + 3628800))/(120543840*m
 + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 + 2637558*m^
6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800) + (B*a^2
*x^2*(d*x)^m*(50292720*m + 50312628*m^2 + 27472724*m^3 + 9261503*m^4 + 2039
016*m^5 + 299271*m^6 + 29076*m^7 + 1797*m^8 + 64*m^9 + m^10 + 19958400))/(1
20543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 13339535*m^5 +
2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11 + 39916800
) + (2*B*b*c*x^8*(d*x)^m*(14444280*m + 17059212*m^2 + 11024858*m^3 + 437135
9*m^4 + 1121022*m^5 + 189567*m^6 + 20982*m^7 + 1461*m^8 + 58*m^9 + m^10 + 4
989600))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730*m^4 + 1333
9535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*m^10 + m^11
 + 39916800) + (2*B*a*b*x^4*(d*x)^m*(27641160*m + 30819204*m^2 + 18609718*m
^3 + 6846503*m^4 + 1623258*m^5 + 253575*m^6 + 25962*m^7 + 1677*m^8 + 62*m^9
 + m^10 + 9979200))/(120543840*m + 150917976*m^2 + 105258076*m^3 + 45995730
*m^4 + 13339535*m^5 + 2637558*m^6 + 357423*m^7 + 32670*m^8 + 1925*m^9 + 66*
m^10 + m^11 + 39916800)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.39 $\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=137

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

```
[Out] a*A*(d*x)^(1+m)/d/(1+m)+a*B*(d*x)^(2+m)/d^2/(2+m)+(A*b+C*a)*(d*x)^(3+m)/d^3
/(3+m)+b*B*(d*x)^(4+m)/d^4/(4+m)+(A*c+C*b)*(d*x)^(5+m)/d^5/(5+m)+B*c*(d*x)^
(6+m)/d^6/(6+m)+c*C*(d*x)^(7+m)/d^7/(7+m)
```

Rubi [A] time = 0.09, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {1628}

$$\frac{(dx)^{m+3}(aC+Ab)}{d^3(m+3)}+\frac{aA(dx)^{m+1}}{d(m+1)}+\frac{aB(dx)^{m+2}}{d^2(m+2)}+\frac{(dx)^{m+5}(Ac+bC)}{d^5(m+5)}+\frac{bB(dx)^{m+4}}{d^4(m+4)}+\frac{Bc(dx)^{m+6}}{d^6(m+6)}+\frac{cC(dx)^{m+7}}{d^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*A*(d*x)^(1 + m))/(d*(1 + m)) + (a*B*(d*x)^(2 + m))/(d^2*(2 + m)) + ((A*b
 + a*C)*(d*x)^(3 + m))/(d^3*(3 + m)) + (b*B*(d*x)^(4 + m))/(d^4*(4 + m)) +
((A*c + b*C)*(d*x)^(5 + m))/(d^5*(5 + m)) + (B*c*(d*x)^(6 + m))/(d^6*(6 + m
)) + (c*C*(d*x)^(7 + m))/(d^7*(7 + m))
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int (dx)^m \left(A+Bx+Cx^2\right)\left(a+bx^2+cx^4\right)\,dx = \int \left(aA(dx)^m+\frac{aB(dx)^{1+m}}{d}+\frac{(Ab+aC)(dx)^{2+m}}{d^2}+\frac{bB(dx)^{3+m}}{d^3}+\frac{(\ldots}{\ldots}\right.$$
$$=\frac{aA(dx)^{1+m}}{d(1+m)}+\frac{aB(dx)^{2+m}}{d^2(2+m)}+\frac{(Ab+aC)(dx)^{3+m}}{d^3(3+m)}+\frac{bB(dx)^{4+m}}{d^4(4+m)}+\frac{(A\ldots}{\ldots}$$

Mathematica [A] time = 0.10, size = 90, normalized size = 0.66

$$x(dx)^m\left(\frac{x^2(aC+Ab)}{m+3}+\frac{aA}{m+1}+\frac{aBx}{m+2}+\frac{x^4(Ac+bC)}{m+5}+\frac{bBx^3}{m+4}+\frac{Bcx^5}{m+6}+\frac{cCx^6}{m+7}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(d*x)^m*((a*A)/(1 + m) + (a*B*x)/(2 + m) + ((A*b + a*C)*x^2)/(3 + m) + (b
*B*x^3)/(4 + m) + ((A*c + b*C)*x^4)/(5 + m) + (B*c*x^5)/(6 + m) + (c*C*x^6)
/(7 + m))
```

fricas [B] time = 1.34, size = 444, normalized size = 3.24

$$\frac{\left(\left(Ccm^6+21\,Ccm^5+175\,Ccm^4+735\,Ccm^3+1624\,Ccm^2+1764\,Ccm+720\,Cc\right)x^7+\left(Bcm^6+22\,Bcm^5+190\,Bc\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((C*c*m^6 + 21*C*c*m^5 + 175*C*c*m^4 + 735*C*c*m^3 + 1624*C*c*m^2 + 1764*C*
c*m + 720*C*c)*x^7 + (B*c*m^6 + 22*B*c*m^5 + 190*B*c*m^4 + 820*B*c*m^3 + 18
49*B*c*m^2 + 2038*B*c*m + 840*B*c)*x^6 + ((C*b + A*c)*m^6 + 23*(C*b + A*c)*
m^5 + 207*(C*b + A*c)*m^4 + 925*(C*b + A*c)*m^3 + 2144*(C*b + A*c)*m^2 + 10
08*C*b + 1008*A*c + 2412*(C*b + A*c)*m)*x^5 + (B*b*m^6 + 24*B*b*m^5 + 226*B
*b*m^4 + 1056*B*b*m^3 + 2545*B*b*m^2 + 2952*B*b*m + 1260*B*b)*x^4 + ((C*a +
 A*b)*m^6 + 25*(C*a + A*b)*m^5 + 247*(C*a + A*b)*m^4 + 1219*(C*a + A*b)*m^3
 + 3112*(C*a + A*b)*m^2 + 1680*C*a + 1680*A*b + 3796*(C*a + A*b)*m)*x^3 + (
B*a*m^6 + 26*B*a*m^5 + 270*B*a*m^4 + 1420*B*a*m^3 + 3929*B*a*m^2 + 5274*B*a
*m + 2520*B*a)*x^2 + (A*a*m^6 + 27*A*a*m^5 + 295*A*a*m^4 + 1665*A*a*m^3 + 5
104*A*a*m^2 + 8028*A*a*m + 5040*A*a)*x)*(d*x)^m/(m^7 + 28*m^6 + 322*m^5 + 1
960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

giac [B] time = 0.53, size = 914, normalized size = 6.67

$$\frac{(dx)^m Ccm^6x^7 + (dx)^m Bcm^6x^6 + 21\,(dx)^m Ccm^5x^7 + (dx)^m Cbm^6x^5 + (dx)^m Acm^6x^5 + 22\,(dx)^m Bcm^5x^6 + 17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((d*x)^m*C*c*m^6*x^7 + (d*x)^m*B*c*m^6*x^6 + 21*(d*x)^m*C*c*m^5*x^7 + (d*x)
^m*C*b*m^6*x^5 + (d*x)^m*A*c*m^6*x^5 + 22*(d*x)^m*B*c*m^5*x^6 + 175*(d*x)^m
*C*c*m^4*x^7 + (d*x)^m*B*b*m^6*x^4 + 23*(d*x)^m*C*b*m^5*x^5 + 23*(d*x)^m*A*
c*m^5*x^5 + 190*(d*x)^m*B*c*m^4*x^6 + 735*(d*x)^m*C*c*m^3*x^7 + (d*x)^m*C*a
*m^6*x^3 + (d*x)^m*A*b*m^6*x^3 + 24*(d*x)^m*B*b*m^5*x^4 + 207*(d*x)^m*C*b*m
^4*x^5 + 207*(d*x)^m*A*c*m^4*x^5 + 820*(d*x)^m*B*c*m^3*x^6 + 1624*(d*x)^m*C
*c*m^2*x^7 + (d*x)^m*B*a*m^6*x^2 + 25*(d*x)^m*C*a*m^5*x^3 + 25*(d*x)^m*A*b*
m^5*x^3 + 226*(d*x)^m*B*b*m^4*x^4 + 925*(d*x)^m*C*b*m^3*x^5 + 925*(d*x)^m*A
*c*m^3*x^5 + 1849*(d*x)^m*B*c*m^2*x^6 + 1764*(d*x)^m*C*c*m*x^7 + (d*x)^m*A*
a*m^6*x + 26*(d*x)^m*B*a*m^5*x^2 + 247*(d*x)^m*C*a*m^4*x^3 + 247*(d*x)^m*A*
b*m^4*x^3 + 1056*(d*x)^m*B*b*m^3*x^4 + 2144*(d*x)^m*C*b*m^2*x^5 + 2144*(d*x
)^m*A*c*m^2*x^5 + 2038*(d*x)^m*B*c*m*x^6 + 720*(d*x)^m*C*c*x^7 + 27*(d*x)^m
*A*a*m^5*x + 270*(d*x)^m*B*a*m^4*x^2 + 1219*(d*x)^m*C*a*m^3*x^3 + 1219*(d*x
)^m*A*b*m^3*x^3 + 2545*(d*x)^m*B*b*m^2*x^4 + 2412*(d*x)^m*C*b*m*x^5 + 2412*
(d*x)^m*A*c*m*x^5 + 840*(d*x)^m*B*c*x^6 + 295*(d*x)^m*A*a*m^4*x + 1420*(d*x
)^m*B*a*m^3*x^2 + 3112*(d*x)^m*C*a*m^2*x^3 + 3112*(d*x)^m*A*b*m^2*x^3 + 295
2*(d*x)^m*B*b*m*x^4 + 1008*(d*x)^m*C*b*x^5 + 1008*(d*x)^m*A*c*x^5 + 1665*(d
*x)^m*A*a*m^3*x + 3929*(d*x)^m*B*a*m^2*x^2 + 3796*(d*x)^m*C*a*m*x^3 + 3796*
(d*x)^m*A*b*m*x^3 + 1260*(d*x)^m*B*b*x^4 + 5104*(d*x)^m*A*a*m^2*x + 5274*(d
*x)^m*B*a*m*x^2 + 1680*(d*x)^m*C*a*x^3 + 1680*(d*x)^m*A*b*x^3 + 8028*(d*x)^
m*A*a*m*x + 2520*(d*x)^m*B*a*x^2 + 5040*(d*x)^m*A*a*x)/(m^7 + 28*m^6 + 322*
m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)
```

maple [B] time = 0.00, size = 585, normalized size = 4.27

$$\frac{\left(Cc\,m^6x^6 + Bc\,m^6x^5 + 21Cc\,m^5x^6 + Ac\,m^6x^4 + 22Bc\,m^5x^5 + Cb\,m^6x^4 + 175Cc\,m^4x^6 + 23Ac\,m^5x^4 + Bb\,m^6x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x)
```

```
[Out] x*(C*c*m^6*x^6+B*c*m^6*x^5+21*C*c*m^5*x^6+A*c*m^6*x^4+22*B*c*m^5*x^5+C*b*m^
6*x^4+175*C*c*m^4*x^6+23*A*c*m^5*x^4+B*b*m^6*x^3+190*B*c*m^4*x^5+23*C*b*m^5
*x^4+735*C*c*m^3*x^6+A*b*m^6*x^2+207*A*c*m^4*x^4+24*B*b*m^5*x^3+820*B*c*m^3
*x^5+C*a*m^6*x^2+207*C*b*m^4*x^4+1624*C*c*m^2*x^6+25*A*b*m^5*x^2+925*A*c*m^
```

```
3*x^4+B*a*m^6*x+226*B*b*m^4*x^3+1849*B*c*m^2*x^5+25*C*a*m^5*x^2+925*C*b*m^3
*x^4+1764*C*c*m*x^6+A*a*m^6+247*A*b*m^4*x^2+2144*A*c*m^2*x^4+26*B*a*m^5*x+1
056*B*b*m^3*x^3+2038*B*c*m*x^5+247*C*a*m^4*x^2+2144*C*b*m^2*x^4+720*C*c*x^6
+27*A*a*m^5+1219*A*b*m^3*x^2+2412*A*c*m*x^4+270*B*a*m^4*x+2545*B*b*m^2*x^3+
840*B*c*x^5+1219*C*a*m^3*x^2+2412*C*b*m*x^4+295*A*a*m^4+3112*A*b*m^2*x^2+10
08*A*c*x^4+1420*B*a*m^3*x+2952*B*b*m*x^3+3112*C*a*m^2*x^2+1008*C*b*x^4+1665
*A*a*m^3+3796*A*b*m*x^2+3929*B*a*m^2*x+1260*B*b*x^3+3796*C*a*m*x^2+5104*A*a
*m^2+1680*A*b*x^2+5274*B*a*m*x+1680*C*a*x^2+8028*A*a*m+2520*B*a*x+5040*A*a)
*(d*x)^m/(m+7)/(m+6)/(m+5)/(m+4)/(m+3)/(m+2)/(m+1)
```

maxima [A] time = 0.82, size = 155, normalized size = 1.13

$$\frac{Ccd^m x^7 x^m}{m+7}+\frac{Bcd^m x^6 x^m}{m+6}+\frac{Cbd^m x^5 x^m}{m+5}+\frac{Acd^m x^5 x^m}{m+5}+\frac{Bbd^m x^4 x^m}{m+4}+\frac{Cad^m x^3 x^m}{m+3}+\frac{Abd^m x^3 x^m}{m+3}+\frac{Bad^m x^2 x^m}{m+2}+\frac{(dx)^{m+1} Aa}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] C*c*d^m*x^7*x^m/(m + 7) + B*c*d^m*x^6*x^m/(m + 6) + C*b*d^m*x^5*x^m/(m + 5)
 + A*c*d^m*x^5*x^m/(m + 5) + B*b*d^m*x^4*x^m/(m + 4) + C*a*d^m*x^3*x^m/(m +
 3) + A*b*d^m*x^3*x^m/(m + 3) + B*a*d^m*x^2*x^m/(m + 2) + (d*x)^(m + 1)*A*a
/(d*(m + 1))
```

mupad [B] time = 1.07, size = 527, normalized size = 3.85

$$\frac{x^3\,{\left(d\,x\right)}^m\,\left(A\,b+C\,a\right)\,\left(m^6+25\,m^5+247\,m^4+1219\,m^3+3112\,m^2+3796\,m+1680\right)}{m^7+28\,m^6+322\,m^5+1960\,m^4+6769\,m^3+13132\,m^2+13068\,m+5040}+\frac{x^5\,{\left(d\,x\right)}^m\,\left(A\,c+C\,b\right)\,\left(m^6 \right.}{m^7+28\,m^6+322\,m^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(A + B*x + C*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (x^3*(d*x)^m*(A*b + C*a)*(3796*m + 3112*m^2 + 1219*m^3 + 247*m^4 + 25*m^5 +
 m^6 + 1680))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6
 + m^7 + 5040) + (x^5*(d*x)^m*(A*c + C*b)*(2412*m + 2144*m^2 + 925*m^3 + 20
7*m^4 + 23*m^5 + m^6 + 1008))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 +
322*m^5 + 28*m^6 + m^7 + 5040) + (A*a*x*(d*x)^m*(8028*m + 5104*m^2 + 1665*m
^3 + 295*m^4 + 27*m^5 + m^6 + 5040))/(13068*m + 13132*m^2 + 6769*m^3 + 1960
*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*a*x^2*(d*x)^m*(5274*m + 3929*m^2
 + 1420*m^3 + 270*m^4 + 26*m^5 + m^6 + 2520))/(13068*m + 13132*m^2 + 6769*m
^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*b*x^4*(d*x)^m*(2952*m +
 2545*m^2 + 1056*m^3 + 226*m^4 + 24*m^5 + m^6 + 1260))/(13068*m + 13132*m^2
 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (B*c*x^6*(d*x)^m*
(2038*m + 1849*m^2 + 820*m^3 + 190*m^4 + 22*m^5 + m^6 + 840))/(13068*m + 13
132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040) + (C*c*x^7*(
d*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6 + 720))/(13068
*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)
```

sympy [A] time = 2.58, size = 3735, normalized size = 27.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-A*a/(6*x**6) - A*b/(4*x**4) - A*c/(2*x**2) - B*a/(5*x**5) - B*
b/(3*x**3) - B*c/x - C*a/(4*x**4) - C*b/(2*x**2) + C*c*log(x))/d**7, Eq(m,
-7)), ((-A*a/(5*x**5) - A*b/(3*x**3) - A*c/x - B*a/(4*x**4) - B*b/(2*x**2)
+ B*c*log(x) - C*a/(3*x**3) - C*b/x + C*c*x)/d**6, Eq(m, -6)), ((-A*a/(4*x*
*4) - A*b/(2*x**2) + A*c*log(x) - B*a/(3*x**3) - B*b/x + B*c*x - C*a/(2*x**
```

```
2) + C*b*log(x) + C*c*x**2/2)/d**5, Eq(m, -5)), ((-A*a/(3*x**3) - A*b/x + A
*c*x - B*a/(2*x**2) + B*b*log(x) + B*c*x**2/2 - C*a/x + C*b*x + C*c*x**3/3)
/d**4, Eq(m, -4)), ((-A*a/(2*x**2) + A*b*log(x) + A*c*x**2/2 - B*a/x + B*b*
x + B*c*x**3/3 + C*a*log(x) + C*b*x**2/2 + C*c*x**4/4)/d**3, Eq(m, -3)), ((
-A*a/x + A*b*x + A*c*x**3/3 + B*a*log(x) + B*b*x**2/2 + B*c*x**4/4 + C*a*x
+ C*b*x**3/3 + C*c*x**5/5)/d**2, Eq(m, -2)), ((A*a*log(x) + A*b*x**2/2 + A*
c*x**4/4 + B*a*x + B*b*x**3/3 + B*c*x**5/5 + C*a*x**2/2 + C*b*x**4/4 + C*c*
x**6/6)/d, Eq(m, -1)), (A*a*d**m*m**6*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1
960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 27*A*a*d**m*m**5*x*x*
*m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*
m + 5040) + 295*A*a*d**m*m**4*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1665*A*a*d**m*m**3*x*x**m/(m*
*7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 50
40) + 5104*A*a*d**m*m**2*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 67
69*m**3 + 13132*m**2 + 13068*m + 5040) + 8028*A*a*d**m*m*x*x**m/(m**7 + 28*
m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 50
40*A*a*d**m*x*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 131
32*m**2 + 13068*m + 5040) + A*b*d**m*m**6*x**3*x**m/(m**7 + 28*m**6 + 322*m
**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 25*A*b*d**m*m*
*5*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**
2 + 13068*m + 5040) + 247*A*b*d**m*m**4*x**3*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1219*A*b*d**m*m*
*3*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**
2 + 13068*m + 5040) + 3112*A*b*d**m*m**2*x**3*x**m/(m**7 + 28*m**6 + 322*m*
*5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3796*A*b*d**m*m
*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
+ 13068*m + 5040) + 1680*A*b*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 19
60*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + A*c*d**m*m**6*x**5*x**
m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
 + 5040) + 23*A*c*d**m*m**5*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**
4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 207*A*c*d**m*m**4*x**5*x**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) + 925*A*c*d**m*m**3*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2144*A*c*d**m*m**2*x**5*x**m/
(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
 5040) + 2412*A*c*d**m*m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1008*A*c*d**m*x**5*x**m/(m**7 +
 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040)
+ B*a*d**m*m**6*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**
3 + 13132*m**2 + 13068*m + 5040) + 26*B*a*d**m*m**5*x**2*x**m/(m**7 + 28*m*
*6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 270*
B*a*d**m*m**4*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
+ 13132*m**2 + 13068*m + 5040) + 1420*B*a*d**m*m**3*x**2*x**m/(m**7 + 28*m*
*6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3929
*B*a*d**m*m**2*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
 + 13132*m**2 + 13068*m + 5040) + 5274*B*a*d**m*m*x**2*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2520*B
*a*d**m*x**2*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 1313
2*m**2 + 13068*m + 5040) + B*b*d**m*m**6*x**4*x**m/(m**7 + 28*m**6 + 322*m*
*5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 24*B*b*d**m*m**
5*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 226*B*b*d**m*m**4*x**4*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1056*B*b*d**m*m**
3*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2
 + 13068*m + 5040) + 2545*B*b*d**m*m**2*x**4*x**m/(m**7 + 28*m**6 + 322*m**
5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2952*B*b*d**m*m*
x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
 13068*m + 5040) + 1260*B*b*d**m*x**4*x**m/(m**7 + 28*m**6 + 322*m**5 + 196
0*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + B*c*d**m*m**6*x**6*x**m
```

```
/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m
+ 5040) + 22*B*c*d**m*m**5*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 190*B*c*d**m*m**4*x**6*x**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 820*B*c*d**m*m**3*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4
+ 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1849*B*c*d**m*m**2*x**6*x**m/(
m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m +
5040) + 2038*B*c*d**m*m*x**6*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 840*B*c*d**m*x**6*x**m/(m**7 + 2
8*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) +
C*a*d**m*m**6*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3
+ 13132*m**2 + 13068*m + 5040) + 25*C*a*d**m*m**5*x**3*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 247*C*
a*d**m*m**4*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
13132*m**2 + 13068*m + 5040) + 1219*C*a*d**m*m**3*x**3*x**m/(m**7 + 28*m**6
 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 3112*C
*a*d**m*m**2*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 +
 13132*m**2 + 13068*m + 5040) + 3796*C*a*d**m*m*x**3*x**m/(m**7 + 28*m**6 +
 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 1680*C*a
*d**m*x**3*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*
m**2 + 13068*m + 5040) + C*b*d**m*m**6*x**5*x**m/(m**7 + 28*m**6 + 322*m**5
 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 23*C*b*d**m*m**5*
x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
 13068*m + 5040) + 207*C*b*d**m*m**4*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 925*C*b*d**m*m**3*x
**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 +
13068*m + 5040) + 2144*C*b*d**m*m**2*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 +
 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + 2412*C*b*d**m*m*x**
5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13
068*m + 5040) + 1008*C*b*d**m*x**5*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m
**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040) + C*c*d**m*m**6*x**7*x**m/(m
**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5
040) + 21*C*c*d**m*m**5*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 +
6769*m**3 + 13132*m**2 + 13068*m + 5040) + 175*C*c*d**m*m**4*x**7*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) + 735*C*c*d**m*m**3*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6
769*m**3 + 13132*m**2 + 13068*m + 5040) + 1624*C*c*d**m*m**2*x**7*x**m/(m**
7 + 28*m**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 504
0) + 1764*C*c*d**m*m*x**7*x**m/(m**7 + 28*m**6 + 322*m**5 + 1960*m**4 + 676
9*m**3 + 13132*m**2 + 13068*m + 5040) + 720*C*c*d**m*x**7*x**m/(m**7 + 28*m
**6 + 322*m**5 + 1960*m**4 + 6769*m**3 + 13132*m**2 + 13068*m + 5040), True
))
```

3.40 $\int \frac{(dx)^m\left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=368

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+$$

```
[Out] (d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))*(C+(2*A*c-C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b-(-4*a*c+b^2)^(1/2))+2*
B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))/d^2/(2+m)/(b-(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(1/2)+(d*x)^(1+m)*hype
rgeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(C+(-2*A*
c+C*b)/(-4*a*c+b^2)^(1/2))/d/(1+m)/(b+(-4*a*c+b^2)^(1/2))-2*B*c*(d*x)^(2+m)
*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))/d^2/(2+m
)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A] time = 0.62, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1662, 1285, 364, 12, 1131}

$$\frac{(dx)^{m+1}\left(\frac{2Ac-bC}{\sqrt{b^2-4ac}}+C\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{d(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(dx)^{m+1}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{d(m+1)\left(\sqrt{b^2-4ac}+b\right)}+$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((C + (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*d*(1 + m)) + ((C - (2*A*c - b*C)/Sqrt[b^2 - 4*a*c])*(d*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*d*(1 + m)) + (2*B*c*(d*x)^(2 + m)*Hypergeometr
ic2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b
^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c])*d^2*(2 + m)) - (2*B*c*(d*x)^(2 + m)*Hyp
ergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
)/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1131

```
Int[((d_.)*(x_))^(m_.)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[(d*x)^m/(b/2 - q/2 + c*x^2), x]
, x] - Dist[c/q, Int[(d*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c
, d, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{a+bx^2+cx^4}\,dx = \frac{\int \frac{B(dx)^{1+m}}{a+bx^2+cx^4}\,dx}{d} + \int \frac{(dx)^m \left(A+Cx^2\right)}{a+bx^2+cx^4}\,dx$$

$$= \frac{1}{2}\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{(dx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}-\frac{1}{2}}$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F}{\left(b+\sqrt{b^2-4\right.}$$

$$= \frac{\left(C+\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)d(1+m)} + \frac{\left(C-\frac{2Ac-bC}{\sqrt{b^2-4ac}}\right)(dx)^{1+m}\,{}_2F}{\left(b+\sqrt{b^2-4\right.}$$

Mathematica [C] time = 0.47, size = 438, normalized size = 1.19

$$\frac{(dx)^m\left(A\left(m^2+3m+2\right)\text{RootSum}\left[\#1^4c+\#1^2b+a\&,\frac{\left(\frac{x}{x-\#1}\right)^{-m}{}_2F_1\left(-m,-m;1-m;-\frac{\#1}{x-\#1}\right)}{2\#1^3c+\#1b}\&\right]+B(m+2)\text{RootSum}\left[\#1^4c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d*x)^m*(A*(2 + 3*m + m^2)*RootSum[a + b*#1^2 + c*#1^4 & , Hypergeometric2
F1[-m, -m, 1 - m, -(#1/(x - #1))]/((x/(x - #1))^m*(b*#1 + 2*c*#1^3)) & ] +
B*(2 + m)*RootSum[a + b*#1^2 + c*#1^4 & , (m*x + (Hypergeometric2F1[-m, -m,
 1 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m + (m*Hypergeometric2F1[-m, -m, 1
 - m, -(#1/(x - #1))]*#1)/(x/(x - #1))^m)/(b*#1 + 2*c*#1^3) & ] + C*RootSum
[a + b*#1^2 + c*#1^4 & , (m*x^2 + m^2*x^2 + 2*m*x*#1 + m^2*x*#1 + (2*Hyperg
eometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (3*m*Hype
rgeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m^2*Hy
pergeometric2F1[-m, -m, 1 - m, -(#1/(x - #1))]*#1^2)/(x/(x - #1))^m + (m*#1
^2)/(x/#1)^m)/(b*#1 + 2*c*#1^3) & ]))/(2*m*(1 + m)*(2 + m))
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(Cx^2+Bx+A\right)(dx)^m}{cx^4+bx^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)(dx)^m}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{\left(C\,x^2 + Bx + A\right)(dx)^m}{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^2 + Bx + A\right)(dx)^m}{cx^4 + bx^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(d\,x)^m\,\left(C\,x^2 + B\,x + A\right)}{c\,x^4 + b\,x^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4),x)
[Out] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx)^m\left(A + Bx + Cx^2\right)}{a + bx^2 + cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a),x)
[Out] Integral((d*x)**m*(A + B*x + C*x**2)/(a + b*x**2 + c*x**4), x)
```

3.41 $\int \frac{(dx)^m \left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2} \, dx$

Optimal. Leaf size=685

$$\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right) {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\right.}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$

```
[Out] 1/2*B*(d*x)^(2+m)*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d^2/(c*x^4+b*x^2+a)+1/
2*(d*x)^(1+m)*(A*(-2*a*c+b^2)-a*b*C+c*(A*b-2*C*a)*x^2)/a/(-4*a*c+b^2)/d/(c*
x^4+b*x^2+a)+1/2*B*c*(d*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],-2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b*m*(b-(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+
b^2)^(3/2)/d^2/(2+m)/(b+(-4*a*c+b^2)^(1/2))-1/2*B*c*(d*x)^(2+m)*hypergeom([
1, 1+1/2*m],[2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(4*a*c*(2-m)+b*m*(b+
(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d^2/(2+m)/(b-(-4*a*c+b^2)^(1/2))-
1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))*(2*a*C*(2*b+(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^2*(1-m)-4*a*c*(3-m)
-b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b+(-4*a*c+b^2)^
(1/2))+1/2*c*(d*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)))*(2*a*C*(2*b-(1-m)*(-4*a*c+b^2)^(1/2))+A*(b^2*(1-m)-4*a*
c*(3-m)+b*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/d/(1+m)/(b-(-4*a*
c+b^2)^(1/2))
```

Rubi [A] time = 2.38, antiderivative size = 670, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1662, 1277, 1285, 364, 12, 1121}

$$\frac{c(dx)^{m+1}\left(A\left(b(1-m)\sqrt{b^2-4ac}-4ac(3-m)+b^2(1-m)\right)+2aC\left(2b-(1-m)\sqrt{b^2-4ac}\right)\right) {}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\right.}{2ad(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(d*x)^(2 + m)*(b^2 - 2*a*c + b*c*x^2))/(2*a*(b^2 - 4*a*c)*d^2*(a + b*x^2
 + c*x^4)) + ((d*x)^(1 + m)*(A*(b^2 - 2*a*c) - a*b*C + c*(A*b - 2*a*C)*x^2)
)/(2*a*(b^2 - 4*a*c)*d*(a + b*x^2 + c*x^4)) + (c*(2*a*C*(2*b - Sqrt[b^2 - 4
*a*c]*(1 - m)) + A*(b^2*(1 - m) + b*Sqrt[b^2 - 4*a*c]*(1 - m) - 4*a*c*(3 -
m)))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*d*
(1 + m)) - (c*(4*a*b*C + A*b^2*(1 - m) - Sqrt[b^2 - 4*a*c]*(A*b - 2*a*C)*(1
 - m) - 4*a*A*c*(3 - m))*(d*x)^(1 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 +
 m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + S
qrt[b^2 - 4*a*c])*d*(1 + m)) - (B*c*(4*a*c*(2 - m) + b*(b + Sqrt[b^2 - 4*a*
c])*m)*(d*x)^(2 + m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*
d^2*(2 + m)) + (B*c*(4*a*c*(2 - m) + b*(b - Sqrt[b^2 - 4*a*c])*m)*(d*x)^(2
+ m)*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*d^2*(2 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1121

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[((d*x)^(m + 1)*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1
))/(2*a*d*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)),
Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m +
 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || In
tegerQ[m])
```

Rule 1277

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)*
(d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2))/(2*a*f*(p + 1)*(b^2 - 4*a*
c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*x^
4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) - a
*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Intege
rQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1285

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{(dx)^m (A + Bx + Cx^2)}{(a + bx^2 + cx^4)^2} dx = \frac{\int \frac{B(dx)^{1+m}}{(a+bx^2+cx^4)^2} dx}{d} + \int \frac{(dx)^m (A + Cx^2)}{(a + bx^2 + cx^4)^2} dx$$

$$= \frac{(dx)^{1+m} (A (b^2 - 2ac) - abC + c(Ab - 2aC)x^2)}{2a (b^2 - 4ac) d (a + bx^2 + cx^4)} - \frac{\int \frac{(dx)^m(-Ab^2(1-m)+2aAc(3-m)-abC(}{a+bx^2+cx^4}}{2a (b^2 - 4}$$

$$= \frac{B(dx)^{2+m} (b^2 - 2ac + bcx^2)}{2a (b^2 - 4ac) d^2 (a + bx^2 + cx^4)} + \frac{(dx)^{1+m} (A (b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a (b^2 - 4ac) d (a + bx^2 + cx^4)}$$

$$= \frac{B(dx)^{2+m} (b^2 - 2ac + bcx^2)}{2a (b^2 - 4ac) d^2 (a + bx^2 + cx^4)} + \frac{(dx)^{1+m} (A (b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a (b^2 - 4ac) d (a + bx^2 + cx^4)}$$

$$= \frac{B(dx)^{2+m} (b^2 - 2ac + bcx^2)}{2a (b^2 - 4ac) d^2 (a + bx^2 + cx^4)} + \frac{(dx)^{1+m} (A (b^2 - 2ac) - abC + c(Ab - 2aC)x^2}{2a (b^2 - 4ac) d (a + bx^2 + cx^4)}$$

Mathematica [C] time = 0.33, size = 242, normalized size = 0.35

$$\frac{x(dx)^m \left(A \left(m^2 + 5m + 6\right) F_1 \left(\frac{m+1}{2}; 2, 2; \frac{m+3}{2}; -\frac{2cx^2}{b+\sqrt{b^2-4ac}}, \frac{2cx^2}{\sqrt{b^2-4ac}-b}\right) + (m+1)x \left(B(m+3)F_1 \left(\frac{m+2}{2}; 2, 2; \frac{m+4}{2}; -\frac{}{b+}\right.\right.\right.}{a^2(m+1)(m+2)(m+3)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(d*x)^m*(A*(6 + 5*m + m^2)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x
*(B*(3 + m)*AppellF1[(2 + m)/2, 2, 2, (4 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + C*(2 + m)*x*AppellF1[(3 + m
)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + S
qrt[b^2 - 4*a*c])])))/(a^2*(1 + m)*(2 + m)*(3 + m))
```

fricas [F] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(Cx^2 + Bx + A)(dx)^m}{c^2x^8 + 2\,bcx^6 + (b^2 + 2\,ac)x^4 + 2\,abx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] integral((C*x^2 + B*x + A)*(d*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4
 + 2*a*b*x^2 + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(Cx^2 + Bx + A)(dx)^m}{(cx^4 + bx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{\left(C\,x^{2}+Bx+A\right)\left(dx\right)^{m}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
[Out] int((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(Cx^{2}+Bx+A\right)\left(dx\right)^{m}}{\left(cx^{4}+bx^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^m*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((C*x^2 + B*x + A)*(d*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d\,x\right)^{m}\,\left(C\,x^{2}+B\,x+A\right)}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
[Out] int(((d*x)^m*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**m*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.42 $\int \frac{x^2(A+Bx+Cx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx = \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4} d}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}dx, x, x^2\right) - \frac{(2Ac-}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b^2}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

Mathematica [A] time = 1.04, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\text{ta}}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 7.04, size = 4440, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
```

```
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.00, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a
+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b
^2)^(1/2)*B*b*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((-
```

```
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*A+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*C*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b-1/
4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^
(1/2)*B*b*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*A*b*c*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)+2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*A*a*c^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*A*b^2*c*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*a*c*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*b^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*a*b
*c*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*b^3*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(C*x^2+B*x+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 0.00, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x + C*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
```

```
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
```

```
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(C*x**2+B*x+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.43 $\int \frac{x(Ax+Bx^2+Cx^3)}{(a+bx^2+cx^4)^2} dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C(4ac+b^2)}{\sqrt{b^2-4ac}}+2Ac-b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{x\left(Ax+Bx^2+Cx^3\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\mathrm{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(}{\sqrt{b^2}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(}{\sqrt{b^2}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(}{\sqrt{b^2}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

Mathematica [A] time = 0.43, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\mathrm{ta}}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 7.13, size = 4440, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) - (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.04, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a
+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b
```

```
^2)^(1/2)*B*b*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*A+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*C*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b-1/
4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^
(1/2)*B*b*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*A*b*c*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)+2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*A*a*c^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*A*b^2*c*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*a*c*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*b^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*a*b
*c*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*b^3*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x^3+B*x^2+A*x)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 1.55, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A*x + B*x^2 + C*x^3))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
```

```
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
```

```
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(C*x**3+B*x**2+A*x)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.44 $\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-b\cdots\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\cdots}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.323, Rules used = {1594, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-b\cdots\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1594

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.))^(n_.), x
_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p) + c*x^(r - p))^n, x] /; FreeQ[{a
, b, c, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && PosQ[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{Ax^2+Bx^3+Cx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-b\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+\right.}{\sqrt{b^2-4\ldots}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4\ldots\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+\right.}{\sqrt{b^2-4\ldots}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4\ldots\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-\left(b^2+\right.}{\sqrt{b^2-4\ldots}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4\ldots\right.}$$

Mathematica [A] time = 0.15, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\text{ta}\ldots}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 6.90, size = 4439, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) + (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.04, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a
+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b
```

```
^2)^(1/2)*B*b*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*A+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*C*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b-1/
4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^
(1/2)*B*b*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*A*b*c*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)+2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*A*a*c^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*A*b^2*c*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*a*c*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*b^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*a*b
*c*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*b^3*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^4+B*x^3+A*x^2)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 1.47, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x^2 + B*x^3 + C*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
```

```
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
```

```
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**4+B*x**3+A*x**2)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.45 $\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{Ax^3+Bx^4+Cx^5}{x\left(a+bx^2+cx^4\right)^2}\,dx=\int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=\int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx+\int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$=-\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx+\frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,dx}{2\left(b^2-4ac\right)}$$

$$=-\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx,x,x^2\right)-\frac{\left(2Ac-\right.}{}$$

$$=\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2Ac-bC-\frac{4Abc-\left(b^2\right.}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4\right.}$$

$$=\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2Ac-bC-\frac{4Abc-\left(b^2\right.}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4\right.}$$

$$=\frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(2Ac-bC-\frac{4Abc-\left(b^2\right.}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4\right.}$$

Mathematica [A] time = 0.15, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\text{ta}}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

[Out] Timed out

giac [B] time = 7.31, size = 4439, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) + (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.03, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a
+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b
```

```
^2)^(1/2)*B*b*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*A+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*C*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b-1/
4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^
(1/2)*B*b*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*A*b*c*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)+2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*A*a*c^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*A*b^2*c*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*a*c*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*b^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*a*b
*c*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*b^3*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2Ac)x^3 + 2Ba + (2Ca - Ab)x}{2\left(\left(b^2c - 4ac^2\right)x^4 + ab^2 - 4a^2c + \left(b^3 - 4abc\right)x^2\right)} - \frac{-\int \frac{2Bbx + (Cb - 2Ac)x^2 - 2Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^5+B*x^4+A*x^3)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 1.39, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x^3 + B*x^4 + C*x^5)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
```

```
z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
 192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
 3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
 + (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192
```

```
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**5+B*x**4+A*x**3)/x/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.46 $\int \frac{Ax^4+Bx^5+Cx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=356

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

```
[Out] 1/2*B*(b*x^2+2*a)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*x*(A*b-2*a*C+(2*A*c-C*b)
*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-b*B*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)^(3/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2))*(2*A*c-b*C+(-4*A*b*c+(4*a*c+b^2)*C)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)
*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-b*C+(4*A*b*c-(4*a*c+b^2)*C)/(-4*a*c+b^
2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1585, 1662, 1275, 1166, 205, 12, 1114, 638, 618, 206}

$$-\frac{x\left(-2aC+x^2(2Ac-bC)+Ab\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\left(-\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-bC\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{4Abc-C\left(4ac+b^2\right)}{\sqrt{b^2-4ac}}+2Ac-b\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (B*(2*a + b*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (x*(A*b - 2*a*C +
 (2*A*c - b*C)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*A*c - b*C
- (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - ((2*A*c - b*C + (4*A*b*c - (b^2 + 4*a*c)*C)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) - (b*B*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1114

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dis
t[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, b, c, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1585

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.
))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p) + c*x^(r - p))^n,
 x] /; FreeQ[{a, b, c, m, p, q, r}, x] && IntegerQ[n] && PosQ[q - p] && Pos
Q[r - p]
```

Rule 1662

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_S
ymbol] :> Module[{q = Expon[Pq, x], k}, Int[(d*x)^m*Sum[Coeff[Pq, x, 2*k]*x
^(2*k), {k, 0, q/2 + 1}]*(a + b*x^2 + c*x^4)^p, x] + Dist[1/d, Int[(d*x)^(m
 + 1)*Sum[Coeff[Pq, x, 2*k + 1]*x^(2*k), {k, 0, (q - 1)/2 + 1}]*(a + b*x^2
+ c*x^4)^p, x], x]] /; FreeQ[{a, b, c, d, m, p}, x] && PolyQ[Pq, x] &&  !Po
lyQ[Pq, x^2]
```

Rubi steps

$$\int \frac{Ax^4+Bx^5+Cx^6}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = \int \frac{x^2\left(A+Bx+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= \int \frac{Bx^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \int \frac{x^2\left(A+Cx^2\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + B\int \frac{x^3}{\left(a+bx^2+cx^4\right)^2}\,dx + \frac{\int \frac{Ab-2aC+(-2Ac+bC)x^2}{a+bx^2+cx^4}\,d}{2\left(b^2-4ac\right)}$$

$$= -\frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{1}{2}B\,\text{Subst}\left(\int \frac{x}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right) - \frac{\left(2Ac-\right.}{}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

$$= \frac{B\left(2a+bx^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{x\left(Ab-2aC+(2Ac-bC)x^2\right)}{2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2Ac-bC-\frac{4Abc-(b}{\sqrt{b^2-}}\right.}{2\sqrt{2}\sqrt{c}\left(b^2-\right.}$$

Mathematica [A] time = 0.15, size = 378, normalized size = 1.06

$$\frac{1}{4}\left(\frac{4a(B+Cx)+2x\left(bx(B+Cx)-A\left(b+2cx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\left(C\left(b\sqrt{b^2-4ac}-4ac-b^2\right)-2Ac\left(\sqrt{b^2-4ac}-2b\right)\right)\text{ta}}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((4*a*(B + C*x) + 2*x*(b*x*(B + C*x) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[2]*(-2*A*c*(-2*b + Sqrt[b^2 - 4*a*c]) + (-b^2 - 4
*a*c + b*Sqrt[b^2 - 4*a*c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (S
qrt[2]*(-2*A*c*(2*b + Sqrt[b^2 - 4*a*c]) + (b^2 + 4*a*c + b*Sqrt[b^2 - 4*a*
c])*C)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b
^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (2*b*B*Log[-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2])/(b^2 - 4*a*c)^(3/2) - (2*b*B*Log[b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/4
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 6.84, size = 4439, normalized size = 12.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(C*b*x^3 - 2*A*c*x^3 + B*b*x^2 + 2*C*a*x - A*b*x + 2*B*a)/((c*x^4 + b*x
^2 + a)*(b^2 - 4*a*c)) - 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 -
 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3
+ 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2
)*(b^2 - 4*a*c)^2*C - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 +
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^
3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) + 4*(sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*
c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*
C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*
a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c
^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*
(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt
((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c
^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^
3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)
*abs(b^2 - 4*a*c)*abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^
2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*
c^2)*(b^2 - 4*a*c)^2*C + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c -
 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3
 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)
*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b
^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^
3)*C*abs(b^2 - 4*a*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 -
 4*a*c)*a*b^2*c^4)*A + (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*
b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 +
32*(b^2 - 4*a*c)*a^2*b*c^4)*C)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c - s
qrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*
a*c^2)))/((a*b^6*c - 12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2
*b^3*c^3 + a*b^4*c^3 - 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c
^5)*abs(b^2 - 4*a*c)*abs(c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*
c^3 + (b^3*c - 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2
- 4*a*c) - (b^6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3
+ b^4*c^3 - 4*a*b^2*c^4 + (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(
b^2 - 4*a*c))*B)*log(x^2 + 1/2*(b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2 - 4*
(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^
2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs
(b^2 - 4*a*c)) + 1/8*((b^4*c - 4*a*b^2*c^2 - 2*b^3*c^2 + b^2*c^3 - (b^3*c -
 4*a*b*c^2 - 2*b^2*c^2 + b*c^3)*sqrt(b^2 - 4*a*c))*B*abs(b^2 - 4*a*c) + (b^
6*c - 8*a*b^4*c^2 - 2*b^5*c^2 + 16*a^2*b^2*c^3 + 8*a*b^3*c^3 + b^4*c^3 - 4*
a*b^2*c^4 - (b^5*c - 4*a*b^3*c^2 - 2*b^4*c^2 + b^3*c^3)*sqrt(b^2 - 4*a*c))*
B)*log(x^2 + 1/2*(b^3 - 4*a*b*c - sqrt((b^3 - 4*a*b*c)^2 - 4*(a*b^2 - 4*a^2
*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*
c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*c^2*abs(b^2 - 4*a*c))
```

maple [B] time = 0.03, size = 1119, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/(4*a*c-b^2)*B*b*x^2+1/2*(2*A*c-C*b)/(4*a*c-b^2)*x^3-1/(4*a*c-b^2)*B*a
+1/2*(A*b-2*C*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)^2*(-4*a*c+b
```

```
^2)^(1/2)*B*b*ln(-2*c*x^2-b+(-4*a*c+b^2)^(1/2))-c/(4*a*c-b^2)^2*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*A*(-4*a*c+b^2)^(1/2)*b-2*c^2/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*A+1/2*c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*A*b^2+c/(4*a*c-b^2)^2*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*C*(-4*a*c+b^2)^(1/2)*a+1/4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*C*(-4*a*c+b^2)^(1/2)*b^2+c/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*C*a*b-1/
4/(4*a*c-b^2)^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*C-1/2/(4*a*c-b^2)^2*(-4*a*c+b^2)^
(1/2)*B*b*ln(2*c*x^2+b+(-4*a*c+b^2)^(1/2))-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*A*b*c*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)+2/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*A*a*c^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/
(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*A*b^2*c*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*a*c*arctan(2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*(-4*a*c+b^2)^(1/2)*C*b^2*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)-1/(4*a*c-b^2)^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*a*b
*c*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/(4*a*c-b^2)^2*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*C*b^3*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{Bbx^2 + (Cb - 2\,Ac)x^3 + 2\,Ba + (2\,Ca - Ab)x}{2\left(\left(b^2c - 4\,ac^2\right)x^4 + ab^2 - 4\,a^2c + \left(b^3 - 4\,abc\right)x^2\right)} - \frac{-\int \frac{2\,Bbx + (Cb - 2\,Ac)x^2 - 2\,Ca + Ab}{cx^4 + bx^2 + a}\,dx}{2\left(b^2 - 4\,ac\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x^6+B*x^5+A*x^4)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(B*b*x^2 + (C*b - 2*A*c)*x^3 + 2*B*a + (2*C*a - A*b)*x)/((b^2*c - 4*a*c
^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-(2*B*b*x
+ (C*b - 2*A*c)*x^2 - 2*C*a + A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

mupad [B] time = 1.41, size = 3835, normalized size = 10.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A*x^4 + B*x^5 + C*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] symsum(log((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*C^2*b^4*c - 3*C^3*a*b^3*c + 4*A
*B^2*b^3*c^2 + 8*A*C^2*a^2*c^3 - 5*A^2*C*b^3*c^2 - 4*C^3*a^2*b*c^2 + 18*A*C
^2*a*b^2*c^2 - 8*B^2*C*a*b^2*c^2 - 28*A^2*C*a*b*c^3)/(8*(b^6 - 64*a^3*c^3 +
 48*a^2*b^2*c^2 - 12*a*b^4*c)) - root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^
6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3
*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 -
6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z
^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2
+ 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^
4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*
b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^
2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z
 - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*
```

z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2
*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*
b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80
*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3
+ 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A
*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^
4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*(root(256*a*b^12*c*z^4 - 157286
4*a^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440
*a^3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*
b^8*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2
*a^5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*
a^5*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^
2*a^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8
192*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*
A^2*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B
*a^3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*
a^2*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^
2 - 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 4
8*A^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*
b^3*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A
^4*a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C
*b^5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^
4 + 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k)*((x*(16*B*b^7*c^2 -
192*B*a*b^5*c^3 - 1024*B*a^3*b*c^5 + 768*B*a^2*b^3*c^4))/(4*(b^6 - 64*a^3*
c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (16*A*b^7*c^2 + 2048*C*a^4*c^5 - 192*
A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*C*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*C
*a^2*b^4*c^3 - 1536*C*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 -
12*a*b^4*c)) + (root(256*a*b^12*c*z^4 - 1572864*a^6*b^2*c^6*z^4 + 983040*a^
5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^3*b^8*c^3*z^4 - 6144*a^2*b
^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8*c*z^2 - 6144*A*C*a^3*b^4*
c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^5*b*c^4*z^2 - 12288*A^2*a^
4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5*c^5*z^2 + 8192*C^2*a^4*b^
3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a^4*b^2*c^4*z^2 - 6144*B^2*
a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192*A^2*a^3*b^3*c^4*z^2 - 153
6*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2*b^9*c*z^2 + 1024*B*C^2*a^
4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^3*b*c^4*z - 192*A^2*B*a*b^
5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2*b^3*c^3*z + 16*A^2*B*b^7*
c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 - 48*A*B^2*C*a*b^4*c + 192*
A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A^2*B^2*a*b^3*c^2 - 96*A^3*
C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3*c^2 - 80*A*C^3*a^2*b^3*c
+ 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*a*b^2*c^3 + 4*B^2*C^2*a*b^
5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^5*c - 6*A*C^3*a*b^5 + 32*A
^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 + 9*A^4*b^4*c^2 + 16*A^4*a^
2*c^4 + A^2*C^2*b^6, z, k)*x*(32*b^9*c^2 - 512*a*b^7*c^3 + 8192*a^4*b*c^6 +
3072*a^2*b^5*c^4 - 8192*a^3*b^3*c^5))/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^
2 - 12*a*b^4*c))) - (16*A*B*b^5*c^2 + 256*B*C*a^2*b^2*c^3 - 256*A*B*a^2*b*c
^4 - 64*B*C*a*b^4*c^2)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c))
+ (x*(2*C^2*b^6*c + 64*A^2*a^2*c^5 + 20*A^2*b^4*c^3 - 8*B^2*b^5*c^2 - 64*C
^2*a^3*c^4 - 12*A*C*b^5*c^2 - 96*A^2*a*b^2*c^4 + 32*B^2*a*b^3*c^3 - 4*C^2*a
*b^4*c^2 + 32*A*C*a*b^3*c^3 + 64*A*C*a^2*b*c^4))/(4*(b^6 - 64*a^3*c^3 + 48*
a^2*b^2*c^2 - 12*a*b^4*c))) + (x*(4*B^3*b^3*c^2 + B*C^2*b^4*c + 8*A^2*B*b^2
*c^3 + 4*B*C^2*a*b^2*c^2 - 6*A*B*C*b^3*c^2 - 8*A*B*C*a*b*c^3))/(4*(b^6 - 64
*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)))*root(256*a*b^12*c*z^4 - 1572864*a
^6*b^2*c^6*z^4 + 983040*a^5*b^4*c^5*z^4 - 327680*a^4*b^6*c^4*z^4 + 61440*a^
3*b^8*c^3*z^4 - 6144*a^2*b^10*c^2*z^4 + 1048576*a^7*c^7*z^4 - 192*A*C*a*b^8
*c*z^2 - 6144*A*C*a^3*b^4*c^3*z^2 + 2048*A*C*a^2*b^6*c^2*z^2 - 12288*C^2*a^
5*b*c^4*z^2 - 12288*A^2*a^4*b*c^5*z^2 - 128*B^2*a*b^8*c*z^2 + 16384*A*C*a^5
*c^5*z^2 + 8192*C^2*a^4*b^3*c^3*z^2 - 1536*C^2*a^3*b^5*c^2*z^2 + 8192*B^2*a
^4*b^2*c^4*z^2 - 6144*B^2*a^3*b^4*c^3*z^2 + 1536*B^2*a^2*b^6*c^2*z^2 + 8192

```
*A^2*a^3*b^3*c^4*z^2 - 1536*A^2*a^2*b^5*c^3*z^2 + 16*C^2*a*b^9*z^2 + 16*A^2
*b^9*c*z^2 + 1024*B*C^2*a^4*b*c^3*z + 192*B*C^2*a^2*b^5*c*z - 1024*A^2*B*a^
3*b*c^4*z - 192*A^2*B*a*b^5*c^2*z - 768*B*C^2*a^3*b^3*c^2*z + 768*A^2*B*a^2
*b^3*c^3*z + 16*A^2*B*b^7*c*z - 16*B*C^2*a*b^7*z - 64*A*B^2*C*a^2*b^2*c^2 -
 48*A*B^2*C*a*b^4*c + 192*A^2*C^2*a^2*b^2*c^2 + 48*B^2*C^2*a^2*b^3*c + 48*A
^2*B^2*a*b^3*c^2 - 96*A^3*C*a^2*b*c^3 - 96*A*C^3*a^3*b*c^2 - 80*A^3*C*a*b^3
*c^2 - 80*A*C^3*a^2*b^3*c + 42*A^2*C^2*a*b^4*c + 24*C^4*a^3*b^2*c + 24*A^4*
a*b^2*c^3 + 4*B^2*C^2*a*b^5 + 4*A^2*B^2*b^5*c + 16*B^4*a*b^4*c - 6*A^3*C*b^
5*c - 6*A*C^3*a*b^5 + 32*A^2*C^2*a^3*c^3 + 16*C^4*a^4*c^2 + 9*C^4*a^2*b^4 +
 9*A^4*b^4*c^2 + 16*A^4*a^2*c^4 + A^2*C^2*b^6, z, k), k, 1, 4) - ((B*a)/(4*
a*c - b^2) - (x*(A*b - 2*C*a))/(2*(4*a*c - b^2)) - (x^3*(2*A*c - C*b))/(2*(
4*a*c - b^2)) + (B*b*x^2)/(2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((C*x**6+B*x**5+A*x**4)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.47 $\int \frac{x^7(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-b^3c(cd-5af)-4ab^2c^2e+abc^2(3cd-5af)+b^5(-f)+b^4ce\right)}{2c^5\sqrt{b^2-4ac}}+\frac{x^4\left(-c(af+be)+b^2f\right.}{4c^3}$$

```
[Out] 1/2*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*x^2/c^4+1/4*(c^2*d+b^2*f-c*(a*
f+b*e))*x^4/c^3+1/6*(-b*f+c*e)*x^6/c^2+1/8*f*x^8/c-1/4*(b^3*c*e-2*a*b*c^2*e
-b^4*f-b^2*c*(-3*a*f+c*d)+a*c^2*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/c^5-1/2*(b^4*
c*e-4*a*b^2*c^2*e+2*a^2*c^3*e-b^5*f-b^3*c*(-5*a*f+c*d)+a*b*c^2*(-5*a*f+3*c*
d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^5/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.85, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2c^3e-4ab^2c^2e-b^3c(cd-5af)+abc^2(3cd-5af)+b^4ce+b^5(-f)\right)}{2c^5\sqrt{b^2-4ac}}+\frac{x^4\left(-c(af+be)+b^2f\right.}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*x^2)/(2*c^4) + ((c^2*d + b
^2*f - c*(b*e + a*f))*x^4)/(4*c^3) + ((c*e - b*f)*x^6)/(6*c^2) + (f*x^8)/(8
*c) - ((b^4*c*e - 4*a*b^2*c^2*e + 2*a^2*c^3*e - b^5*f - b^3*c*(c*d - 5*a*f)
 + a*b*c^2*(3*c*d - 5*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
5*Sqrt[b^2 - 4*a*c]) - ((b^3*c*e - 2*a*b*c^2*e - b^4*f - b^2*c*(c*d - 3*a*f
) + a*c^2*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^3\left(d+ex+fx^2\right)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{b^2ce-ac^2e-b^3f-bc(cd-2af)}{c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x}{c^3} + \frac{(ce \ldots\right.\right.$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} \ldots$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} \ldots$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} \ldots$$

$$= \frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af)\right)x^2}{2c^4} + \frac{\left(c^2d+b^2f-c(be+af)\right)x^4}{4c^3} + \frac{(ce-bf)x^6}{6c^2} \ldots$$

Mathematica [A] time = 0.20, size = 260, normalized size = 0.95

$$-\frac{12\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-2a^2c^3e+b^3c(cd-5af)+4ab^2c^2e+abc^2(5af-3cd)+b^5f-b^4ce\right)}{\sqrt{4ac-b^2}} + 6c^2x^4\left(-c(af+be)+b^2f+c^2d\right) - 12cx^2\left(bc(\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-12*c*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*x^2 + 6*c^2*(c^2*
d + b^2*f - c*(b*e + a*f))*x^4 + 4*c^3*(c*e - b*f)*x^6 + 3*c^4*f*x^8 - (12*
(-(b^4*c*e) + 4*a*b^2*c^2*e - 2*a^2*c^3*e + b^5*f + b^3*c*(c*d - 5*a*f) + a
*b*c^2*(-3*c*d + 5*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^
2 + 4*a*c] + 6*(-(b^3*c*e) + 2*a*b*c^2*e + b^4*f + b^2*c*(c*d - 3*a*f) + a*
c^2*(-(c*d) + a*f))*Log[a + b*x^2 + c*x^4])/(24*c^5)
```

fricas [A] time = 1.85, size = 900, normalized size = 3.30

$$\left[\frac{3\left(b^2c^4-4\,ac^5\right)fx^8+4\left(\left(b^2c^4-4\,ac^5\right)e-\left(b^3c^3-4\,abc^4\right)f\right)x^6+6\left(\left(b^2c^4-4\,ac^5\right)d-\left(b^3c^3-4\,abc^4\right)e+\left(b^4c^2 \ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c^4 - 4*a*c^5)*e - (b^3*c^3 -
4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e + (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*((b^3*c^3 - 4*a*b*c^4)*d - (b
^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*f
)*x^2 + 6*sqrt(b^2 - 4*a*c)*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2
 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
+ 6*((b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b
*c^3)*e + (b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x
^2 + a))/(b^2*c^5 - 4*a*c^6), 1/24*(3*(b^2*c^4 - 4*a*c^5)*f*x^8 + 4*((b^2*c
^4 - 4*a*c^5)*e - (b^3*c^3 - 4*a*b*c^4)*f)*x^6 + 6*((b^2*c^4 - 4*a*c^5)*d -
 (b^3*c^3 - 4*a*b*c^4)*e + (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*f)*x^4 - 12*
((b^3*c^3 - 4*a*b*c^4)*d - (b^4*c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*e + (b^5*c -
 6*a*b^3*c^2 + 8*a^2*b*c^3)*f)*x^2 + 12*sqrt(-b^2 + 4*a*c)*((b^3*c^2 - 3*a*
b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e + (b^5 - 5*a*b^3*c + 5*a^2*b
*c^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 6*((b^4*
c^2 - 5*a*b^2*c^3 + 4*a^2*c^4)*d - (b^5*c - 6*a*b^3*c^2 + 8*a^2*b*c^3)*e +
(b^6 - 7*a*b^4*c + 13*a^2*b^2*c^2 - 4*a^3*c^3)*f)*log(c*x^4 + b*x^2 + a))/(
b^2*c^5 - 4*a*c^6)]
```

giac [A] time = 1.87, size = 306, normalized size = 1.12

$$\frac{3\,c^3 f x^8 - 4\,bc^2 f x^6 + 4\,c^3 x^6 e + 6\,c^3 d x^4 + 6\,b^2 c f x^4 - 6\,ac^2 f x^4 - 6\,bc^2 x^4 e - 12\,bc^2 d x^2 - 12\,b^3 f x^2 + 24\,abcf x^2 + 12}{24\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/24*(3*c^3*f*x^8 - 4*b*c^2*f*x^6 + 4*c^3*x^6*e + 6*c^3*d*x^4 + 6*b^2*c*f*x
^4 - 6*a*c^2*f*x^4 - 6*b*c^2*x^4*e - 12*b*c^2*d*x^2 - 12*b^3*f*x^2 + 24*a*b
*c*f*x^2 + 12*b^2*c*x^2*e - 12*a*c^2*x^2*e)/c^4 + 1/4*(b^2*c^2*d - a*c^3*d
+ b^4*f - 3*a*b^2*c*f + a^2*c^2*f - b^3*c*e + 2*a*b*c^2*e)*log(c*x^4 + b*x^
2 + a)/c^5 - 1/2*(b^3*c^2*d - 3*a*b*c^3*d + b^5*f - 5*a*b^3*c*f + 5*a^2*b*c
^2*f - b^4*c*e + 4*a*b^2*c^2*e - 2*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^
2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^5)
```

maple [B] time = 0.01, size = 622, normalized size = 2.28

$$\frac{f x^8}{8c} - \frac{bf x^6}{6c^2} + \frac{e x^6}{6c} - \frac{af x^4}{4c^2} + \frac{b^2 f x^4}{4c^3} - \frac{be x^4}{4c^2} + \frac{d x^4}{4c} - \frac{5a^2 bf \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c^3} + \frac{a^2 e \arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c^2} + \frac{5ab^3 f \arctan\left(\frac{2}{\sqrt{}}\right)}{2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/c^3*x^2*a*b*f+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*b^4*e+1/8*f*x^8/c-1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-
b^2)^(1/2))*b^3*d+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*a^2*e-1/2/c^5/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
b^5*f-3/4/c^4*ln(c*x^4+b*x^2+a)*a*b^2*f+1/2/c^3*ln(c*x^4+b*x^2+a)*a*b*e+1/6
/c*x^6*e+1/4/c*x^4*d+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*a*b*d+5/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*a*b^3*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*
a*b^2*e-5/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2
*b*f-1/2/c^2*x^2*a*e-1/6/c^2*x^6*b*f-1/4/c^2*x^4*a*f-1/2/c^4*x^2*b^3*f+1/2/
c^3*x^2*b^2*e-1/2/c^2*x^2*b*d+1/4/c^3*x^4*b^2*f-1/4/c^2*x^4*b*e+1/4/c^3*ln(
```

```
c*x^4+b*x^2+a)*b^2*d+1/4/c^3*ln(c*x^4+b*x^2+a)*a^2*f-1/4/c^2*ln(c*x^4+b*x^2
+a)*a*d+1/4/c^5*ln(c*x^4+b*x^2+a)*b^4*f-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.60, size = 2972, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^6*(e/(6*c) - (b*f)/(6*c^2)) - x^4*((b*(e/c - (b*f)/c^2))/(4*c) - d/(4*c)
+ (a*f)/(4*c^2)) - x^2*((a*(e/c - (b*f)/c^2))/(2*c) - (b*((b*(e/c - (b*f)/c
^2))/c - d/c + (a*f)/c^2))/(2*c)) + (f*x^8)/(8*c) - (log(a + b*x^2 + c*x^4)
*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^
2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))
/(2*(16*a*c^6 - 4*b^2*c^5)) + (atan((2*c^8*(4*a*c - b^2)*(x^2*(((((4*a^2*c^
8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d - 16*a*b^2*c^7
*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*f + 8*a^2*c^4*d
 + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f
- 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5)
)*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f +
4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) - (b*(b^5*f - 2
*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*
e + 5*a^2*b*c^2*f)*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b
^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e
- 16*a^2*b*c^3*e))/(2*c^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a -
(b*((((4*a^2*c^8*e - 6*b^3*c^7*d + 6*b^4*c^6*e - 6*b^5*c^5*f + 10*a*b*c^8*d
 - 16*a*b^2*c^7*e + 22*a*b^3*c^6*f - 14*a^2*b*c^7*f)/c^8 - (4*b*c^2*(2*b^6*
f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f
- 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c
^6 - 4*b^2*c^5))*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5
*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e -
16*a^2*b*c^3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) - (b^9*f^2 + b^5*c^4*d^2 + b^7*
c^2*e^2 - 3*a*b^3*c^5*d^2 + 2*a^2*b*c^6*d^2 - 5*a*b^5*c^3*e^2 - 2*a^3*b*c^5
*e^2 + 3*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 7*a^2*b^3*c^4*e^2 + 16*a^2*b^5*c^2*f
^2 - 13*a^3*b^3*c^3*f^2 - 7*a*b^7*c*f^2 + a^3*c^6*d*e - 2*b^6*c^3*d*e - a^4
*c^5*e*f + 2*b^7*c^2*d*f + 8*a*b^4*c^4*d*e - 10*a*b^5*c^3*d*f - 5*a^3*b*c^5
*d*f + 12*a*b^6*c^2*e*f - 8*a^2*b^2*c^5*d*e + 14*a^2*b^3*c^4*d*f - 22*a^2*b
^4*c^3*e*f + 12*a^3*b^2*c^4*e*f)/c^8 + (b*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d
- b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)^2)/(
2*c^8*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a^3*c^7*f - 8*a^2
*c^8*d - 24*a^2*b^2*c^6*f + 8*a*b^2*c^7*d - 8*a*b^3*c^6*e + 16*a^2*b*c^7*e
+ 8*a*b^4*c^5*f)/c^8 + (8*a*c^2*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^
3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12
*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 - 4*b^2*c^5))*(b^5*f - 2*a^2*c^3*
e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2
*b*c^2*f))/(8*c^5*(4*a*c - b^2)^(1/2)) + (a*(b^5*f - 2*a^2*c^3*e + b^3*c^2*
d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)*(2
```

```
*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c
^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(c
^3*(4*a*c - b^2)^(1/2)*(16*a*c^6 - 4*b^2*c^5)))/a + (b*((a*b^8*f^2 + a^3*c^
6*d^2 + a^5*c^4*f^2 + a*b^4*c^4*d^2 + a*b^6*c^2*e^2 - 6*a^2*b^6*c*f^2 - 2*a
^2*b^2*c^5*d^2 - 4*a^2*b^4*c^3*e^2 + 4*a^3*b^2*c^4*e^2 + 11*a^3*b^4*c^2*f^2
 - 6*a^4*b^2*c^3*f^2 - 2*a^4*c^5*d*f - 2*a*b^5*c^3*d*e - 4*a^3*b*c^5*d*e +
2*a*b^6*c^2*d*f + 4*a^4*b*c^4*e*f + 6*a^2*b^3*c^4*d*e - 8*a^2*b^4*c^3*d*f +
 8*a^3*b^2*c^4*d*f + 10*a^2*b^5*c^2*e*f - 14*a^3*b^3*c^3*e*f - 2*a*b^7*c*e*
f)/c^8 + (((8*a^3*c^7*f - 8*a^2*c^8*d - 24*a^2*b^2*c^6*f + 8*a*b^2*c^7*d -
8*a*b^3*c^6*e + 16*a^2*b*c^7*e + 8*a*b^4*c^5*f)/c^8 + (8*a*c^2*(2*b^6*f + 8
*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e + 26*a^2*b^2*c^2*f - 14*
a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^2*b*c^3*e))/(16*a*c^6 -
4*b^2*c^5))*(2*b^6*f + 8*a^2*c^4*d + 2*b^4*c^2*d - 8*a^3*c^3*f - 2*b^5*c*e
+ 26*a^2*b^2*c^2*f - 14*a*b^4*c*f - 10*a*b^2*c^3*d + 12*a*b^3*c^2*e - 16*a^
2*b*c^3*e))/(2*(16*a*c^6 - 4*b^2*c^5)) - (a*(b^5*f - 2*a^2*c^3*e + b^3*c^2*
d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f)^2)
/(c^8*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(b^10*f^2 + 4*a^4*c^6*e^
2 + b^6*c^4*d^2 + b^8*c^2*e^2 - 6*a*b^4*c^5*d^2 - 8*a*b^6*c^3*e^2 - 2*b^9*c
*e*f + 9*a^2*b^2*c^6*d^2 + 20*a^2*b^4*c^4*e^2 - 16*a^3*b^2*c^5*e^2 + 35*a^2
*b^6*c^2*f^2 - 50*a^3*b^4*c^3*f^2 + 25*a^4*b^2*c^4*f^2 - 10*a*b^8*c*f^2 - 2
*b^7*c^3*d*e + 2*b^8*c^2*d*f + 14*a*b^5*c^4*d*e + 12*a^3*b*c^6*d*e - 16*a*b
^6*c^3*d*f + 18*a*b^7*c^2*e*f - 20*a^4*b*c^5*e*f - 28*a^2*b^3*c^5*d*e + 40*
a^2*b^4*c^4*d*f - 30*a^3*b^2*c^5*d*f - 54*a^2*b^5*c^3*e*f + 60*a^3*b^3*c^4*
e*f))*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*
f + 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c^5*(4*a*c - b^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.48 $\int \frac{x^5\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=203

$$\frac{x^2\left(-c(af+be)+b^2f+c^2d\right)}{2c^3}+\frac{\log\left(a+bx^2+cx^4\right)\left(-bc(cd-2af)-ac^2e+b^3(-f)+b^2ce\right)}{4c^4}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{}$$

```
[Out] 1/2*(c^2*d+b^2*f-c*(a*f+b*e))*x^2/c^3+1/4*(-b*f+c*e)*x^4/c^2+1/6*f*x^6/c+1/
4*(b^2*c*e-a*c^2*e-b^3*f-b*c*(-2*a*f+c*d))*ln(c*x^4+b*x^2+a)/c^4+1/2*(b^3*c
*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))*arctanh((2*c*x^
2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{x^2\left(-c(af+be)+b^2f+c^2d\right)}{2c^3}+\frac{\log\left(a+bx^2+cx^4\right)\left(-bc(cd-2af)-ac^2e+b^2ce+b^3(-f)\right)}{4c^4}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x^2)/(2*c^3) + ((c*e - b*f)*x^4)/(4*c^2) +
 (f*x^6)/(6*c) + ((b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*
a*c^2*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*Sqrt[b^
2 - 4*a*c]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f))*Log[a + b*x^
2 + c*x^4])/(4*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2\left(d+ex+fx^2\right)}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int\left(\frac{c^2d+b^2f-c(be+af)}{c^3}+\frac{(ce-bf)x}{c^2}+\frac{fx^2}{c}-\frac{a\left(c^2d+b^2f-c(be+af)\right)}{c^3}\right.\right.$$

$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}-\frac{\text{Subst}\left(\int \frac{a\left(c^2d+b^2f-c(be+af)\right)-\left(b^2ce-a\right.}{a+bx+cx}\right.}{2c^3}$$

$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af\right)}{4c^4}$$

$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{\left(b^2ce-ac^2e-b^3f-bc(cd-2af\right)}{4c^4}$$

$$= \frac{\left(c^2d+b^2f-c(be+af)\right)x^2}{2c^3}+\frac{(ce-bf)x^4}{4c^2}+\frac{fx^6}{6c}+\frac{\left(b^3ce-3abc^2e-b^4f-b^2c(cd-4\right.}{2c^4}$$

Mathematica [A] time = 0.14, size = 193, normalized size = 0.95

$$\frac{6cx^2\left(-c(af+be)+b^2f+c^2d\right)-3\log\left(a+bx^2+cx^4\right)\left(bc(cd-2af)+ac^2e+b^3f-b^2ce\right)+\frac{6\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(b^2c(cd-\right.}{}}{12c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*c*(c^2*d + b^2*f - c*(b*e + a*f))*x^2 + 3*c^2*(c*e - b*f)*x^4 + 2*c^3*f*
x^6 + (6*(-(b^3*c*e) + 3*a*b*c^2*e + b^4*f + b^2*c*(c*d - 4*a*f) + 2*a*c^2*
(-(c*d) + a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] - 3*(-(b^2*c*e) + a*c^2*e + b^3*f + b*c*(c*d - 2*a*f))*Log[a + b*x^2 + c*
x^4])/(12*c^4)
```

fricas [A] time = 1.96, size = 677, normalized size = 3.33

$$\left[\frac{2\left(b^2c^3-4ac^4\right)fx^6+3\left(\left(b^2c^3-4ac^4\right)e-\left(b^3c^2-4abc^3\right)f\right)x^4+6\left(\left(b^2c^3-4ac^4\right)d-\left(b^3c^2-4abc^3\right)e+\left(b^4c-5\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e - (b^3*c^2 -
4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*b*c^3)*e + (b
^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 + 3*sqrt(b^2 - 4*a*c)*((b^2*c^2 - 2*
a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c^2)*f)*log((2*
c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
 + b*x^2 + a)) - 3*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*
c^3)*e + (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^
4 - 4*a*c^5), 1/12*(2*(b^2*c^3 - 4*a*c^4)*f*x^6 + 3*((b^2*c^3 - 4*a*c^4)*e
- (b^3*c^2 - 4*a*b*c^3)*f)*x^4 + 6*((b^2*c^3 - 4*a*c^4)*d - (b^3*c^2 - 4*a*
b*c^3)*e + (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*f)*x^2 - 6*sqrt(-b^2 + 4*a*c)*
((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2*a^2*c
^2)*f)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 3*((b^3*c^
2 - 4*a*b*c^3)*d - (b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*e + (b^5 - 6*a*b^3*c +
 8*a^2*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^4 - 4*a*c^5)]
```

giac [A] time = 2.00, size = 214, normalized size = 1.05

$$\frac{2\,c^2fx^6-3\,bcfx^4+3\,c^2x^4e+6\,c^2dx^2+6\,b^2fx^2-6\,acfx^2-6\,bcx^2e}{12\,c^3}-\frac{\left(bc^2d+b^3f-2\,abcf-b^2ce+ac^2e\right)\log(}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*(2*c^2*f*x^6 - 3*b*c*f*x^4 + 3*c^2*x^4*e + 6*c^2*d*x^2 + 6*b^2*f*x^2 -
 6*a*c*f*x^2 - 6*b*c*x^2*e)/c^3 - 1/4*(b*c^2*d + b^3*f - 2*a*b*c*f - b^2*c*
e + a*c^2*e)*log(c*x^4 + b*x^2 + a)/c^4 + 1/2*(b^2*c^2*d - 2*a*c^3*d + b^4*
f - 4*a*b^2*c*f + 2*a^2*c^2*f - b^3*c*e + 3*a*b*c^2*e)*arctan((2*c*x^2 + b)
/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^4)
```

maple [B] time = 0.01, size = 474, normalized size = 2.33

$$\frac{f\,x^6}{6c}-\frac{bf\,x^4}{4c^2}+\frac{e\,x^4}{4c}+\frac{a^2f\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^2}-\frac{2a\,b^2f\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c^3}+\frac{3abe\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2}-\frac{ad\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/6*f*x^6/c-1/4/c^2*x^4*b*f+1/4/c*x^4*e-1/2/c^2*x^2*a*f+1/2/c^3*x^2*b^2*f-1
/2/c^2*x^2*b*e+1/2/c*x^2*d+1/2/c^3*ln(c*x^4+b*x^2+a)*a*b*f-1/4/c^2*ln(c*x^4
+b*x^2+a)*a*e-1/4/c^4*ln(c*x^4+b*x^2+a)*b^3*f+1/4/c^3*ln(c*x^4+b*x^2+a)*b^2
*e-1/4/c^2*ln(c*x^4+b*x^2+a)*b*d+1/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*a^2*f-2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c
-b^2)^(1/2))*a*b^2*f+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*a*b*e-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
*a*d+1/2/c^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*f-
1/2/c^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e+1/2/c
^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.63, size = 2295, normalized size = 11.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^4*(e/(4*c) - (b*f)/(4*c^2)) - x^2*((b*(e/c - (b*f)/c^2))/(2*c) - d/(2*c)
+ (a*f)/(2*c^2)) + (log(a + b*x^2 + c*x^4)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c
^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c
^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)) + (f*x^6)/(6*c) + (atan((2*c^6*(4*a*c - b
^2)*(x^2*(((((6*b^2*c^6*d + 4*a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c
^7*d + 10*a*b*c^6*e - 16*a*b^2*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e
 + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e +
16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f -
 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(8*c^4*(4*a*c - b^2)^(1/
2)) + (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2
*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*
c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*c^2*(4*a*c - b^
2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a - (b*((b^7*f^2 + b^3*c^4*d^2 + b^5*c^2*
e^2 - 3*a*b^3*c^3*e^2 + 2*a^2*b*c^4*e^2 - 2*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 7
*a^2*b^3*c^2*f^2 - a*b*c^5*d^2 - 5*a*b^5*c*f^2 - a^2*c^5*d*e - 2*b^4*c^3*d*
e + a^3*c^4*e*f + 2*b^5*c^2*d*f + 4*a*b^2*c^4*d*e - 6*a*b^3*c^3*d*f + 3*a^2
*b*c^4*d*f + 8*a*b^4*c^2*e*f - 8*a^2*b^2*c^3*e*f)/c^6 + (((6*b^2*c^6*d + 4*
a^2*c^6*f - 6*b^3*c^5*e + 6*b^4*c^4*f - 4*a*c^7*d + 10*a*b*c^6*e - 16*a*b^2
*c^5*f)/c^6 + (4*b*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8
*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4
*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d -
 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(2*(16*a*c^5 - 4*b^2*c^4)
) - (b*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e
 - 4*a*b^2*c*f)^2)/(2*c^6*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (((
(8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e + 8*a*b^3*c^4*f - 16*a^2*b*c^5*f
)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c
^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/(16*a*c^5 - 4*b^2*c
^4))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e -
 4*a*b^2*c*f))/(8*c^4*(4*a*c - b^2)^(1/2)) + (a*(b^4*f + b^2*c^2*d + 2*a^2*
c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)*(2*b^5*f - 8*a^2*c
^3*e + 2*b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*
e + 16*a^2*b*c^2*f))/(c^2*(4*a*c - b^2)^(1/2)*(16*a*c^5 - 4*b^2*c^4)))/a -
(b*((a*b^6*f^2 + a^3*c^4*e^2 + a*b^2*c^4*d^2 + a*b^4*c^2*e^2 - 4*a^2*b^4*c*
f^2 - 2*a^2*b^2*c^3*e^2 + 4*a^3*b^2*c^2*f^2 - 2*a*b^3*c^3*d*e + 2*a^2*b*c^4
*d*e + 2*a*b^4*c^2*d*f - 4*a^3*b*c^3*e*f - 4*a^2*b^2*c^3*d*f + 6*a^2*b^3*c^
2*e*f - 2*a*b^5*c*e*f)/c^6 + (((8*a^2*c^6*e + 8*a*b*c^6*d - 8*a*b^2*c^5*e +
 8*a*b^3*c^4*f - 16*a^2*b*c^5*f)/c^6 + (8*a*c^2*(2*b^5*f - 8*a^2*c^3*e + 2*
b^3*c^2*d - 2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^
2*b*c^2*f))/(16*a*c^5 - 4*b^2*c^4))*(2*b^5*f - 8*a^2*c^3*e + 2*b^3*c^2*d -
2*b^4*c*e - 8*a*b*c^3*d - 12*a*b^3*c*f + 10*a*b^2*c^2*e + 16*a^2*b*c^2*f))/
(2*(16*a*c^5 - 4*b^2*c^4)) - (a*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*
d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f)^2)/(c^6*(4*a*c - b^2))))/(2*a*(4*a
*c - b^2)^(1/2))))/(b^8*f^2 + 4*a^2*c^6*d^2 + b^4*c^4*d^2 + 4*a^4*c^4*f^2 +
 b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*
c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 8*a^3*c
^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 12*a^2*b*c^5*d*
e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 12*a^3*b*c^4*e*f + 20*a^2*b^2*c^4
*d*f - 28*a^2*b^3*c^3*e*f))*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d -
b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*c^4*(4*a*c - b^2)^(1/2))
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.49 $\int \frac{x^3(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=144

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(c\ldots}{2\ldots}$$

```
[Out] 1/2*(-b*f+c*e)*x^2/c^2+1/4*f*x^4/c+1/4*(c^2*d+b^2*f-c*(a*f+b*e))*ln(c*x^4+b
*x^2+a)/c^3-1/2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d))*arctanh((2*c*x^2
+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)}{2c^3\sqrt{b^2-4ac}}+\frac{x^2(c\ldots}{2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x^2)/(2*c^2) + (f*x^4)/(4*c) - ((b^2*c*e - 2*a*c^2*e - b^3*f -
 b*c*(c*d - 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b
^2 - 4*a*c]) + ((c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*
c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\left(d+ex+fx^2\right)}{a+bx+cx^2}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{ce-bf}{c^2}+\frac{fx}{c}-\frac{a(ce-bf)-\left(c^2d-bce+b^2f-acf\right)x}{c^2\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\text{Subst}\left(\int \frac{a(ce-bf)-\left(c^2d-bce+b^2f-acf\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\left(-c^2d+bce-b^2f+acf\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}+\frac{\left(b^2c\right.}{}$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}+\frac{\left(c^2d-bce+b^2f-acf\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{\left(b^2ce-2ac^2e\right.}{}$$

$$= \frac{(ce-bf)x^2}{2c^2}+\frac{fx^4}{4c}-\frac{\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\left(c^2d\right.}{}$$

Mathematica [A] time = 0.10, size = 136, normalized size = 0.94

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(af+be)+b^2f+c^2d\right)-\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(bc(cd-3af)+2ac^2e+b^3f-b^2ce\right)}{\sqrt{4ac-b^2}}+2cx^2(ce-bf)+c^2fx}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*e - b*f)*x^2 + c^2*f*x^4 - (2*(-(b^2*c*e) + 2*a*c^2*e + b^3*f + b*c
*(c*d - 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c
] + (c^2*d + b^2*f - c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

fricas [A] time = 1.50, size = 473, normalized size = 3.28

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)fx^4+2\left(\left(b^2c^2-4\,ac^3\right)e-\left(b^3c-4\,abc^2\right)f\right)x^2-\left(bc^2d-\left(b^2c-2\,ac^2\right)e+\left(b^3-3\,abc\right)f\right)\sqrt{b^2-4\,a}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b
*c^2)*f)*x^2 - (b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c
^2)*e + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 -
```

```
 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*f*x^4 + 2*((b^2*c^2 - 4*a*c^3)*e - (b^3
*c - 4*a*b*c^2)*f)*x^2 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)
*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*
c)) + ((b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 5*a*b^2*c + 4
*a^2*c^2)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

giac [**A**] time = 1.99, size = 141, normalized size = 0.98

$$\frac{cfx^4 - 2\,bfx^2 + 2\,cx^2e}{4\,c^2} + \frac{\left(c^2d + b^2f - acf - bce\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{\left(bc^2d + b^3f - 3\,abcf - b^2ce + 2\,ac^2e\right)\operatorname{arcta}}{2\,\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*f*x^4 - 2*b*f*x^2 + 2*c*x^2*e)/c^2 + 1/4*(c^2*d + b^2*f - a*c*f - b*
c*e)*log(c*x^4 + b*x^2 + a)/c^3 - 1/2*(b*c^2*d + b^3*f - 3*a*b*c*f - b^2*c*
e + 2*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*c^3)
```

maple [**B**] time = 0.01, size = 321, normalized size = 2.23

$$\frac{f\,x^4}{4c} + \frac{3abf\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c^2} - \frac{ae\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\,c} - \frac{b^3f\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c^3} + \frac{b^2e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\,c^2} - \frac{bd\arctan\left(\frac{2c\,x}{\sqrt{4a}}\right.}{2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*f*x^4/c-1/2/c^2*x^2*b*f+1/2/c*x^2*e-1/4/c^2*ln(c*x^4+b*x^2+a)*a*f+1/4/c
^3*ln(c*x^4+b*x^2+a)*b^2*f-1/4/c^2*ln(c*x^4+b*x^2+a)*b*e+1/4/c*ln(c*x^4+b*x
^2+a)*d+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b
*f-1/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/2/c^3/
(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*f+1/2/c^2/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e-1/2/c/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 1.30, size = 1689, normalized size = 11.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(e/(2*c) - (b*f)/(2*c^2)) + (f*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 1
0*a*b^2*c*f))/(2*(16*a*c^4 - 4*b^2*c^3)) - (atan((2*c^4*(4*a*c - b^2)*(x^2*
```

```
(((((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^4*f)/c^4
+ (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8
*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*e + b*
c^2*d - b^2*c*e - 3*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) + (b*(b^3*f + 2*a
*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*
f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(2*c*(4*a*c - b^2)
^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((b^5*f^2 + b*c^4*d^2 + b^3*c^2*e^2
+ 2*a^2*b*c^2*f^2 + a*c^4*d*e - 2*b^4*c*e*f - a*b*c^3*e^2 - 3*a*b^3*c*f^2 -
 2*b^2*c^3*d*e - a^2*c^3*e*f + 2*b^3*c^2*d*f + 4*a*b^2*c^2*e*f - 3*a*b*c^3*
d*f)/c^4 + (((6*b^3*c^3*f - 6*b^2*c^4*e + 4*a*c^5*e + 6*b*c^5*d - 10*a*b*c^
4*f)/c^4 + (4*b*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^
3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*f + 2*b
^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f
))/(2*(16*a*c^4 - 4*b^2*c^3)) - (b*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e -
 3*a*b*c*f)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a
^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f)/c^4 - (8*a*c^2*(2*b^4*f
 + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b
^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3
*a*b*c*f))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(b^3*f + 2*a*c^2*e + b*c^2*d -
b^2*c*e - 3*a*b*c*f)*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b
^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*
b^2*c^3)))/a + (b*((((8*a^2*c^4*f - 8*a*c^5*d + 8*a*b*c^4*e - 8*a*b^2*c^3*f
)/c^4 - (8*a*c^2*(2*b^4*f + 2*b^2*c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c
*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*f + 2*b^2*
c^2*d + 8*a^2*c^2*f - 8*a*c^3*d - 2*b^3*c*e + 8*a*b*c^2*e - 10*a*b^2*c*f))/
(2*(16*a*c^4 - 4*b^2*c^3)) - (a*c^4*d^2 + a*b^4*f^2 + a^3*c^2*f^2 + a*b^2*c
^2*e^2 - 2*a^2*b^2*c*f^2 - 2*a^2*c^3*d*f + 2*a*b^2*c^2*d*f + 2*a^2*b*c^2*e*
f - 2*a*b*c^3*d*e - 2*a*b^3*c*e*f)/c^4 + (a*(b^3*f + 2*a*c^2*e + b*c^2*d -
b^2*c*e - 3*a*b*c*f)^2)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(
b^6*f^2 + 4*a^2*c^4*e^2 + b^2*c^4*d^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b
^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 - 2*b^3*c^3*d*e + 2*b^4*c^2*d*
f - 6*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 12*a^2*b*c^3*e*f + 4*a*b*c^4*d*e))
*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^3*(4*a*c - b^2)^
(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.50 $\int \frac{x(d+ex^2+fx^4)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2acf+b^2f-bce+2c^2d\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{fx^2}{2c}$$

```
[Out] 1/2*f*x^2/c+1/4*(-b*f+c*e)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-2*a*c*f+b^2*f-b*c*e+
2*c^2*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1663, 1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2acf+b^2f-bce+2c^2d\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{fx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x^2)/(2*c) - ((2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*ArcTanh[(b + 2*c*x^2)/
Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) + ((c*e - b*f)*Log[a + b*x^2
+ c*x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex+fx^2}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{f}{c}+\frac{cd-af+(ce-bf)x}{c\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\ &= \frac{fx^2}{2c}+\frac{\text{Subst}\left(\int \frac{cd-af+(ce-bf)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\ &= \frac{fx^2}{2c}+\frac{(ce-bf)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{\left(2c^2d-bce+b^2f-2acf\right)\text{Subst}\left(\int\right.}{4c^2}\\ &= \frac{fx^2}{2c}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{\left(2c^2d-bce+b^2f-2acf\right)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\right.}{2c^2}\\ &= \frac{fx^2}{2c}-\frac{\left(2c^2d-bce+b^2f-2acf\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}+\frac{(ce-bf)\log\left(a+bx^2+cx^4\right)}{4c^2}\end{aligned}$$

Mathematica [A] time = 0.07, size = 100, normalized size = 0.97

$$\frac{\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-c(2af+be)+b^2f+2c^2d\right)}{\sqrt{4ac-b^2}}+(ce-bf)\log\left(a+bx^2+cx^4\right)+2cfx^2}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*f*x^2 + (2*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*ArcTan[(b + 2*c*x^2)/Sq
rt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (c*e - b*f)*Log[a + b*x^2 + c*x^4])
/(4*c^2)
```

fricas [A] time = 1.34, size = 318, normalized size = 3.09

$$\left[\frac{2\left(b^2c-4ac^2\right)fx^2-\left(2c^2d-bce+\left(b^2-2ac\right)f\right)\sqrt{b^2-4ac}\,\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac+\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)+\left(\left(b^2c-\right.\right.}{4\left(b^2c^2-4ac^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*f*x^2 - (2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(
b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(
b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c - 4*a*c^2)*e - (b^3 - 4*a*b*c)
*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(b^2*c - 4*a*c^2)*f
*x^2 - 2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*
c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c - 4*a*c^2)*e - (b^3
- 4*a*b*c)*f)*log(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

giac [A] time = 1.78, size = 99, normalized size = 0.96

$$\frac{fx^2}{2\,c} - \frac{(bf - ce)\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{\left(2\,c^2d + b^2f - 2\,acf - bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/c - 1/4*(b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(2*c^2*d + b
^2*f - 2*a*c*f - b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
 + 4*a*c)*c^2)
```

maple [B] time = 0.00, size = 211, normalized size = 2.05

$$-\frac{af\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c}+\frac{b^2f\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2}-\frac{be\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}+\frac{f\,x^2}{2c}+\frac{d\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}-\frac{bf\ln\left(c\,x^4+b\,x^2\right.}{4c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/2*f*x^2/c-1/4/c^2*ln(c*x^4+b*x^2+a)*b*f+1/4/c*ln(c*x^4+b*x^2+a)*e-1/c/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*f+1/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*d+1/2/c^2/(4*a*c-b^2)^(1/2)*arctan(
(2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*f-1/2/c/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.83, size = 1081, normalized size = 10.50

$$\frac{f\,x^2}{2\,c}+\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,f\,b^3-2\,e\,b^2\,c-8\,a\,f\,b\,c+8\,a\,e\,c^2\right)}{2\,\left(16\,a\,c^3-4\,b^2\,c^2\right)}+\frac{\mathrm{atan}\left(\frac{2\,c^2\,\left(4\,a\,c-b^2\right)\,\left(x^2\,\left(\frac{\frac{6\,f\,b^2\,c^2-6\,e\,b\,c^3+4\,d\,c^4-4\,a\,f\,c^3}{c^2}+\frac{4\,b\,c^2\,(\ldots}{\ldots}}{\ldots}\right.\right.}{\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e -
8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) + (atan((2*c^2*(4*a*c - b^2)*(x^2*((
(((4*c^4*d + 6*b^2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f +
 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2
*f - 2*a*c*f - b*c*e))/(8*c^2*(4*a*c - b^2)^(1/2)) + (b*(2*c^2*d + b^2*f -
2*a*c*f - b*c*e)*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/(2*(4*a*c -
 b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a - (b*((b^3*f^2 + b*c^2*e^2 - c^3*d*e
 - a*b*c*f^2 + a*c^2*e*f + b*c^2*d*f - 2*b^2*c*e*f)/c^2 + (((4*c^4*d + 6*b^
2*c^2*f - 4*a*c^3*f - 6*b*c^3*e)/c^2 + (4*b*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^
2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*
e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (b*(2*c^2*d + b^2*f - 2*a*c*f
- b*c*e)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) - ((((8*a*c^
3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e - 2*b^2*c*e - 8*a*b*
c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(8*c^2*(
4*a*c - b^2)^(1/2)) - (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)*(2*b^3*f + 8*a
*c^2*e - 2*b^2*c*e - 8*a*b*c*f))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2
)))/a + (b*((((8*a*c^3*e - 8*a*b*c^2*f)/c^2 - (8*a*c^2*(2*b^3*f + 8*a*c^2*e
 - 2*b^2*c*e - 8*a*b*c*f))/(16*a*c^3 - 4*b^2*c^2))*(2*b^3*f + 8*a*c^2*e - 2
*b^2*c*e - 8*a*b*c*f))/(2*(16*a*c^3 - 4*b^2*c^2)) - (a*b^2*f^2 + a*c^2*e^2
- 2*a*b*c*e*f)/c^2 + (a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e)^2)/(c^2*(4*a*c
- b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(4*c^4*d^2 + b^4*f^2 + 4*a^2*c^2*f^2
+ b^2*c^2*e^2 - 8*a*c^3*d*f - 4*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 4
*b^2*c^2*d*f + 4*a*b*c^2*e*f))*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*
(4*a*c - b^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.51 $\int \frac{d+ex^2+fx^4}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=97

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

```
[Out] d*ln(x)/a-1/4*(-a*f+c*d)*ln(c*x^4+b*x^2+a)/a/c+1/2*(a*b*f-2*a*c*e+b*c*d)*ar
ctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1663, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(abf-2ace+bcd)}{2ac\sqrt{b^2-4ac}}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{d\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - 2*a*c*e + a*b*f)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*c
*Sqrt[b^2 - 4*a*c]) + (d*Log[x])/a - ((c*d - a*f)*Log[a + b*x^2 + c*x^4])/(
4*a*c)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{d+ex+fx^2}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\operatorname{Subst}\left(\int \left(\frac{d}{ax}+\frac{-bd+ae-(cd-af)x}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)$$

$$= \frac{d\log(x)}{a}+\frac{\operatorname{Subst}\left(\int \frac{-bd+ae-(cd-af)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}$$

$$= \frac{d\log(x)}{a}-\frac{(cd-af)\operatorname{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4ac}-\frac{(bcd-2ace+abf)\operatorname{Subst}\left(\int \frac{}{a+b\ldots}\right.}{4ac}$$

$$= \frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}+\frac{(bcd-2ace+abf)\operatorname{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx\ldots\right.}{2ac}$$

$$= \frac{(bcd-2ace+abf)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2ac\sqrt{b^2-4ac}}+\frac{d\log(x)}{a}-\frac{(cd-af)\log\left(a+bx^2+cx^4\right)}{4ac}$$

Mathematica [A] time = 0.14, size = 178, normalized size = 1.84

$$\frac{-\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+abf-2ace+bcd\right)+\log\left(\sqrt{b^2-4ac}+b+2c\ldots\right.}{4ac\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*c*Sqrt[b^2 - 4*a*c]*d*Log[x] - (b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e
+ a*b*f - a*Sqrt[b^2 - 4*a*c]*f)*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (b*
c*d - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*Log[
b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*c*Sqrt[b^2 - 4*a*c])
```

fricas [A] time = 1.41, size = 309, normalized size = 3.19

$$\left[\frac{4\left(b^2c-4\,ac^2\right)d\log(x)+\left(bcd-2\,ace+abf\right)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(\left(b^2c-4\,a\ldots\right.\right.}{4\left(ab^2c-4\,a^2c^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(b^2*c - 4*a*c^2)*d*log(x) + (b*c*d - 2*a*c*e + a*b*f)*sqrt(b^2 - 4
*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4
*a*c))/(c*x^4 + b*x^2 + a)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*c)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2), 1/4*(4*(b^2*c - 4*a*c^2)*d*lo
g(x) + 2*(b*c*d - 2*a*c*e + a*b*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^2*c - 4*a*c^2)*d - (a*b^2 - 4*a^2*
c)*f)*log(c*x^4 + b*x^2 + a))/(a*b^2*c - 4*a^2*c^2)]
```

giac [A] time = 1.90, size = 97, normalized size = 1.00

$$\frac{d\log\left(x^2\right)}{2\,a} - \frac{\left(cd - af\right)\log\left(cx^4 + bx^2 + a\right)}{4\,ac} - \frac{\left(bcd + abf - 2\,ace\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2 + 4\,ac}\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)/a - 1/4*(c*d - a*f)*log(c*x^4 + b*x^2 + a)/(a*c) - 1/2*(b*c*
d + a*b*f - 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 +
4*a*c)*a*c)
```

maple [A] time = 0.01, size = 165, normalized size = 1.70

$$-\frac{bd\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}-\frac{bf\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c}+\frac{e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{d\ln(x)}{a}-\frac{d\ln\left(c\,x^4+b\,x^2+a\right)}{4a}+\frac{f\ln\left(c\,x^4+b\,x^2\right.}{4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x)
```

```
[Out] d*ln(x)/a+1/4/c*ln(c*x^4+b*x^2+a)*f-1/4/a*ln(c*x^4+b*x^2+a)*d+1/(4*a*c-b^2)
^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*e-1/2/a/(4*a*c-b^2)^(1/2)*arct
an((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*d-1/2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b/c*f
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 8.88, size = 3927, normalized size = 40.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d*log(x))/a - (log((b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2
 - b*e*f - c*d*f) + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^
2*(4*a*c - b^2)))^(1/2))*(a*b^2*f^2 - x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*
e + 11*a*b*c*f^2 - 9*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) + a*c^2*e^2 - 4
*b*c^2*d*e + 4*b^2*c*d*f + ((c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/
(a^2*c^2*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d
- 4*b^2*c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (b*c*
(c*d - a*f + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/
2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) - 2*a*b*c*e*f))/(4*a*c) - 2
*b*c*d*e*f)*(b^2*d*f^2 + c^2*d*e^2 - x^2*(b*f - c*e)*(a*f^2 + c*e^2 - b*e*f
 - c*d*f) + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c
 - b^2)))^(1/2))*(x^2*(b*c^2*e^2 - 3*b^3*f^2 + 5*c^3*d*e + 11*a*b*c*f^2 - 9
*a*c^2*e*f - 7*b*c^2*d*f + 2*b^2*c*e*f) - a*b^2*f^2 - a*c^2*e^2 + 4*b*c^2*d
```

```
*e - 4*b^2*c*d*f + ((a*f - c*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2
*(4*a*c - b^2)))^(1/2))*(2*c*x^2*(6*b^3*f + 10*a*c^2*e + 5*b*c^2*d - 4*b^2*
c*e - 19*a*b*c*f) + 4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f - (b*c*(a*f - c
*d + a*c*(-(a*b*f - 2*a*c*e + b*c*d)^2/(a^2*c^2*(4*a*c - b^2)))^(1/2))*(a*b
 + 3*b^2*x^2 - 10*a*c*x^2))/a))/(4*a*c) + 2*a*b*c*e*f))/(4*a*c) - 2*b*c*d*e
*f))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*
b^2*c)) + (atan(((4*a*c - b^2)*(((((a*b*f - 2*a*c*e + b*c*d)*(4*b^2*c^2*d -
 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*
d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a*b^2*c)))/(4*a*c*(4*a*c - b^2)^(1/2)) + (b
^2*c*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c
*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(8*a*c^2*d + 2*a*b^2
*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c
*e + b*c*d)*(a*b^2*f^2 + a*c^2*e^2 + ((4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*
c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2
*c^2 - 4*a*b^2*c))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*
a^2*c^2 - 4*a*b^2*c)) - 4*b*c^2*d*e + 4*b^2*c*d*f - 2*a*b*c*e*f))/(4*a*c*(4
*a*c - b^2)^(1/2)) - (b^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(16*a^2*c*(4*a*c - b
^2)^(3/2)))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 4*a^3*c*f -
 28*a*b^2*c*d + 6*a^2*b*c*e))/(c*(a^2*b^2*f^2 + 4*a^2*c^2*e^2 + b^2*c^2*d^2
 - 4*a*b*c^2*d*e + 2*a*b^2*c*d*f - 4*a^2*b*c*e*f)*(a^3*f^2 + 25*a*c^2*d^2 +
 a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d
*e)) + (16*a^3*c*x^2*(((3*b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 8*a*b*c*d)*
(c^2*e^3 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(3*b^3*f^2 - b*
c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f)*(((12*b^3*c^2 -
40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2
 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*f - 38*a*
b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d*e - 11*a*b*c*f^2 + 9*a*c^2
*e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + b^2*e*f^2
 - a*b*f^3 + a*c*e*f^2 + b*c*d*f^2 - 2*b*c*e^2*f - c^2*d*e*f - ((((a*b*f -
2*a*c*e + b*c*d)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2
*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e
+ 10*b*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^(1/2)) + ((
12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f -
 2*b^2*c*d - 8*a^2*c*f))/(8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2
)))*(a*b*f - 2*a*c*e + b*c*d))/(4*a*c*(4*a*c - b^2)^(1/2)) - ((12*b^3*c^2 -
 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d
 - 8*a^2*c*f))/(32*a^2*c^2*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2))))/(8*a^3
*c^2*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^2*
b*e*f - 10*a^2*c*d*f - a*b*c*d*e)) + ((((((a*b*f - 2*a*c*e + b*c*d)*(((12*b
^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(1
6*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b*c^3*d + 12*b^3*c*
f - 38*a*b*c^2*f))/(4*a*c*(4*a*c - b^2)^(1/2)) + ((12*b^3*c^2 - 40*a*b*c^3)
*(a*b*f - 2*a*c*e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))
/(8*a*c*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(8*a*c^2*d + 2*a*b^2
*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) + ((a*b*f - 2*a*c
*e + b*c*d)*(3*b^3*f^2 - b*c^2*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d -
8*a^2*c*f)*(((12*b^3*c^2 - 40*a*b*c^3)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d -
 8*a^2*c*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 8*b^2*c^2*e + 20*a*c^3*e + 10*b
*c^3*d + 12*b^3*c*f - 38*a*b*c^2*f))/(2*(16*a^2*c^2 - 4*a*b^2*c)) - 5*c^3*d
*e - 11*a*b*c*f^2 + 9*a*c^2*e*f + 7*b*c^2*d*f - 2*b^2*c*e*f))/(4*a*c*(4*a*c
 - b^2)^(1/2)) - ((12*b^3*c^2 - 40*a*b*c^3)*(a*b*f - 2*a*c*e + b*c*d)^3)/(6
4*a^3*c^3*(4*a*c - b^2)^(3/2)))*(6*b^4*d + 20*a^2*c^2*d + 2*a^2*b^2*f - 2*a
*b^3*e - 4*a^3*c*f - 28*a*b^2*c*d + 6*a^2*b*c*e))/(16*a^3*c^2*(4*a*c - b^2)
^(1/2)*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c*d^2 + 3*a*b^2*d*f - a^
2*b*e*f - 10*a^2*c*d*f - a*b*c*d*e)))*(4*a*c - b^2)^(3/2))/(a^2*b^2*f^2 + 4
*a^2*c^2*e^2 + b^2*c^2*d^2 - 4*a*b*c^2*d*e + 2*a*b^2*c*d*f - 4*a^2*b*c*e*f)
 + (2*(4*a*c - b^2)^(3/2)*(3*b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 8*a*b*c*
d)*(b^2*d*f^2 + c^2*d*e^2 + ((8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f
)*(a*b^2*f^2 + a*c^2*e^2 + ((4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b^2*c*f + (2*a
```

```
*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*a^2*c^2 - 4*a
*b^2*c))*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2 -
4*a*b^2*c)) - 4*b*c^2*d*e + 4*b^2*c*d*f - 2*a*b*c*e*f))/(2*(16*a^2*c^2 - 4*
a*b^2*c)) - ((((a*b*f - 2*a*c*e + b*c*d)*(4*b^2*c^2*d - 4*a*b*c^2*e + 4*a*b
^2*c*f + (2*a*b^2*c^2*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(16*
a^2*c^2 - 4*a*b^2*c)))/(4*a*c*(4*a*c - b^2)^(1/2)) + (b^2*c*(a*b*f - 2*a*c*
e + b*c*d)*(8*a*c^2*d + 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(2*(16*a^2*c^2
- 4*a*b^2*c)*(4*a*c - b^2)^(1/2)))*(a*b*f - 2*a*c*e + b*c*d))/(4*a*c*(4*a*c
 - b^2)^(1/2)) - 2*b*c*d*e*f - (b^2*(a*b*f - 2*a*c*e + b*c*d)^2*(8*a*c^2*d
+ 2*a*b^2*f - 2*b^2*c*d - 8*a^2*c*f))/(8*a*(16*a^2*c^2 - 4*a*b^2*c)*(4*a*c
- b^2))))/(c*(a^2*b^2*f^2 + 4*a^2*c^2*e^2 + b^2*c^2*d^2 - 4*a*b*c^2*d*e + 2
*a*b^2*c*d*f - 4*a^2*b*c*e*f)*(a^3*f^2 + 25*a*c^2*d^2 + a^2*c*e^2 - 6*b^2*c
*d^2 + 3*a*b^2*d*f - a^2*b*e*f - 10*a^2*c*d*f - a*b*c*d*e)))*(a*b*f - 2*a*c
*e + b*c*d))/(2*a*c*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.52 $\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=118

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

```
[Out] -1/2*d/a/x^2-(-a*e+b*d)*ln(x)/a^2+1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)/a^2-1/2*
(b^2*d-a*b*e-2*a*(-a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-
4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-abe-2a(cd-af)+b^2d\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(bd-ae)}{a^2}-\frac{d}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(2*a*x^2) - ((b^2*d - a*b*e - 2*a*(c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqr
t[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((b*d - a*e)*Log[x])/a^2 + ((b
*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^2}+\frac{-bd+ae}{a^2x}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a^2\left(a+bx+cx^2\right)}\right)dx, x, x^2\right)$$

$$= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}$$

$$= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{\left(b^2d-abe-2a(c\right.}{}$$

$$= -\frac{d}{2ax^2}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\left(b^2d-abe-2a(cd-af)\right)}{}$$

$$= -\frac{d}{2ax^2}-\frac{\left(b^2d-abe-2a(cd-af)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(bd-ae)\log(x)}{a^2}+\frac{(bd-ae)\,\mathrm{lo}}{}$$

Mathematica [A] time = 0.15, size = 203, normalized size = 1.72

$$\frac{\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(a\left(-e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}-ae\right)+b^2d\right)}{\sqrt{b^2-4ac}}+\frac{\log\left(\sqrt{b^2-4ac}+b+2cx^2\right)\left(-a\left(e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}+ae\right.\right.}{\sqrt{b^2-4ac}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^2 + 4*(-(b*d) + a*e)*Log[x] + ((b^2*d + b*(Sqrt[b^2 - 4*a*c]*d
- a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b - Sqrt[b^2 - 4*a*c
] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + ((-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d + a*e
) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2])/Sqrt[b^2 - 4*a*c])/(4*a^2)
```

fricas [A] time = 1.64, size = 399, normalized size = 3.38

$$\left[-\frac{\left(abe-2a^2f-\left(b^2-2ac\right)d\right)\sqrt{b^2-4ac}\,x^2\log\left(\frac{2c^2x^4+2bcx^2+b^2-2ac-\left(2cx^2+b\right)\sqrt{b^2-4ac}}{cx^4+bx^2+a}\right)-\left(\left(b^3-4abc\right)d-\left(ab^2-4a^{\cdots}\right.\right.}{4\left(a^2b^2-4a^3c\right)x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2
*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 +
b*x^2 + a)) - ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b*x
^2 + a) + 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) + 2*(a*b^2
```

```
 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(a*b*e - 2*a^2*f - (b^2 -
2*a*c)*d)*sqrt(-b^2 + 4*a*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(
b^2 - 4*a*c)) + ((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(c*x^4 + b
*x^2 + a) - 4*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e)*x^2*log(x) - 2*(a*b
^2 - 4*a^2*c)*d)/((a^2*b^2 - 4*a^3*c)*x^2)]
```

giac [A] time = 1.78, size = 135, normalized size = 1.14

$$\frac{(bd-ae)\log\left(cx^4+bx^2+a\right)}{4\,a^2}-\frac{(bd-ae)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2d-2\,acd+2\,a^2f-abe\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a^2}+\frac{bdx^2-ax^2e}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 +
 1/2*(b^2*d - 2*a*c*d + 2*a^2*f - a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4
*a*c))/(sqrt(-b^2 + 4*a*c)*a^2) + 1/2*(b*d*x^2 - a*x^2*e - a*d)/(a^2*x^2)
```

maple [B] time = 0.01, size = 227, normalized size = 1.92

$$-\frac{be\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a}-\frac{cd\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{b^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}+\frac{f\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}+\frac{e\ln(x)}{a}-\frac{e\ln\left(cx^4+b\right.}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/2*d/a/x^2+1/a*ln(x)*e-1/a^2*ln(x)*b*d-1/4/a*ln(c*x^4+b*x^2+a)*e+1/4/a^2*
ln(c*x^4+b*x^2+a)*b*d+1/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1
/2))*f-1/2/a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*e-1/
a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d+1/2/a^2/(4*a*
c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 7.86, size = 4437, normalized size = 37.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(a*e - b*d))/a^2 - d/(2*a*x^2) - (log(((c^2*(a*e - b*d)*(a*f - c*d)
^2)/a^3 - ((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4
*a*c - b^2)))^(1/2))*(((b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*
d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*(10*a*c^2*d + 4*a*b^2*f + b^2*
c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(b^2*d + a^2*f - a*b*e - a*c*d)
)/a + (b*c^2*(b*d - a*e + a^2*(-(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*
```

```
(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2))/(4*a^2) + (c^2
*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c*d))/a^2 - (c^2*x^2*(a*f - c*d
)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2) + (c^2*x^2*(a*f - c*d)^3)/a^3)
*((c^2*(a*e - b*d)*(a*f - c*d)^2)/a^3 - ((a*e - b*d + a^2*(-(b^2*d + 2*a^2*
f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(((a*e - b*d + a^2*(-(b^
2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*((2*c^2*x^2*
(10*a*c^2*d + 4*a*b^2*f + b^2*c*d - 10*a^2*c*f - 5*a*b*c*e))/a + (4*b*c^2*(
b^2*d + a^2*f - a*b*e - a*c*d))/a - (b*c^2*(a*e - b*d + a^2*(-(b^2*d + 2*a^
2*f - a*b*e - 2*a*c*d)^2/(a^4*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 - 10*
a*c*x^2))/a^2))/(4*a^2) - (c^2*(a*f - c*d)*(4*b^2*d + a^2*f - 4*a*b*e - a*c
*d))/a^2 + (c^2*x^2*(a*f - c*d)*(a*b*f + 5*a*c*e - 6*b*c*d))/a^2))/(4*a^2)
+ (c^2*x^2*(a*f - c*d)^3)/a^3))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*
d))/(2*(16*a^3*c - 4*a^2*b^2)) - (atan((16*a^6*(4*a*c - b^2)^(3/2)*(x^2*(((
(c^5*d^3 - a^3*c^2*f^3 + 3*a^2*c^3*d*f^2 - 3*a*c^4*d^2*f)/a^3 + (((a^3*b*c^
2*f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2*b*c^3*d*f)/a^
3 + (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10
*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2*a*b^2*e +
 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2
*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2
*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((((20*a^3*c^4*d
 - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*c^2*f - 10*a^3*b*c^3*e)/a^3 +
 ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*
c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/
(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(b^2*d + 2*a
^2*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(8*a
^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e -
2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(b
^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*
b*c*d))/(32*a^7*(4*a*c - b^2)*(16*a^3*c - 4*a^2*b^2)))*(3*b^4*d + a^2*c^2*d
 + a^2*b^2*f - 3*a*b^3*e - a^3*c*f - 9*a*b^2*c*d + 8*a^2*b*c*e))/(8*a^3*c^2
*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b
^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b*
c*d*e)) - ((((((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*b^2*
c^2*f - 10*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d - 2
*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(b^2*d +
 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 -
 12*a^3*b^3*c^2)*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e +
 8*a^2*c*e - 8*a*b*c*d))/(8*a^5*(4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2))
)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2))
 + (((a^3*b*c^2*f^2 + 6*a*b*c^4*d^2 - 5*a^2*c^4*d*e + 5*a^3*c^3*e*f - 7*a^2
*b*c^3*d*f)/a^3 + (((20*a^3*c^4*d - 20*a^4*c^3*f + 2*a^2*b^2*c^3*d + 8*a^3*
b^2*c^2*f - 10*a^3*b*c^3*e)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*b^3*d
 - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*b
^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(b^2
*d + 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - ((40*a^4*b*c
^3 - 12*a^3*b^3*c^2)*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^3)/(64*a^9*(4*a*c
- b^2)^(3/2)))*(6*b^5*d + 2*a^2*b^3*f - 20*a^3*c^2*e - 6*a*b^4*e - 30*a*b^3
*c*d - 6*a^3*b*c*f + 26*a^2*b*c^2*d + 28*a^2*b^2*c*e))/(16*a^3*c^2*(4*a*c -
 b^2)^(1/2)*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2 + a^2*c^2*d
^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a^2*b^2*d*f
- 49*a^2*b*c*d*e))) + (((b*c^4*d^3 - a^3*c^2*e*f^2 - a*c^4*d^2*e - 2*a*b*c^
3*d^2*f + 2*a^2*c^3*d*e*f + a^2*b*c^2*d*f^2)/a^3 - (((a^2*c^4*d^2 + a^4*c^2
*f^2 - 4*a*b^2*c^3*d^2 - 2*a^3*c^3*d*f + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f
+ 4*a^2*b^2*c^2*d*f)/a^3 - (((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c
^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e -
 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a
*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a
*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*
e - 4*a^3*b*c^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e
```

```
+ 8*a^2*c*e - 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(b^2*d + 2*a^2*f - a*b*e
- 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e
 - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a*(4*a*c - b^
2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/(4*a
^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^2*(2
*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(8*a^3*(4*a*c - b^2)*(16*a^3*c
 - 4*a^2*b^2)))*(3*b^4*d + a^2*c^2*d + a^2*b^2*f - 3*a*b^3*e - a^3*c*f - 9*
a*b^2*c*d + 8*a^2*b*c*e))/(8*a^3*c^2*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 -
6*a^2*b^2*e^2 + a^2*c^2*d^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a
*b^2*c*d^2 + a^2*b^2*d*f - 49*a^2*b*c*d*e)) - (((((((4*a^2*b^3*c^2*d - 4*a^
3*b^2*c^2*e - 4*a^3*b*c^3*d + 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d -
2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(16*a^3*c - 4*a^2*b^2))*(b^2*d + 2*a^2*
f - a*b*e - 2*a*c*d))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^2*d + 2*a^2
*f - a*b*e - 2*a*c*d)*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d))/(2*a*(
4*a*c - b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*
e - 8*a*b*c*d))/(2*(16*a^3*c - 4*a^2*b^2)) - (((a^2*c^4*d^2 + a^4*c^2*f^2 -
 4*a*b^2*c^3*d^2 - 2*a^3*c^3*d*f + 4*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f + 4*a^
2*b^2*c^2*d*f)/a^3 - (((4*a^2*b^3*c^2*d - 4*a^3*b^2*c^2*e - 4*a^3*b*c^3*d +
 4*a^4*b*c^2*f)/a^3 - (2*a*b^2*c^2*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b
*c*d))/(16*a^3*c - 4*a^2*b^2))*(2*b^3*d - 2*a*b^2*e + 8*a^2*c*e - 8*a*b*c*d
))/(2*(16*a^3*c - 4*a^2*b^2)))*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/(4*a^2*
(4*a*c - b^2)^(1/2)) + (b^2*c^2*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)^3)/(16*
a^5*(4*a*c - b^2)^(3/2)))*(6*b^5*d + 2*a^2*b^3*f - 20*a^3*c^2*e - 6*a*b^4*e
 - 30*a*b^3*c*d - 6*a^3*b*c*f + 26*a^2*b*c^2*d + 28*a^2*b^2*c*e))/(16*a^3*c
^2*(4*a*c - b^2)^(1/2)*(a^4*f^2 - 6*b^4*d^2 + 25*a^3*c*e^2 - 6*a^2*b^2*e^2
+ a^2*c^2*d^2 + 12*a*b^3*d*e - a^3*b*e*f - 2*a^3*c*d*f + 24*a*b^2*c*d^2 + a
^2*b^2*d*f - 49*a^2*b*c*d*e))))/(4*a^2*c^4*d^2 + b^4*c^2*d^2 + 4*a^4*c^2*f^
2 - 4*a*b^2*c^3*d^2 + a^2*b^2*c^2*e^2 - 8*a^3*c^3*d*f - 2*a*b^3*c^2*d*e + 4
*a^2*b*c^3*d*e - 4*a^3*b*c^2*e*f + 4*a^2*b^2*c^2*d*f))*(b^2*d + 2*a^2*f - a
*b*e - 2*a*c*d))/(2*a^2*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.53 $\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=174

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{4a^3}+\frac{\log(x)\left(-abe-a(cd-af)+b^2d\right)}{a^3}+\frac{bd-ae}{2a^2x^2}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(\cdots}{\cdots}$$

```
[Out] -1/4*d/a/x^4+1/2*(-a*e+b*d)/a^2/x^2+(b^2*d-a*b*e-a*(-a*f+c*d))*ln(x)/a^3-1/
4*(b^2*d-a*b*e-a*(-a*f+c*d))*ln(c*x^4+b*x^2+a)/a^3+1/2*(b^3*d-a*b^2*e+2*a^2
*c*e-a*b*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+
b^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-abe-a(cd-af)+b^2d\right)}{4a^3}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\log(x)\,(\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(4*a*x^4) + (b*d - a*e)/(2*a^2*x^2) + ((b^3*d - a*b^2*e + 2*a^2*c*e - a*
b*(3*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*Sqrt[b^2
- 4*a*c]) + ((b^2*d - a*b*e - a*(c*d - a*f))*Log[x])/a^3 - ((b^2*d - a*b*e
- a*(c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
```

```
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)}dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3\left(a+bx+cx^2\right)}dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d}{ax^3}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)}{a^3x}+\frac{-b^3d+ab^2e-a^2ce+ab}{}\right.\right.$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}+\frac{\text{Subst}\left(\int \frac{-b^3d+ab^2e-a^2ce+ab(2c}{a-}\right.}{}$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}-\frac{\left(b^2d-abe-a(cd-af)\right)\text{Sul}}{4a^3}$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^2d-abe-a(cd-af)\right)\log(x)}{a^3}-\frac{\left(b^2d-abe-a(cd-af)\right)\log}{4a^3}$$

$$= -\frac{d}{4ax^4}+\frac{bd-ae}{2a^2x^2}+\frac{\left(b^3d-ab^2e+2a^2ce-ab(3cd-af)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}}+\frac{\left(b^2d\right.}{}$$

Mathematica [A] time = 0.35, size = 314, normalized size = 1.80

$$-\frac{\frac{a^2d}{x^4}-4\log(x)\left(-abe+a(af-cd)+b^2d\right)+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(ab\left(-e\sqrt{b^2-4ac}+af-3cd\right)+a\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+2a\right.\right.}{\sqrt{b^2-4ac}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*((a^2*d)/x^4 + (2*a*(-(b*d) + a*e))/x^2 - 4*(b^2*d - a*b*e + a*(-(c*d)
 + a*f))*Log[x] + ((b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) + a*b*(-3*c*d -
 Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*a*c*e + a*Sqr
t[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]
+ ((-(b^3*d) + b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*b*(-3*c*d + Sqrt[b^2 - 4
*a*c]*e + a*f) + a*(-(c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e)) + a*Sqrt[b^2 - 4*a*c
]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c])/a^3
```

fricas [A] time = 2.54, size = 609, normalized size = 3.50

$$\left[\frac{\left(a^2bf+\left(b^3-3\,abc\right)d-\left(ab^2-2\,a^2c\right)e\right)\sqrt{b^2-4\,ac}\,x^4\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-\left(\left(b^4-5\,ab^2c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*b*f + (b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(b^2 - 4*a*c)
*x^4*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*
a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^4 -
5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x
^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^2*b^
2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4), 1/4*(2*(a^2*b*f + (b^3 - 3*a*b*c
)*d - (a*b^2 - 2*a^2*c)*e)*sqrt(-b^2 + 4*a*c)*x^4*arctan(-(2*c*x^2 + b)*sqr
t(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3
- 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*f)*x^4*log(c*x^4 + b*x^2 + a) + 4*((b^
4 - 5*a*b^2*c + 4*a^2*c^2)*d - (a*b^3 - 4*a^2*b*c)*e + (a^2*b^2 - 4*a^3*c)*
f)*x^4*log(x) + 2*((a*b^3 - 4*a^2*b*c)*d - (a^2*b^2 - 4*a^3*c)*e)*x^2 - (a^
2*b^2 - 4*a^3*c)*d)/((a^3*b^2 - 4*a^4*c)*x^4)]
```

giac [A] time = 1.72, size = 212, normalized size = 1.22

$$-\frac{\left(b^2 d - acd + a^2 f - abe\right)\log\left(cx^4 + bx^2 + a\right)}{4\,a^3} + \frac{\left(b^2 d - acd + a^2 f - abe\right)\log\left(x^2\right)}{2\,a^3} - \frac{\left(b^3 d - 3\,abcd + a^2 bf - ab^2 e + \cdots\right.}{2\,\sqrt{-b^2 + 4 \cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*d - a*c*d + a^2*f - a*b*e)*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*(b^2*
d - a*c*d + a^2*f - a*b*e)*log(x^2)/a^3 - 1/2*(b^3*d - 3*a*b*c*d + a^2*b*f
- a*b^2*e + 2*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2
+ 4*a*c)*a^3) - 1/4*(3*b^2*d*x^4 - 3*a*c*d*x^4 + 3*a^2*f*x^4 - 3*a*b*x^4*e
- 2*a*b*d*x^2 + 2*a^2*x^2*e + a^2*d)/(a^3*x^4)
```

maple [B] time = 0.01, size = 356, normalized size = 2.05

$$-\frac{bf\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a} - \frac{ce\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a} + \frac{b^2 e\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2} + \frac{3bcd\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2} - \frac{b^3 d\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^3} \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/4*d/a/x^4-1/2/a/x^2*e+1/2/a^2/x^2*b*d+1/a*ln(x)*f-1/a^2*ln(x)*b*e-1/a^2*
ln(x)*c*d+1/a^3*ln(x)*b^2*d-1/4/a*ln(c*x^4+b*x^2+a)*f+1/4/a^2*ln(c*x^4+b*x^
2+a)*b*e+1/4/a^2*c*ln(c*x^4+b*x^2+a)*d-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2*d-1/2/
a/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*f-1/a/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*e+1/2/a^2/(4*a*c-b^2)^(1/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^2*e+3/2/a^2/(4*a*c-b^2)^(1/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c*d-1/2/a^3/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 9.92, size = 6187, normalized size = 35.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(x)*(b^2*d + a^2*f - a*b*e - a*c*d))/a^3 - (d/(4*a) + (x^2*(a*e - b*d))
/(2*a^2))/x^4 + (log((((((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*
c*e + 5*a*b*c*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a
*b*c*d))/a^2 + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^2*d + a^2*f + a^3*(
-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))
^(1/2) - a*b*e - a*c*d))/a^3)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2
*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d)
)/(4*a^3) + (c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5
*a*b*c*d))/a^4 + (c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*
d))/a^4)*(b^2*d + a^2*f + a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*
a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a*b*e - a*c*d))/(4*a^3) + (c^4*(a*e
 - b*d)^2*(b^2*d + a^2*f - a*b*e - a*c*d))/a^6 - (c^5*x^2*(a*e - b*d)^3)/a^
6)*((((c^3*(a*e - b*d)*(4*b^3*d - 4*a*b^2*e + 4*a^2*b*f + a^2*c*e - 5*a*b*c
*d))/a^4 - (((2*c^3*x^2*(b^3*d - a*b^2*e + 5*a^2*b*f - 10*a^2*c*e + 5*a*b*c
*d))/a^2 + (4*b*c^2*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^2
- (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f +
 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*
e + a*c*d))/a^3)*(a^3*(-(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)
^2/(a^6*(4*a*c - b^2)))^(1/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) + (
c^4*x^2*(a*e - b*d)*(6*b^2*d + 5*a^2*f - 6*a*b*e - 5*a*c*d))/a^4)*(a^3*(-(b
^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2/(a^6*(4*a*c - b^2)))^(1
/2) - a^2*f - b^2*d + a*b*e + a*c*d))/(4*a^3) - (c^4*(a*e - b*d)^2*(b^2*d +
 a^2*f - a*b*e - a*c*d))/a^6 + (c^5*x^2*(a*e - b*d)^3)/a^6))*(2*b^4*d + 8*a
^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e
))/(2*(16*a^4*c - 4*a^3*b^2)) - (atan((16*a^9*(4*a*c - b^2)^(3/2)*(x^2*((((
a^3*c^5*e^3 - b^3*c^5*d^3 + 3*a*b^2*c^5*d^2*e - 3*a^2*b*c^5*d*e^2)/a^6 - ((
(6*a^4*b*c^4*e^2 - 5*a^3*b*c^5*d^2 + 6*a^2*b^3*c^4*d^2 + 5*a^4*c^5*d*e - 5*
a^5*c^4*e*f + 5*a^4*b*c^4*d*f - 12*a^3*b^2*c^4*d*e)/a^6 + (((2*a^4*b^3*c^3*
d - 20*a^6*c^4*e - 2*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 +
 ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*
a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3
*b^2)))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a
*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d
 + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(1
6*a^4*c - 4*a^3*b^2)) + (((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^2*c^3
*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^3*c^2
)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c
*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b
*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40*a^7*b*c^3 -
 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)*(2*b^4
*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a
^2*b*c*e))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40
*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d)^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a
*b^2*c*d + 8*a^2*b*c*e))/(32*a^12*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3
*b^5*d + 3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f +
 9*a^2*b*c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a
^2*b^4*e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^
2 + 12*a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 1
2*a^3*b^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a
^3*b^2*c*d*f - 49*a^4*b*c*e*f)) + (((((((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2
```

```
*a^5*b^2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12
*a^6*b^3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f
 - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b
^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + ((40
*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b
^2*c*d + 8*a^2*b*c*e))/(8*a^9*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^2)))*
(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d
 + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)) + (((6*a^4*b*c^4*e^2 - 5*a^3*b*
c^5*d^2 + 6*a^2*b^3*c^4*d^2 + 5*a^4*c^5*d*e - 5*a^5*c^4*e*f + 5*a^4*b*c^4*d
*f - 12*a^3*b^2*c^4*d*e)/a^6 + (((2*a^4*b^3*c^3*d - 20*a^6*c^4*e - 2*a^5*b^
2*c^3*e + 10*a^5*b*c^4*d + 10*a^6*b*c^3*f)/a^6 + ((40*a^7*b*c^3 - 12*a^6*b^
3*c^2)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*
b^2*c*d + 8*a^2*b*c*e))/(2*a^6*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^
2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2
*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*
d))/(4*a^3*(4*a*c - b^2)^(1/2)) - ((40*a^7*b*c^3 - 12*a^6*b^3*c^2)*(b^3*d -
 a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^3)/(64*a^15*(4*a*c - b^2)^(3/2)
))*(6*b^6*d - 20*a^3*c^3*d + 6*a^2*b^4*f + 20*a^4*c^2*f - 6*a*b^5*e + 54*a^
2*b^2*c^2*d - 36*a*b^4*c*d + 30*a^2*b^3*c*e - 26*a^3*b*c^2*e - 28*a^3*b^2*c
*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*
e^2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*
a*b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b
^3*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*
c*d*f - 49*a^4*b*c*e*f))) - (((b^4*c^4*d^3 - a*b^2*c^5*d^3 - a^3*b*c^4*e^3
 - a^3*c^5*d*e^2 + a^4*c^4*e^2*f - 3*a*b^3*c^4*d^2*e + 2*a^2*b*c^5*d^2*e + 3
*a^2*b^2*c^4*d*e^2 + a^2*b^2*c^4*d^2*f - 2*a^3*b*c^4*d*e*f)/a^6 - (((a^5*c^
4*e^2 - 4*a^2*b^4*c^3*d^2 + 5*a^3*b^2*c^4*d^2 - 4*a^4*b^2*c^3*e^2 - 6*a^4*b
*c^4*d*e + 4*a^5*b*c^3*e*f + 8*a^3*b^3*c^3*d*e - 4*a^4*b^2*c^3*d*f)/a^6 - (
((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4*a^6*b^2*c^2*f + 4
*a^6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a
*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(16*a^4*c - 4*a^3*b^2))*(
2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d
+ 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*
b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c -
 4*a^3*b^2)) - (((((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4
*a^6*b^2*c^2*f + 4*a^6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d +
 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(16*a^4
*c - 4*a^3*b^2))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^
3*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e -
3*a*b*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 1
0*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*c - 4*a^3*b^
2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b
^2)^(1/2)) + (b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^2
*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*
d + 8*a^2*b*c*e))/(8*a^5*(4*a*c - b^2)*(16*a^4*c - 4*a^3*b^2)))*(3*b^5*d +
3*a^2*b^3*f - a^3*c^2*e - 3*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 9*a^2*b*
c^2*d + 9*a^2*b^2*c*e))/(8*a^3*c^2*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^
2 + 25*a^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*
b^5*d*e - 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3
*e*f - 50*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*
d*f - 49*a^4*b*c*e*f)) + ((((((4*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b
^3*c^2*e + 4*a^6*b^2*c^2*f + 4*a^6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8
*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c
*e))/(16*a^4*c - 4*a^3*b^2))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 3*a*b
*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^3*d - a*b^2*e + a^2*b*f +
2*a^2*c*e - 3*a*b*c*d)*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8
*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*a^2*(4*a*c - b^2)^(1/2)*(16*a^4*
c - 4*a^3*b^2)))*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c
```

```
*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)) - (((a^5*c^4*e
^2 - 4*a^2*b^4*c^3*d^2 + 5*a^3*b^2*c^4*d^2 - 4*a^4*b^2*c^3*e^2 - 6*a^4*b*c^
4*d*e + 4*a^5*b*c^3*e*f + 8*a^3*b^3*c^3*d*e - 4*a^4*b^2*c^3*d*f)/a^6 - (((4
*a^4*b^4*c^2*d - 8*a^5*b^2*c^3*d - 4*a^5*b^3*c^2*e + 4*a^6*b^2*c^2*f + 4*a^
6*b*c^3*e)/a^6 - (2*a*b^2*c^2*(2*b^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^
3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8*a^2*b*c*e))/(16*a^4*c - 4*a^3*b^2))*(2*b
^4*d + 8*a^2*c^2*d + 2*a^2*b^2*f - 2*a*b^3*e - 8*a^3*c*f - 10*a*b^2*c*d + 8
*a^2*b*c*e))/(2*(16*a^4*c - 4*a^3*b^2)))*(b^3*d - a*b^2*e + a^2*b*f + 2*a^2
*c*e - 3*a*b*c*d))/(4*a^3*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(b^3*d - a*b^2*e
+ a^2*b*f + 2*a^2*c*e - 3*a*b*c*d)^3)/(16*a^8*(4*a*c - b^2)^(3/2)))*(6*b^6*
d - 20*a^3*c^3*d + 6*a^2*b^4*f + 20*a^4*c^2*f - 6*a*b^5*e + 54*a^2*b^2*c^2*
d - 36*a*b^4*c*d + 30*a^2*b^3*c*e - 26*a^3*b*c^2*e - 28*a^3*b^2*c*f))/(16*a
^3*c^2*(4*a*c - b^2)^(1/2)*(25*a^5*c*f^2 - 6*b^6*d^2 - 6*a^2*b^4*e^2 + 25*a
^3*c^3*d^2 - 6*a^4*b^2*f^2 + a^4*c^2*e^2 + 24*a^3*b^2*c*e^2 + 12*a*b^5*d*e
- 54*a^2*b^2*c^2*d^2 + 36*a*b^4*c*d^2 - 12*a^2*b^4*d*f + 12*a^3*b^3*e*f - 5
0*a^4*c^2*d*f - 60*a^2*b^3*c*d*e + 47*a^3*b*c^2*d*e + 61*a^3*b^2*c*d*f - 49
*a^4*b*c*e*f))))/(4*a^4*c^4*e^2 + b^6*c^2*d^2 - 6*a*b^4*c^3*d^2 + 9*a^2*b^2
*c^4*d^2 + a^2*b^4*c^2*e^2 - 4*a^3*b^2*c^3*e^2 + a^4*b^2*c^2*f^2 - 2*a*b^5*
c^2*d*e - 12*a^3*b*c^4*d*e + 4*a^4*b*c^3*e*f + 10*a^2*b^3*c^3*d*e + 2*a^2*b
^4*c^2*d*f - 6*a^3*b^2*c^3*d*f - 2*a^3*b^3*c^2*e*f))*(b^3*d - a*b^2*e + a^2
*b*f + 2*a^2*c*e - 3*a*b*c*d))/(2*a^3*(4*a*c - b^2)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.54 $\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=244

$$-\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{bd-ae}{4a^2x^4}+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4}-\frac{\log(x)\left(a^2ce-ab^2e\right.}{}$$

```
[Out] -1/6*d/a/x^6+1/4*(-a*e+b*d)/a^2/x^4+1/2*(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x^2
-(b^3*d-a*b^2*e+a^2*c*e-a*b*(-a*f+2*c*d))*ln(x)/a^4+1/4*(b^3*d-a*b^2*e+a^2*
c*e-a*b*(-a*f+2*c*d))*ln(c*x^4+b*x^2+a)/a^4-1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+
2*a^2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/
2))/a^4/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1663, 1628, 634, 618, 206, 628}

$$-\frac{-abe-a(cd-af)+b^2d}{2a^3x^2}+\frac{\log\left(a+bx^2+cx^4\right)\left(a^2ce-ab^2e-ab(2cd-af)+b^3d\right)}{4a^4}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(3a^2bce+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(6*a*x^6) + (b*d - a*e)/(4*a^2*x^4) - (b^2*d - a*b*e - a*(c*d - a*f))/(2
*a^3*x^2) - ((b^4*d - a*b^3*e + 3*a^2*b*c*e + 2*a^2*c*(c*d - a*f) - a*b^2*(
4*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*Sqrt[b^2 - 4
*a*c]) - ((b^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[x])/a^4 + ((b
^3*d - a*b^2*e + a^2*c*e - a*b*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^
4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^7\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex+fx^2}{x^4\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{d}{ax^4}+\frac{-bd+ae}{a^2x^3}+\frac{b^2d-abe-a(cd-af)}{a^3x^2}+\frac{-b^3d+ab^2e-a^2ce+ab}{a^4x}\right.\right.$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\text{lo}}{a^4}$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\text{lo}}{a^4}$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^3d-ab^2e+a^2ce-ab(2cd-af)\right)\text{lo}}{a^4}$$

$$= -\frac{d}{6ax^6}+\frac{bd-ae}{4a^2x^4}-\frac{b^2d-abe-a(cd-af)}{2a^3x^2}-\frac{\left(b^4d-ab^3e+3a^2bce+2a^2c(cd-af\right)}{2a^4\sqrt{b^2}}$$

Mathematica [A] time = 0.35, size = 416, normalized size = 1.70

$$\frac{-\frac{2a^3d}{x^6}-12\log(x)\left(a^2ce-ab^2e+ab(af-2cd)+b^3d\right)+\frac{3\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(a^2c\left(e\sqrt{b^2-4ac}-2af+2cd\right)+ab^2\left(-e\sqrt{b^2-4ac}\right.\right.}{\sqrt{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a^3*d)/x^6 + (3*a^2*(b*d - a*e))/x^4 + (6*a*(-(b^2*d) + a*b*e + a*(c*d
 - a*f)))/x^2 - 12*(b^3*d - a*b^2*e + a^2*c*e + a*b*(-2*c*d + a*f))*Log[x]
+ (3*(b^4*d + b^3*(Sqrt[b^2 - 4*a*c]*d - a*e) + a^2*c*(2*c*d + Sqrt[b^2 - 4
*a*c]*e - 2*a*f) + a*b^2*(-4*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-2*c*S
qrt[b^2 - 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c] + (3*(-(b^4*d) + b^3*(Sqrt[b^2 - 4*a*c]
*d + a*e) - a*b^2*(-4*c*d + Sqrt[b^2 - 4*a*c]*e + a*f) + a^2*c*(-2*c*d + Sq
rt[b^2 - 4*a*c]*e + 2*a*f) + a*b*(-2*c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e + a*Sq
rt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c]
)/(12*a^4)
```

fricas [A] time = 5.31, size = 834, normalized size = 3.42

$$\left[-\frac{3\sqrt{b^2-4\,ac}\left(\left(b^4-4\,ab^2c+2\,a^2c^2\right)d-\left(ab^3-3\,a^2bc\right)e+\left(a^2b^2-2\,a^3c\right)f\right)x^6\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+\left(2\,cx^2+b\right)}{cx^4+bx^2+a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(b^2 - 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a
^2*b*c)*e + (a^2*b^2 - 2*a^3*c)*f)*x^6*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 3*((b^5 - 6*
a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 -
 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b^5 - 6*a*b^3*c + 8*a^2*b*
c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6
*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 - 4*a^3*b*c)*e
+ (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3*b*c)*d - (a^3*b^2 - 4*a^
4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 - 4*a^5*c)*x^6), -1/12*(6*
sqrt(-b^2 + 4*a*c)*((b^4 - 4*a*b^2*c + 2*a^2*c^2)*d - (a*b^3 - 3*a^2*b*c)*e
 + (a^2*b^2 - 2*a^3*c)*f)*x^6*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2
 - 4*a*c)) - 3*((b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c +
4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^6*log(c*x^4 + b*x^2 + a) + 12*((b
^5 - 6*a*b^3*c + 8*a^2*b*c^2)*d - (a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*e + (a^
2*b^3 - 4*a^3*b*c)*f)*x^6*log(x) + 6*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d -
 (a^2*b^3 - 4*a^3*b*c)*e + (a^3*b^2 - 4*a^4*c)*f)*x^4 - 3*((a^2*b^3 - 4*a^3
*b*c)*d - (a^3*b^2 - 4*a^4*c)*e)*x^2 + 2*(a^3*b^2 - 4*a^4*c)*d)/((a^4*b^2 -
 4*a^5*c)*x^6)]
```

giac [A] time = 1.94, size = 313, normalized size = 1.28

$$\frac{\left(b^3d-2\,abcd+a^2bf-ab^2e+a^2ce\right)\log\left(cx^4+bx^2+a\right)}{4\,a^4}-\frac{\left(b^3d-2\,abcd+a^2bf-ab^2e+a^2ce\right)\log\left(x^2\right)}{2\,a^4}+\frac{\left(b^4d-4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a
)/a^4 - 1/2*(b^3*d - 2*a*b*c*d + a^2*b*f - a*b^2*e + a^2*c*e)*log(x^2)/a^4
+ 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d + a^2*b^2*f - 2*a^3*c*f - a*b^3*e
+ 3*a^2*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)
*a^4) + 1/12*(11*b^3*d*x^6 - 22*a*b*c*d*x^6 + 11*a^2*b*f*x^6 - 11*a*b^2*x^6
*e + 11*a^2*c*x^6*e - 6*a*b^2*d*x^4 + 6*a^2*c*d*x^4 - 6*a^3*f*x^4 + 6*a^2*b
*x^4*e + 3*a^2*b*d*x^2 - 3*a^3*x^2*e - 2*a^3*d)/(a^4*x^6)
```

maple [B] time = 0.01, size = 523, normalized size = 2.14

$$-\frac{cf\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a}+\frac{b^2f\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}+\frac{3bce\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^2}+\frac{c^2d\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;a^2}-\frac{b^3e\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/6*d/a/x^6-1/4/a/x^4*e+1/4/a^2/x^4*b*d-1/2/a/x^2*f+1/2/a^2/x^2*b*e+1/2/a^
2/x^2*c*d-1/2/a^3/x^2*b^2*d-1/a^2*ln(x)*b*f-1/a^2*ln(x)*c*e+1/a^3*ln(x)*b^2
*e+2/a^3*ln(x)*b*c*d-1/a^4*ln(x)*b^3*d+1/4/a^2*ln(c*x^4+b*x^2+a)*b*f+1/4/a^
2*c*ln(c*x^4+b*x^2+a)*e-1/4/a^3*ln(c*x^4+b*x^2+a)*b^2*e-1/2/a^3*c*ln(c*x^4+
b*x^2+a)*b*d+1/4/a^4*ln(c*x^4+b*x^2+a)*b^3*d-1/a/(4*a*c-b^2)^(1/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*f+1/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2))*b^2*f+3/2/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*b*c*e+1/a^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b
^2)^(1/2))*c^2*d-1/2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(
1/2))*b^3*e-2/a^3/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b
```

```
^2*c*d+1/2/a^4/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*
d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^7/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 13.83, size = 9141, normalized size = 37.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^7*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (atan((16*a^12*(4*a*c - b^2)^(3/2)*(x^2*((((a^3*c^8*d^3 - b^6*c^5*d^3 - a^6
*c^5*f^3 + 3*a*b^4*c^6*d^3 - 3*a^4*c^7*d^2*f + 3*a^5*c^6*d*f^2 - 3*a^2*b^2*
c^7*d^3 + a^3*b^3*c^5*e^3 + 3*a*b^5*c^5*d^2*e + 3*a^3*b*c^7*d^2*e + 3*a^5*b
*c^5*e*f^2 - 6*a^2*b^3*c^6*d^2*e - 3*a^2*b^4*c^5*d*e^2 + 3*a^3*b^2*c^6*d*e^
2 - 3*a^2*b^4*c^5*d^2*f + 6*a^3*b^2*c^6*d^2*f - 3*a^4*b^2*c^5*d*f^2 - 3*a^4
*b^2*c^5*e^2*f - 6*a^4*b*c^6*d*e*f + 6*a^3*b^3*c^5*d*e*f)/a^9 - (((11*a^5*b
*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 17*a^4*b
^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17*a^6*b*c
^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*d*f - 12*
a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8
*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 +
((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*
a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2
*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^
4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16
*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12
*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c -
 4*a^4*b^2)) + (((((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*d + 8*a^7*b
^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a^9 + ((40*a
^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*
e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*a^9*(
16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*
c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) + ((40*a^10*b
*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c
*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*
b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(8*a
^13*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2
*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
b^2)^(1/2)) + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*
b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2*(2*b^5*d + 2*a^2
*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^
2*d + 10*a^2*b^2*c*e))/(32*a^17*(4*a*c - b^2)*(16*a^5*c - 4*a^4*b^2)))*(3*b
^6*d - a^3*c^3*d + 3*a^2*b^4*f + a^4*c^2*f - 3*a*b^5*e + 18*a^2*b^2*c^2*d -
 15*a*b^4*c*d + 12*a^2*b^3*c*e - 9*a^3*b*c^2*e - 9*a^3*b^2*c*f))/(8*a^3*c^2
*(a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2
+ a^6*c^2*f^2 + 36*a^3*b^4*c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^
2*b^4*c^2*d^2 + 96*a^3*b^2*c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 -
12*a^2*b^6*d*f + 12*a^3*b^5*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4
```

*b*c^3*d*e + 72*a^3*b^4*c*d*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a
^3*b^3*c^2*d*e - 95*a^4*b^2*c^2*d*f)) + (((((((20*a^9*c^4*f - 20*a^8*c^5*d
+ 2*a^6*b^4*c^3*d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 1
0*a^8*b*c^4*e)/a^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3
*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d
+ 10*a^2*b^2*c*e))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a
^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c
- b^2)^(1/2)) + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^
2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2
*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^
2*d + 10*a^2*b^2*c*e))/(8*a^13*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))
*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*
c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) + (((11*
a^5*b*c^6*d^2 - 5*a^6*b*c^5*e^2 + 6*a^7*b*c^4*f^2 + 6*a^3*b^5*c^4*d^2 - 17*
a^4*b^3*c^5*d^2 + 6*a^5*b^3*c^4*e^2 - 5*a^6*c^6*d*e + 5*a^7*c^5*e*f - 17*a^
6*b*c^5*d*f - 12*a^4*b^4*c^4*d*e + 22*a^5*b^2*c^5*d*e + 12*a^5*b^3*c^4*d*f
- 12*a^6*b^2*c^4*e*f)/a^9 + (((20*a^9*c^4*f - 20*a^8*c^5*d + 2*a^6*b^4*c^3*
d + 8*a^7*b^2*c^4*d - 2*a^7*b^3*c^3*e + 2*a^8*b^2*c^3*f - 10*a^8*b*c^4*e)/a
^9 + ((40*a^10*b*c^3 - 12*a^9*b^3*c^2)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e
- 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e
))/(2*a^9*(16*a^5*c - 4*a^4*b^2)))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2
*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(
2*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a
^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) - ((40*a^1
0*b*c^3 - 12*a^9*b^3*c^2)*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^
3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^3)/(64*a^21*(4*a*c - b^2)^(3/2)))*(6*b^7
*d + 6*a^2*b^5*f + 20*a^4*c^3*e - 6*a*b^6*e + 84*a^2*b^3*c^2*d - 54*a^3*b^2
*c^2*e - 42*a*b^5*c*d - 46*a^3*b*c^3*d + 36*a^2*b^4*c*e - 30*a^3*b^3*c*f +
26*a^4*b*c^2*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^4*c^4*d^2 - 6*a^2*b^6*e
^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*
c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*
c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5
*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d
*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2
*c^2*d*f))) - (((b^7*c^4*d^3 - 4*a*b^5*c^5*d^3 - 2*a^3*b*c^7*d^3 + a^6*b*c^
4*f^3 + a^4*c^7*d^2*e + a^6*c^5*e*f^2 + 5*a^2*b^3*c^6*d^3 - a^3*b^4*c^4*e^3
+ a^4*b^2*c^5*e^3 - 2*a^5*c^6*d*e*f - 3*a*b^6*c^4*d^2*e + 2*a^4*b*c^6*d*e^
2 + 5*a^4*b*c^6*d^2*f - 4*a^5*b*c^5*d*f^2 - 2*a^5*b*c^5*e^2*f + 9*a^2*b^4*c
^5*d^2*e + 3*a^2*b^5*c^4*d*e^2 - 7*a^3*b^2*c^6*d^2*e - 6*a^3*b^3*c^5*d*e^2
+ 3*a^2*b^5*c^4*d^2*f - 8*a^3*b^3*c^5*d^2*f + 3*a^4*b^3*c^4*d*f^2 + 3*a^4*b
^3*c^4*e^2*f - 3*a^5*b^2*c^4*e*f^2 - 6*a^3*b^4*c^4*d*e*f + 10*a^4*b^2*c^5*d
*e*f)/a^9 - (((a^6*c^6*d^2 + a^8*c^4*f^2 - 4*a^3*b^6*c^3*d^2 + 13*a^4*b^4*c
^4*d^2 - 10*a^5*b^2*c^5*d^2 - 4*a^5*b^4*c^3*e^2 + 5*a^6*b^2*c^4*e^2 - 4*a^7
*b^2*c^3*f^2 - 2*a^7*c^5*d*f + 6*a^6*b*c^5*d*e - 6*a^7*b*c^4*e*f + 8*a^4*b^
5*c^3*d*e - 18*a^5*b^3*c^4*d*e - 8*a^5*b^4*c^3*d*f + 14*a^6*b^2*c^4*d*f + 8
*a^6*b^3*c^3*e*f)/a^9 - (((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c
^2*e + 8*a^8*b^2*c^3*e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a
^9 - (2*a*b^2*c^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b
^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*
b^2))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a
^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)))*(2
*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f
+ 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) - (((((4*a^
6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e + 8*a^8*b^2*c^3*e + 4*a^8*
b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2*a*b^2*c^2*(2*b^5*d + 2*
a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b
*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2))*(b^4*d + 2*a^2*c^2*d + a^
2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
b^2)^(1/2)) - (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*

```
c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a
*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*
a^3*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d + a^2
*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c -
b^2)^(1/2)) + (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c
*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*
a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(8
*a^7*(4*a*c - b^2)*(16*a^5*c - 4*a^4*b^2)))*(3*b^6*d - a^3*c^3*d + 3*a^2*b^
4*f + a^4*c^2*f - 3*a*b^5*e + 18*a^2*b^2*c^2*d - 15*a*b^4*c*d + 12*a^2*b^3*
c*e - 9*a^3*b*c^2*e - 9*a^3*b^2*c*f))/(8*a^3*c^2*(a^4*c^4*d^2 - 6*a^2*b^6*e
^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*
c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*
c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5
*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d
*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2
*c^2*d*f)) + (((((((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e +
8*a^8*b^2*c^3*e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2
*a*b^2*c^2*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d
- 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2))*(
b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2
*b*c*e))/(4*a^4*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*
b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)*(2*b^5*d + 2*a^2*b
^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*
d + 10*a^2*b^2*c*e))/(2*a^3*(4*a*c - b^2)^(1/2)*(16*a^5*c - 4*a^4*b^2)))*(2
*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f
 + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)) - (((a^6*c^
6*d^2 + a^8*c^4*f^2 - 4*a^3*b^6*c^3*d^2 + 13*a^4*b^4*c^4*d^2 - 10*a^5*b^2*c
^5*d^2 - 4*a^5*b^4*c^3*e^2 + 5*a^6*b^2*c^4*e^2 - 4*a^7*b^2*c^3*f^2 - 2*a^7*
c^5*d*f + 6*a^6*b*c^5*d*e - 6*a^7*b*c^4*e*f + 8*a^4*b^5*c^3*d*e - 18*a^5*b^
3*c^4*d*e - 8*a^5*b^4*c^3*d*f + 14*a^6*b^2*c^4*d*f + 8*a^6*b^3*c^3*e*f)/a^9
 - (((4*a^6*b^5*c^2*d - 12*a^7*b^3*c^3*d - 4*a^7*b^4*c^2*e + 8*a^8*b^2*c^3*
e + 4*a^8*b^3*c^2*f + 4*a^8*b*c^4*d - 4*a^9*b*c^3*f)/a^9 - (2*a*b^2*c^2*(2*
b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f
+ 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(16*a^5*c - 4*a^4*b^2))*(2*b^5*d + 2*a^
2*b^3*f - 8*a^3*c^2*e - 2*a*b^4*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c
^2*d + 10*a^2*b^2*c*e))/(2*(16*a^5*c - 4*a^4*b^2)))*(b^4*d + 2*a^2*c^2*d +
a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(4*a^4*(4*a*c
 - b^2)^(1/2)) + (b^2*c^2*(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^
3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^3)/(16*a^11*(4*a*c - b^2)^(3/2)))*(6*b^7
*d + 6*a^2*b^5*f + 20*a^4*c^3*e - 6*a*b^6*e + 84*a^2*b^3*c^2*d - 54*a^3*b^2
*c^2*e - 42*a*b^5*c*d - 46*a^3*b*c^3*d + 36*a^2*b^4*c*e - 30*a^3*b^3*c*f +
26*a^4*b*c^2*f))/(16*a^3*c^2*(4*a*c - b^2)^(1/2)*(a^4*c^4*d^2 - 6*a^2*b^6*e
^2 - 6*b^8*d^2 - 6*a^4*b^4*f^2 + 25*a^5*c^3*e^2 + a^6*c^2*f^2 + 36*a^3*b^4*
c*e^2 + 24*a^5*b^2*c*f^2 + 12*a*b^7*d*e - 120*a^2*b^4*c^2*d^2 + 96*a^3*b^2*
c^3*d^2 - 54*a^4*b^2*c^2*e^2 + 48*a*b^6*c*d^2 - 12*a^2*b^6*d*f + 12*a^3*b^5
*e*f - 2*a^5*c^3*d*f - 84*a^2*b^5*c*d*e - 97*a^4*b*c^3*d*e + 72*a^3*b^4*c*d
*f - 60*a^4*b^3*c*e*f + 47*a^5*b*c^2*e*f + 168*a^3*b^3*c^2*d*e - 95*a^4*b^2
*c^2*d*f))))/(4*a^4*c^6*d^2 + b^8*c^2*d^2 + 4*a^6*c^4*f^2 - 8*a*b^6*c^3*d^2
 + 20*a^2*b^4*c^4*d^2 - 16*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 6*a^3*b^4*c^
3*e^2 + 9*a^4*b^2*c^4*e^2 + a^4*b^4*c^2*f^2 - 4*a^5*b^2*c^3*f^2 - 8*a^5*c^5
*d*f - 2*a*b^7*c^2*d*e + 12*a^4*b*c^5*d*e - 12*a^5*b*c^4*e*f + 14*a^2*b^5*c
^3*d*e - 28*a^3*b^3*c^4*d*e + 2*a^2*b^6*c^2*d*f - 12*a^3*b^4*c^3*d*f + 20*a
^4*b^2*c^4*d*f - 2*a^3*b^5*c^2*e*f + 10*a^4*b^3*c^3*e*f))*(b^4*d + 2*a^2*c^
2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e))/(2*a^4*
(4*a*c - b^2)^(1/2)) - (log(((c^4*(b^2*d + a^2*f - a*b*e - a*c*d)^2*(b^3*d
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^9 - (((c^3*(4*b^6*d^2 - a^5*c
*f^2 + 4*a^2*b^4*e^2 - a^3*c^3*d^2 + 4*a^4*b^2*f^2 - 5*a^3*b^2*c*e^2 - 8*a*
b^5*d*e + 10*a^2*b^2*c^2*d^2 - 13*a*b^4*c*d^2 + 8*a^2*b^4*d*f - 8*a^3*b^3*e
*f + 2*a^4*c^2*d*f + 18*a^2*b^3*c*d*e - 6*a^3*b*c^2*d*e - 14*a^3*b^2*c*d*f
```

```
+ 6*a^4*b*c*e*f))/a^6 - (((4*b*c^2*(b^4*d + a^2*c^2*d + a^2*b^2*f - a*b^3*e
 - a^3*c*f - 3*a*b^2*c*d + 2*a^2*b*c*e))/a^3 + (2*c^3*x^2*(b^4*d - 10*a^2*c
^2*d + a^2*b^2*f - a*b^3*e + 10*a^3*c*f + 4*a*b^2*c*d - 5*a^2*b*c*e))/a^3 +
 (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^3*d + a^4*(-(b^4*d + 2*a^2*c^2*d
+ a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*
c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^4)*(b^3*d +
a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d
+ 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e -
 2*a*b*c*d))/(4*a^4) + (c^4*x^2*(6*b^5*d^2 + 6*a^4*b*f^2 + 6*a^2*b^3*e^2 +
11*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 5*a^4*c*e*f - 17*a*b^3*c*d^2 - 5*a^3*b*c*
e^2 + 12*a^2*b^3*d*f - 5*a^3*c^2*d*e - 12*a^3*b^2*e*f + 22*a^2*b^2*c*d*e -
17*a^3*b*c*d*f))/a^6)*(b^3*d + a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b
^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2)
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/(4*a^4) + (c^5*x^2*(b^2*d + a^2
*f - a*b*e - a*c*d)^3)/a^9)*((c^4*(b^2*d + a^2*f - a*b*e - a*c*d)^2*(b^3*d
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^9 - (((c^3*(4*b^6*d^2 - a^5*c
*f^2 + 4*a^2*b^4*e^2 - a^3*c^3*d^2 + 4*a^4*b^2*f^2 - 5*a^3*b^2*c*e^2 - 8*a*
b^5*d*e + 10*a^2*b^2*c^2*d^2 - 13*a*b^4*c*d^2 + 8*a^2*b^4*d*f - 8*a^3*b^3*e
*f + 2*a^4*c^2*d*f + 18*a^2*b^3*c*d*e - 6*a^3*b*c^2*d*e - 14*a^3*b^2*c*d*f
+ 6*a^4*b*c*e*f))/a^6 - (((4*b*c^2*(b^4*d + a^2*c^2*d + a^2*b^2*f - a*b^3*e
 - a^3*c*f - 3*a*b^2*c*d + 2*a^2*b*c*e))/a^3 + (2*c^3*x^2*(b^4*d - 10*a^2*c
^2*d + a^2*b^2*f - a*b^3*e + 10*a^3*c*f + 4*a*b^2*c*d - 5*a^2*b*c*e))/a^3 +
 (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(b^3*d - a^4*(-(b^4*d + 2*a^2*c^2*d
+ a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*
c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/a^4)*(b^3*d -
a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b^3*e - 2*a^3*c*f - 4*a*b^2*c*d
+ 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2) - a*b^2*e + a^2*b*f + a^2*c*e -
 2*a*b*c*d))/(4*a^4) + (c^4*x^2*(6*b^5*d^2 + 6*a^4*b*f^2 + 6*a^2*b^3*e^2 +
11*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 5*a^4*c*e*f - 17*a*b^3*c*d^2 - 5*a^3*b*c*
e^2 + 12*a^2*b^3*d*f - 5*a^3*c^2*d*e - 12*a^3*b^2*e*f + 22*a^2*b^2*c*d*e -
17*a^3*b*c*d*f))/a^6)*(b^3*d - a^4*(-(b^4*d + 2*a^2*c^2*d + a^2*b^2*f - a*b
^3*e - 2*a^3*c*f - 4*a*b^2*c*d + 3*a^2*b*c*e)^2/(a^8*(4*a*c - b^2)))^(1/2)
- a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*c*d))/(4*a^4) + (c^5*x^2*(b^2*d + a^2
*f - a*b*e - a*c*d)^3)/a^9))*(2*b^5*d + 2*a^2*b^3*f - 8*a^3*c^2*e - 2*a*b^4
*e - 12*a*b^3*c*d - 8*a^3*b*c*f + 16*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(2*(16*
a^5*c - 4*a^4*b^2)) - (log(x)*(b^3*d - a*b^2*e + a^2*b*f + a^2*c*e - 2*a*b*
c*d))/a^4 - (d/(6*a) + (x^4*(b^2*d + a^2*f - a*b*e - a*c*d))/(2*a^3) + (x^2
*(a*e - b*d))/(4*a^2))/x^6
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**7/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.55 $\int \frac{x^4(d+ex^2+fx^4)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=369

$$\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^4(-f)+b^3ce}{\sqrt{b^2-4ac}}-bc(cd-2af)-a\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (c^2*d+b^2*f-c*(a*f+b*e))*x/c^3+1/3*(-b*f+c*e)*x^3/c^2+1/5*f*x^5/c+1/2*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^2*e-b^3*f-b
*c*(-2*a*f+c*d)+(-b^3*c*e+3*a*b*c^2*e+b^4*f+b^2*c*(-4*a*f+c*d)-2*a*c^2*(-a*
f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-a*c^2*e-b
^3*f-b*c*(-2*a*f+c*d)+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2
*(-a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/
2)
```

Rubi [A] time = 4.58, antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1664, 1166, 205}

$$\frac{x\left(-c(af+be)+b^2f+c^2d\right)}{c^3}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-b^2c(cd-4af)-3abc^2e+2ac^2(cd-af)+b^3ce+b^4(-f)}{\sqrt{b^2-4ac}}-bc(cd-2af)-a\right.}{\sqrt{2}c^{7/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d - 2*a*f) - (b^3*c*e - 3*a
*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(c*d - a*f))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(
7/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - a*c^2*e - b^3*f - b*c*(c*d
- 2*a*f) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*f) + 2*a*c^2*(
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4 \left(d + ex^2 + fx^4\right)}{a + bx^2 + cx^4} dx = \int \left(\frac{c^2 d + b^2 f - c(be + af)}{c^3} + \frac{(ce - bf)x^2}{c^2} + \frac{fx^4}{c} - \frac{a\left(c^2 d + b^2 f - c(be + af)\right) - \left(b^2 ce \right.}{c^3 \left(a + bx\right.} \right.$$

$$= \frac{\left(c^2 d + b^2 f - c(be + af)\right) x}{c^3} + \frac{(ce - bf)x^3}{3c^2} + \frac{fx^5}{5c} - \frac{\int \frac{a\left(c^2 d + b^2 f - c(be + af)\right) + \left(-b^2 ce + ac^2 e + b^3 f + \right.}{a + bx^2 + cx^4}}{c^3}$$

$$= \frac{\left(c^2 d + b^2 f - c(be + af)\right) x}{c^3} + \frac{(ce - bf)x^3}{3c^2} + \frac{fx^5}{5c} + \frac{\left(b^2 ce - ac^2 e - b^3 f - bc(cd - 2af)\right.}{}$$

$$= \frac{\left(c^2 d + b^2 f - c(be + af)\right) x}{c^3} + \frac{(ce - bf)x^3}{3c^2} + \frac{fx^5}{5c} + \frac{\left(b^2 ce - ac^2 e - b^3 f - bc(cd - 2af)\right.}{}$$

Mathematica [A] time = 0.51, size = 456, normalized size = 1.24

$$\frac{x\left(-c(af + be) + b^2 f + c^2 d\right)}{c^3} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b - \sqrt{b^2 - 4ac}}}\right)\left(ac^2\left(e\sqrt{b^2 - 4ac} - 2af + 2cd\right) - b^2 c\left(e\sqrt{b^2 - 4ac} - 4af + cd\right)\right.}{\sqrt{2}\, c^{7/2}\sqrt{b^2 - 4ac}\,\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c^2*d + b^2*f - c*(b*e + a*f))*x)/c^3 + ((c*e - b*f)*x^3)/(3*c^2) + (f*x^
5)/(5*c) - ((-(b^4*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e - 4*a*f) + a*c^2*(
2*c*d + Sqrt[b^2 - 4*a*c]*e - 2*a*f) + b^3*(c*e + Sqrt[b^2 - 4*a*c]*f) + b*
c*(c*Sqrt[b^2 - 4*a*c]*d - 3*a*c*e - 2*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(7/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^4*f + b^2*c*(c*d - Sqrt[b^2 - 4*a*c
]*e - 4*a*f) + a*c^2*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b^3*(-(c*e) +
 Sqrt[b^2 - 4*a*c]*f) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e - 2*a*Sqrt[b^2
 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqr
t[2]*c^(7/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

fricas [B] time = 35.65, size = 15467, normalized size = 41.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/30*(6*c^2*f*x^5 - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(
b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c
^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^
2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2
*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^1
0)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*
a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 +
7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^
6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 6
2*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c
^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^
7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3
```

```
*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6
- 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^
5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66
*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^
6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*
c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*
a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16
*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15))
)/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5
*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^
2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^
4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2
*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*
b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3
 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^
3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2
 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*
b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*
d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*x + sqrt(1/2
)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20
*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)
*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10
 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^
5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*
a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28
*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c
^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e
+ (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*
e^2)*f - ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e
 + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 +
 a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c
^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^
5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^
2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*
((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6
- a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22
*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*
b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 4
8*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*
c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2
+ 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 1
5*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c
^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*
c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*
a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a
^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c
+ 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c
^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 -
 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b
^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8
- a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8
)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8
)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b
^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^
2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3
+ 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^
8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*
```

a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c
^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2
*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^
3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^
7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c
^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) + 1
5*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4
 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*
b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a
^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2
*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 -
 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^
2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3
*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a
^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22
*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*
b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e
)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*
a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^
6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3
*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 +
 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7
 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*
b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3
*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8)
)*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3
*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c
^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*
b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c
^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2
 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*
a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e
 + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*
a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5
)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^
2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^4*c^6 - 5*a*b^2
*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (
3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7
*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10 - 10*a*b^8*c + 35*a^
2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2
 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c
 - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 +
((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^
3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b
^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*e^2)*f - ((b^3*c^9 -
4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3
*c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b
^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 1
2*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7
 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2
*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*
c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*
c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*
b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*
c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^
3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4
 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b

^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f + (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))) - 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 + a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 + 8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*x + sqrt(1/2)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6 - 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a

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^2*b^4*c^4 - 59*a^3*b^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*
a^2*b^5*c^3 - 87*a^3*b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^
4*c^5 + 35*a^2*b^2*c^6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46
*a^2*b^3*c^5 - 24*a^3*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c
^4 - 41*a^3*b^2*c^5 + 4*a^4*c^6)*e^2)*f + ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*
c^8 - 6*a*b^2*c^9 + 8*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*
f)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8
+ 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c
^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3
 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 +
 (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4
- 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*
a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3
*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2
- 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c
^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27
*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^
8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8
)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4
*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*
a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2
 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a
^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((
b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c
^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a
^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6
 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*
c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*
b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c
^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((
b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 -
a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a
^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^
4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*
a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^
3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 +
4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*
a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5
 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^
4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*
c^15)))/(b^2*c^7 - 4*a*c^8))) + 15*sqrt(1/2)*c^3*sqrt(-((b^3*c^4 - 3*a*b*c^
5)*d^2 - 2*(b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3
 + 5*a^2*b*c^4)*e^2 + (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2
+ 2*((b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2
*b^2*c^3 - 2*a^3*c^4)*e)*f - (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c
^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b
^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*
a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 1
1*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2
*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4
+ 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*
c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4
+ 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*
a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4
 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*
b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*
f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5
 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b
```

```
^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*
b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14
- 4*a*c^15)))/(b^2*c^7 - 4*a*c^8))*log(-2*((a*b^2*c^6 - a^2*c^7)*d^4 - (3*a
*b^3*c^5 - 5*a^2*b*c^6)*d^3*e + 3*(a*b^4*c^4 - 2*a^2*b^2*c^5)*d^2*e^2 - (a*
b^5*c^3 - a^2*b^3*c^4 - 3*a^3*b*c^5)*d*e^3 + (a^2*b^4*c^3 - 3*a^3*b^2*c^4 +
 a^4*c^5)*e^4 + (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*f^4 + ((a
*b^8 - 7*a^2*b^6*c + 18*a^3*b^4*c^2 - 19*a^4*b^2*c^3 + 4*a^5*c^4)*d - (a^2*
b^7 - 3*a^3*b^5*c - 2*a^4*b^3*c^2 + 5*a^5*b*c^3)*e)*f^3 + 3*((a*b^6*c^2 - 5
*a^2*b^4*c^3 + 7*a^3*b^2*c^4 - 2*a^4*c^5)*d^2 - (a*b^7*c - 5*a^2*b^5*c^2 +
8*a^3*b^3*c^3 - 5*a^4*b*c^4)*d*e + (a^2*b^6*c - 4*a^3*b^4*c^2 + 3*a^4*b^2*c
^3)*e^2)*f^2 + ((3*a*b^4*c^4 - 9*a^2*b^2*c^5 + 4*a^3*c^6)*d^3 - 3*(2*a*b^5*
c^3 - 7*a^2*b^3*c^4 + 5*a^3*b*c^5)*d^2*e + 3*(a*b^6*c^2 - 3*a^2*b^4*c^3 + a
^3*b^2*c^4)*d*e^2 - (3*a^2*b^5*c^2 - 11*a^3*b^3*c^3 + 7*a^4*b*c^4)*e^3)*f)*
x - sqrt(1/2)*((b^4*c^6 - 5*a*b^2*c^7 + 4*a^2*c^8)*d^3 - (3*b^5*c^5 - 17*a*
b^3*c^6 + 20*a^2*b*c^7)*d^2*e + (3*b^6*c^4 - 19*a*b^4*c^5 + 29*a^2*b^2*c^6
- 4*a^3*c^7)*d*e^2 - (b^7*c^3 - 7*a*b^5*c^4 + 13*a^2*b^3*c^5 - 4*a^3*b*c^6)
*e^3 + (b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^
4 - 4*a^5*c^5)*f^3 + ((3*b^8*c^2 - 25*a*b^6*c^3 + 66*a^2*b^4*c^4 - 59*a^3*b
^2*c^5 + 12*a^4*c^6)*d - (3*b^9*c - 27*a*b^7*c^2 + 80*a^2*b^5*c^3 - 87*a^3*
b^3*c^4 + 28*a^4*b*c^5)*e)*f^2 + ((3*b^6*c^4 - 20*a*b^4*c^5 + 35*a^2*b^2*c^
6 - 12*a^3*c^7)*d^2 - 2*(3*b^7*c^3 - 22*a*b^5*c^4 + 46*a^2*b^3*c^5 - 24*a^3
*b*c^6)*d*e + (3*b^8*c^2 - 24*a*b^6*c^3 + 58*a^2*b^4*c^4 - 41*a^3*b^2*c^5 +
 4*a^4*c^6)*e^2)*f + ((b^3*c^9 - 4*a*b*c^10)*d - (b^4*c^8 - 6*a*b^2*c^9 + 8
*a^2*c^10)*e + (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*f)*sqrt(((b^4*c^8 - 2
*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e
+ 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*
c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6
*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c
+ 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c
^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*
a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*
b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c
^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a
*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2
 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 - 72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^
7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3
*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4
- 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 18*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^
3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 - 13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b
^2*c^14 - 4*a*c^15)))*sqrt(-((b^3*c^4 - 3*a*b*c^5)*d^2 - 2*(b^4*c^3 - 4*a*b
^2*c^4 + 2*a^2*c^5)*d*e + (b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*e^2 + (b^7
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*f^2 + 2*((b^5*c^2 - 5*a*b^3*c^3
 + 5*a^2*b*c^4)*d - (b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*e)*f
- (b^2*c^7 - 4*a*c^8)*sqrt(((b^4*c^8 - 2*a*b^2*c^9 + a^2*c^10)*d^4 - 4*(b^5
*c^7 - 3*a*b^3*c^8 + 2*a^2*b*c^9)*d^3*e + 2*(3*b^6*c^6 - 12*a*b^4*c^7 + 12*
a^2*b^2*c^8 - a^3*c^9)*d^2*e^2 - 4*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 -
 2*a^3*b*c^8)*d*e^3 + (b^8*c^4 - 6*a*b^6*c^5 + 11*a^2*b^4*c^6 - 6*a^3*b^2*c
^7 + a^4*c^8)*e^4 + (b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*f^4 + 4*((b^10*c^2 - 8*a*b^8*c^
3 + 22*a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6 - a^5*c^7)*d - (b^11*c
- 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*
c^6)*e)*f^3 + 2*((3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 19*a^3*b^2*c^
7 + 3*a^4*c^8)*d^2 - 2*(3*b^9*c^3 - 21*a*b^7*c^4 + 48*a^2*b^5*c^5 - 39*a^3*
b^3*c^6 + 8*a^4*b*c^7)*d*e + (3*b^10*c^2 - 24*a*b^8*c^3 + 66*a^2*b^6*c^4 -
72*a^3*b^4*c^5 + 27*a^4*b^2*c^6 - a^5*c^7)*e^2)*f^2 + 4*((b^6*c^6 - 4*a*b^4
*c^7 + 4*a^2*b^2*c^8 - a^3*c^9)*d^3 - (3*b^7*c^5 - 15*a*b^5*c^6 + 21*a^2*b^
3*c^7 - 7*a^3*b*c^8)*d^2*e + (3*b^8*c^4 - 18*a*b^6*c^5 + 33*a^2*b^4*c^6 - 1
8*a^3*b^2*c^7 + a^4*c^8)*d*e^2 - (b^9*c^3 - 7*a*b^7*c^4 + 16*a^2*b^5*c^5 -
13*a^3*b^3*c^6 + 3*a^4*b*c^7)*e^3)*f)/(b^2*c^14 - 4*a*c^15)))/(b^2*c^7 - 4*
```

```
a*c^8))) + 10*(c^2*e - b*c*f)*x^3 + 30*(c^2*d - b*c*e + (b^2 - a*c)*f)*x)/c
^3
```

giac [B] time = 5.03, size = 7243, normalized size = 19.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d +
 (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3
- 16*(b^2 - 4*a*c)*a^2*b*c^4)*c^2*f - (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^
2*c^5 - 32*a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^6*c + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^
2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^
3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*
c^3 + 10*(b^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*e + 2*(sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^5 + 2*a*b^4*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^6 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^2*c^7 + 32*a^3*c^7 - 2*(b^2 - 4*a*c)*a*b^2*c^5 + 8*(b^2 -
4*a*c)*a^2*c^6)*d*abs(c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6
*c^2 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 18*
a^2*b^4*c^4 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b^2*c^5 + 48*a^3*b^2*c^5 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*c^6 - 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 10*(b^2 - 4*a
*c)*a^2*b^2*c^4 - 8*(b^2 - 4*a*c)*a^3*c^5)*f*abs(c) - 2*(sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^
5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sq
```

```
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)
*a^2*b*c^5)*abs(c)*e - (2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*
a*b*c^7)*d - (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2 - 4*
a*c)*a*b^3*c^5 - 4*(b^2 - 4*a*c)*a^2*b*c^6)*f + (2*b^6*c^5 - 14*a*b^4*c^6 +
 24*a^2*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*b^6*c^3 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^4
 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5
 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^2 - 4
*a*c)*b^4*c^5 + 6*(b^2 - 4*a*c)*a*b^2*c^6)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*
c^5 + sqrt(b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^
3*c^6 + 16*a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) + 1/8*((2*b^
5*c^4 - 16*a*b^3*c^5 + 32*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*
c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*c^2*d + (2*b^7*c^2
 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*c)*a*b^3*c^3 - 16*(b^2 -
 4*a*c)*a^2*b*c^4)*c^2*f - (2*b^6*c^3 - 18*a*b^4*c^4 + 48*a^2*b^2*c^5 - 32*
a^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c +
 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 24*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 10*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 16*sqrt(2)*sqrt(
```

```
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 5*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^3 + 10*(b
^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*c^2*e - 2*(sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*a
*b^4*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^6 + 8*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^6 + 16*a^2*b^2*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c^7 - 32*a^3*c^7 + 2*(b^2 - 4*a*c)*a*b^2*c^5 - 8*(b^2 - 4*a*c)*a^2*
c^6)*d*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^2 - 9*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 2*a*b^6*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^2*c^4 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^3*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 18*a^2*b^4*c^4
 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^5 - 8*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^5 - 48*a^3*b^2*c^5 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^6 + 32*a^4*c^6 + 2*(b^2 - 4*a*c)*a*b^4*c^3 - 10*(b^2 - 4*a*c)*a^2*b^2
*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*f*abs(c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 2*a*b^5*c^4 + 16*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^5 + 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^6 - 32*a^3*b*c^6 + 2*(b^2 - 4*a*c)*a*b^3*c^4 - 8*(b^2 - 4*a*c)*a^2*b*c^5)
*abs(c)*e - (2*b^5*c^6 - 12*a*b^3*c^7 + 16*a^2*b*c^8 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^7 - 2*(b^2 - 4*a*c)*b^3*c^6 + 4*(b^2 - 4*a*c)*a*b*c^7)*d
- (2*b^7*c^4 - 16*a*b^5*c^5 + 36*a^2*b^3*c^6 - 16*a^3*b*c^7 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^2 + 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^5*c^4 + 8*(b^2 - 4*a*c)*a*b^3*
c^5 - 4*(b^2 - 4*a*c)*a^2*b*c^6)*f + (2*b^6*c^5 - 14*a*b^4*c^6 + 24*a^2*b^2
*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^3 +
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^4 - 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 6*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^5 + 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^6 - 2*(b^2 - 4*a*c)*b^4*c
^5 + 6*(b^2 - 4*a*c)*a*b^2*c^6)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c^5 - sqrt(
b^2*c^10 - 4*a*c^11))/c^6))/((a*b^4*c^5 - 8*a^2*b^2*c^6 - 2*a*b^3*c^6 + 16*
a^3*c^7 + 8*a^2*b*c^7 + a*b^2*c^7 - 4*a^2*c^8)*c^2) + 1/15*(3*c^4*f*x^5 - 5
*b*c^3*f*x^3 + 5*c^4*x^3*e + 15*c^4*d*x + 15*b^2*c^2*f*x - 15*a*c^3*f*x - 1
5*b*c^3*x*e)/c^5
```

maple [B] time = 0.04, size = 1450, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/5*f*x^5/c-1/3/c^2*x^3*b*f-1/c^2*a*f*x+1/c^3*b^2*f*x-1/c^2*b*e*x-1/c^2*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b*f-1/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*f
-1/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*f+1/2/c^2/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^2*d+1/c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*f-1/c/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*a^2*f-1/2/c^3/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*f+1/2/c
^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e-1/2/c/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*b^2*d-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*a*d+1/3/c*x^3*e+1/c*d*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e+1/2
/c^3*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*f-1/2/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/2/c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b*d+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d-1/
2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a*e-1/2/c^3*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*f+1/2/c^2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b^2*e+2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*f-3/
2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e+2/c^2/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*a*b^2*f-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b
*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\,c^2 f x^5 + 5\left(c^2 e - bcf\right)x^3 + 15\left(c^2 d - bce + \left(b^2 - ac\right)f\right)x}{15\,c^3} + \frac{-\int \frac{ac^2d - abce + \left(bc^2d - \left(b^2c - ac^2\right)e + \left(b^3 - 2\,abc\right)f\right)x^2 + \left(ab^2 - a^2c\right)f}{cx^4 + bx^2 + a}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/15*(3*c^2*f*x^5 + 5*(c^2*e - b*c*f)*x^3 + 15*(c^2*d - b*c*e + (b^2 - a*c)
*f)*x)/c^3 + integrate(-(a*c^2*d - a*b*c*e + (b*c^2*d - (b^2*c - a*c^2)*e +
 (b^3 - 2*a*b*c)*f)*x^2 + (a*b^2 - a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/c^3
```

mupad [B] time = 4.91, size = 23332, normalized size = 63.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^3*(e/(3*c) - (b*f)/(3*c^2)) - x*((b*(e/c - (b*f)/c^2))/c - d/c + (a*f)/c^
2) + atan(((((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*
d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 -
 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^
2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*
c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4
*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a
^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*
c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6
*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5
*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*
c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f
+ 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^
3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*
e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/
2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*
f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c
^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*
d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2
*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e
 + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*
d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*
b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(
1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) -
 (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^
2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b
^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^
3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5
*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*
c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2
+ b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a
*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28
*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 -
 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-
(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e -
16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a
^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*
a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c -
 b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*
```

```
f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) -
6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c
^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*
c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5
 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b
^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^
5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^
4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63
*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*
a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*
b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2
*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1
/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^
2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f +
 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*
e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^
(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7
- 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*
c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3
*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*
c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^
5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3
)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*
a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) + (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^
4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2
*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 -
8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^
4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c
^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^
4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 +
12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 2
0*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2
*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1
/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*
e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*
a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c
- b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^
2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
 - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*
c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*
e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8
```

```
*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*f - 16*a^2
*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e
 + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d
^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*
a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a
^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^
4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*
a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2)
 + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e -
 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b
^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b
^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(
-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c
^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) -
66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c -
 b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 +
 b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*
b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b
*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*
b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b
^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b
^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c
^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^
3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d
*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a
^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(
4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*
a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 -
 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 1
6*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c
^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*
c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)
*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a
^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*
c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(
-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c
*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16
*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2
*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^
(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^
2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4
*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*
d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4
*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^
(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(a^4
*b^3*f^3 + a^4*c^3*e^3 + a^2*b*c^4*d^3 + a^2*b^5*d*f^2 + a^3*c^4*d^2*e - a^
3*b^4*e*f^2 + a^5*c^2*e*f^2 - a^3*b^2*c^2*e^3 - 2*a^5*b*c*f^3 - 2*a^4*c^3*d
*e*f - 4*a^3*b*c^3*d^2*f - 4*a^3*b^3*c*d*f^2 + 5*a^4*b*c^2*d*f^2 + 2*a^3*b^
```

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3*c*e^2*f - 3*a^4*b*c^2*e^2*f + a^4*b^2*c*e*f^2 - 2*a^2*b^2*c^3*d^2*e + a^2
*b^3*c^2*d*e^2 + 2*a^2*b^3*c^2*d^2*f - 2*a^2*b^4*c*d*e*f + 4*a^3*b^2*c^2*d*
e*f))/c^5 + (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6
*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 + (2*x*(4*b^3*c^7
 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8
*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^
4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 -
a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7
*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^
6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^
5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3
*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f
 + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)
^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3
*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1
/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f^2*(-(4*a*c - b^2)^
3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e
*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^4*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 - a^3*
c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 1
1*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2
*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^
2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2*c^2*f^2*(-(4*a*c -
 b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*
e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b^3*c^3
*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f + 2
*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^
(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^3*d*f*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b*c^3*e*f
*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)
+ (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f
^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*
b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a
^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^
5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2
*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2
 + b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 -
a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 2
8*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2
- 63*a^3*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e -
 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*
a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6
*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c
- b^2)^3)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e
*f + 76*a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) -
 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*
c^7 - 8*a*b^2*c^8)))^(1/2)))*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 + b^6*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 - a*c^5*d^
```

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2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*
c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 + a^2*c^4*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3
*b^3*c^3*f^2 - a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^4*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*
c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^
5*d*f + 20*a*b^6*c^2*e*f - 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f + 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3
)^(1/2) - 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*
a^3*b^2*c^4*e*f + 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c^3*e^2*
(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2
) - 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2)*2i + atan(((((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c
^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5
- (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^
6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5
*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4
*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)
^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*
a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a
^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b
*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*
b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2
)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f +
76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*
b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 -
 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c
^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^
(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*
b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c
^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5
*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*
c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a
^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*
c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2)
 + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a
*b^2*c^8)))^(1/2) - (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4
*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*
b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8
*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4
*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^
4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4
*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 1
2*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20
*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*
c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 4
2*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/
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2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e
 - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a
*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c -
 b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2
*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3
)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e
*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*
(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*f - 16*a^2*
c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e +
 4*a*b^4*c^4*f)/c^5 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^
2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a
^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^
3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4
*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a
^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2)
- b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e -
2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^
5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^
2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-
(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^
4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6
6*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(
1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*
(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 +
b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b
*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*
c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2
*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b
^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^
4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^
6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^
3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^
2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)
^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) + (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a
^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 -
6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16
*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^
2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c
^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*
(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) -
9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^
2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-
(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*
f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*
a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*
b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(
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1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2
*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*
a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d
*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(
1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i)/((((16
*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*
e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5 - (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(
b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b
^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*
a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2
 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b
^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5
*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2
) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^
3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c
 - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*
(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*
f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5
*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c
^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 1
6*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c
^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e
*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) +
5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^
4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4
*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) -
8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^
3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*x*(b^8*f^2 +
 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2
- 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*
a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 - 8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^
5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*
c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2
*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 -
 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*
f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f +
 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 2
0*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*
(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*
b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) +
 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c
^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c
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- b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*
(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2
*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8
)))^(1/2) - (2*(a^4*b^3*f^3 + a^4*c^3*e^3 + a^2*b*c^4*d^3 + a^2*b^5*d*f^2 +
 a^3*c^4*d^2*e - a^3*b^4*e*f^2 + a^5*c^2*e*f^2 - a^3*b^2*c^2*e^3 - 2*a^5*b*
c*f^3 - 2*a^4*c^3*d*e*f - 4*a^3*b*c^3*d^2*f - 4*a^3*b^3*c*d*f^2 + 5*a^4*b*c
^2*d*f^2 + 2*a^3*b^3*c*e^2*f - 3*a^4*b*c^2*e^2*f + a^4*b^2*c*e*f^2 - 2*a^2*
b^2*c^3*d^2*e + a^2*b^3*c^2*d*e^2 + 2*a^2*b^3*c^2*d^2*f - 2*a^2*b^4*c*d*e*f
 + 4*a^3*b^2*c^2*d*e*f))/c^5 + (((16*a^3*c^6*f - 16*a^2*c^7*d - 20*a^2*b^2*
c^5*f + 4*a*b^2*c^6*d - 4*a*b^3*c^5*e + 16*a^2*b*c^6*e + 4*a*b^4*c^4*f)/c^5
 + (2*x*(4*b^3*c^7 - 16*a*b*c^8)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b
^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^
5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^
4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2
)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63
*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*
a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*
b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2
*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1
/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^
2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f +
 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a
*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^
(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7
- 8*a*b^2*c^8)))^(1/2))/c^5)*(-(b^9*f^2 + b^5*c^4*d^2 + b^7*c^2*e^2 - b^6*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b*c^6*d^2 + a*c^5*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*c^5*e^2 + 28*a^4*b*
c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2*(-(4*a*c - b^2)^3)
^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*f^2 - 63*a^3
*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*e^2*(-(4*a*c
- b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^6*c^3*d*e - 16*a^4*
c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^3*d*f - 40*a^3*b*c^
5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*f*(-(4*a*c - b^2)^3
)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*e*f + 76*
a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c^3*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*
a*b^2*c^8)))^(1/2) + (2*x*(b^8*f^2 + 2*a^2*c^6*d^2 - 2*a^3*c^5*e^2 + b^4*c^
4*d^2 + 2*a^4*c^4*f^2 + b^6*c^2*e^2 - 4*a*b^2*c^5*d^2 - 6*a*b^4*c^3*e^2 - 2
*b^7*c*e*f + 9*a^2*b^2*c^4*e^2 + 20*a^2*b^4*c^2*f^2 - 16*a^3*b^2*c^3*f^2 -
8*a*b^6*c*f^2 - 4*a^3*c^5*d*f - 2*b^5*c^3*d*e + 2*b^6*c^2*d*f + 10*a*b^3*c^
4*d*e - 10*a^2*b*c^5*d*e - 12*a*b^4*c^3*d*f + 14*a*b^5*c^2*e*f + 14*a^3*b*c
^4*e*f + 18*a^2*b^2*c^4*d*f - 28*a^2*b^3*c^3*e*f))/c^5)*(-(b^9*f^2 + b^5*c^
4*d^2 + b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 +
12*a^2*b*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 2
0*a^3*b*c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2
*c^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
42*a^2*b^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1
/2) - b^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*
e - 2*b^6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*
a*b^5*c^3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c
- b^2)^3)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^
2*c^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
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- 66*a^2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*
c - b^2)^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*
e*f*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)))*(-(b^9*f^2 + b^5*c^4*d^2 +
b^7*c^2*e^2 - b^6*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^5*d^2 + 12*a^2*b
*c^6*d^2 + a*c^5*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*e^2 - 20*a^3*b*
c^5*e^2 + 28*a^4*b*c^4*f^2 - 2*b^8*c*e*f + 25*a^2*b^3*c^4*e^2 - a^2*c^4*e^2
*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b
^5*c^2*f^2 - 63*a^3*b^3*c^3*f^2 + a^3*c^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^
4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*f^2 + 16*a^3*c^6*d*e - 2*b^
6*c^3*d*e - 16*a^4*c^5*e*f + 2*b^7*c^2*d*f + 16*a*b^4*c^4*d*e - 18*a*b^5*c^
3*d*f - 40*a^3*b*c^5*d*f + 20*a*b^6*c^2*e*f + 2*b^5*c*e*f*(-(4*a*c - b^2)^3
)^(1/2) - 6*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^5*d*e + 50*a^2*b^3*c^4*d*f - 2*a^2*c^4*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 2*b^3*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) - 66*a^
2*b^4*c^3*e*f + 76*a^3*b^2*c^4*e*f - 2*b^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2)
 + 3*a*b^2*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d*e*(-(4*a*c - b^2)
^3)^(1/2) + 6*a*b^2*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*e*f*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^2*b*c^3*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*2i + (f*x^5)/(5*c)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.56 $\int \frac{x^2\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=282

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2a}{\sqrt{b^2}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{}}$$

```
[Out] (-b*f+c*e)*x/c^2+1/3*f*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*(c^2*d-b*c*e+b^2*f-a*c*f+(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+
c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c^2*d-b*c*e+b^2*f-a
*c*f+(-b^2*c*e+2*a*c^2*e+b^3*f+b*c*(-3*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2
)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 3.59, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1664, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)}{\sqrt{b^2-4ac}}-acf+b^2f-bce+c^2d\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-bc(cd-3af)-2a}{\sqrt{b^2}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*e - b*f)*x)/c^2 + (f*x^3)/(3*c) + ((c^2*d - b*c*e + b^2*f - a*c*f + (b^
2*c*e - 2*a*c^2*e - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) + ((c^2*d - b*c*e + b^2*f - a*c*f - (b^2*c*e - 2*a*c^2*e
 - b^3*f - b*c*(c*d - 3*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2+fx^4\right)}{a+bx^2+cx^4}\,dx = \int \left(\frac{ce-bf}{c^2}+\frac{fx^2}{c}-\frac{a(ce-bf)-\left(c^2d-bce+b^2f-acf\right)x^2}{c^2\left(a+bx^2+cx^4\right)}\right)dx$$

$$= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}-\frac{\int \frac{a(ce-bf)+\left(-c^2d+bce-b^2f+acf\right)x^2}{a+bx^2+cx^4}\,dx}{c^2}$$

$$= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf-\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx}}{2c^2}$$

$$= \frac{(ce-bf)x}{c^2}+\frac{fx^3}{3c}+\frac{\left(c^2d-bce+b^2f-acf+\frac{b^2ce-2ac^2e-b^3f-bc(cd-3af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-}}}\right.}{\sqrt{2}c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A] time = 0.49, size = 365, normalized size = 1.29

$$\frac{\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-bc\left(e\sqrt{b^2-4ac}-3af+cd\right)+c\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}-2ace\right)+b^2\left(f\sqrt{b^2-4ac}+ce\right)+b^3(-f)\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\right.}{}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (6*Sqrt[c]*(c*e - b*f)*x + 2*c^(3/2)*f*x^3 + (3*Sqrt[2]*(-(b^3*f) - b*c*(c*
d + Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) + c*(c*S
qrt[b^2 - 4*a*c]*d - 2*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt
[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (3*Sqrt[2]*(b^3*f + b*c*(c*d - Sqrt[b^2 - 4*a*c]*e - 3*a*f) + b
^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f) + c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*
Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*c^(5/2))
```

fricas [B] time = 8.05, size = 9364, normalized size = 33.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(2*c*f*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d
*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((
b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5
- 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 -
4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8
- 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 -
 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^
3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2
- 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3
 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c
^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^
3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4
*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*
b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2
)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3
```

```
*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^
5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 +
 ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2
- a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + sqrt(1/2)*((b^
2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2
*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^
3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 +
29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*
c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c
^4)*e^2)*f + (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5
- 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6
- a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a
^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*
f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a
*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^
5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6
*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*
c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c
^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c
^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)
*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (
b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 -
 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^
3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c
^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c
^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^
3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c
^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^
5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3
*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*
c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^
2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2
+ (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c
 - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*
c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (
b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 -
a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2
*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 +
5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^
2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*
c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*
e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c
^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2
*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a
^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((
b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3
)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3
*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c
^2 - 5*a^2*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3
*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 -
7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 +
 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*
f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*
d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f + (2*(b^2*c^7 - 4*a*
c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*s
qrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 -
 a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3
```

```
 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^
3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c
^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^
2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c
^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*
c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b*c^4*d^2 - 2*(b^
2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2
*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3
)*e)*f + (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 -
 a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^
2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f
^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*
b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5
 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*
c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c
^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^
6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^
11)))/(b^2*c^5 - 4*a*c^6))) + 3*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3
 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^
2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*
f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^
7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6
)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 +
4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c
^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*
a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 -
 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d
^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*
e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))
/(b^2*c^5 - 4*a*c^6))*log(2*(c^6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 -
(b^3*c^3 + a*b*c^4)*d*e^3 + (a*b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^
2*c + a^4*c^2)*f^4 + ((b^6 - 5*a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*
b^5 - a^2*b^3*c - 3*a^3*b*c^2)*e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c
^4)*d^2 - (b^5*c - 3*a*b^3*c^2 + 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^
2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e
+ 3*(b^4*c^2 - a*b^2*c^3)*d*e^2 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x + s
qrt(1/2)*((b^2*c^5 - 4*a*c^6)*d^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*
c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*
a^3*b*c^3)*f^3 - (2*(b^5*c^2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19
*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^
2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4 + 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3
 + 20*a^2*b*c^4)*e^2)*f - (2*(b^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*
e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e +
2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*
a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d -
 (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4
 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)
*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*(
(b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^
2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2
*c^10 - 4*a*c^11)))*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2
 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a
*b*c^3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sq
rt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 -
a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c +
 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3
+ 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3
*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^
```

```
3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2
*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^
6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c
^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3
*sqrt(1/2)*c^2*sqrt(-(b*c^4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*
a*b*c^3)*e^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^
3)*d - (b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c
^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c
^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a
^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^
4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9
*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4
- a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^
2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 +
2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(2*(c^
6*d^4 - 3*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 - (b^3*c^3 + a*b*c^4)*d*e^3 + (a*
b^2*c^3 - a^2*c^4)*e^4 + (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*f^4 + ((b^6 - 5*
a*b^4*c + 9*a^2*b^2*c^2 - 4*a^3*c^3)*d - (a*b^5 - a^2*b^3*c - 3*a^3*b*c^2)*
e)*f^3 + 3*((b^4*c^2 - 3*a*b^2*c^3 + 2*a^2*c^4)*d^2 - (b^5*c - 3*a*b^3*c^2
+ 3*a^2*b*c^3)*d*e + (a*b^4*c - 2*a^2*b^2*c^2)*e^2)*f^2 + ((3*b^2*c^4 - 4*a
*c^5)*d^3 - 3*(2*b^3*c^3 - 3*a*b*c^4)*d^2*e + 3*(b^4*c^2 - a*b^2*c^3)*d*e^2
 - (3*a*b^3*c^2 - 5*a^2*b*c^3)*e^3)*f)*x - sqrt(1/2)*((b^2*c^5 - 4*a*c^6)*d
^2*e - 2*(b^3*c^4 - 4*a*b*c^5)*d*e^2 + (b^4*c^3 - 5*a*b^2*c^4 + 4*a^2*c^5)*
e^3 - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*f^3 - (2*(b^5*c^2 -
5*a*b^3*c^3 + 4*a^2*b*c^4)*d - (3*b^6*c - 19*a*b^4*c^2 + 29*a^2*b^2*c^3 - 4
*a^3*c^4)*e)*f^2 - ((b^3*c^4 - 4*a*b*c^5)*d^2 - 2*(2*b^4*c^3 - 9*a*b^2*c^4
+ 4*a^2*c^5)*d*e + (3*b^5*c^2 - 17*a*b^3*c^3 + 20*a^2*b*c^4)*e^2)*f - (2*(b
^2*c^7 - 4*a*c^8)*d - (b^3*c^6 - 4*a*b*c^7)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*
a^2*c^7)*f)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2*(3*b^2*c^6 - a*c^7)*d^2*e^2 -
 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*e^4 + (b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*f^4 + 4*((b^6*c^2
- 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d - (b^7*c - 5*a*b^5*c^2 + 7*a^2*b
^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4 - 7*a*b^2*c^5 + 3*a^2*c^6)*d^2
 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*d*e + (3*b^6*c^2 - 12*a*b^4*c^
3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((b^2*c^6 - a*c^7)*d^3 - (3*b^3*
c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2*c^5 + a^2*c^6)*d*e^2 - (b^5*c
^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(b*c^
4*d^2 - 2*(b^2*c^3 - 2*a*c^4)*d*e + (b^3*c^2 - 3*a*b*c^3)*e^2 + (b^5 - 5*a*
b^3*c + 5*a^2*b*c^2)*f^2 + 2*((b^3*c^2 - 3*a*b*c^3)*d - (b^4*c - 4*a*b^2*c^
2 + 2*a^2*c^3)*e)*f - (b^2*c^5 - 4*a*c^6)*sqrt((c^8*d^4 - 4*b*c^7*d^3*e + 2
*(3*b^2*c^6 - a*c^7)*d^2*e^2 - 4*(b^3*c^5 - a*b*c^6)*d*e^3 + (b^4*c^4 - 2*a
*b^2*c^5 + a^2*c^6)*e^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
 + a^4*c^4)*f^4 + 4*((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4 - a^3*c^5)*d -
(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*e)*f^3 + 2*((3*b^4*c^4
- 7*a*b^2*c^5 + 3*a^2*c^6)*d^2 - 2*(3*b^5*c^3 - 9*a*b^3*c^4 + 5*a^2*b*c^5)*
d*e + (3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*e^2)*f^2 + 4*((
b^2*c^6 - a*c^7)*d^3 - (3*b^3*c^5 - 4*a*b*c^6)*d^2*e + (3*b^4*c^4 - 6*a*b^2
*c^5 + a^2*c^6)*d*e^2 - (b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*e^3)*f)/(b^2*
c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 6*(c*e - b*f)*x)/c^2
```

giac [B] time = 4.75, size = 5461, normalized size = 19.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^4*c^4 - 16*a*b^2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

```
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d + (2*b^6*c^
2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2
*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4
- 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^
2*c^4)*c^2*f - (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*
c^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c
)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(c) + 2*(sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*
c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*
abs(c)*e - (2*b^4*c^6 - 8*a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^6)*d - (2*b^6*c^4 - 14*a*b^4*c^5 + 24
*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 -
12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 -
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*
c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5)*f + (2*b^5*c^5 - 12*a*b^3*c^6 + 16*
a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c
^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c
^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^
```

```
2*c^6 - 4*a*c^7))/c^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*
c^6 + 8*a^2*b*c^6 + a*b^2*c^6 - 4*a^2*c^7)*c^2) - 1/8*((2*b^4*c^4 - 16*a*b^
2*c^5 + 32*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b
^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*c^2*d + (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*
b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c -
24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2
+ 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*c^2*f - (2*b^5*c^3
- 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*
(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a
*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a
*c)*a^2*b*c^4)*f*abs(c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*
c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*
c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b
^2 - 4*a*c)*a*b^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*abs(c)*e - (2*b^4*c^6 - 8*
a*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^6 - 2*(b^2 - 4
*a*c)*b^2*c^6)*d - (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*
c)*a*b^2*c^5)*f + (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
```

```
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c
^6)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 - sqrt(b^2*c^6 - 4*a*c^7))/c^4))/((
a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 + a*b^2*
c^6 - 4*a^2*c^7)*c^2) + 1/3*(c^2*f*x^3 - 3*b*c*f*x + 3*c^2*x*e)/c^3
```

maple [B] time = 0.03, size = 1035, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/3*f*x^3/c-1/c^2*b*f*x+1/c*e*x+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f-1/2/c^2*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*b^2*f+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/2*d*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*f+1/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b^3*f-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f+1/2/
c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2*f-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/2*d*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)-3/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*f+1/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a*e+1/2/c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^
3*f-1/2/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/2/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(c*f*x^3 + 3*(c*e - b*f)*x)/c^2 - integrate((a*c*e - a*b*f - (c^2*d - b
*c*e + (b^2 - a*c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

mupad [B] time = 3.36, size = 15674, normalized size = 55.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4),x)

[Out] x*(e/c - (b*f)/c^2) - atan((((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f
 - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4
*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
+ 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5
*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f +
12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*
f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*
b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2) - (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^
3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2
 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*
c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7
*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^
2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e +
 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*
a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c
 - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2)
+ 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d
*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*a^2*c^5*e - 4*a*b^2*c
^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*
(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 -
 b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*
b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3
*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f
 + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2
*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4
*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
+ 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^
```

```
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2
+ b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f
 + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^
4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c
^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*
b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c
*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d
*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14
*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) +
2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*
a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*a
^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^
3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 1
2*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2
*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^
2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2
*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*
e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1
/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f
*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2
*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*
(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 -
 b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*
b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3
*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f
 + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^
(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2
*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6*f^2 - 2*a*c^5*
d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c
^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f -
2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2
*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a
*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b
^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3
*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)
^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*
c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 1
2*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f
 + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6))
)^(1/2) + (((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)
/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(
4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*
a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b
```

^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b
^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f
+ 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4
*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*
e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^
3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^
6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3)^(1/2
) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a
^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^
6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^
5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e -
14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2)
+ 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-
(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^
2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c
- b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b
^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^
2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 +
4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3
*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2
- c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)
^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^
2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c
^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f +
2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*
a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) +
16*a*b^4*c^2*e*f + 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-
(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)
^3)^(1/2) - 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))^(1/2) - (2*(a*c^4*d^3 - a^4*c*f^3 + a^3*b^2*f^3 - a^2*b*
c^2*e^3 + a^2*c^3*d*e^2 - a^2*b^3*e*f^2 - 3*a^2*c^3*d^2*f + 3*a^3*c^2*d*f^2
- a^3*c^2*e^2*f + a*b^4*d*f^2 - 2*a*b*c^3*d^2*e + a*b^2*c^2*d*e^2 + 2*a*b^
2*c^2*d^2*f - 3*a^2*b^2*c*d*f^2 + 2*a^2*b^2*c*e^2*f - 2*a*b^3*c*d*e*f + 2*a
^2*b*c^2*d*e*f))/c^3))*(-(b^7*f^2 + b^3*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^3
)^(1/2) + b^5*c^2*e^2 - b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2
+ 12*a^2*b*c^4*e^2 + a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2
- 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 - a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) -
b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*
a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4
*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f + 2*a*c^3*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f + 2*b^3*c*
e*f*(-(4*a*c - b^2)^3)^(1/2) + 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*
a^2*b^2*c^3*e*f - 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*e*f*(-
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i
- atan(((((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c
^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5
*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f +
12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*
f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*
b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)

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))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2
*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*
c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*
d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 1
4*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) -
 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4
*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*
c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c -
 b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6
*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*
e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4
*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c
^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 +
 c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3
)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)
^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*
(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5
*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*
b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*
c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 1
6*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4
*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3
)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))^(1/2)*1i - (((16*a^2*c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f
 - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4
*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
- 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*
b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^
3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5
*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f +
12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a
*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*
f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*
b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)
))^(1/2) + (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^
3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2
 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*
c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7
*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^
2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e +
 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*
a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c
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 - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d
*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*a^2*c^5*e - 4*a*b^2*c
^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*
(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 +
 b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*
c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*
b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3
*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f
 + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^
(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2
*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(b^7*f^2 + b^3*c^4
*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b
^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*
f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f
- 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(b^6*f^2 - 2*a*c^5*d^2 + 2*a^2*c^4*e^2
+ b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*e^2 - 2*b^5*c*e*f
 + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b^3*c^3*d*e + 2*b^
4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*c^3*e*f + 6*a*b*c
^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) +
 b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*
b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c
*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*
e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d
*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14
*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) -
2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*
a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c
^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((16*a^2*
c^5*e - 4*a*b^2*c^4*e + 4*a*b^3*c^3*f - 16*a^2*b*c^4*f)/c^3 + (2*x*(4*b^3*c
^5 - 16*a*b*c^6)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2
) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a
^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^
6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c
^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^
5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e -
 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2)
 - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-
(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^
2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(
b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^2 + b^
4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3
*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*
e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*d*f +
```

```
24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/
2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^
2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(b^6*f^2 - 2*a*c^5*d^2
 + 2*a^2*c^4*e^2 + b^2*c^4*d^2 - 2*a^3*c^3*f^2 + b^4*c^2*e^2 - 4*a*b^2*c^3*
e^2 - 2*b^5*c*e*f + 9*a^2*b^2*c^2*f^2 - 6*a*b^4*c*f^2 + 4*a^2*c^4*d*f - 2*b
^3*c^3*d*e + 2*b^4*c^2*d*f - 8*a*b^2*c^3*d*f + 10*a*b^3*c^2*e*f - 10*a^2*b*
c^3*e*f + 6*a*b*c^4*d*e))/c^3)*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c
- b^2)^3)^(1/2) + b^5*c^2*e^2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*
c^3*e^2 + 12*a^2*b*c^4*e^2 - a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*
c^3*f^2 - 2*b^6*c*e*f + 25*a^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f
^2 - 16*a^2*c^5*d*e - 2*b^4*c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a
*b^2*c^4*d*e - 14*a*b^3*c^3*d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c -
 b^2)^3)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f -
2*b^3*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^2*b^2*c^3*e*f + 2*b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^
2*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2) - (2*(a*c^4*d^3 - a^4*c*f^3 + a^3*b^2*f^3 - a^2*b*c^2*e^3 + a^2*c^3*d*
e^2 - a^2*b^3*e*f^2 - 3*a^2*c^3*d^2*f + 3*a^3*c^2*d*f^2 - a^3*c^2*e^2*f + a
*b^4*d*f^2 - 2*a*b*c^3*d^2*e + a*b^2*c^2*d*e^2 + 2*a*b^2*c^2*d^2*f - 3*a^2*
b^2*c*d*f^2 + 2*a^2*b^2*c*e^2*f - 2*a*b^3*c*d*e*f + 2*a^2*b*c^2*d*e*f))/c^3
))*(-(b^7*f^2 + b^3*c^4*d^2 + c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + b^5*c^2*e^
2 + b^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 -
 a*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*f^2 - 2*b^6*c*e*f + 25*a
^2*b^3*c^2*f^2 + a^2*c^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 4*a*b*c^5*d^2 - 9*a*b^5*c*f^2 - 16*a^2*c^5*d*e - 2*b^4*
c^3*d*e + 16*a^3*c^4*e*f + 2*b^5*c^2*d*f + 12*a*b^2*c^4*d*e - 14*a*b^3*c^3*
d*f + 24*a^2*b*c^4*d*f - 2*a*c^3*d*f*(-(4*a*c - b^2)^3)^(1/2) - 2*b*c^3*d*e
*(-(4*a*c - b^2)^3)^(1/2) + 16*a*b^4*c^2*e*f - 2*b^3*c*e*f*(-(4*a*c - b^2)^
3)^(1/2) - 3*a*b^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*e*f + 2*
b^2*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a*b*c^2*e*f*(-(4*a*c - b^2)^3)^(1/
2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*2i + (f*x^3)/(3*c)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.57 $\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4} dx$

Optimal. Leaf size=219

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

```
[Out] f*x/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*e-b*f+(
2*c^2*d+b^2*f-c*(2*a*f+b*e))/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2
))*(c*e-b*f+(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1676, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-c(2af+be)+b^2f+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{-2acf+b^2f-bce+2c^2d}{\sqrt{b^2-4ac}}-bf+ce\right)}{\sqrt{2}c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{fx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (f*x)/c + ((c*e - b*f + (2*c^2*d + b^2*f - c*(b*e + 2*a*f))/Sqrt[b^2 - 4*a*
c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((c*e - b*f - (2*c^2*d - b*c*e + b^2*f -
2*a*c*f)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{a+bx^2+cx^4}\,dx = \int \left(\frac{f}{c}+\frac{cd-af+(ce-bf)x^2}{c\left(a+bx^2+cx^4\right)}\right)dx$$
$$= \frac{fx}{c}+\frac{\int \frac{cd-af+(ce-bf)x^2}{a+bx^2+cx^4}\,dx}{c}$$
$$= \frac{fx}{c}+\frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}\dots}}{2c}$$
$$= \frac{fx}{c}+\frac{\left(ce-bf+\frac{2c^2d+b^2f-c(be+2af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(ce-bf-\frac{2c^2d-bce+b^2f-2acf}{\sqrt{b^2-4ac}}\right)\tan\dots}{\sqrt{2}c^{3/2}\sqrt{b+\sqrt{b^2-\dots}}}$$

Mathematica [**A**] time = 0.33, size = 258, normalized size = 1.18

$$\frac{\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(c\left(e\sqrt{b^2-4ac}-2af-be\right)+bf\left(b-\sqrt{b^2-4ac}\right)+2c^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-c\left(e\sqrt{b^2-4ac}+2af+be\right)+bf\left(\sqrt{b^2-4ac}+b\right)+2c\dots\right)}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}\dots}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*f*x + (Sqrt[2]*(2*c^2*d + b*(b - Sqrt[b^2 - 4*a*c])*f + c*(-(b*e
) + Sqrt[b^2 - 4*a*c]*e - 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*
(2*c^2*d + b*(b + Sqrt[b^2 - 4*a*c])*f - c*(b*e + Sqrt[b^2 - 4*a*c]*e + 2*a
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4
*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*c^(3/2))
```

fricas [**B**] time = 4.49, size = 5788, normalized size = 26.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3
*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*
a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b
^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2
)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^
2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*
b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c
^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4
- ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^
2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2
+ (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c
^4)*d^3)*f)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4
)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c
^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4
*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)
*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a
^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4
```

```
+ (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a
^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)
*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e -
a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*
d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d -
(a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^
5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b
^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e +
 (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^
5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*
a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c
^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c -
2*a^2*c^2)*e)*f + (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 +
 a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^
3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3
 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b
*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/
(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2
*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (
a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2
- (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3
*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - sqrt(1/2)*((b^2*c^4 - 4*a*
c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c
^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*
b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3
)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f - ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2
*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4
 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 +
4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*
c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2
- 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2
*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 -
 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f + (a*b^2*c^3 -
4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3
*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c
^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*
c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^
2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) + sqr
t(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)
*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*
sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^
4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3
- 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c
^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3
)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))*log(2*(c^5*d^4 -
b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4 - (a^3*b^2 - a^4*c)*f^4 - ((a*b^4
 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 + a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e
^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*
c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*
f)*x + sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3 - (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 +
 (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 - ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a
^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)
*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f +
 ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^
3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^
4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c
- a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3
*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d
*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a
```

```
*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c
- 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2
 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 -
a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c
^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a
*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7))
)/(a*b^2*c^3 - 4*a^2*c^4))) - sqrt(1/2)*c*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e +
a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2
)*e)*f - (a*b^2*c^3 - 4*a^2*c^4)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*
e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d
- (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*
c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*
e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^
3 - 4*a^2*c^4))*log(2*(c^5*d^4 - b*c^4*d^3*e + a*b*c^3*d*e^3 - a^2*c^3*e^4
- (a^3*b^2 - a^4*c)*f^4 - ((a*b^4 - 3*a^2*b^2*c + 4*a^3*c^2)*d - (a^2*b^3 +
 a^3*b*c)*e)*f^3 - 3*(a^2*b^2*c*e^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^2 - (a*b^3*
c - a^2*b*c^2)*d*e)*f^2 + (3*a*b*c^3*d^2*e - 3*a*b^2*c^2*d*e^2 + 3*a^2*b*c^
2*e^3 + (b^2*c^3 - 4*a*c^4)*d^3)*f)*x - sqrt(1/2)*((b^2*c^4 - 4*a*c^5)*d^3
- (a*b^2*c^3 - 4*a^2*c^4)*d*e^2 + (a^2*b^4 - 5*a^3*b^2*c + 4*a^4*c^2)*f^3 -
 ((a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d + 2*(a^2*b^3*c - 4*a^3*b*c^2)*e)
*f^2 - (3*(a*b^2*c^3 - 4*a^2*c^4)*d^2 - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d*e - (
a^2*b^2*c^2 - 4*a^3*c^3)*e^2)*f + ((a*b^3*c^4 - 4*a^2*b*c^5)*d - 2*(a^2*b^2
*c^4 - 4*a^3*c^5)*e + (a^2*b^3*c^3 - 4*a^3*b*c^4)*f)*sqrt((c^6*d^4 - 2*a*c^
5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*f^4 + 4*((a^2*b
^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^3 - 2*(4*a^2*b*c^3*d*e +
 (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3*c^3)*e^2)*f^2 - 4*(a*c^
5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^2*c^6 - 4*
a^3*c^7)))*sqrt(-(b*c^3*d^2 - 4*a*c^3*d*e + a*b*c^2*e^2 + (a*b^3 - 3*a^2*b*
c)*f^2 + 2*(a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e)*f - (a*b^2*c^3 - 4*a^2*c^4
)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 2*a^3*b^2*c +
a^4*c^2)*f^4 + 4*((a^2*b^2*c^2 - a^3*c^3)*d - (a^2*b^3*c - a^3*b*c^2)*e)*f^
3 - 2*(4*a^2*b*c^3*d*e + (a*b^2*c^3 - 3*a^2*c^4)*d^2 - (3*a^2*b^2*c^2 - a^3
*c^3)*e^2)*f^2 - 4*(a*c^5*d^3 - a*b*c^4*d^2*e - a^2*c^4*d*e^2 + a^2*b*c^3*e
^3)*f)/(a^2*b^2*c^6 - 4*a^3*c^7)))/(a*b^2*c^3 - 4*a^2*c^4))) - 2*f*x)/c
```

giac [B] time = 3.91, size = 4086, normalized size = 18.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] f*x/c - 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*c^2*f -
(2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 -
 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e - 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*b^4*c^
```

```
4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^2*c^5 - 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^6 + 3
2*a^2*c^6 - 2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*d*abs(c) + 2*(
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^3*c^3 + 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^
4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^
2 - 4*a*c)*a^2*c^4)*f*abs(c) - 2*(2*b^3*c^6 - 8*a*b*c^7 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d - (2*b^5*c^4 - 12*
a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2
 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*f + (2*b^4*c^5 - 8*a*b^2*c^6 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c
^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sqrt(b^2*c^2 - 4*a*c^3))/c^2))/((a*
b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^
5 - 4*a^2*c^6)*c^2) + 1/8*((2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b
*c^3)*c^2*f - (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*c^2*e + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c^4 - 2*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 8*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c^5 + 16*a*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*c^6 - 32*a^2*c^6 + 2*(b^2 - 4*a*c)*b^2*c^4 - 8*(b^2 - 4*a*c)*a*c^5)*
d*abs(c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b
^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*f*abs(c) - 2*(2*b^3*c^6 - 8*a*b*c^7 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 4*sqrt(2)*
```

```
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b*c^6)*d - (2
*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*f + (2*b^4*c^5 -
 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^
4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2
- 4*a*c)*b^2*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((b*c - sqrt(b^2*c^2 - 4*a*c^
3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16*a^3*c^5 + 8*a^2*b*
c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2)
```

maple [B] time = 0.03, size = 676, normalized size = 3.09

$$\frac{\sqrt{2}\, af \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\sqrt{2}\, af \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}-\frac{\sqrt{2}\, b^2 f \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] f*x/c+1/2/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f-1/2/(-4*a*c+b^2)^(1/2)/c*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b^2*f+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/(-4*a*c+
b^2)^(1/2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/2/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f+1/2*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*e+1/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f-1/2/(-4*a*c+b^2
)^(1/2)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b*e-1/(-4*a*c+b^2)^(1/2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{fx}{c}-\frac{-\int \frac{\left(ce-bf\right)x^2+cd-af}{cx^4+bx^2+a}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] f*x/c - integrate(-((c*e - b*f)*x^2 + c*d - a*f)/(c*x^4 + b*x^2 + a), x)/c
```

mupad [B] time = 3.36, size = 10209, normalized size = 46.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x)/c - atan(((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)
/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 +
12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*
a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*
c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b
^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3
 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b
^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^
3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c
^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d
*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*
e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*
a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*
f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2
 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4
*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2
 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 +
16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2
*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*
a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*
c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*
d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3
*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 +
 a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4
*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*
c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b
^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a
^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2
- c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^
2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2
*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*
c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d
*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^
2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^
5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^
3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e
*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*
c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^
2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b
^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((((4*b^2*c^3*d + 16*a^2
*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a
*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 -
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4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*(a*c^2*e^3 - a^2*b*f^3 - b^3*d*f^2 + c^3*d^2*e + a*b^2*e*f^2 - b*c^2*d*e^2 - b*c^2*d^2*f + a^2*c*e*f^2 + 2*a*b*c*d*f^2 - 2*a*b*c*e^2*f - 2*a*c^2*d*e*f + 2*b^2*c*d*e*f))/c + (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)))*(-(a*b^5*f^2 + b^3*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 - a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f + 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*2i - atan(((((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -

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 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3
*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*
b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2
 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b
^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^
2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*
d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c
^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c
^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c
^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*
e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3
*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e
^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
 - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*
b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a
^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i - (((4*b^2*c^3*d + 16*a^
2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(
a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2
 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2
*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5
*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a
^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*
d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1
/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2
 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*
c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*
(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e
^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*
a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*
b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^
2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*1i)/((((4
*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c^4*d - 4*a*b^2*c^2*f)/c - (2*x*(4*b^3*c^3
 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2
) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) -
a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a
^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3
*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e
*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^
(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a
*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*
f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*
e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-
```

```
(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)
 - (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 -
4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6
*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(
1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2)
 - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2
+ a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*
a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2
*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*
c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)
))^(1/2) - (2*(a*c^2*e^3 - a^2*b*f^3 - b^3*d*f^2 + c^3*d^2*e + a*b^2*e*f^2
- b*c^2*d*e^2 - b*c^2*d^2*f + a^2*c*e*f^2 + 2*a*b*c*d*f^2 - 2*a*b*c*e^2*f -
 2*a*c^2*d*e*f + 2*b^2*c*d*e*f))/c + (((4*b^2*c^3*d + 16*a^2*c^3*f - 16*a*c
^4*d - 4*a*b^2*c^2*f)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(a*b^5*f^2 + b^3*
c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^
2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b
^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2))/c)*(-(a*b^5*f^2 + b^3*c^3*d
^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 - a
*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c^
2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^2
*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^3
*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2) + (2*x*(2*c^4*d^2 + b^4*f^2 - 2*a
*c^3*e^2 + 2*a^2*c^2*f^2 + b^2*c^2*e^2 - 4*a*c^3*d*f - 2*b*c^3*d*e - 2*b^3*
c*e*f - 4*a*b^2*c*f^2 + 2*b^2*c^2*d*f + 6*a*b*c^2*e*f))/c)*(-(a*b^5*f^2 + b
^3*c^3*d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3
*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 7*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/
2) - 4*a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*
a*b^3*c^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12
*a^2*b^2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)))*(-(a*b^5*f^2 + b^3*c^3*
d^2 + c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + a*b^3*c^2*e^2 - 4*a^2*b*c^3*e^2 -
a*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7
*a^2*b^3*c*f^2 + 12*a^3*b*c^2*f^2 + a^2*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^4*d^2 + 16*a^2*c^4*d*e - 16*a^3*c^3*e*f - 4*a*b^2*c^3*d*e + 2*a*b^3*c
^2*d*f - 8*a^2*b*c^3*d*f - 2*a*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b^
2*c^2*e*f - 2*a*b^4*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
3*c^5 + a*b^4*c^3 - 8*a^2*b^2*c^4)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{d+ex^2+fx^4}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=213

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

```
[Out] -d/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-a*f+
(a*b*f-2*a*c*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)
^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c
*d-a*f+(-a*b*f+2*a*c*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1664, 1166, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{abf-2ace+bcd}{\sqrt{b^2-4ac}}-af+cd\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{d}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(d/(a*x)) - ((c*d - a*f + (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b^2 - 4*a*c])*Arc
Tan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[c]*Sq
rt[b - Sqrt[b^2 - 4*a*c]]) - ((c*d - a*f - (b*c*d - 2*a*c*e + a*b*f)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{d}{ax^2}+\frac{-bd+ae-(cd-af)x^2}{a\left(a+bx^2+cx^4\right)}\right)dx$$

$$= -\frac{d}{ax}+\frac{\int \frac{-bd+ae+(-cd+af)x^2}{a+bx^2+cx^4}\,dx}{a}$$

$$= -\frac{d}{ax}-\frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}+\frac{\left(-cd+af+\frac{2ace-b(cd+af)}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\cdots}}{2a}$$

$$= -\frac{d}{ax}-\frac{\left(cd-af-\frac{2ace-b(cd+af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(cd-af-\frac{bcd-2ace+abf}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right)}{\sqrt{2}\,a\sqrt{c}\sqrt{b+\sqrt{b^2-4\cdots}}}$$

Mathematica [**A**] time = 0.30, size = 253, normalized size = 1.19

$$\frac{-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(cd\sqrt{b^2-4ac}-af\sqrt{b^2-4ac}+abf-2ace+bcd\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+abf-2ace+bcd\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}-\cdots}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*d)/x - (Sqrt[2]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f - a*S
qrt[b^2 - 4*a*c]*f)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b*c*d
 - c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + a*b*f + a*Sqrt[b^2 - 4*a*c]*f)*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c
]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(2*a)
```

fricas [**B**] time = 2.26, size = 5930, normalized size = 27.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^
2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^3*b^2*c
- 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f
^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*
e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a
^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^
2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4
*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*
f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b
^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 +
(3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 -
3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x + sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a
^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3
*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b
*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^
3*b^2*c - 4*a^4*c^2)*d*e)*f - ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d -
(a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c
```

```
^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 +
 a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^
3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^
2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/
(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b
*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f + (a^
3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3
 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c
^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*
e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e
^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c
 - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b^3*c - 3
*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*c*e)*f +
 (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d
*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2
*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^
5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2
*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b
^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*e^3 + a^3*c^
2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d^3*e + (a^4*
b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c - 2*a^3*c^2)
*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2*c^2 + 4*a^2
*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - sqrt(1/2)*((b^5*c - 5*a*b
^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^2*e
+ 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e^3 - ((a^3*b
^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c - 4*a^3*b*c^
2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f - ((a^3*b^4*c - 6*a^4*b^2*c^2 + 8*a
^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*c^2)*f)*sqrt
(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2
*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*
c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*
c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c
^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2 + (b
^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*a^3*
c*e)*f + (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4
*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c
^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*
d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e -
 a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3))
)/(a^3*b^2*c - 4*a^4*c^2))) + sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2 + a^3*b*f^2
+ (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*c*d - 2*
a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4
 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b
^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*
b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2
*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c
^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*a^2*b*c^2*d*
e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2 + a*b*c^3)*d
^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e - (a^2*b^2*c
- 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4*c - 3*a*b^2
*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x + sqrt(1/2)*((b
^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2*c^2 + 4*a^3
*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c - 4*a^4*c^2)*e
^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*((a^2*b^3*c
- 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f + ((a^3*b^4*c - 6*a^4*b
^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^2*c - 4*a^6*
c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 -
(b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2
*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^
```

```
2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2
*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*b*c*e^2 + a^
3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^2*b*
c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4
*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4
+ 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2
*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b
*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 -
 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) - sqrt(1/2)*a*x*sqrt(-(a^2*b*c*e^2
+ a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e + 2*(a^
2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2*d*e^3 -
 a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*
d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2
 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a
^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c
^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))*log(-2*(3*a*b^2*c^2*d^2*e^2 - 3*
a^2*b*c^2*d*e^3 + a^3*c^2*e^4 - a^5*f^4 + (b^2*c^3 - a*c^4)*d^4 - (b^3*c^2
+ a*b*c^3)*d^3*e + (a^4*b*e - (a^3*b^2 - 4*a^4*c)*d)*f^3 - 3*(a^3*b*c*d*e -
 (a^2*b^2*c - 2*a^3*c^2)*d^2)*f^2 + (3*a^2*b^2*c*d*e^2 - a^3*b*c*e^3 + (b^4
*c - 3*a*b^2*c^2 + 4*a^2*c^3)*d^3 - 3*(a*b^3*c - a^2*b*c^2)*d^2*e)*f)*x - s
qrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^3 - (3*a*b^4*c - 13*a^2*b^2
*c^2 + 4*a^3*c^3)*d^2*e + 3*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^2 - (a^3*b^2*c -
4*a^4*c^2)*e^3 - ((a^3*b^3 - 4*a^4*b*c)*d - (a^4*b^2 - 4*a^5*c)*e)*f^2 + 2*
((a^2*b^3*c - 4*a^3*b*c^2)*d^2 - (a^3*b^2*c - 4*a^4*c^2)*d*e)*f + ((a^3*b^4
*c - 6*a^4*b^2*c^2 + 8*a^5*c^3)*d - (a^4*b^3*c - 4*a^5*b*c^2)*e + 2*(a^5*b^
2*c - 4*a^6*c^2)*f)*sqrt(-(4*a^3*b*c^2*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3
- a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^
3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e
^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2 + 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^
2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a^6*b^2*c^2 - 4*a^7*c^3)))*sqrt(-(a^2*
b*c*e^2 + a^3*b*f^2 + (b^3*c - 3*a*b*c^2)*d^2 - 2*(a*b^2*c - 2*a^2*c^2)*d*e
 + 2*(a^2*b*c*d - 2*a^3*c*e)*f - (a^3*b^2*c - 4*a^4*c^2)*sqrt(-(4*a^3*b*c^2
*d*e^3 - a^4*c^2*e^4 + 4*a^5*c*d*f^3 - a^6*f^4 - (b^4*c^2 - 2*a*b^2*c^3 + a
^2*c^4)*d^4 + 4*(a*b^3*c^2 - a^2*b*c^3)*d^3*e - 2*(3*a^2*b^2*c^2 - a^3*c^3)
*d^2*e^2 - 2*(2*a^4*b*c*d*e - a^5*c*e^2 - (a^3*b^2*c - 3*a^4*c^2)*d^2)*f^2
+ 4*(2*a^3*b*c^2*d^2*e - a^4*c^2*d*e^2 - (a^2*b^2*c^2 - a^3*c^3)*d^3)*f)/(a
^6*b^2*c^2 - 4*a^7*c^3)))/(a^3*b^2*c - 4*a^4*c^2))) + 2*d)/(a*x)
```

giac [B] time = 5.94, size = 3988, normalized size = 18.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -d/(a*x) - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*a^2*d - (
2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
```

```
a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*a^2*f + 2*(s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a*c)*a*b^3*c^2 -
8*(b^2 - 4*a*c)*a^2*b*c^3)*d*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 + 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 -
 32*a^4*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*abs(a)
*e + (2*a^2*b^4*c^3 - 8*a^3*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c^3)*d + (2*a^3*b^4*c^2
 - 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2
 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*f - 2*(2*a^3*b^3*c^3 - 8*a^4*b*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c + 4*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^2 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a^3*
b*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))
/((a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2 + 16*a^5*c^3 + 8*a^4*b*c^3 + a
^3*b^2*c^3 - 4*a^4*c^4)*abs(a)*abs(c)) + 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 3
2*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c
 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*
(b^2 - 4*a*c)*a*c^4)*a^2*d - (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2
- 4*a*c)*a^2*c^3)*a^2*f - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*
c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 16*a^2*b^
3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 32*a^3*b*c^4
- 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*d*abs(a) + 2*(sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^3*c^2 + 2*a^2*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^
3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 32*a^4*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 + 8
```

```
*(b^2 - 4*a*c)*a^3*c^3)*abs(a)*e + (2*a^2*b^4*c^3 - 8*a^3*b^2*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 4*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*(b^2 - 4*a*c)*a
^2*b^2*c^3)*d + (2*a^3*b^4*c^2 - 8*a^4*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2)*f - 2*(2*a^3*
b^3*c^3 - 8*a^4*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^3 - 2*(b^2 - 4*a*c)*a^3*b*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a*b - sq
rt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4*c - 8*a^4*b^2*c^2 - 2*a^3*b^3*c^2
+ 16*a^5*c^3 + 8*a^4*b*c^3 + a^3*b^2*c^3 - 4*a^4*c^4)*abs(a)*abs(c))
```

maple [B] time = 0.02, size = 563, normalized size = 2.64

$$\frac{\sqrt{2}\, bcd \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bcd \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bf \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+\cdots}\right.}}\right.}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x)
```

```
[Out] -d/a/x-1/2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*e+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+1/2*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*f-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b*f-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/2/a*c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*b*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(cd-af\right)x^2+bd-ae}{cx^4+bx^2+a}\,dx}{a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((c*d - a*f)*x^2 + b*d - a*e)/(c*x^4 + b*x^2 + a), x)/a - d/(a*x
)
```

mupad [B] time = 3.52, size = 10170, normalized size = 47.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2)*1i + (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2*d - 4*a^5*b^2*c^2*e - 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2)*1i)/((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2
 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) +
 (-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^
2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e
^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a
^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c -
```

b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
+ 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2*d + 4*a^5*b^2*c^2*e + 16*a^
5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) -
7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) +
a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^
2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/
2) - (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*c^2*f^2 - 2*a^5*b^2*c*f^2 -
2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3*d*e + 4*a^5*b*c^2*e*f) + (-
(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d
^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2
- 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c
- b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*
b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*
b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2
)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*(x*(32*a^6*
b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)
^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(
1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e
+ 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c
- b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2
*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2
*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2*d - 4*a^5*b^2*c^2*e - 16*a^5*b
*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*
a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2
*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^2*c*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e
*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/
2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e + 2*a*b*c*d*e*(-(
4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)
- 2*a^6*c*f^3 + 2*a^3*c^4*d^3 + 2*a^4*c^3*d*e^2 - 6*a^4*c^3*d^2*f + 6*a^5*c
^2*d*f^2 - 2*a^5*c^2*e^2*f + 2*a^5*b*c*e*f^2 - 2*a^3*b*c^3*d^2*e - 2*a^4*b^
2*c*d*f^2 + 2*a^3*b^2*c^2*d^2*f))*(-(b^5*c*d^2 + a^3*b^3*f^2 + a^3*f^2*(-(4
*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 + a*c^2*d^2*(-(4*
a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 - a^2*c*e^2*(-(4*a*c
- b^2)^3)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a
^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f - 2*a^2*c*d
f(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^
4*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c
- 8*a^4*b^2*c^2)))^(1/2)*2i - atan(((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a
^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*
c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c -
b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^
3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-
(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d
*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a
^4*b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^

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3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d
^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 +
 a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b
*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*
b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2
*d + 4*a^5*b^2*c^2*e + 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2
- 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*
b^4*c - 8*a^4*b^2*c^2)))^(1/2)*1i + (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a
^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*
c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^
3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-
(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d
*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a
^4*b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^
3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d
^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 +
 a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b
*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*
b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2
*d - 4*a^5*b^2*c^2*e - 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2
- 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*
b^4*c - 8*a^4*b^2*c^2)))^(1/2)*1i)/((x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a
^6*c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*
c^3*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2
)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^
3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-
(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d
*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a
^4*b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^
3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d
^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 +
 a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b
*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*
b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*
a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) - 16*a^6*c^3*e - 4*a^4*b^3*c^2
*d + 4*a^5*b^2*c^2*e + 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2
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- 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a
^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e -
2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*
b^4*c - 8*a^4*b^2*c^2)))^(1/2) - (x*(4*a^4*c^4*d^2 - 4*a^5*c^3*e^2 + 4*a^6*
c^2*f^2 - 2*a^5*b^2*c*f^2 - 2*a^3*b^2*c^3*d^2 - 8*a^5*c^3*d*f + 4*a^4*b*c^3
*d*e + 4*a^5*b*c^2*e*f) + (-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(-(4*a*c - b
^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d
*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e
- 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*
b^2*c^2)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(b^5*c*d^2 + a^3*b^3*f
^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2
- a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^
2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a
^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^
2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2
*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5
*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2) + 16*a^6*c^3*e + 4*a^4*b^3*c^2*d
- 4*a^5*b^2*c^2*e - 16*a^5*b*c^3*d))*(-(b^5*c*d^2 + a^3*b^3*f^2 - a^3*f^2*(
-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2*b*c^3*d^2 - a*c^2*d^2*(-
(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c^2*e^2 + a^2*c*e^2*(-(4*a
*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*f^2 - 1
6*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f - 8*a^3*b*c^2*d*f + 2*a^2*
c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f + 12*a^2*b^2*c^2*d*e - 2*a
*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^5*c^3 + a^3*b^4
*c - 8*a^4*b^2*c^2)))^(1/2) - 2*a^6*c*f^3 + 2*a^3*c^4*d^3 + 2*a^4*c^3*d*e^2
 - 6*a^4*c^3*d^2*f + 6*a^5*c^2*d*f^2 - 2*a^5*c^2*e^2*f + 2*a^5*b*c*e*f^2 -
2*a^3*b*c^3*d^2*e - 2*a^4*b^2*c*d*f^2 + 2*a^3*b^2*c^2*d^2*f))*(-(b^5*c*d^2
+ a^3*b^3*f^2 - a^3*f^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^2*d^2 + 12*a^2
*b*c^3*d^2 - a*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*b^3*c*e^2 - 4*a^3*b*c
^2*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^3)^
(1/2) - 4*a^4*b*c*f^2 - 16*a^3*c^3*d*e + 16*a^4*c^2*e*f + 2*a^2*b^3*c*d*f -
 8*a^3*b*c^2*d*f + 2*a^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b^2*c*e*f +
 12*a^2*b^2*c^2*d*e - 2*a*b^4*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(16*a^5*c^3 + a^3*b^4*c - 8*a^4*b^2*c^2)))^(1/2)*2i - d/(a*x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=267

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}}}$$

```
[Out] -1/3*d/a/x^3+(-a*e+b*d)/a^2/x+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*c^(1/2)*(b*d-a*e+(b^2*d-a*b*e-2*a*(-a*f+c*d))/(-4*a*c+b^2)^(1
/2))/a^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-b*(a*e+d*(-4*a*c+b^2)^(1/2))-a
*(2*c*d-2*a*f-e*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.07, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1664, 1166, 205}

$$\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-abe-2a(cd-af)+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}a^2\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(-e\sqrt{b^2-4ac}-2af+2cd\right.\right.}{\sqrt{2}a^2\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(3*a*x^3) + (b*d - a*e)/(a^2*x) + (Sqrt[c]*(b*d - a*e + (b^2*d - a*b*e -
 2*a*(c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(b^
2*d - b*(Sqrt[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - Sqrt[b^2 - 4*a*c]*e - 2*a*
f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{d}{ax^4}+\frac{-bd+ae}{a^2x^2}+\frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a^2\left(a+bx^2+cx^4\right)}\right)dx$$

$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\int \frac{b^2d-abe-a(cd-af)+c(bd-ae)x^2}{a+bx^2+cx^4}\,dx}{a^2}$$

$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\left(c\left(bd-ae-\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a^2}+\frac{\left(c\left(bd-ae\right.\right.}{}$$

$$= -\frac{d}{3ax^3}+\frac{bd-ae}{a^2x}+\frac{\sqrt{c}\left(bd-ae+\frac{b^2d-abe-2a(cd-af)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(bd-a\right.}{}$$

Mathematica [A] time = 0.34, size = 284, normalized size = 1.06

$$\frac{\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(a\left(-e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b^2-4ac}-ae\right)+b^2d\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-a\left(e\sqrt{b^2-4ac}+2af-2cd\right)+b\left(d\sqrt{b\right.\right.}{\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b}}}{6a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*d)/x^3 + (6*b*d - 6*a*e)/x + (3*Sqrt[2]*Sqrt[c]*(b^2*d + b*(Sqrt[b^2
 - 4*a*c]*d - a*e) + a*(-2*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-(b^2*d) + b*(Sqrt[b^2 - 4*a*c]*d +
 a*e) - a*(-2*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]]))/(6*a^2)
```

fricas [B] time = 10.54, size = 9850, normalized size = 36.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2
)*d^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2
 + 2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6
*c)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2
*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5
 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4
 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*
a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2
 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 -
9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^
2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)
)*log(2*(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b
^3*c^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^
2*b^3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e
 - (3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3
```

```
*b^2*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c
- 5*a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2
 + 3*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^
4*b*c^2)*e^3)*f)*x + sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*
b^2*c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^
4*b*c^3)*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*
e^2 - (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 +
 3*((a^4*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 +
((3*a^2*b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^
5 - 16*a^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^
2)*e^2)*f - ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^
2*c + 8*a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^
6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*
c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a
^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e
^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)
*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a
^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*
a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e
+ (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(
a^10*b^2 - 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d
^2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 +
2*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)
*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^
4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3
*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 -
3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 -
4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6
*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 +
4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a
^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 -
 (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)))
- 3*sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^
2 - 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2
*((a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*
sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*
a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3
*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4
*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*
c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4
*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^
4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 -
(a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*lo
g(2*(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c
^3 - 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^
3*c^2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (
3*a^4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2
*c^2 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*
a*b^4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3
*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*
c^2)*e^3)*f)*x - sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*
c^3 + 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*
c^3)*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2
- (a^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*(
(a^4*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*
a^2*b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 -
16*a^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)*e
^2)*f - ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c
```

```
+ 8*a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c +
7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b
^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 +
 (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*
f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b
*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*
b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3
*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10
*b^2 - 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 -
 2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((
a^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f + (a^5*b^2 - 4*a^6*c)*sqr
t((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d
^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2
*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^
4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a
^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2
)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((
a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b
^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^
5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) + 3*
sqrt(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 -
2*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a
^2*b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt
((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^
4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*
b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4
*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^
7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)
*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a
^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^
3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5
*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*
(a^6*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c^3 -
 3*a^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^3*c^
2 - 5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (3*a^
4*b^2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2*c^2
 + 2*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*a*b^
4*c^2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3*a^3
*b*c^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*c^2)
*e^3)*f)*x + sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3
+ 4*a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*c^3)
*d^2*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2 - (a
^3*b^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*((a^4
*b^4 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*a^2*
b^6 - 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 - 16*a
^4*b^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)*e^2)*
f + ((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c + 8*
a^8*c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^
3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c
^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^
4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3
+ 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*
d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*
c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4
*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2
 - 4*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(
a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*
```

```
b^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a
^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 -
 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6
 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^
3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b
*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^
2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*
b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c
 + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^
3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt
(1/2)*a^2*x^3*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a
*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b
^3 - 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a^
8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 -
4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6
- 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3
*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*
e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2
 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b
^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c
+ 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3
 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(a^6
*c*f^4 + (b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*d^4 - (b^5*c^2 - a*b^3*c^3 - 3*a
^2*b*c^4)*d^3*e + 3*(a*b^4*c^2 - 2*a^2*b^2*c^3)*d^2*e^2 - (3*a^2*b^3*c^2 -
5*a^3*b*c^3)*d*e^3 + (a^3*b^2*c^2 - a^4*c^3)*e^4 - (3*a^5*b*c*e - (3*a^4*b^
2*c - 4*a^5*c^2)*d)*f^3 + 3*(a^4*b^2*c*e^2 + (a^2*b^4*c - 3*a^3*b^2*c^2 + 2
*a^4*c^3)*d^2 - (2*a^3*b^3*c - 3*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 5*a*b^4*c^
2 + 9*a^2*b^2*c^3 - 4*a^3*c^4)*d^3 - 3*(a*b^5*c - 3*a^2*b^3*c^2 + 3*a^3*b*c
^3)*d^2*e + 3*(a^2*b^4*c - a^3*b^2*c^2)*d*e^2 - (a^3*b^3*c + a^4*b*c^2)*e^3
)*f)*x - sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*
a^4*c^4)*d^3 - (3*a*b^7 - 21*a^2*b^5*c + 41*a^3*b^3*c^2 - 20*a^4*b*c^3)*d^2
*e + (3*a^2*b^6 - 18*a^3*b^4*c + 25*a^4*b^2*c^2 - 4*a^5*c^3)*d*e^2 - (a^3*b
^5 - 5*a^4*b^3*c + 4*a^5*b*c^2)*e^3 + (a^6*b^2 - 4*a^7*c)*f^3 + 3*((a^4*b^4
 - 5*a^5*b^2*c + 4*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*f^2 + ((3*a^2*b^6
- 19*a^3*b^4*c + 31*a^4*b^2*c^2 - 12*a^5*c^3)*d^2 - 2*(3*a^3*b^5 - 16*a^4*b
^3*c + 16*a^5*b*c^2)*d*e + (3*a^4*b^4 - 13*a^5*b^2*c + 4*a^6*c^2)*e^2)*f +
((a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*d - (a^6*b^4 - 6*a^7*b^2*c + 8*a^8*
c^2)*e + (a^7*b^3 - 4*a^8*b*c)*f)*sqrt((a^8*f^4 + (b^8 - 6*a*b^6*c + 11*a^2
*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^
3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 -
 a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^
4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e - (a^6*b^2 - a^7*c)*d)*f^3 + 2*
((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e
+ (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2
- a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5*a^5*b*c^2)*d^2*e + (3*a^4*b^4
 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 - a^6*b*c)*e^3)*f)/(a^10*b^2 - 4
*a^11*c)))*sqrt(-(a^4*b*f^2 + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^
4 - 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + 2*((a^2*b^3
- 3*a^3*b*c)*d - (a^3*b^2 - 2*a^4*c)*e)*f - (a^5*b^2 - 4*a^6*c)*sqrt((a^8*f
^4 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(
a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 1
2*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c
+ 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4 - 4*(a^7*b*e -
 (a^6*b^2 - a^7*c)*d)*f^3 + 2*((3*a^4*b^4 - 7*a^5*b^2*c + 3*a^6*c^2)*d^2 -
2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (3*a^6*b^2 - a^7*c)*e^2)*f^2 + 4*((a^2*b^6
- 4*a^3*b^4*c + 4*a^4*b^2*c^2 - a^5*c^3)*d^3 - (3*a^3*b^5 - 9*a^4*b^3*c + 5
*a^5*b*c^2)*d^2*e + (3*a^4*b^4 - 6*a^5*b^2*c + a^6*c^2)*d*e^2 - (a^5*b^3 -
a^6*b*c)*e^3)*f)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 6*(b*d - a
*e)*x^2 + 2*a*d)/(a^2*x^3)
```

giac [B] time = 3.44, size = 3813, normalized size = 14.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*
c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32
*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c
 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 -
 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c
)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4
*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*d + (sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^
2 + 16*a^3*b^2*c^2 + 2*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*c^3 - 32*a^4*c^3 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2 -
2*(b^2 - 4*a*c)*a^2*b*c^2)*f - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2
 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a
^3*b*c^3 - 12*a^2*b^2*c^3 + 16*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)
*a*b^2*c^2 + 4*(b^2 - 4*a*c)*a^2*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b +
 sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c +
 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*((sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c
+ 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b
^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 - 2
```

```
4*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5
 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 12*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10
*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*c^
3 - 6*(b^2 - 4*a*c)*a*b*c^3)*d + (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 16*a^3*b
^2*c^2 - 2*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3
+ 32*a^4*c^3 + 8*a^3*b*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2 + 2*(b^2 - 4*a
*c)*a^2*b*c^2)*f - (sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 2*a*b^4*c
^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 + 1
2*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3
*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^
2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2
 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 -
 4*(b^2 - 4*a*c)*a^2*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b - sqrt(a^4*b^
2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2
+ 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/3*(3*b*d*x^2 - 3*a*x^2
*e - a*d)/(a^2*x^3)
```

maple [B] time = 0.03, size = 727, normalized size = 2.72

$$\frac{\sqrt{2}\, bce \operatorname{arctanh}\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, bce \arctan\left(\frac{\sqrt{2}\, cx}{\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}}\right)}{2\sqrt{-4ac+b^2}\,\sqrt{\left(b+\sqrt{-4ac+b^2}\right)c}\; a}+\frac{\sqrt{2}\, c^2 d \operatorname{arctanh}\left(\frac{\sqrt{2}}{\sqrt{\left(-b+\sqrt{-}\right.}}\right.}{\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4a}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/3*d/a/x^3-1/a/x*e+1/a^2/x*b*d+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/a^2*c*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*b*d-c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f+1/2/
a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/a*c^2/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*d-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
```

```
*b^2*d-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-c/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b*e+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-1/2/a^2*c/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((a*b*e - a^2*f - (b*c*d - a*c*e)*x^2 - (b^2 - a*c)*d)/(c*x^4 + b
*x^2 + a), x)/a^2 + 1/3*(3*(b*d - a*e)*x^2 - a*d)/(a^2*x^3)
```

mupad [B] time = 4.76, size = 15505, normalized size = 58.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^
2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*
e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 +
 a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(
4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e
^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2
+ a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3
*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b
^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2
)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*
b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(
4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4
 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-
(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a
^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a
^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^
3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d
*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4
*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*
e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b
^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^
3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2
*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3
)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) +
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a
^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*
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(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c
^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)
^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*
a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i
+ (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 -
8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e -
4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2
*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*
c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 -
 a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^
2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4
*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c
*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)
^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7
*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f
^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b
*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^
2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e -
 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16
*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c
 - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f
- 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d
*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16*a^11*c^3*f
+ 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10*b^2*c^2*f +
 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1
/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7
*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*
a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2
*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b
^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4
*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d
*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*
b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a
*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i)/((x
*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^
7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^
9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2*b^5
*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c -
b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^
2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f
 - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e
 - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/
2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^
2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^
7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7*d^2
 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(
-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2
*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^
```

2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16*a^11*c^3*f + 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10*b^2*c^2*f + 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f - 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 2*a^8*c^4*e^3 - 2*a^6*b*c^5*d^3 + 2*a^7*c^5*d^2*e + 2*a^9*c^3*e*f^2 - 4*a^8*c^4*d*e*f - 4*a^7*b*c^4*d*e^2 + 4*a^7*b*c^4*d^2*f - 2*a^8*b*c^3*d*f^2 - 2*a^8*b*c^3*e^2*f

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+ 2*a^6*b^2*c^4*d^2*e - 2*a^6*b^3*c^3*d^2*f + 4*a^7*b^2*c^3*d*e*f))*(-(b^7*
d^2 + a^2*b^5*e^2 + b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 + a^4*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*
c^2*e^2 - a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2
*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^2*c^2*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e -
2*a^3*b^4*e*f - 16*a^5*c^2*e*f - 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*
a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f - 2*a^3*b*e*f*(-(4*a*c
- b^2)^3)^(1/2) - 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f -
 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e + 2*a^2*b^2*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i - (d/(3*a) + (x^2*(a*e - b*d))
/a^2)/x^3 + atan(((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^
6*b^4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*
a^8*b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) -
(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 -
 a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12
*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*
b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c -
 b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3
*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-
(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*
c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2
*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))
/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10
*b^3*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4
*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f +
16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^
3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 1
2*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d
*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)
^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d
+ 16*a^11*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*
a^10*b^2*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*
a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a
^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2
)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b
*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f +
2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f
+ 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(
4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^
2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)
))^(1/2)*1i + (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^
4*c^3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*
b*c^4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b
^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4
*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4
*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*
c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2
)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e
 - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) +
16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a
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*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*
f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2
*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3
*c^2)*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3
*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^
2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 2
5*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a
^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3
)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*
e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^
4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e -
 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^
(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16
*a^11*c^3*f + 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10
*b^2*c^2*f + 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b
*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f
^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*
b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24
*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*
c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*
c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(
1/2)*1i)/((x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^
3*d^2 - 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^
4*d*e - 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d
^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c
^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*
d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2
*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a
^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c -
 b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f +
3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f
*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5
*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2
)*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2
 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 +
12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^
2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c
 - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c
^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1
/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*
(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^
2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a
^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) - 16*a^10*c^4*d + 16*a^1
1*c^3*f - 4*a^8*b^4*c^2*d + 20*a^9*b^2*c^3*d + 4*a^9*b^3*c^2*e - 4*a^10*b^2
*c^2*f - 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^
2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3
*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/
2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 +
 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*
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d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4
*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c -
b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*
e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)
 - (x*(4*a^8*c^5*d^2 - 4*a^9*c^4*e^2 + 4*a^10*c^3*f^2 + 2*a^6*b^4*c^3*d^2 -
 8*a^7*b^2*c^4*d^2 + 2*a^8*b^2*c^3*e^2 - 8*a^9*c^4*d*f + 12*a^8*b*c^4*d*e -
 4*a^9*b*c^3*e*f - 4*a^7*b^3*c^3*d*e + 4*a^8*b^2*c^3*d*f) - (-(b^7*d^2 + a^
2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a
*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2
+ a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a
^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^
4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*
c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3
)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a
*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 +
16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(b^
7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*f^2 - a^4*
f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*
b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c
^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)
^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^4*c^3*d*e
- 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 1
6*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e*f*(-(4*a*
c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4*b^2*c*e*f
 + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 2*a^2*b^2*
d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(
a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*a^10*c^4*d - 16*a^11*c^3*f
 + 4*a^8*b^4*c^2*d - 20*a^9*b^2*c^3*d - 4*a^9*b^3*c^2*e + 4*a^10*b^2*c^2*f
+ 16*a^10*b*c^3*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(
1/2) + a^4*b^3*f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 -
7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2
*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^
2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*
b^5*d*f + 16*a^4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*
d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^4*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3
*b^2*c^2*d*e - 2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 2*a^
8*c^4*e^3 - 2*a^6*b*c^5*d^3 + 2*a^7*c^5*d^2*e + 2*a^9*c^3*e*f^2 - 4*a^8*c^4
*d*e*f - 4*a^7*b*c^4*d*e^2 + 4*a^7*b*c^4*d^2*f - 2*a^8*b*c^3*d*f^2 - 2*a^8*
b*c^3*e^2*f + 2*a^6*b^2*c^4*d^2*e - 2*a^6*b^3*c^3*d^2*f + 4*a^7*b^2*c^3*d*e
*f))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^3*
f^2 - a^4*f^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2
 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25
*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 - 4*a^5*b*c*f^2 + 2*a^2*b^5*d*f + 16*a^
4*c^3*d*e - 2*a^3*b^4*e*f - 16*a^5*c^2*e*f + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 16*a^2*b^4*c*d*e - 14*a^3*b^3*c*d*f + 24*a^4*b*c^2*d*f + 2*a^3*b*e
*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^3*c*d*f*(-(4*a*c - b^2)^3)^(1/2) + 12*a^4
*b^2*c*e*f + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e -
2*a^2*b^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

3.60 $\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=329

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}+\frac{bd-ae}{3a^2x^3}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/5*d/a/x^5+1/3*(-a*e+b*d)/a^2/x^3+(-b^2*d+a*b*e+a*(-a*f+c*d))/a^3/x-1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*e-
a*(-a*f+c*d)+(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))
/a^3*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(
-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b^2*d-a*b*e-a*(-a*f+c*d)+(-b^3*d+a*b^2*e
-2*a^2*c*e+a*b*(-a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a^3*2^(1/2)/(b+(-4*a*c+b^2
)^(1/2))^(1/2)
```

Rubi [A] time = 1.94, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1664, 1166, 205}

$$-\frac{-abe-a(cd-af)+b^2d}{a^3x}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2a^2ce-ab^2e-ab(3cd-af)+b^3d}{\sqrt{b^2-4ac}}-abe-a(cd-af)+b^2d\right)}{\sqrt{2}a^3\sqrt{b-\sqrt{b^2-4ac}}}-\sqrt{c}\tan\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -d/(5*a*x^5) + (b*d - a*e)/(3*a^2*x^3) - (b^2*d - a*b*e - a*(c*d - a*f))/(a
^3*x) - (Sqrt[c]*(b^2*d - a*b*e - a*(c*d - a*f) + (b^3*d - a*b^2*e + 2*a^2*
c*e - a*b*(3*c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt
[c]*(b^2*d - a*b*e - a*(c*d - a*f) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*
c*d - a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/(Sqrt[2]*a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^6\left(a+bx^2+cx^4\right)}\,dx = \int\left(\frac{d}{ax^6}+\frac{-bd+ae}{a^2x^4}+\frac{b^2d-abe-a(cd-af)}{a^3x^2}+\frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-}{a^3\left(a+bx^2+\right.}\right.$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}+\frac{\int\frac{-b^3d+ab^2e-a^2ce+ab(2cd-af)-c\left(b^2d-abe-a(cd-a\right.}{a+bx^2+cx^4}}{a^3}$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}-\frac{\left(c\left(b^2d-abe-a(cd-af)-\frac{b^3d-ab^2e+2a^2}{\sqrt{b^2-}}\right.\right.}{2a^3}$$

$$= -\frac{d}{5ax^5}+\frac{bd-ae}{3a^2x^3}-\frac{b^2d-abe-a(cd-af)}{a^3x}-\frac{\sqrt{c}\left(b^2d-abe-a(cd-af)+\frac{b^3d-ab^2e+2a}{\sqrt{b}}\right.}{\sqrt{2}\,a^3\sqrt{b-\sqrt{b}}}$$

Mathematica [A] time = 0.55, size = 394, normalized size = 1.20

$$\frac{-\frac{6a^2d}{x^5}+\frac{30\left(abe+a(cd-af)+b^2(-d)\right)}{x}-\frac{15\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab\left(-e\sqrt{b^2-4ac}+af-3cd\right)+a\left(-cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+2ace\right)+b^2\left(d\sqrt{b^2-4a}\right.\right.}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}}{30}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-6*a^2*d)/x^5 + (10*a*(b*d - a*e))/x^3 + (30*(-(b^2*d) + a*b*e + a*(c*d -
 a*f)))/x - (15*Sqrt[2]*Sqrt[c]*(b^3*d + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) +
a*b*(-3*c*d - Sqrt[b^2 - 4*a*c]*e + a*f) + a*(-(c*Sqrt[b^2 - 4*a*c]*d) + 2*
a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (15*Sqrt[2]
*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-3*c*d + Sqrt[b^2
- 4*a*c]*e + a*f) + a*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e - a*Sqrt[b^2 - 4*a*c
]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(30*a^3)
```

fricas [B] time = 38.59, size = 15830, normalized size = 48.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/30*(15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^
3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a
^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a
^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)
*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62
*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 -
 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c
^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3
+ 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5
*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^
6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^
2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5
- 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6
*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c +
```

```
21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*
c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*
a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48
*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^
6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b
^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^
2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4
 - (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c
^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c
^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^
6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d -
 (3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7
*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^
4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 +
 18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^
5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3
 + a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x
 + sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4
*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c^2 -
151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b
^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8
*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 -
5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^
2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^
3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 +
20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b
^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a
^7*b*c^3)*e^2)*f - ((a^7*b^6 - 8*a^8*b^4*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d
 - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*
a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c +
29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(
3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^
4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b
^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a
^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^
6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c +
 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a
^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2
- 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*
e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 -
39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^
4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^
3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*
c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*
c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3
 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 -
 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 2
2*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*
b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 -
 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*
b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^
4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2
 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^
8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*
a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 1
```

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2*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8
*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^
3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e
 + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d
*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*
b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7
- 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c +
9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2
+ (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d -
(a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12
- 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b
^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b
^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c
 + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*
(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e
^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4
 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a
^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*
((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d
^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a
^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 -
 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d
^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*
c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 +
a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)
*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4
*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^
5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*
e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 -
 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c
^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 +
3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b
^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e
^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4
*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2
*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a
^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^
2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10
- 30*a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6
*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 +
 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2
*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4
*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^
6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c +
 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^
4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 2
1*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*c^3)*e^2)*f - ((a^7*b^6 - 8*a^8*b^4
*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^
2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5
*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c
^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^
4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 -
 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2
*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^
9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 1
8*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b
^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7
```

*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c +
22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9
 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3
*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2
- (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 -
 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 -
2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*
b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*
b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f + (a^7*b^
2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +
46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c +
29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(
3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^
4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b
^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a
^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^
6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c +
 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a
^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2
- 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*
e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9
*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 -
39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^
4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^
3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c)))
+ 15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c
^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^
5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^
5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f
 - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*
b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^
2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d
^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*
a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2
- 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4
*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^
4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a
^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*
c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^
7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 -
 a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b
^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*
b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c +
 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c
+ 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4
*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5 + 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b
^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 -
4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 - (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 +
7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2
*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 - 9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a
^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*
b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*
e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a
^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3
 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e + 3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^
4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x + sq
rt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*
c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a
^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c

+ 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*
b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3 + 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7
*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7 - 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 1
6*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)
*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^
6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^
3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*
c^3)*e^2)*f + ((a^7*b^6 - 8*a^8*b^4*c + 18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a
^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e + (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*
c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^
3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2
*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a
^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^
3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^
2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6
 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^
9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^
2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a
^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*
f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*
b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^
6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2
 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2
 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 1
4*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 -
 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*
a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^
4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5
*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c
^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^
4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 -
 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2
*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^
9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 1
8*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b
^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7
*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c +
22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9
 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3
*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2
- (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 -
 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 15*sqrt(1/2)*a^3*x^5*sqrt(-((b^7 - 7*a
*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3
*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + (a^
4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*d - (a^3*
b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10*
a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^
5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^
3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66
*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*
b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 +
(a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a
^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^
2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a
^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 -
2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^

```
6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^
3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 -
(3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*
d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c
^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(
a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(-2*((b^6*c^4 - 5*a*b^4*c^5
+ 6*a^2*b^2*c^6 - a^3*c^7)*d^4 - (b^7*c^3 - 3*a*b^5*c^4 - 2*a^2*b^3*c^5 + 5
*a^3*b*c^6)*d^3*e + 3*(a*b^6*c^3 - 4*a^2*b^4*c^4 + 3*a^3*b^2*c^5)*d^2*e^2 -
 (3*a^2*b^5*c^3 - 11*a^3*b^3*c^4 + 7*a^4*b*c^5)*d*e^3 + (a^3*b^4*c^3 - 3*a^
4*b^2*c^4 + a^5*c^5)*e^4 + (a^6*b^2*c^2 - a^7*c^3)*f^4 + ((3*a^4*b^4*c^2 -
9*a^5*b^2*c^3 + 4*a^6*c^4)*d - (3*a^5*b^3*c^2 - 5*a^6*b*c^3)*e)*f^3 + 3*((a
^2*b^6*c^2 - 5*a^3*b^4*c^3 + 7*a^4*b^2*c^4 - 2*a^5*c^5)*d^2 - (2*a^3*b^5*c^
2 - 7*a^4*b^3*c^3 + 5*a^5*b*c^4)*d*e + (a^4*b^4*c^2 - 2*a^5*b^2*c^3)*e^2)*f
^2 + ((b^8*c^2 - 7*a*b^6*c^3 + 18*a^2*b^4*c^4 - 19*a^3*b^2*c^5 + 4*a^4*c^6)
*d^3 - 3*(a*b^7*c^2 - 5*a^2*b^5*c^3 + 8*a^3*b^3*c^4 - 5*a^4*b*c^5)*d^2*e +
3*(a^2*b^6*c^2 - 3*a^3*b^4*c^3 + a^4*b^2*c^4)*d*e^2 - (a^3*b^5*c^2 - a^4*b^
3*c^3 - 3*a^5*b*c^4)*e^3)*f)*x - sqrt(1/2)*((b^11 - 11*a*b^9*c + 44*a^2*b^7
*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c^5)*d^3 - (3*a*b^10 - 30*
a^2*b^8*c + 105*a^3*b^6*c^2 - 151*a^4*b^4*c^3 + 77*a^5*b^2*c^4 - 4*a^6*c^5)
*d^2*e + (3*a^2*b^9 - 27*a^3*b^7*c + 81*a^4*b^5*c^2 - 92*a^5*b^3*c^3 + 32*a
^6*b*c^4)*d*e^2 - (a^3*b^8 - 8*a^4*b^6*c + 20*a^5*b^4*c^2 - 17*a^6*b^2*c^3
+ 4*a^7*c^4)*e^3 + (a^6*b^5 - 5*a^7*b^3*c + 4*a^8*b*c^2)*f^3 + ((3*a^4*b^7
- 21*a^5*b^5*c + 40*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (3*a^5*b^6 - 18*a^6*b^4
*c + 25*a^7*b^2*c^2 - 4*a^8*c^3)*e)*f^2 + ((3*a^2*b^9 - 27*a^3*b^7*c + 80*a
^4*b^5*c^2 - 85*a^5*b^3*c^3 + 20*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 24*a^4*b^6
*c + 59*a^5*b^4*c^2 - 45*a^6*b^2*c^3 + 4*a^7*c^4)*d*e + (3*a^4*b^7 - 21*a^5
*b^5*c + 41*a^6*b^3*c^2 - 20*a^7*b*c^3)*e^2)*f + ((a^7*b^6 - 8*a^8*b^4*c +
18*a^9*b^2*c^2 - 8*a^10*c^3)*d - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*e
+ (a^9*b^4 - 6*a^10*b^2*c + 8*a^11*c^2)*f)*sqrt(((b^12 - 10*a*b^10*c + 37*a
^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^
4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*
c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 -
72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7
*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^
5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*
b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3
)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5
*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 -
15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*
c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^
4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21
*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*
b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^
5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^
15*c)))*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*
b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c
 + 5*a^4*b*c^2)*e^2 + (a^4*b^3 - 3*a^5*b*c)*f^2 + 2*((a^2*b^5 - 5*a^3*b^3*c
 + 5*a^4*b*c^2)*d - (a^3*b^4 - 4*a^4*b^2*c + 2*a^5*c^2)*e)*f - (a^7*b^2 - 4
*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^
4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^
3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2
*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a
^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^
3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^
2*c^3 + a^8*c^4)*e^4 + (a^8*b^4 - 2*a^9*b^2*c + a^10*c^2)*f^4 + 4*((a^6*b^6
 - 4*a^7*b^4*c + 4*a^8*b^2*c^2 - a^9*c^3)*d - (a^7*b^5 - 3*a^8*b^3*c + 2*a^
9*b*c^2)*e)*f^3 + 2*((3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2 - 19*a^7*b^
2*c^3 + 3*a^8*c^4)*d^2 - 2*(3*a^5*b^7 - 15*a^6*b^5*c + 21*a^7*b^3*c^2 - 7*a
^8*b*c^3)*d*e + (3*a^6*b^6 - 12*a^7*b^4*c + 12*a^8*b^2*c^2 - a^9*c^3)*e^2)*
```

```
f^2 + 4*((a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*
b^2*c^4 - a^7*c^5)*d^3 - (3*a^3*b^9 - 21*a^4*b^7*c + 48*a^5*b^5*c^2 - 39*a^
6*b^3*c^3 + 8*a^7*b*c^4)*d^2*e + (3*a^4*b^8 - 18*a^5*b^6*c + 33*a^6*b^4*c^2
 - 18*a^7*b^2*c^3 + a^8*c^4)*d*e^2 - (a^5*b^7 - 5*a^6*b^5*c + 7*a^7*b^3*c^2
 - 2*a^8*b*c^3)*e^3)*f)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))) - 30*
(a*b*e - a^2*f - (b^2 - a*c)*d)*x^4 + 6*a^2*d - 10*(a*b*d - a^2*e)*x^2)/(a^
3*x^5)
```

giac [B] time = 7.02, size = 6718, normalized size = 20.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(
b^2 - 4*a*c)*a^2*c^4)*a^2*d + (2*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 32*a^4*c^4
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 2*(b^2 - 4*a*c)*a^2*b^2*c^
2 + 8*(b^2 - 4*a*c)*a^3*c^3)*a^2*f - (2*a*b^5*c^2 - 16*a^2*b^3*c^3 + 32*a^3
*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 16*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b
^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^7 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 32*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 12*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c
^2 + 20*a^2*b^5*c^2 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3
- 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 6*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 64*a^3*b^3*c^3 + 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*a^4*b*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c
- 12*(b^2 - 4*a*c)*a^2*b^3*c^2 + 16*(b^2 - 4*a*c)*a^3*b*c^3)*d*abs(a) + 2*(
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b^4*c - 2*a^3*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^2
+ 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 16*a^4*b^3*c^2 - 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 32*a^5*b*c^3 + 2*(b^2 - 4*a*c)*a^3*b^3*c -
 8*(b^2 - 4*a*c)*a^4*b*c^2)*f*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 2*sq
```

```
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*a^2*b^6*c + 24*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + 10*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
*c^2 + 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 5*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 48*a^4*b^2*c^3 + 4*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 32*a^5*c^4 + 2*(b^2 - 4*a*c)*a^2*b^4*c - 10*
(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 - 4*a*c)*a^4*c^3)*abs(a)*e + (2*a^2*b^6*
c^2 - 14*a^3*b^4*c^3 + 24*a^4*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b^6 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^4*c^2 + 6*(b^2 - 4*a*c)*a^3*b^2
*c^3)*d + (2*a^4*b^4*c^2 - 8*a^5*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^2 - 2*(b^2 - 4*a*c)*a^4*b^2*c^2)*f - (2*a^3*b^5*c^2
- 12*a^4*b^3*c^3 + 16*a^5*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^5*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^4*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b*c^3 - 2*(b^2 - 4*a*c)*a^3*b^3*c^2 + 4*(b^2 - 4*a*c)*a^4*b*c^3)*e)*arct
an(2*sqrt(1/2)*x/sqrt((a^3*b + sqrt(a^6*b^2 - 4*a^7*c))/(a^3*c)))/((a^5*b^4
 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b*c^2 + a^5*b^2*c^2 - 4*a
^6*c^3)*abs(a)*abs(c)) + 1/8*((2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 -
32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6
+ 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2
- 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*a^2*d + (2*a^2*b^4*c^2 - 16*a
^3*b^2*c^3 + 32*a^4*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 2*
(b^2 - 4*a*c)*a^2*b^2*c^2 + 8*(b^2 - 4*a*c)*a^3*c^3)*a^2*f - (2*a*b^5*c^2 -
 16*a^2*b^3*c^3 + 32*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c
^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^3)*a^2*e - 2*(sqrt
```

```
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c +
 2*a*b^7*c + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 12*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^5*c^2 - 20*a^2*b^5*c^2 - 32*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^4*b*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
2*c^3 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 64*a^3*b^3*
c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 64*a^4*b*c^4 -
2*(b^2 - 4*a*c)*a*b^5*c + 12*(b^2 - 4*a*c)*a^2*b^3*c^2 - 16*(b^2 - 4*a*c)*a
^3*b*c^3)*d*abs(a) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5 - 8
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c - 2*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^4*c + 2*a^3*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^5*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 16*a^4*b^3*c^2
 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 32*a^5*b*c^3 - 2*(
b^2 - 4*a*c)*a^3*b^3*c + 8*(b^2 - 4*a*c)*a^4*b*c^2)*f*abs(a) + 2*(sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c + 2
*a^2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 + 10*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 18*a^3*b^4*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c
^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 48*a^4*b^2*c^3
 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^4 - 2*(b^2
- 4*a*c)*a^2*b^4*c + 10*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3
)*abs(a)*e + (2*a^2*b^6*c^2 - 14*a^3*b^4*c^3 + 24*a^4*b^2*c^4 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6 + 7*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 12*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 2*(b^2 - 4*a*c)*a^2*b^4*c^2 +
 6*(b^2 - 4*a*c)*a^3*b^2*c^3)*d + (2*a^4*b^4*c^2 - 8*a^5*b^2*c^3 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4 + 4*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*(b^2 - 4*a*c)*a^4*b^2*
c^2)*f - (2*a^3*b^5*c^2 - 12*a^4*b^3*c^3 + 16*a^5*b*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5 + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 2*(b^2 - 4*a*c)*a^3*b^3*c^2 + 4*(b^2 - 4
*a*c)*a^4*b*c^3)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b - sqrt(a^6*b^2 - 4*a^7
*c))/(a^3*c)))/((a^5*b^4 - 8*a^6*b^2*c - 2*a^5*b^3*c + 16*a^7*c^2 + 8*a^6*b
*c^2 + a^5*b^2*c^2 - 4*a^6*c^3)*abs(a)*abs(c)) - 1/15*(15*b^2*d*x^4 - 15*a*
c*d*x^4 + 15*a^2*f*x^4 - 15*a*b*x^4*e - 5*a*b*d*x^2 + 5*a^2*x^2*e + 3*a^2*d
)/(a^3*x^5)
```

maple [B] time = 0.03, size = 1121, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x)
```

```
[Out] -1/5*d/a/x^5+1/3/a^2/x^3*b*d+1/a^2/x*b*e+1/a^2/x*c*d-1/a^3/x*b^2*d+1/2/a^3*
```

```
c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*e+1/2/a^2*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/2/a^3*c*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1
/a*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/2/a^2*c*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b*e-1/3/a/x^3*e-1/a/x*f+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
*f+1/2/a^3*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-3/2/a^2*c^2/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-1/2/a*c*2^(1/2)/((b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f+1/2/
a^2*c^2*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/2/a*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f-1/2/a^2*c^2*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*d-3/2/a^2*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d
+1/2/a^3*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-1/2/a^2*c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e-1/2/a^2*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b^2*e+1/2/a*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*b*f - (a*b*c*e - a^2*c*f - (b^2*c - a*c^2)*d)*x^2 + (b^3 -
2*a*b*c)*d - (a*b^2 - a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/a^3 + 1/15*(15*(a*b
*e - a^2*f - (b^2 - a*c)*d)*x^4 - 3*a^2*d + 5*(a*b*d - a^2*e)*x^2)/(a^3*x^5
)
```

mupad [B] time = 6.25, size = 23019, normalized size = 69.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^6*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*
d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^
12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e +
12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c
^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e
^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*
a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5
*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2)
 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f -
```

```
16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^
2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a
^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b
^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a
*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^
2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c -
 b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a
^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c
^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f
^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^
2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 4
2*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/
2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^
2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^
7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f -
 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f
 - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76
*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*
d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2
)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
+ 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a
^15*c^4*e + 4*a^12*b^5*c^2*d - 24*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^
14*b^2*c^3*e + 4*a^14*b^3*c^2*f + 32*a^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^
9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a
^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^
6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5
*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*
c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*
c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 +
 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*
d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5
*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*
c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a
^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2
) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b
*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2
)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i + (x*(4*a^13
*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^
4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 +
 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*
f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12
*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(
4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 -
20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a
^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e -
 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20
*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6
*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3
)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c
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- b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/
2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^
2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) +
 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)
^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*
c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 +
 a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^
4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2
*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^
2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2
*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2
)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*
b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*
d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b
^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e
 + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*
e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*
a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*
(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3
*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1
/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^
12*b^5*c^2*d + 24*a^13*b^3*c^3*d + 4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4
*a^14*b^3*c^2*f - 32*a^14*b*c^4*d + 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*
e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9
*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^
5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^
3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2
) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f -
 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b
^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*
a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*
b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*
a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c
^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c
- b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(
a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i)/((x*(4*a^13*c^5*e^2 - 4*a^1
2*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a
^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*
f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c
^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4
*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^
(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2
 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*
a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2
*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3
)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f +
 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e -
 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3
*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2)
+ 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d
*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c
 - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^
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3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*
c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b
^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^
5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c
^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4
*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5
*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^
(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4
*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f
*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b
^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f
 - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^
3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*
a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4
 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a^15*c^4*e + 4*a^12*b^5*c^2*d - 2
4*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2*c^3*e + 4*a^14*b^3*c^2*f
+ 32*a^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-
(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 -
 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c
 - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 +
a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e
- 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 2
0*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f +
6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c
 - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1
/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b
^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2)
+ 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2
)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9
*c^2 - 8*a^8*b^2*c)))^(1/2) - (x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*
c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2
*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*
f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^
3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f)
- (-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2
 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2
+ 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*
a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*
(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*
c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2
*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f -
 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a
^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*
f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^
3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) +
4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)
^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*
c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a
^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b
^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*
c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)
```

^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^
2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*
b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^
6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*
c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*
c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) -
66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)
^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^
4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^
3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a
^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^12*b^5*c^2*d + 24*a^13*b^3*c^3*d +
4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*b^3*c^2*f - 32*a^14*b*c^4*d +
16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1
/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 -
7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) -
2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*
c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e
^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 1
6*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 1
8*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b
^4*c^2*d*e + 76*a^4*b^2*c^3*d*e + 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) +
50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c -
b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(
-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*
b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)
))^(1/2) - 2*a^10*c^7*d^3 + 2*a^13*c^4*f^3 - 2*a^11*b*c^5*e^3 - 2*a^11*c^6*
d*e^2 + 6*a^11*c^6*d^2*f - 6*a^12*c^5*d*f^2 + 2*a^12*c^5*e^2*f + 2*a^9*b^2*
c^6*d^3 - 4*a^12*b*c^4*e*f^2 - 2*a^9*b^3*c^5*d^2*e + 4*a^10*b^2*c^5*d*e^2 +
2*a^9*b^4*c^4*d^2*f - 6*a^10*b^2*c^5*d^2*f + 4*a^11*b^2*c^4*d*f^2 + 2*a^11
*b^2*c^4*e^2*f + 4*a^11*b*c^5*d*e*f - 4*a^10*b^3*c^4*d*e*f))*(-(b^9*d^2 + a
^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*
d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f
^2 - a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2
- 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(
-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^
3)^(1/2) + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^
7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f - 2*a*b^5*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*
f + 16*a^4*b^4*c*e*f + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4
*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e +
2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f - 2*a^3*b^3*e*
f*(-(4*a*c - b^2)^3)^(1/2) + 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^
5*b^2*c^2*e*f - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 4*a^4*b*c*e*f*(-
(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b
*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2
))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*2i - (d/(5*a) + (x^4*(b^
2*d + a^2*f - a*b*e - a*c*d))/a^3 + (x^2*(a*e - b*d))/(3*a^2))/x^5 + atan((
(x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 -
12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2
*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^1
3*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f
- 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 - b
^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^

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5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c
^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)
^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4
*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5
*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^
(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4
*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f
*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b
^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f
 + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^
3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4
 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-
(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 2
8*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12
*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*
b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a
^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4
*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^
2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b
^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*
a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/
2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b
^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f +
2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(
1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^
4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/
2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c -
b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a^15*c^
4*e + 4*a^12*b^5*c^2*d - 24*a^13*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2
*c^3*e + 4*a^14*b^3*c^2*f + 32*a^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^9*d^2
+ a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c
^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^
2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d
^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^
2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2
*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-
(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3
*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*
b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*
e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3
*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36
*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f
*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^
3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(
1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i + (x*(4*a^13*c^5*e
^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*
d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^1
3*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*
a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c
^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c
- b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5
*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4
*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a
^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*
```

c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3
*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b
^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b
^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2
) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)
^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2
*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^
2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2
*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1
/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 -
8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b
^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2
- 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 +
a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63
*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*
a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(
1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*
f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c
- b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f +
16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d
^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a
^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-
(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^
2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a
*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^12*b^5
*c^2*d + 24*a^13*b^3*c^3*d + 4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*
b^3*c^2*f - 32*a^14*b*c^4*d + 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 -
b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b
^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^
2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*
c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2
)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^
4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^
5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)
^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^
4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a
*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*
f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c -
b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f
+ 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)
^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4
*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^
4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*1i)/((x*(4*a^13*c^5*e^2 - 4*a^12*c^6*
d^2 - 4*a^14*c^4*f^2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^
2*c^5*d^2 + 2*a^11*b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 +
8*a^13*c^5*d*f - 20*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e
+ 20*a^11*b^3*c^4*d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*
b^3*c^3*e*f) - (-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a
^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*
b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c -
b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 -
a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2
) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^
6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^
3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*
a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c

```
^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a
^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(
4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2
)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*
a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*
c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(
1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2
*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^
2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*
a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2
 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d
*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2)
+ 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f
 - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^
2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*
a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)
^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^
3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/
2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c -
b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*
a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 16*a^15*c^4*e + 4*a^12*b^5*c^2*d - 24*a^13
*b^3*c^3*d - 4*a^13*b^4*c^2*e + 20*a^14*b^2*c^3*e + 4*a^14*b^3*c^2*f + 32*a
^14*b*c^4*d - 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c
 - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^
5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2
)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^
4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*
a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a
*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^
3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*
b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*
b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/
2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2
)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) -
2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e
^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^
2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(
1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 -
 8*a^8*b^2*c)))^(1/2) - (x*(4*a^13*c^5*e^2 - 4*a^12*c^6*d^2 - 4*a^14*c^4*f^
2 + 2*a^9*b^6*c^3*d^2 - 12*a^10*b^4*c^4*d^2 + 18*a^11*b^2*c^5*d^2 + 2*a^11*
b^4*c^3*e^2 - 8*a^12*b^2*c^4*e^2 + 2*a^13*b^2*c^3*f^2 + 8*a^13*c^5*d*f - 20
*a^12*b*c^5*d*e + 12*a^13*b*c^4*e*f - 4*a^10*b^5*c^3*d*e + 20*a^11*b^3*c^4*
d*e + 4*a^11*b^4*c^3*d*f - 16*a^12*b^2*c^4*d*f - 4*a^12*b^3*c^3*e*f) - (-(b
^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*
a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a
^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^
5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a
*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2
+ 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5
*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^
5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
 + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2
*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*
a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/
2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*
b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
```

+ 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*(x*(32*a^16*b*c^3 - 8*a^15*b^3*c^2)*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) + 16*a^15*c^4*e - 4*a^12*b^5*c^2*d + 24*a^13*b^3*c^3*d + 4*a^13*b^4*c^2*e - 20*a^14*b^2*c^3*e - 4*a^14*b^3*c^2*f - 32*a^14*b*c^4*d + 16*a^15*b*c^3*f))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2) - 2*a^10*c^7*d^3 + 2*a^13*c^4*f^3 - 2*a^11*b*c^5*e^3 - 2*a^11*c^6*d*e^2 + 6*a^11*c^6*d^2*f - 6*a^12*c^5*d*f^2 + 2*a^12*c^5*e^2*f + 2*a^9*b^2*c^6*d^3 - 4*a^12*b*c^4*e*f^2 - 2*a^9*b^3*c^5*d^2*e + 4*a^10*b^2*c^5*d*e^2 + 2*a^9*b^4*c^4*d^2*f - 6*a^10*b^2*c^5*d^2*f + 4*a^11*b^2*c^4*d*f^2 + 2*a^11*b^2*c^4*e^2*f + 4*a^11*b*c^5*d*e*f - 4*a^10*b^3*c^4*d*e*f))*(-(b^9*d^2 + a^2*b^7*e^2 - b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + a^4*b^5*f^2 + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 7*a^5*b^3*c*f^2 + 12*a^6*b*c^2*f^2 + a^5*c*f^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 - a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) + a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^3)^(1/2) - a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 + 2*a^2*b^7*d*f - 16*a^5*c^4*d*e - 2*a^3*b^6*e*f + 16*a^6*c^3*e*f + 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e - 18*a^3*b^5*c*d*f - 40*a^5*b*c^3*d*f + 16*a^4*b^4*c*e*f - 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 2*a^2*b^4*d*f*(-(4*a*c - b^2)^3)^(1/2) + 50*a^4*b^3*c^2*d*f + 2*a^3*b^3*e*f*(-(4*a*c - b^2)^3)^(1/2) - 2*a^4*c^2*d*f*(-(4*a*c - b^2)^3)^(1/2) - 36*a^5*b^2*c^2*e*f + 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^4*b*c*e*f*(-(4*a*c - b^2)^3)^(1/2) - 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^3*b^2*c*d*f*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4 + 16*a^9*c^2 - 8*a^8*b^2*c)))^(1/2)*2i

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**6/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.61 $\int \frac{x^7(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=320

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-b^3c(cd-20af)-12ab^2c^2e+6abc^2(cd-5af)-3b^5f+2b^4ce\right)}{2c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(-2c(4af+b\right.}{4c^2\left(b\right.}$$

```
[Out] 1/2*(2*b^2*c*e-6*a*c^2*e-3*b^3*f-b*c*(-11*a*f+c*d))*x^2/c^3/(-4*a*c+b^2)+1/
4*(4*c^2*d+3*b^2*f-2*c*(4*a*f+b*e))*x^4/c^2/(-4*a*c+b^2)+1/2*x^6*(2*a*c*e-b
*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)-1/2*(2*b^4*c*e-12*a*b^2*c^2*e+12*a^2*c^3*e-3*b^5*f-b^3*c*(-20*a*f+c*d)+6
*a*b*c^2*(-5*a*f+c*d))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^4/(-4*a*c+
b^2)^(3/2)+1/4*(c^2*d+3*b^2*f-2*c*(a*f+b*e))*ln(c*x^4+b*x^2+a)/c^4
```

Rubi [A] time = 1.23, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1644, 800, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^3e-12ab^2c^2e-b^3c(cd-20af)+6abc^2(cd-5af)+2b^4ce-3b^5f\right)}{2c^4\left(b^2-4ac\right)^{3/2}}+\frac{x^6\left(x^2\left(-\left(-2ac\right.\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*c*e - 6*a*c^2*e - 3*b^3*f - b*c*(c*d - 11*a*f))*x^2)/(2*c^3*(b^2 -
4*a*c)) + ((4*c^2*d + 3*b^2*f - 2*c*(b*e + 4*a*f))*x^4)/(4*c^2*(b^2 - 4*a*c
)) + (x^6*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^
2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*b^4*c*e - 12*a*b^2*c^2*e
+ 12*a^2*c^3*e - 3*b^5*f - b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(c*d - 5*a*f))*
ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^2 - 4*a*c)^(3/2)) + ((c
^2*d + 3*b^2*f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{x^3\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^6\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{x^2\left(3\left(2ae-\frac{b(cd+af)}{c}\right)\cdots}{a+\cdots}\right.}{2\left(b^2\cdots\right.}$$

$$= \frac{x^6\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int\left(-\frac{2b^2ce-6ac^2e-3b^3\cdots}{c^3\cdots}\right.\right.}{\cdots}$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \cdots$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \cdots$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \cdots$$

$$= \frac{\left(2b^2ce-6ac^2e-3b^3f-bc(cd-11af)\right)x^2}{2c^3\left(b^2-4ac\right)} + \frac{\left(4c^2d+3b^2f-2c(be+4af)\right)x^4}{4c^2\left(b^2-4ac\right)} + \cdots$$

Mathematica [A] time = 0.50, size = 309, normalized size = 0.97

$$\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-12a^2c^3e+b^3c(cd-20af)+12ab^2c^2e+6abc^2(5af-cd)+3b^5f-2b^4ce\right)}{\left(4ac-b^2\right)^{3/2}} + \frac{2\left(2a^3c^2f+a^2c\left(-4b^2f+bc\left(3e+5fx^2\right)-2c^2\left(d+ex^2\right)\right)+ab\left(b^3\cdots\right.\right.}{\left(b^2-4ac\right)\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*(c*e - 2*b*f)*x^2 + c^2*f*x^4 + (2*(2*a^3*c^2*f + b^3*(c^2*d - b*c*e +
 b^2*f)*x^2 + a*b*(b^3*f - 3*c^3*d*x^2 + b*c^2*(d + 4*e*x^2) - b^2*c*(e + 5
*f*x^2)) + a^2*c*(-4*b^2*f - 2*c^2*(d + e*x^2) + b*c*(3*e + 5*f*x^2))))/((b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(-2*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2
*c^3*e + 3*b^5*f + b^3*c*(c*d - 20*a*f) + 6*a*b*c^2*(-(c*d) + 5*a*f))*ArcTa
n[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c^2*d + 3*b^2*
f - 2*c*(b*e + a*f))*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

fricas [B] time = 1.66, size = 2111, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^
4 + 16*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b
^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b
^2*c^3 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d -
```

```
(b^6*c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c
+ 41*a^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 - (((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^
4*c^2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3
)*f)*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^
3)*e + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*
c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*
c + 30*a^3*b*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(a*b^4*c^2
 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c - 7*a^2*b^3*c^2 + 12*a^3*b*c^3
)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^2 - 8*a^4*c^3)*f + (((b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e +
 (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*f)*x^4 + ((b^5*c^2
- 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c - 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*
e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*f)*x^2 + (a*b^4*c^
2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*f)*log(c*x^4
 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6 + (b^4*c^5 - 8*a*b^2
*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*x^2), 1/4*(
(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*f*x^8 + (2*(b^4*c^3 - 8*a*b^2*c^4 + 16
*a^2*c^5)*e - 3*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*f)*x^6 + (2*(b^5*c^2
 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*e - (4*b^6*c - 33*a*b^4*c^2 + 72*a^2*b^2*c^3
 - 16*a^3*c^4)*f)*x^4 + 2*((b^5*c^2 - 7*a*b^3*c^3 + 12*a^2*b*c^4)*d - (b^6*
c - 9*a*b^4*c^2 + 26*a^2*b^2*c^3 - 24*a^3*c^4)*e + (b^7 - 11*a*b^5*c + 41*a
^2*b^3*c^2 - 52*a^3*b*c^3)*f)*x^2 + 2*(((b^3*c^3 - 6*a*b*c^4)*d - 2*(b^4*c^
2 - 6*a*b^2*c^3 + 6*a^2*c^4)*e + (3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*f)
*x^4 + ((b^4*c^2 - 6*a*b^2*c^3)*d - 2*(b^5*c - 6*a*b^3*c^2 + 6*a^2*b*c^3)*e
 + (3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*f)*x^2 + (a*b^3*c^2 - 6*a^2*b*c^3)
*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e + (3*a*b^5 - 20*a^2*b^3*c +
30*a^3*b*c^2)*f)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c
)/(b^2 - 4*a*c)) + 2*(a*b^4*c^2 - 6*a^2*b^2*c^3 + 8*a^3*c^4)*d - 2*(a*b^5*c
 - 7*a^2*b^3*c^2 + 12*a^3*b*c^3)*e + 2*(a*b^6 - 8*a^2*b^4*c + 18*a^3*b^2*c^
2 - 8*a^4*c^3)*f + (((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*d - 2*(b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4)*e + (3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 -
32*a^3*c^4)*f)*x^4 + ((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*d - 2*(b^6*c -
 8*a*b^4*c^2 + 16*a^2*b^2*c^3)*e + (3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 3
2*a^3*b*c^3)*f)*x^2 + (a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*d - 2*(a*b^5
*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2
*c^2 - 32*a^4*c^3)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^4 - 8*a^2*b^2*c^5 +
16*a^3*c^6 + (b^4*c^5 - 8*a*b^2*c^6 + 16*a^2*c^7)*x^4 + (b^5*c^4 - 8*a*b^3*
c^5 + 16*a^2*b*c^6)*x^2)]
```

giac [A] time = 1.95, size = 424, normalized size = 1.32

$$-\frac{\left(b^3c^2d - 6\,abc^3d + 3\,b^5f - 20\,ab^3cf + 30\,a^2bc^2f - 2\,b^4ce + 12\,ab^2c^2e - 12\,a^2c^3e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(b^2c^4 - 4\,ac^5\right)\sqrt{-b^2+4\,ac}} - \frac{b^2c^3dx^4 - }{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*c^2*d - 6*a*b*c^3*d + 3*b^5*f - 20*a*b^3*c*f + 30*a^2*b*c^2*f - 2
*b^4*c*e + 12*a*b^2*c^2*e - 12*a^2*c^3*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((b^2*c^4 - 4*a*c^5)*sqrt(-b^2 + 4*a*c)) - 1/4*(b^2*c^3*d*x^4 - 4*a
*c^4*d*x^4 + 3*b^4*c*f*x^4 - 14*a*b^2*c^2*f*x^4 + 8*a^2*c^3*f*x^4 - 2*b^3*c
^2*x^4*e + 8*a*b*c^3*x^4*e - b^3*c^2*d*x^2 + 2*a*b*c^3*d*x^2 + b^5*f*x^2 -
4*a*b^3*c*f*x^2 - 2*a^2*b*c^2*f*x^2 + 4*a^2*c^3*x^2*e - a*b^2*c^2*d + a*b^4
*f - 6*a^2*b^2*c*f + 4*a^3*c^2*f + 2*a^2*b*c^2*e)/((b^2*c^4 - 4*a*c^5)*(c*x
^4 + b*x^2 + a)) + 1/4*(c^2*d + 3*b^2*f - 2*a*c*f - 2*b*c*e)*log(c*x^4 + b*
x^2 + a)/c^4 + 1/4*(c^2*f*x^4 - 4*b*c*f*x^2 + 2*c^2*x^2*e)/c^4
```

maple [B] time = 0.02, size = 1167, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^2*e+3/2/c/(c*x^4+b*x^2+a)/(4*a*c
-b^2)*x^2*a*b*d+5/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b^3*f-5/2/c^2/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*b*f-1/c^3*x^2*b*f-1/c^2/(c*x^4+b*x^2+a)*a^
3/(4*a*c-b^2)*f+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*d+15/c^2/(4*a*c-b^2)^(3
/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*b*f-10/c^3/(4*a*c-b^2)^(3/2)*
arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^3*f+6/c^2/(4*a*c-b^2)^(3/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b^2*e-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*a*b*d+7/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b^2
*f-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b*e-1/2/c^4/(c*x^4+b*x^2+a)/(4*a*c
-b^2)*x^2*b^5*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^3*d-3/2/c^2/(c*x^
4+b*x^2+a)*a^2/(4*a*c-b^2)*b*e-1/2/c^4/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^4*f-
1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*d+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*x^2*b^4*e+2/c^3/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b^2*f+1/4/c^2*x^4*f+1/
2/c^2*x^2*e+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*e+1/c/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*x^2*a^2*e-1/c^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^4*e+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*e-2/c^2/(4*a*c-b^
2)*ln(c*x^4+b*x^2+a)*a^2*f+1/c/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*d-3/4/c^4/(4
*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^4*f-1/4/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2
*d-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a^2*e+3/2/c^
4/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*f+1/2/c^2/(4*
a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.33, size = 3499, normalized size = 10.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x^2*(e/(2*c^2) - (b*f)/c^3) - ((2*a^3*c^2*f - 2*a^2*c^3*d + a*b^4*f - a*b^3
*c*e + a*b^2*c^2*d + 3*a^2*b*c^2*e - 4*a^2*b^2*c*f)/(2*c*(4*a*c - b^2)) + (
x^2*(b^5*f - 2*a^2*c^3*e + b^3*c^2*d - b^4*c*e - 3*a*b*c^3*d - 5*a*b^3*c*f
+ 4*a*b^2*c^2*e + 5*a^2*b*c^2*f))/(2*c*(4*a*c - b^2)))/(a*c^3 + c^4*x^4 + b
*c^3*x^2) - (log(a + b*x^2 + c*x^4)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d
+ 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^
2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*
c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*
a^2*b^2*c^6)) + (f*x^4)/(4*c^2) + (atan(((8*a*c^7*(4*a*c - b^2)^3 - 2*b^2*c
^6*(4*a*c - b^2)^3)*(((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2
*c^4*f)/c^6 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4
```

```
*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f -
 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a
^3*b*c^4*e))/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6))*(3
*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f
+ 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) - (a*(3*b^5
*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12
*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256
*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4
*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e
 + 256*a^3*b*c^4*e))/(c^2*(4*a*c - b^2)^(3/2)*(256*a^3*c^7 - 4*b^6*c^4 + 48
*a*b^4*c^5 - 192*a^2*b^2*c^6)))/(a*(4*a*c - b^2)) - x^2*(((((24*a^2*c^7*e -
 6*b^3*c^6*d + 12*b^4*c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e
+ 96*a*b^3*c^5*f - 92*a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a
*b*c^9)*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e
+ 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^
3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2
*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*
c^6)))*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a
*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f))/(8*c^4*(4*a*c - b^2)^(3/2)) -
((8*b^3*c^8 - 32*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e -
 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c^2*f)*(6*b^8*f - 1
28*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d -
 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f -
 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(16*c^4*(4*a*c - b^2)^
(3/2)*(4*a*c^7 - b^2*c^6)*(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2
*b^2*c^6)))/(a*(4*a*c - b^2)) + (b*((((24*a^2*c^7*e - 6*b^3*c^6*d + 12*b^4*
c^5*e - 18*b^5*c^4*f + 28*a*b*c^7*d - 56*a*b^2*c^6*e + 96*a*b^3*c^5*f - 92*
a^2*b*c^6*f)/(4*a*c^7 - b^2*c^6) - ((8*b^3*c^8 - 32*a*b*c^9)*(6*b^8*f - 128
*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 1
92*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 2
4*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(4*a*c^7 - b^2*c^6)*(
256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)))*(6*b^8*f - 128*
a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d - 19
2*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f - 24
*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6*c
^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (9*b^7*f^2 + b^3*c^4*d^2 + 4*b^5*c^
2*e^2 - 20*a*b^3*c^3*e^2 + 12*a^2*b*c^4*e^2 - 38*a^3*b*c^3*f^2 - 12*b^6*c*e
*f + 91*a^2*b^3*c^2*f^2 - 5*a*b*c^5*d^2 - 57*a*b^5*c*f^2 - 6*a^2*c^5*d*e -
4*b^4*c^3*d*e + 12*a^3*c^4*e*f + 6*b^5*c^2*d*f + 20*a*b^2*c^4*d*e - 34*a*b^
3*c^3*d*f + 29*a^2*b*c^4*d*f + 68*a*b^4*c^2*e*f - 76*a^2*b^2*c^3*e*f)/(4*a*
c^7 - b^2*c^6) + (((b^3*c^8)/2 - 2*a*b*c^9)*(3*b^5*f - 12*a^2*c^3*e + b^3*c
^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2*c^2*e + 30*a^2*b*c
^2*f)^2)/(c^8*(4*a*c - b^2)^3*(4*a*c^7 - b^2*c^6))))/(2*a*(4*a*c - b^2)^(3/
2))) + (b*((((16*a^2*c^5*f - 8*a*c^6*d + 16*a*b*c^5*e - 24*a*b^2*c^4*f)/c^6
 - (8*a*c^2*(6*b^8*f - 128*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*
c*e + 96*a^2*b^2*c^4*d - 192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^
2*c^3*f - 76*a*b^6*c*f - 24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e)
)/(256*a^3*c^7 - 4*b^6*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6))*(6*b^8*f - 12
8*a^3*c^5*d + 2*b^6*c^2*d + 256*a^4*c^4*f - 4*b^7*c*e + 96*a^2*b^2*c^4*d -
192*a^2*b^3*c^3*e + 336*a^2*b^4*c^2*f - 576*a^3*b^2*c^3*f - 76*a*b^6*c*f -
24*a*b^4*c^3*d + 48*a*b^5*c^2*e + 256*a^3*b*c^4*e))/(2*(256*a^3*c^7 - 4*b^6
*c^4 + 48*a*b^4*c^5 - 192*a^2*b^2*c^6)) - (a*c^4*d^2 + 9*a*b^4*f^2 + 4*a^3*
c^2*f^2 + 4*a*b^2*c^2*e^2 - 12*a^2*b^2*c*f^2 - 4*a^2*c^3*d*f + 6*a*b^2*c^2*
d*f + 8*a^2*b*c^2*e*f - 4*a*b*c^3*d*e - 12*a*b^3*c*e*f)/c^6 + (a*(3*b^5*f -
 12*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b
^2*c^2*e + 30*a^2*b*c^2*f)^2)/(c^6*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3
/2))))/(9*b^10*f^2 + 144*a^4*c^6*e^2 + b^6*c^4*d^2 + 4*b^8*c^2*e^2 - 12*a*b
^4*c^5*d^2 - 48*a*b^6*c^3*e^2 - 12*b^9*c*e*f + 36*a^2*b^2*c^6*d^2 + 192*a^2
*b^4*c^4*e^2 - 288*a^3*b^2*c^5*e^2 + 580*a^2*b^6*c^2*f^2 - 1200*a^3*b^4*c^3
```

```
*f^2 + 900*a^4*b^2*c^4*f^2 - 120*a*b^8*c*f^2 - 4*b^7*c^3*d*e + 6*b^8*c^2*d*
f + 48*a*b^5*c^4*d*e + 144*a^3*b*c^6*d*e - 76*a*b^6*c^3*d*f + 152*a*b^7*c^2
*e*f - 720*a^4*b*c^5*e*f - 168*a^2*b^3*c^5*d*e + 300*a^2*b^4*c^4*d*f - 360*
a^3*b^2*c^5*d*f - 672*a^2*b^5*c^3*e*f + 1200*a^3*b^3*c^4*e*f))*(3*b^5*f - 1
2*a^2*c^3*e + b^3*c^2*d - 2*b^4*c*e - 6*a*b*c^3*d - 20*a*b^3*c*f + 12*a*b^2
*c^2*e + 30*a^2*b*c^2*f))/(2*c^4*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.62 $\int \frac{x^5\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)-\left(b^3(ce-2bf)\right)\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^2\left(-c(6af+be)+2b^2f+2c^2d\right)}{2c^2\left(b^2-4ac\right)}+\frac{x^4 \cdots}{\cdots}$$

```
[Out] 1/2*(2*c^2*d+2*b^2*f-c*(6*a*f+b*e))*x^2/c^2/(-4*a*c+b^2)+1/2*x^4*(2*a*c*e-b
*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+
a)-1/2*(12*a^2*c^2*f-b^3*(-2*b*f+c*e)-2*a*c*(6*b^2*f-3*b*c*e+2*c^2*d))*arct
anh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(3/2)+1/4*(-2*b*f+c*e)
*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A] time = 0.44, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1663, 1644, 773, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(-(ce-2bf))\right)}{2c^3\left(b^2-4ac\right)^{3/2}}+\frac{x^4\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2 \cdots\right.\right.\right.}{2c\left(b^2-4ac\right)\left(a+b \cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*c^2*d + 2*b^2*f - c*(b*e + 6*a*f))*x^2)/(2*c^2*(b^2 - 4*a*c)) + (x^4*(2
*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2*c*(b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((12*a^2*c^2*f - b^3*(c*e - 2*b*f) - 2*a*
c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/
(2*c^3*(b^2 - 4*a*c)^(3/2)) + ((c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])/(4*c^3
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 773

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{x\left(2\left(2ae-\frac{b(cd+af)}{c}\right)-\right.}{a+b}\right.}{2\left(b^2\right.}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2ac\right.\right.}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2ac\right.\right.}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2ac\right.\right.}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{\left(2c^2d+2b^2f-c(be+6af)\right)x^2}{2c^2\left(b^2-4ac\right)} + \frac{x^4\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2ac\right.\right.}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 0.36, size = 236, normalized size = 1.00

$$\frac{-\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(12a^2c^2f-2ac\left(6b^2f-3bce+2c^2d\right)+b^3(2bf-ce)\right)}{\left(4ac-b^2\right)^{3/2}}-\frac{2\left(a^2c\left(2c\left(e+fx^2\right)-3bf\right)+a\left(b^3f-b^2c\left(e+4fx^2\right)+bc^2\left(d+3ex^2\right)-2c^3dx^2\right)+b^2x^2\left(b^2f-\right.\right.}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*c*f*x^2 - (2*(b^2*(c^2*d - b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e +
 f*x^2)) + a*(b^3*f - 2*c^3*d*x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^
2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (2*(12*a^2*c^2*f + b^3*(-(c*e)
+ 2*b*f) - 2*a*c*(2*c^2*d - 3*b*c*e + 6*b^2*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-
b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (c*e - 2*b*f)*Log[a + b*x^2 + c*x^4])
/(4*c^3)
```

fricas [B] time = 1.01, size = 1455, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d - (b^5*c
- 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*a^2*b^2*c^2 - 24*a^
3*c^3)*f)*x^2 + (4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 - 6*a*b*c^3)*e - 2*(b^
4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d + (b^4*c - 6*a*b^2*c^2
)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^3*c - 6*a^2*b*c^2)*e
- 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 +
 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
+ a)) - 2*(a*b^3*c^2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*
c^3)*e - 2*(a*b^5 - 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^
3 + 16*a^2*c^4)*e - 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c
 - 8*a*b^3*c^2 + 16*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*
x^2 + (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 1
6*a^3*b*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3
*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 +
16*a^2*b*c^5)*x^2), 1/4*(2*(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*f*x^6 + 2*(
b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f*x^4 - 2*((b^4*c^2 - 6*a*b^2*c^3 + 8*a
^2*c^4)*d - (b^5*c - 7*a*b^3*c^2 + 12*a^2*b*c^3)*e + (b^6 - 9*a*b^4*c + 26*
a^2*b^2*c^2 - 24*a^3*c^3)*f)*x^2 + 2*(4*a^2*c^3*d + (4*a*c^4*d + (b^3*c^2 -
 6*a*b*c^3)*e - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*f)*x^4 + (4*a*b*c^3*d +
 (b^4*c - 6*a*b^2*c^2)*e - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)*f)*x^2 + (a*b^
3*c - 6*a^2*b*c^2)*e - 2*(a*b^4 - 6*a^2*b^2*c + 6*a^3*c^2)*f)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(a*b^3*c^
2 - 4*a^2*b*c^3)*d + 2*(a*b^4*c - 6*a^2*b^2*c^2 + 8*a^3*c^3)*e - 2*(a*b^5 -
 7*a^2*b^3*c + 12*a^3*b*c^2)*f + (((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*e -
 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*f)*x^4 + ((b^5*c - 8*a*b^3*c^2 + 16
*a^2*b*c^3)*e - 2*(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*f)*x^2 + (a*b^4*c - 8*
a^2*b^2*c^2 + 16*a^3*c^3)*e - 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*f)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

giac [A] time = 1.86, size = 279, normalized size = 1.18

$$\frac{fx^2}{2c^2}-\frac{\left(4ac^3d-2b^4f+12ab^2cf-12a^2c^2f+b^3ce-6abc^2e\right)\arctan\left(\frac{2cx^2+b}{\sqrt{-b^2+4ac}}\right)}{2\left(b^2c^3-4ac^4\right)\sqrt{-b^2+4ac}}+\frac{2b^3fx^4-8abcfx^4-b^2cx^4e+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*f*x^2/c^2 - 1/2*(4*a*c^3*d - 2*b^4*f + 12*a*b^2*c*f - 12*a^2*c^2*f + b^
3*c*e - 6*a*b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4
*a*c^4)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*b^3*f*x^4 - 8*a*b*c*f*x^4 - b^2*c*x^4*
e + 4*a*c^2*x^4*e - 2*b^2*c*d*x^2 + 4*a*c^2*d*x^2 - 4*a^2*c*f*x^2 + b^3*x^2
*e - 2*a*b*c*x^2*e - 2*a*b*c*d - 2*a^2*b*f + a*b^2*e)/((c*x^4 + b*x^2 + a)*
(b^2*c^2 - 4*a*c^3)) - 1/4*(2*b*f - c*e)*log(c*x^4 + b*x^2 + a)/c^3
```

maple [B] time = 0.02, size = 832, normalized size = 3.53

$$\frac{a^2 f x^2}{\left(c x^4+b x^2+a\right)\left(4ac-b^2\right)c}-\frac{2a b^2 f x^2}{\left(c x^4+b x^2+a\right)\left(4ac-b^2\right)c^2}+\frac{3abe x^2}{2\left(c x^4+b x^2+a\right)\left(4ac-b^2\right)c}-\frac{ad x^2}{\left(c x^4+b x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*f*x^2/c^2+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a^2*f-2/c^2/(c*x^4+b*x^2+
a)/(4*a*c-b^2)*x^2*a*b^2*f+3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*b*e-1/(c
*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*a*d+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b
^4*f-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b^3*e+1/2/c/(c*x^4+b*x^2+a)/(4
*a*c-b^2)*x^2*b^2*d-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*b*f+1/c/(c*x^4+
b*x^2+a)*a^2/(4*a*c-b^2)*e+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^3*f-1/2/
c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*b^2*e+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)
*b*d-2/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*a*b*f+1/c/(4*a*c-b^2)*ln(c*x^4+b*x
^2+a)*a*e+1/2/c^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*f-1/4/c^2/(4*a*c-b^2)*l
n(c*x^4+b*x^2+a)*b^2*e-6/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))*a^2*f+6/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
*a*b^2*f-3/c/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*e+
2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*d-1/c^3/(4*a*c-
b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^4*f+1/2/c^2/(4*a*c-b^2)^
(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.81, size = 2450, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(b^3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c*(4*a*c - b^2)
) + (x^2*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*d - b^3*c*e + 3*a*b*c^2
*e - 4*a*b^2*c*f))/(2*c*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (f*
x^2)/(2*c^2) + (log(a + b*x^2 + c*x^4)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e
```

```
 - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 2
56*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c
^5)) - (atan(((8*a*c^5*(4*a*c - b^2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*((
(((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*c^6*d + 28*a*b*c^5*e - 5
6*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*b^7*f + 1
28*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*
c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^
6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(2*b^4*f + 12*a^2*c^2*f -
 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f))/(8*c^3*(4*a*c - b^2)^(3
/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*
c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96
*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^
3*b*c^3*f))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 -
4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((4*b^
5*f^2 + b^3*c^2*e^2 + 12*a^2*b*c^2*f^2 + 2*a*c^4*d*e - 4*b^4*c*e*f - 5*a*b*
c^3*e^2 - 20*a*b^3*c*f^2 - 6*a^2*c^3*e*f + 20*a*b^2*c^2*e*f - 4*a*b*c^3*d*f
)/(4*a*c^5 - b^2*c^4) + (((24*a^2*c^5*f - 6*b^3*c^4*e + 12*b^4*c^3*f - 8*a*
c^6*d + 28*a*b*c^5*e - 56*a*b^2*c^4*f)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 -
32*a*b*c^7)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a
^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(4*a*c^
5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(
4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f
- 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6
*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*b^4
*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)^2)/(c
^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4))))/(2*a*(4*a*c - b^2)^(3/2))) + ((((
8*a*c^4*e - 16*a*b*c^3*f)/c^4 - (8*a*c^2*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c
*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e -
 256*a^3*b*c^3*f))/(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^
5))*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*
c*f))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d
- b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*
e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e -
256*a^3*b*c^3*f))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^
4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((((8*a*c^4*e - 16*a*b*c^
3*f)/c^4 - (8*a*c^2*(4*b^7*f + 128*a^3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e
 + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f + 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2
56*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*b^7*f + 128*a^
3*c^4*e - 2*b^6*c*e - 96*a^2*b^2*c^3*e + 192*a^2*b^3*c^2*f - 48*a*b^5*c*f +
 24*a*b^4*c^2*e - 256*a^3*b*c^3*f))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*
c^4 - 192*a^2*b^2*c^5)) - (4*a*b^2*f^2 + a*c^2*e^2 - 4*a*b*c*e*f)/c^4 + (a*
(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e - 12*a*b^2*c*f)
^2)/(c^4*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(4*b^8*f^2 + 16*a^2
*c^6*d^2 + 144*a^4*c^4*f^2 + b^6*c^2*e^2 - 12*a*b^4*c^3*e^2 - 4*b^7*c*e*f +
 36*a^2*b^2*c^4*e^2 + 192*a^2*b^4*c^2*f^2 - 288*a^3*b^2*c^3*f^2 - 48*a*b^6*
c*f^2 - 96*a^3*c^5*d*f + 8*a*b^3*c^4*d*e - 48*a^2*b*c^5*d*e - 16*a*b^4*c^3*
d*f + 48*a*b^5*c^2*e*f + 144*a^3*b*c^4*e*f + 96*a^2*b^2*c^4*d*f - 168*a^2*b
^3*c^3*e*f))*(2*b^4*f + 12*a^2*c^2*f - 4*a*c^3*d - b^3*c*e + 6*a*b*c^2*e -
12*a*b^2*c*f))/(2*c^3*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

3.63 $\int \frac{x^3(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=165

$$\frac{x^2\left(-\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right.}{2c^2\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*x^2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+
b^2)/(c*x^4+b*x^2+a)+1/2*(4*a*c^2*e+b^3*f-2*b*c*(3*a*f+c*d))*arctanh((2*c*x
^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*f*ln(c*x^4+b*x^2+a)/c^
2
```

Rubi [A] time = 0.29, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1663, 1644, 634, 618, 206, 628}

$$\frac{x^2\left(x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right.}{2c^2\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x^2*(2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(
2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((4*a*c^2*e + b^3*f - 2*b*c*(c*d +
 3*a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/
2)) + (f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1644

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f =
```

```
Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[Polynom
ialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((d + e*x)^m*(a + b*x +
c*x^2)^(p + 1)*(f*b - 2*a*g + (2*c*f - b*g)*x))/((p + 1)*(b^2 - 4*a*c)), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^
(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*(d + e*x)*Q + g*(2*a*e*m + b*d*(2
*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(m + 2*p + 3)*x, x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (Integer
Q[p] ||  !IntegerQ[m] ||  !RationalQ[a, b, c, d, e]) &&  !(IGtQ[m, 0] && Ra
tionalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\left(d+ex+fx^2\right)}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{2ae-\frac{b(cd+af)}{c}-\frac{\left(b^2-4ac\right)fx}{c}}{a+bx+cx^2}\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{f\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right.}{4c^2}$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{f\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{(4a}{}$$

$$= \frac{x^2\left(2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4ac^2e+b^3f-2bc(cd+3af\right.}{2c^2\left(b^2-4ac\right.}$$

Mathematica [A] time = 0.25, size = 175, normalized size = 1.06

$$\frac{\frac{2\left(-2a^2cf+a\left(b^2f-bc\left(e+3fx^2\right)+2c^2\left(d+ex^2\right)\right)+bx^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-2bc(3af+cd)+4ac^2e+b^3f\right)}{\left(4ac-b^2\right)^{3/2}}+f\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 + a*(b^2*f + 2*c^2*(d + e*x
^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(4*a*c^
2*e + b^3*f - 2*b*c*(c*d + 3*a*f))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]]
)/(-b^2 + 4*a*c)^(3/2) + f*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

fricas [B] time = 1.06, size = 970, normalized size = 5.88

$$\left[\frac{2\left(\left(b^3c^2-4\,abc^3\right)d-\left(b^4c-6\,ab^2c^2+8\,a^2c^3\right)e+\left(b^5-7\,ab^3c+12\,a^2bc^2\right)f\right)x^2-\left(2\,abc^2d-4\,a^2c^2e+\left(2\,bc^3d-\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*e + (b
^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - (2*a*b*c^2*d - 4*a^2*c^2*e + (2*b*c
^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^2*c^2*d - 4*a*b*c^2*e
- (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*sqrt(b^2 - 4*a*c)*log((
2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x
^4 + b*x^2 + a)) + 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*
e + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*
c^3)*f*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c
+ 16*a^3*c^2)*f)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^
3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 +
 16*a^2*b*c^4)*x^2), 1/4*(2*((b^3*c^2 - 4*a*b*c^3)*d - (b^4*c - 6*a*b^2*c^2
 + 8*a^2*c^3)*e + (b^5 - 7*a*b^3*c + 12*a^2*b*c^2)*f)*x^2 - 2*(2*a*b*c^2*d
- 4*a^2*c^2*e + (2*b*c^3*d - 4*a*c^3*e - (b^3*c - 6*a*b*c^2)*f)*x^4 + (2*b^
2*c^2*d - 4*a*b*c^2*e - (b^4 - 6*a*b^2*c)*f)*x^2 - (a*b^3 - 6*a^2*b*c)*f)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 4*(a*b^2*c^2 - 4*a^2*c^3)*d - 2*(a*b^3*c - 4*a^2*b*c^2)*e + 2*(a*b^4 - 6*a
^2*b^2*c + 8*a^3*c^2)*f + ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*f*x^4 + (b^5
- 8*a*b^3*c + 16*a^2*b*c^2)*f*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*f)*l
og(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 -
 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2
)]
```

giac [A] time = 1.84, size = 195, normalized size = 1.18

$$\frac{\left(2\,bc^2d-b^3f+6\,abcf-4\,ac^2e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^2c^2-4\,ac^3\right)\sqrt{-b^2+4\,ac}}+\frac{f\log\left(cx^4+bx^2+a\right)}{4\,c^2}+\frac{2\,ac^2d+ab^2f-2\,a^2cf-abce+\left(\right.}{2\,\left(cx^4+bx^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b*c^2*d - b^3*f + 6*a*b*c*f - 4*a*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-
b^2 + 4*a*c))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*f*log(c*x^4 +
b*x^2 + a)/c^2 + 1/2*(2*a*c^2*d + a*b^2*f - 2*a^2*c*f - a*b*c*e + (b*c^2*d
+ b^3*f - 3*a*b*c*f - b^2*c*e + 2*a*c^2*e)*x^2)/((c*x^4 + b*x^2 + a)*(b^2 -
 4*a*c)*c^2)
```

maple [B] time = 0.02, size = 336, normalized size = 2.04

$$-\frac{3abf\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}c}+\frac{2ae\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{b^3f\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\left(4ac-b^2\right)^{\frac{3}{2}}c^2}-\frac{bd\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}}+\frac{af\ln\left(c\,x^4+b\,x\right.}{\left(4ac-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*((3*a*b*c*f-2*a*c^2*e-b^3*f+b^2*c*e-b*c^2*d)/(4*a*c-b^2)/c^2*x^2+a*(2*a
*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c^2)/(c*x^4+b*x^2+a)+1/c/(4*a*c-b^2)*
```

```
ln(c*x^4+b*x^2+a)*a*f-1/4/c^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*f-3/c/(4*a*
c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b*f+2/(4*a*c-b^2)^(3/2
)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*e-1/(4*a*c-b^2)^(3/2)*arctan((2*c
*x^2+b)/(4*a*c-b^2)^(1/2))*b*d+1/2/c^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2))*b^3*f
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 2.72, size = 1651, normalized size = 10.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((a*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c^2*(4*a*c - b^2)) + (x^2*(b^
3*f + 2*a*c^2*e + b*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*c^2*(4*a*c - b^2)))/(a
 + b*x^2 + c*x^4) - (log(a + b*x^2 + c*x^4)*(2*b^6*f - 128*a^3*c^3*f + 96*a
^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 -
192*a^2*b^2*c^4)) - (atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^
2)^3)*((((8*a*f + (8*a*c^2*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24
*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(b
^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a
*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c
^2*e - 2*b*c^2*d - 6*a*b*c*f))/((4*a*c - b^2)^(3/2)*(256*a^3*c^5 - 4*b^6*c^
2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) - x^2*(((((6*b^3*c^
2*f + 8*a*c^4*e - 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c
^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f
))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2
*b^2*c^4)))*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(8*c^2*(4*a*c - b^
2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2
*c^2*f - 24*a*b^4*c*f)*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(16*c^2
*(4*a*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^
4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((b^3*f^2 - 5*a*b*c*f^2 +
 2*a*c^2*e*f - b*c^2*d*f)/(4*a*c^3 - b^2*c^2) + (((6*b^3*c^2*f + 8*a*c^4*e
- 4*b*c^4*d - 28*a*b*c^3*f)/(4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)
*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(4*a*c^3 -
 b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(2*b
^6*f - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 -
4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) - (((b^3*c^4)/2 - 2*a*b*c^5)*(
b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3
 - b^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2))) + (b*(((8*a*f + (8*a*c^2*(2*b^6*f
 - 128*a^3*c^3*f + 96*a^2*b^2*c^2*f - 24*a*b^4*c*f))/(256*a^3*c^5 - 4*b^6*c
^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*b^6*f - 128*a^3*c^3*f + 96*a^2*b^2
*c^2*f - 24*a*b^4*c*f))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^
2*b^2*c^4)) + (a*f^2)/c^2 - (a*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f)^
2)/(c^2*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(b^6*f^2 + 16*a^2*c^
4*e^2 + 4*b^2*c^4*d^2 + 36*a^2*b^2*c^2*f^2 - 12*a*b^4*c*f^2 - 4*b^4*c^2*d*f
 + 24*a*b^2*c^3*d*f + 8*a*b^3*c^2*e*f - 48*a^2*b*c^3*e*f - 16*a*b*c^4*d*e))
*(b^3*f + 4*a*c^2*e - 2*b*c^2*d - 6*a*b*c*f))/(2*c^2*(4*a*c - b^2)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.64 $\int \frac{x\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=123

$$\frac{-\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(2*a*c*e-b*(a*f+c*d)-(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+(2*a*f-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
(-4*a*c+b^2)^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1663, 1660, 12, 618, 206}

$$\frac{x^2\left(-\left(-2acf+b^2f-bce+2c^2d\right)\right)-b(af+cd)+2ace}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)(2af-be+2cd)}{\left(b^2-4ac\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*a*c*e - b*(c*d + a*f) - (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2)/(2*c*(b
^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((2*c*d - b*e + 2*a*f)*ArcTanh[(b + 2*c*
x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\,\mathrm{Subst}\left(\int \frac{d+ex+fx^2}{\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\mathrm{Subst}\left(\int \frac{2cd-be+2af}{a+bx+cx^2}\,dx, x, x^2\right)}{2\left(b^2-4ac\right)}$$

$$= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2cd-be+2af)\,\mathrm{Subst}\left(\int \frac{1}{a+bx}\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(2cd-be+2af)\,\mathrm{Subst}\left(\int \frac{}{b^2-4}\right.}{b^2-4ac}$$

$$= \frac{2ace-b(cd+af)-\left(2c^2d-bce+b^2f-2acf\right)x^2}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{(2cd-be+2af)\tanh^{-1}\left(\frac{b+2c}{\sqrt{b^2-4}}\right.}{\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A] time = 0.10, size = 130, normalized size = 1.06

$$\frac{abf-2ac\left(e+fx^2\right)+b^2fx^2+bc\left(d-ex^2\right)+2c^2dx^2}{2c\left(4ac-b^2\right)\left(a+bx^2+cx^4\right)} - \frac{\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)(-2af+be-2cd)}{\left(4ac-b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(e + f*x^2))/(2*
c*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)) - ((-2*c*d + b*e - 2*a*f)*ArcTan[(b +
 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2)
```

fricas [B] time = 0.89, size = 650, normalized size = 5.28

$$\left[-\frac{\left(2\left(b^2c^2-4\,ac^3\right)d-\left(b^3c-4\,abc^2\right)e+\left(b^4-6\,ab^2c+8\,a^2c^2\right)f\right)x^2+\left(\left(2\,c^3d-bc^2e+2\,ac^2f\right)x^4+2\,ac^2d-abc\right.}{2\left(ab^4c-8\,a^2b^2c^2+16\,a^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
+ 8*a^2*c^2)*f)*x^2 + ((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d - a*
b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(b^2 - 4*a*c
)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c
))/(c*x^4 + b*x^2 + a)) + (b^3*c - 4*a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e
 + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2
- 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)
```

```
, -1/2*((2*(b^2*c^2 - 4*a*c^3)*d - (b^3*c - 4*a*b*c^2)*e + (b^4 - 6*a*b^2*c
 + 8*a^2*c^2)*f)*x^2 - 2*((2*c^3*d - b*c^2*e + 2*a*c^2*f)*x^4 + 2*a*c^2*d -
 a*b*c*e + 2*a^2*c*f + (2*b*c^2*d - b^2*c*e + 2*a*b*c*f)*x^2)*sqrt(-b^2 + 4
*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (b^3*c - 4*
a*b*c^2)*d - 2*(a*b^2*c - 4*a^2*c^2)*e + (a*b^3 - 4*a^2*b*c)*f)/(a*b^4*c -
8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^
5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2)]
```

giac [A]   time = 2.17, size = 140, normalized size = 1.14

$$-\frac{\left(2\,cd + 2\,af - be\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(b^2 - 4\,ac\right)\sqrt{-b^2 + 4\,ac}} - \frac{2\,c^2dx^2 + b^2fx^2 - 2\,acfx^2 - bcx^2e + bcd + abf - 2\,ace}{2\,\left(cx^4 + bx^2 + a\right)\left(b^2c - 4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*c*d + 2*a*f - b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a
*c)*sqrt(-b^2 + 4*a*c)) - 1/2*(2*c^2*d*x^2 + b^2*f*x^2 - 2*a*c*f*x^2 - b*c*
x^2*e + b*c*d + a*b*f - 2*a*c*e)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

maple [A]   time = 0.01, size = 205, normalized size = 1.67

$$\frac{2af\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} - \frac{be\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} + \frac{2cd\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}} + \frac{-\frac{\left(2acf-b^2f+bce-2c^2d\right)x^2}{\left(4ac-b^2\right)c} + \frac{abf-2ace+bcd}{\left(4ac-b^2\right)c}}{2c\,x^4 + 2b\,x^2 + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 1/2*(-(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c*x^2+1/c*(a*b*f-2*a*c*e+b*
c*d)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*a*f-1/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/
2))*b*e+2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*d
```

maxima [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B]   time = 0.38, size = 342, normalized size = 2.78

$$\frac{\frac{a\,b\,f-2\,a\,c\,e+b\,c\,d}{2\,c\,\left(4\,a\,c-b^2\right)} + \frac{x^2\,\left(f\,b^2-e\,b\,c+2\,d\,c^2-2\,a\,f\,c\right)}{2\,c\,\left(4\,a\,c-b^2\right)}}{c\,x^4 + b\,x^2 + a} + \frac{\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x^2\,\left(\frac{\left(2\,c^3\,d+2\,a\,c^2\,f-b\,c^2\,e\right)\,\left(2\,a\,f-b\,e+2\,c\,d\right)}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}} + \frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,\left(b^3-4\,a\,b\,c\right)\,\left(2\,a\,f-b\,e+2\,c\,d\right.}{2\,a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right.\right.}{8\,a^2\,c^2\,f^2-8\,a\,b\,c^2\,e\,f+16\,a\,c^3\,d\,f+2\,b^2\,c^2\,e^2-8\,b\,c^3\,d\,e+8\,c^{\cdots}}\right.}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*b*f - 2*a*c*e + b*c*d)/(2*c*(4*a*c - b^2)) + (x^2*(2*c^2*d + b^2*f - 2*
a*c*f - b*c*e))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (atan(((4*a*c -
b^2)^4*(x^2*(((2*c^3*d + 2*a*c^2*f - b*c^2*e)*(2*a*f - b*e + 2*c*d))/(a*(4*
a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(b^3 - 4*a*b*c)*(2*a*f - b*e +
 2*c*d)^2)/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(b^3 - 4*a*b*c)*(2*a*f - b*
e + 2*c*d)^2)/(4*a*c - b^2)^(11/2)))/(8*c^4*d^2 + 8*a^2*c^2*f^2 + 2*b^2*c^2
*e^2 + 16*a*c^3*d*f - 8*b*c^3*d*e - 8*a*b*c^2*e*f))*(2*a*f - b*e + 2*c*d))/
(4*a*c - b^2)^(3/2)
```

sympy [**B**] time = 38.03, size = 474, normalized size = 3.85

$$-\frac{\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2af-be+2cd\right)\log\left(x^2+\frac{-16a^2c^2\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2af-be+2cd\right)+8ab^2c\sqrt{-\frac{1}{\left(4ac-b^2\right)^3}}\left(2af-be+2cd\right)+2abf-b^4\sqrt{-\frac{1}{\left(4ac-\right.}}}{4acf-2bce+4c^2d}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (-16*a**2*c**2
*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 8*a*b**2*c*sqrt(-1/(4*a
*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f - b**4*sqrt(-1/(4*a*c - b**2
)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2*b*c*e + 4*c**2
*d))/2 + sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d)*log(x**2 + (16*a*
*2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) - 8*a*b**2*c*sqrt(
-1/(4*a*c - b**2)**3)*(2*a*f - b*e + 2*c*d) + 2*a*b*f + b**4*sqrt(-1/(4*a*c
 - b**2)**3)*(2*a*f - b*e + 2*c*d) - b**2*e + 2*b*c*d)/(4*a*c*f - 2*b*c*e +
 4*c**2*d))/2 + (a*b*f - 2*a*c*e + b*c*d + x**2*(-2*a*c*f + b**2*f - b*c*e
+ 2*c**2*d))/(8*a**2*c**2 - 2*a*b**2*c + x**4*(8*a*c**3 - 2*b**2*c**2) + x*
*2*(8*a*b*c**2 - 2*b**3*c))
```

3.65 $\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=166

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(4a^2ce-2ab(af+3cd)+b^3d\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{d\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{d\log(x)}{a^2}+\frac{x^2(abf-2ace+bcd)-abe-}{2a\left(b^2-4ac\right)\left(a+b\right.}$$

```
[Out] 1/2*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)/(
c*x^4+b*x^2+a)+1/2*(b^3*d+4*a^2*c*e-2*a*b*(a*f+3*c*d))*arctanh((2*c*x^2+b)/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^(3/2)+d*ln(x)/a^2-1/4*d*ln(c*x^4+b*x^2
+a)/a^2
```

Rubi [A] time = 0.39, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1646, 800, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(4a^2ce-2ab(af+3cd)+b^3d\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{d\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{d\log(x)}{a^2}+\frac{x^2(abf-2ace+bcd)-abe-}{2a\left(b^2-4ac\right)\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] (b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2)/(2*a*(b^2
 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^3*d + 4*a^2*c*e - 2*a*b*(3*c*d + a*f))
*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (d
*Log[x])/a^2 - (d*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
```

```
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\left(\frac{b^2}{a}-4c\right)d-\frac{(bcd-2ace+abf)x}{a}}{x\left(a+bx+cx^2\right)}\,d\cdots\right.}{2\left(b^2-4ac\right)}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\left(\frac{\left(-b^2+4ac\right)d}{a^2x}+\frac{b^3d+2a^2ce-\cdots}{\cdots}\right.\right.}{2\left(b^2-\cdots\right.}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{b^3d+2a^2ce-ab\cdots}{\cdots}\right.}{2a^2\cdots}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,d\cdots\right.}{4a^2}$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{d\log(x)}{a^2} - \frac{d\log\left(a+bx^2+cx^4\right)}{4a^2}\cdots$$

$$= \frac{b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b^3d+4a^2ce-2ab(3cd+af)\right)\text{t}\cdots}{2a^2\left(b^2-4ac\right)^{3/2}\cdots}$$

Mathematica [A] time = 0.45, size = 268, normalized size = 1.61

$$-\frac{-\frac{2a\left(b\left(-ae+afx^2+cdx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(4ac\left(ae-d\sqrt{b^2-4ac}\right)+b^2d\sqrt{b^2-4ac}-2ab(af+3cd)+b^3d\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(\cdots\right.}{\cdots}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((-2*a*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x
^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - 4*d*Log[x] + ((b^3*d + b^2*Sqr
t[b^2 - 4*a*c]*d + 4*a*c*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) - 2*a*b*(3*c*d + a*
f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-(b^3*d)
+ b^2*Sqrt[b^2 - 4*a*c]*d - 4*a*c*(Sqrt[b^2 - 4*a*c]*d + a*e) + 2*a*b*(3*c*
d + a*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/a^2
```

fricas [B] time = 3.26, size = 1103, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((a*b^3*c - 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3
 - 4*a^3*b*c)*f)*x^2 + (4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f
+ (b^3*c - 6*a*b*c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*
c)*d)*x^2 + (a*b^3 - 6*a^2*b*c)*d)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c
*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
+ 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*a^3*b*c)*e + 4*(a^
3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(c*
x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*d*x^4 + (b^5 - 8*a
*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*d)*log(x)
)/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), 1/4*(2*((a*b^3*c -
 4*a^2*b*c^2)*d - 2*(a^2*b^2*c - 4*a^3*c^2)*e + (a^2*b^3 - 4*a^3*b*c)*f)*x^
2 + 2*(4*a^3*c*e - 2*a^3*b*f + (4*a^2*c^2*e - 2*a^2*b*c*f + (b^3*c - 6*a*b*
c^2)*d)*x^4 + (4*a^2*b*c*e - 2*a^2*b^2*f + (b^4 - 6*a*b^2*c)*d)*x^2 + (a*b^
3 - 6*a^2*b*c)*d)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*
c)/(b^2 - 4*a*c)) + 2*(a*b^4 - 6*a^2*b^2*c + 8*a^3*c^2)*d - 2*(a^2*b^3 - 4*
a^3*b*c)*e + 4*(a^3*b^2 - 4*a^4*c)*f - ((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(c*x^4 + b*x^2 + a) + 4*((b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*
d*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)*d*x^2 + (a*b^4 - 8*a^2*b^2*c + 16*
a^3*c^2)*d)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^
3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)]
```

giac [A] time = 2.00, size = 227, normalized size = 1.37

$$-\frac{\left(b^3d-6\,abcd-2\,a^2bf+4\,a^2ce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^2b^2-4\,a^3c\right)\sqrt{-b^2+4\,ac}}-\frac{d\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{d\log\left(x^2\right)}{2\,a^2}+\frac{b^2cdx^4-4\,ac^2dx^4+b^3d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^3*d - 6*a*b*c*d - 2*a^2*b*f + 4*a^2*c*e)*arctan((2*c*x^2 + b)/sqrt(
-b^2 + 4*a*c))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*d*log(c*x^4 +
 b*x^2 + a)/a^2 + 1/2*d*log(x^2)/a^2 + 1/4*(b^2*c*d*x^4 - 4*a*c^2*d*x^4 + b
^3*d*x^2 - 2*a*b*c*d*x^2 + 2*a^2*b*f*x^2 - 4*a^2*c*x^2*e + 3*a*b^2*d - 8*a^
2*c*d + 4*a^3*f - 2*a^2*b*e)/((c*x^4 + b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

maple [B] time = 0.02, size = 462, normalized size = 2.78

$$-\frac{bcd\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{bf\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)}+\frac{ce\,x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)}-\frac{3bcd\arctan\left(\frac{2c\,x^2+}{\sqrt{4ac-}}\right)}{\left(4ac-b^2\right)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] d*ln(x)/a^2-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*f+c/(c*x^4+b*x^2+a)/(4*a*
c-b^2)*x^2*e-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^2*b*c*d-a/(c*x^4+b*x^2+a)/
(4*a*c-b^2)*f+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*e+1/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*c*d-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*d-1/a/(4*a*c-b^2)*c*ln(c*x^4
+b*x^2+a)*d+1/4/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*d-1/(4*a*c-b^2)^(3/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*f+2/(4*a*c-b^2)^(3/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2))*c*e-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*b*c*d+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^3*d
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [**B**] time = 11.85, size = 8706, normalized size = 52.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (d*log(x))/a^2 - ((b^2*d + 2*a^2*f - a*b*e - 2*a*c*d)/(2*a*(4*a*c - b^2)) +
 (x^2*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
 (log((((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*
c - b^2)^3))^(1/2))*(((d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d
)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(20*a^2*c^2*e + 4*a*b^3*f - b
^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c*f))/(a*(4*a*c - b^2)) + (b
*c^2*(d + a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c -
 b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 - (4*b*c^2*(b^3*d - a^
2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2))))/(4*a^2) + (c^2*(a^3*b^2
*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^2*d^2 - 4*a*b^4*d*f - 36*a^
2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e - 4*a^3*b*c*e*f))/(a^2*(4*a*
c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*d^2 + 4*a^2*b*c^2*e^2 - 20*
a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e - 20*a^2*b*c^2*d*f - 4*a^2*
b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) - (c^2*x^2*(a*b
*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) + (c^2*d*(a*b*f - 2*a*c*e +
b*c*d)^2)/(a^3*(4*a*c - b^2)^2))*(((d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*
e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((d - a^2*(-(b^3*d - 2*a^2*
b*f + 4*a^2*c*e - 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((2*c^2*x^2*(2
0*a^2*c^2*e + 4*a*b^3*f - b^3*c*d + 10*a*b*c^2*d - 8*a*b^2*c*e - 10*a^2*b*c
*f))/(a*(4*a*c - b^2)) + (b*c^2*(d - a^2*(-(b^3*d - 2*a^2*b*f + 4*a^2*c*e -
 6*a*b*c*d)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))
/a^2 - (4*b*c^2*(b^3*d - a^2*b*f + 2*a^2*c*e - 5*a*b*c*d))/(a*(4*a*c - b^2)
)))/(4*a^2) + (c^2*(a^3*b^2*f^2 - 4*b^4*c*d^2 + 4*a^3*c^2*e^2 + 17*a*b^2*c^
2*d^2 - 4*a*b^4*d*f - 36*a^2*b*c^2*d*e + 18*a^2*b^2*c*d*f + 8*a*b^3*c*d*e -
 4*a^3*b*c*e*f))/(a^2*(4*a*c - b^2)^2) - (c^2*x^2*(a^2*b^3*f^2 + 6*b^3*c^2*
d^2 + 4*a^2*b*c^2*e^2 - 20*a*b*c^3*d^2 + 40*a^2*c^3*d*e - 14*a*b^2*c^2*d*e
- 20*a^2*b*c^2*d*f - 4*a^2*b^2*c*e*f + 7*a*b^3*c*d*f))/(a^2*(4*a*c - b^2)^2
```

```
)))/(4*a^2) - (c^2*x^2*(a*b*f - 2*a*c*e + b*c*d)^3)/(a^3*(4*a*c - b^2)^3) +
 (c^2*d*(a*b*f - 2*a*c*e + b*c*d)^2)/(a^3*(4*a*c - b^2)^2)))*(2*b^6*d - 128
*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3
- 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((x^2*((((b^3*c^5*d^3 - 8*a^3*c^5
*e^3 + a^3*b^3*c^2*f^3 - 6*a*b^2*c^5*d^2*e + 12*a^2*b*c^5*d*e^2 + 3*a*b^3*c
^4*d^2*f + 12*a^3*b*c^4*e^2*f + 3*a^2*b^3*c^3*d*f^2 - 6*a^3*b^2*c^3*e*f^2 -
 12*a^2*b^2*c^4*d*e*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2) - (((6*a*b^5*c^4*d^2 + 80*a^3*b*c^6*d^2 - 16*a^4*b*c^5*e^2 - 44*a^2*b^3*
c^5*d^2 + 4*a^3*b^3*c^4*e^2 + a^3*b^5*c^2*f^2 - 4*a^4*b^3*c^3*f^2 - 160*a^4
*c^6*d*e + 80*a^4*b*c^5*d*f - 14*a^2*b^4*c^4*d*e + 96*a^3*b^2*c^5*d*e + 7*a
^2*b^5*c^3*d*f - 48*a^3*b^3*c^4*d*f - 4*a^3*b^4*c^3*e*f + 16*a^4*b^2*c^4*e*
f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((640*a^6*c^6*
e - 2*a^2*b^7*c^3*d + 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3
*e + 168*a^4*b^4*c^4*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c
^3*f + 288*a^5*b^3*c^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64
*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*
a^2*b^2*c^2*d - 24*a*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^
7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12
*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*
a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))
/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d -
 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*
c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*a^6*c^6*e - 2*a^2*b^7*c^3
*d + 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*
c^4*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^
3*c^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4
*b^4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 2
4*a*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*
b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a
^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b
^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((
2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b
*f + 4*a^2*c*e - 6*a*b*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*
c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3
*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3
 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b
*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2
*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2*(2560*
a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*
b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*
(3*b^5*d - a^2*b^3*f - 2*a^3*c^2*e - 21*a*b^3*c*d + a^3*b*c*f + 33*a^2*b*c^
2*d + 2*a^2*b^2*c*e))/(8*a^3*c^2*(4*a*c - b^2)^3*(400*a^3*c^3*d^2 - 6*b^6*d
^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2 - a
^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^4*b
*c*e*f)) + (((((((640*a^6*c^6*e - 2*a^2*b^7*c^3*d + 36*a^3*b^5*c^4*d - 192*
a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*c^4*e - 576*a^5*b^2*c^5*e +
8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^3*c^4*f + 320*a^5*b*c^6*d -
320*a^6*b*c^5*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (
(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(2560*a^7*b*c^6
 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))
/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256
*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e
 - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((2*b^6*d - 128*a^3*c^3*d + 96
*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)*
(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 268
8*a^6*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b
^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^
2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4
```

```
*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((6*a*b^5*c^4*
d^2 + 80*a^3*b*c^6*d^2 - 16*a^4*b*c^5*e^2 - 44*a^2*b^3*c^5*d^2 + 4*a^3*b^3*
c^4*e^2 + a^3*b^5*c^2*f^2 - 4*a^4*b^3*c^3*f^2 - 160*a^4*c^6*d*e + 80*a^4*b*
c^5*d*f - 14*a^2*b^4*c^4*d*e + 96*a^3*b^2*c^5*d*e + 7*a^2*b^5*c^3*d*f - 48*
a^3*b^3*c^4*d*f - 4*a^3*b^4*c^3*e*f + 16*a^4*b^2*c^4*e*f)/(a^3*b^6 - 64*a^6
*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((640*a^6*c^6*e - 2*a^2*b^7*c^3*d
+ 36*a^3*b^5*c^4*d - 192*a^4*b^3*c^5*d - 16*a^3*b^6*c^3*e + 168*a^4*b^4*c^4
*e - 576*a^5*b^2*c^5*e + 8*a^3*b^7*c^2*f - 84*a^4*b^5*c^3*f + 288*a^5*b^3*c
^4*f + 320*a^5*b*c^6*d - 320*a^6*b*c^5*f)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^
4*c + 48*a^5*b^2*c^2) - ((2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a
*b^4*c*d)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5
*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*
b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^
6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256
*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e
 - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) + ((b^3*d - 2*a^2*b*f + 4*a^2*c*
e - 6*a*b*c*d)^3*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*
a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(64*a^6*(4*a*c - b^2)^(9/2)*(a^3*b^6 - 64*
a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)))*(96*b^6*d - 1280*a^3*c^3*d - 32*
a^2*b^4*f + 2208*a^2*b^2*c^2*d - 864*a*b^4*c*d + 64*a^2*b^3*c*e - 192*a^3*b
*c^2*e + 96*a^3*b^2*c*f))/(256*a^3*c^2*(4*a*c - b^2)^(7/2)*(400*a^3*c^3*d^2
 - 6*b^6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4
*c*d^2 - a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f
 - 4*a^4*b*c*e*f)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) - 1024*a^9*c^3*(4*a*c -
 b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c -
b^2)^(9/2)))/(16*a^4*c^4*e^2 + b^6*c^2*d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*
c^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b
^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3*b^2*c^3*d*f) + ((16*a^6*b^6*(4*a*c
- b^2)^(9/2) - 1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^
2)^(9/2) + 768*a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*((b^2*c^4*d^3 + 4*a^2*c^4*d
*e^2 - 4*a*b*c^4*d^2*e + 2*a*b^2*c^3*d^2*f + a^2*b^2*c^2*d*f^2 - 4*a^2*b*c^
3*d*e*f)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^4*c^4*e^2 - 4*a*b^4*
c^3*d^2 + 17*a^2*b^2*c^4*d^2 + a^4*b^2*c^2*f^2 - 36*a^3*b*c^4*d*e - 4*a^4*b
*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 18*a^3*b^2*c^3*d*f)/(a^3
*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d +
80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 3
2*a^5*b*c^4*e)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*
a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d -
 24*a*b^4*c*d))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^
5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2
*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 19
2*a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d
))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (((((4*
a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d + 80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a
^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 32*a^5*b*c^4*e)/(a^3*b^4 + 16*a^5*c^2 - 8
*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d -
 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(a^3*b^4 + 16*a^5*c^2
 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2))
)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2))
+ ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3
*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*
b*c*d))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*
a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*
f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 -
 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2
*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^2)/(32*a^4*(
4*a*c - b^2)^3*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^
3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(3*b^5*d - a^2*b^3*f - 2*a^3*c^2*e -
21*a*b^3*c*d + a^3*b*c*f + 33*a^2*b*c^2*d + 2*a^2*b^2*c*e))/(8*a^3*c^2*(4*a
```

```
*c - b^2)^3*(16*a^4*c^4*e^2 + b^6*c^2*d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*c
^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b^
3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3*b^2*c^3*d*f)*(400*a^3*c^3*d^2 - 6*b^
6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 - 291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2
- a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^
4*b*c*e*f)) - ((((((4*a^2*b^6*c^2*d - 36*a^3*b^4*c^3*d + 80*a^4*b^2*c^4*d
+ 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*c^3*f - 32*a^5*b*c^4*e)/(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*
a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(
2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^
4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a^
2*(4*a*c - b^2)^(3/2)) + ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)
*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d)*(b^3*d - 2*a^2
*b*f + 4*a^2*c*e - 6*a*b*c*d))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^4 + 16*a^5
*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)))*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) + (((4*a^4*c^4*e^2 -
 4*a*b^4*c^3*d^2 + 17*a^2*b^2*c^4*d^2 + a^4*b^2*c^2*f^2 - 36*a^3*b*c^4*d*e
- 4*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 18*a^3*b^2*c^3*
d*f)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + (((4*a^2*b^6*c^2*d - 36*a^3*b^4
*c^3*d + 80*a^4*b^2*c^4*d + 8*a^4*b^3*c^3*e - 4*a^4*b^4*c^2*f + 16*a^5*b^2*
c^3*f - 32*a^5*b*c^4*e)/(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c) + ((4*a^4*b^6*
c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4)*(2*b^6*d - 128*a^3*c^3*d + 96*a^2*b^
2*c^2*d - 24*a*b^4*c*d))/(2*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)*(4*a^2*b^6
 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*b^6*d - 128*a^3*c^3*d
 + 96*a^2*b^2*c^2*d - 24*a*b^4*c*d))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b
^4*c + 192*a^4*b^2*c^2)))*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d))/(4*a
^2*(4*a*c - b^2)^(3/2)) - ((4*a^4*b^6*c^2 - 32*a^5*b^4*c^3 + 64*a^6*b^2*c^4
)*(b^3*d - 2*a^2*b*f + 4*a^2*c*e - 6*a*b*c*d)^3)/(64*a^6*(4*a*c - b^2)^(9/2
)*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))*(16*a^6*b^6*(4*a*c - b^2)^(9/2) -
1024*a^9*c^3*(4*a*c - b^2)^(9/2) - 192*a^7*b^4*c*(4*a*c - b^2)^(9/2) + 768*
a^8*b^2*c^2*(4*a*c - b^2)^(9/2))*(96*b^6*d - 1280*a^3*c^3*d - 32*a^2*b^4*f
+ 2208*a^2*b^2*c^2*d - 864*a*b^4*c*d + 64*a^2*b^3*c*e - 192*a^3*b*c^2*e + 9
6*a^3*b^2*c*f))/(256*a^3*c^2*(4*a*c - b^2)^(7/2)*(16*a^4*c^4*e^2 + b^6*c^2*
d^2 - 12*a*b^4*c^3*d^2 + 36*a^2*b^2*c^4*d^2 + 4*a^4*b^2*c^2*f^2 - 48*a^3*b*
c^4*d*e - 16*a^4*b*c^3*e*f + 8*a^2*b^3*c^3*d*e - 4*a^2*b^4*c^2*d*f + 24*a^3
*b^2*c^3*d*f)*(400*a^3*c^3*d^2 - 6*b^6*d^2 + a^4*b^2*f^2 + 4*a^4*c^2*e^2 -
291*a^2*b^2*c^2*d^2 + 72*a*b^4*c*d^2 - a^2*b^4*d*f + 2*a^2*b^3*c*d*e - 12*a
^3*b*c^2*d*e + 6*a^3*b^2*c*d*f - 4*a^4*b*c*e*f)))*(b^3*d - 2*a^2*b*f + 4*a^
2*c*e - 6*a*b*c*d))/(2*a^2*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.66 $\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=234

$$\frac{(2bd-ae)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2bd-ae)}{a^3}-\frac{2a^2ce+cx^2\left(-abe-2a(cd-af)+b^2d\right)-ab^2e-ab(3cd-a}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -1/2*d/a^2/x^2+1/2*(-b^3*d+a*b^2*e-2*a^2*c*e+a*b*(-a*f+3*c*d)-c*(b^2*d-a*b*
e-2*a*(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(2*b^4*d-12*a*b
^2*c*d-a*b^3*e+6*a^2*b*c*e+4*a^2*c*(-a*f+3*c*d))*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/a^3/(-4*a*c+b^2)^(3/2)-(-a*e+2*b*d)*ln(x)/a^3+1/4*(-a*e+2*b*d
)*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A] time = 0.73, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1663, 1646, 1628, 634, 618, 206, 628}

$$-\frac{2a^2ce+cx^2\left(-abe-2a(cd-af)+b^2d\right)-ab^2e-ab(3cd-af)+b^3d}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(6a^2bce+4a^2c(3cd-\right.}{2a^3\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(2*a^2*x^2) - (b^3*d - a*b^2*e + 2*a^2*c*e - a*b*(3*c*d - a*f) + c*(b^2*
d - a*b*e - 2*a*(c*d - a*f))*x^2)/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
 - ((2*b^4*d - 12*a*b^2*c*d - a*b^3*e + 6*a^2*b*c*e + 4*a^2*c*(3*c*d - a*f)
)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - (
(2*b*d - a*e)*Log[x])/a^3 + ((2*b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\left(\frac{b^2}{a}\right.}{}\right.}{}$$

$$= -\frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int\left(\left(-\right.\right.\right.}{}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bd}{}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bd}{}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2bd}{}$$

$$= -\frac{d}{2a^2x^2} - \frac{b^3d-ab^2e+2a^2ce-ab(3cd-af)+c\left(b^2d-abe-2a(cd-af)\right)x^2}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(2b^4}{}$$

Mathematica [A] time = 0.66, size = 403, normalized size = 1.72

$$\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(4a^2c\left(e\sqrt{b^2-4ac}-af+3cd\right)-ab^2\left(e\sqrt{b^2-4ac}+12cd\right)+2abc\left(3ae-4d\sqrt{b^2-4ac}\right)+b^3\left(2d\sqrt{b^2-4ac}-ae\right)+2b^4d\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\log\left(\sqrt{b^2-4a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*d)/x^2 - (2*a*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d +
e*x^2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) + 4*(-2*b*d + a*e)*Log[x] + ((2*b^4*d + b^3*(2*Sqrt[b^2 - 4*a*c]*d
- a*e) + 2*a*b*c*(-4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(12*c*d + Sqrt[b^
2 - 4*a*c]*e) + 4*a^2*c*(3*c*d + Sqrt[b^2 - 4*a*c]*e - a*f))*Log[b - Sqrt[b
^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2) + ((-2*b^4*d + b^3*(2*Sqrt[b^2
- 4*a*c]*d + a*e) - 2*a*b*c*(4*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(12*c*d
 - Sqrt[b^2 - 4*a*c]*e) + 4*a^2*c*(-3*c*d + Sqrt[b^2 - 4*a*c]*e + a*f))*Log
[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3/2))/(4*a^3)
```

fricas [B] time = 7.13, size = 1764, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b^3*c - 4*a^3*b
*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15*a^2*b^3*c + 2
8*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3*b^3 - 4*a^4*b
*c)*f)*x^2 + ((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2*c^3)*d + (a*b^3
*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*c + 6*a^2*b*c^2)
*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 - 6*a^2*b^2*c + 6
*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^
4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x
^2 + a)) + 2*(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*
c^2 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*
(b^6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)
*e)*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*
c + 16*a^4*c^2)*e)*x^2)*log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2
 + 16*a^2*b*c^3)*d - (a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^
6 - 8*a*b^4*c + 16*a^2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)
*x^4 + (2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c +
 16*a^4*c^2)*e)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6
+ (a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*
a^6*c^2)*x^2), -1/4*(2*(2*(a*b^4*c - 7*a^2*b^2*c^2 + 12*a^3*c^3)*d - (a^2*b
^3*c - 4*a^3*b*c^2)*e + 2*(a^3*b^2*c - 4*a^4*c^2)*f)*x^4 + 2*((2*a*b^5 - 15
*a^2*b^3*c + 28*a^3*b*c^2)*d - (a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*e + (a^3
*b^3 - 4*a^4*b*c)*f)*x^2 - 2*((4*a^3*c^2*f - 2*(b^4*c - 6*a*b^2*c^2 + 6*a^2
*c^3)*d + (a*b^3*c - 6*a^2*b*c^2)*e)*x^6 + (4*a^3*b*c*f - 2*(b^5 - 6*a*b^3*
c + 6*a^2*b*c^2)*d + (a*b^4 - 6*a^2*b^2*c)*e)*x^4 + (4*a^4*c*f - 2*(a*b^4 -
 6*a^2*b^2*c + 6*a^3*c^2)*d + (a^2*b^3 - 6*a^3*b*c)*e)*x^2)*sqrt(-b^2 + 4*a
*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(a^2*b^4 -
8*a^3*b^2*c + 16*a^4*c^2)*d - ((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d -
(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^
2*b^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*
a^2*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*
log(c*x^4 + b*x^2 + a) + 4*((2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*d - (a*
b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3)*e)*x^6 + (2*(b^6 - 8*a*b^4*c + 16*a^2*b
^2*c^2)*d - (a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*e)*x^4 + (2*(a*b^5 - 8*a^2
```

```
*b^3*c + 16*a^3*b*c^2)*d - (a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)*e)*x^2)*log
(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

giac [A] time = 1.85, size = 287, normalized size = 1.23

$$\frac{\left(2\,b^4d-12\,ab^2cd+12\,a^2c^2d-4\,a^3cf-ab^3e+6\,a^2bce\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}-\frac{2\,b^2cdx^4-6\,ac^2dx^4+2\,a^2cfx^4-abc}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*d - 12*a*b^2*c*d + 12*a^2*c^2*d - 4*a^3*c*f - a*b^3*e + 6*a^2*b*
c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^
2 + 4*a*c)) - 1/2*(2*b^2*c*d*x^4 - 6*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a*b*c*x^
4*e + 2*b^3*d*x^2 - 7*a*b*c*d*x^2 + a^2*b*f*x^2 - a*b^2*x^2*e + 2*a^2*c*x^2
*e + a*b^2*d - 4*a^2*c*d)/((c*x^6 + b*x^4 + a*x^2)*(a^2*b^2 - 4*a^3*c)) + 1
/4*(2*b*d - a*e)*log(c*x^4 + b*x^2 + a)/a^3 - 1/2*(2*b*d - a*e)*log(x^2)/a^
3
```

maple [B] time = 0.02, size = 722, normalized size = 3.09

$$-\frac{bce\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{c^2d\,x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}+\frac{b^2cd\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^2}+\frac{cf\,x}{\left(c\,x^4+b\,x^2+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/2*d/a^2/x^2+1/a^2*ln(x)*e-2/a^3*ln(x)*b*d+1/(c*x^4+b*x^2+a)*c/(4*a*c-b^2
)*x^2*f-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b*e-1/a/(c*x^4+b*x^2+a)*c^2
/(4*a*c-b^2)*x^2*d+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^2*d+1/2/(c*x
^4+b*x^2+a)/(4*a*c-b^2)*b*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*e-1/2/a/(c*x^4+
b*x^2+a)/(4*a*c-b^2)*b^2*e-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*c*d+1/2/a^2/
(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^3*d-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*e+1/4
/a^2/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^2*e+2/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2
+a)*b*d-1/2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*d+2/(4*a*c-b^2)^(3/2)*arc
tan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c*f-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))*b*c*e-6/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a
*c-b^2)^(1/2))*c^2*d+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^
2)^(1/2))*b^3*e+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2
))*b^2*c*d-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^
4*d
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 12.98, size = 11879, normalized size = 50.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(2*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 7*a*b*c*d))/(2*a^2*(4*a*c
- b^2)) - d/(2*a) + (c*x^4*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d))/(2*a^2*(4
*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (log(x)*(a*e - 2*b*d))/a^3 + (log((
((((a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*
a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((2*c^3*x^2*(2*b^4
*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e + 20*a^3*c*f + 4*a*b^2*c*d + 10*a
^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2*(2*b^4*d + 6*a^2*c^2*d - a*b^3*e
- 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (b*c^2*(a*
e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*
c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*
c*x^2))/a^3))/(4*a^3) + (c^3*(4*a^5*c*f^2 - 16*b^6*d^2 - 4*a^2*b^4*e^2 + 36
*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^5*d*e - 216*a^2*b^2*c^2*d^2 + 116*
a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*e*f - 24*a^4*c^2*d*f - 92*a^2*b^3*
c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*d*f - 36*a^4*b*c*e*f))/(a^4*(4*a*c
 - b^2)^2) - (2*c^4*x^2*(12*b^5*d^2 + 2*a^4*b*f^2 + 3*a^2*b^3*e^2 + 138*a^2
*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f - 82*a*b^3*c*d^2 - 10*a^3*b*c*e^2
+ 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3*b^2*e*f + 61*a^2*b^2*c*d*e - 52*a
^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(a*e - 2*b*d + a^3*(-(2*b^4*d + 12*a^2*
c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b
^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b*d)*(2*b^2*d + 2*a^2*f - a*b*e - 6
*a*c*d)^2)/(a^6*(4*a*c - b^2)^2) + (c^5*x^2*(2*b^2*d + 2*a^2*f - a*b*e - 6*
a*c*d)^3)/(a^6*(4*a*c - b^2)^3))*(((((2*b*d - a*e + a^3*(-(2*b^4*d + 12*a^2
*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c -
b^2)^3))^(1/2))*((2*c^3*x^2*(2*b^4*d - 60*a^2*c^2*d - 8*a^2*b^2*f - a*b^3*e
 + 20*a^3*c*f + 4*a*b^2*c*d + 10*a^2*b*c*e))/(a^2*(4*a*c - b^2)) + (4*b*c^2
*(2*b^4*d + 6*a^2*c^2*d - a*b^3*e - 2*a^3*c*f - 10*a*b^2*c*d + 5*a^2*b*c*e)
)/(a^2*(4*a*c - b^2)) - (b*c^2*(2*b*d - a*e + a^3*(-(2*b^4*d + 12*a^2*c^2*d
 - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3
))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3))/(4*a^3) - (c^3*(4*a^5*c*f^2
 - 16*b^6*d^2 - 4*a^2*b^4*e^2 + 36*a^3*c^3*d^2 + 17*a^3*b^2*c*e^2 + 16*a*b^
5*d*e - 216*a^2*b^2*c^2*d^2 + 116*a*b^4*c*d^2 - 16*a^2*b^4*d*f + 8*a^3*b^3*
e*f - 24*a^4*c^2*d*f - 92*a^2*b^3*c*d*e + 108*a^3*b*c^2*d*e + 72*a^3*b^2*c*
d*f - 36*a^4*b*c*e*f))/(a^4*(4*a*c - b^2)^2) + (2*c^4*x^2*(12*b^5*d^2 + 2*a
^4*b*f^2 + 3*a^2*b^3*e^2 + 138*a^2*b*c^2*d^2 - 12*a*b^4*d*e + 20*a^4*c*e*f
- 82*a*b^3*c*d^2 - 10*a^3*b*c*e^2 + 14*a^2*b^3*d*f - 60*a^3*c^2*d*e - 7*a^3
*b^2*e*f + 61*a^2*b^2*c*d*e - 52*a^3*b*c*d*f))/(a^4*(4*a*c - b^2)^2))*(2*b*
d - a*e + a^3*(-(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*
d + 6*a^2*b*c*e)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^4*(a*e - 2*b
*d)*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d)^2)/(a^6*(4*a*c - b^2)^2) + (c^5*x
^2*(2*b^2*d + 2*a^2*f - a*b*e - 6*a*c*d)^3)/(a^6*(4*a*c - b^2)^3)))*(4*b^7*
d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a
*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 -
 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (atan((x^2*((((216*a^3*c^8*d^3 - 8*b^6*
c^5*d^3 - 8*a^6*c^5*f^3 + 72*a*b^4*c^6*d^3 - 216*a^4*c^7*d^2*f + 72*a^5*c^6
*d*f^2 - 216*a^2*b^2*c^7*d^3 + a^3*b^3*c^5*e^3 + 12*a*b^5*c^5*d^2*e + 108*a
^3*b*c^7*d^2*e + 12*a^5*b*c^5*e*f^2 - 72*a^2*b^3*c^6*d^2*e - 6*a^2*b^4*c^5*
d*e^2 + 18*a^3*b^2*c^6*d*e^2 - 24*a^2*b^4*c^5*d^2*f + 144*a^3*b^2*c^6*d^2*f
 - 24*a^4*b^2*c^5*d*f^2 - 6*a^4*b^2*c^5*e^2*f - 72*a^4*b*c^6*d*e*f + 24*a^3
*b^3*c^5*d*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + ((
(80*a^6*b*c^6*e^2 - 1104*a^5*b*c^7*d^2 - 16*a^7*b*c^5*f^2 + 24*a^2*b^7*c^4*
d^2 - 260*a^3*b^5*c^5*d^2 + 932*a^4*b^3*c^6*d^2 + 6*a^4*b^5*c^4*e^2 - 44*a^
5*b^3*c^5*e^2 + 4*a^6*b^3*c^4*f^2 + 480*a^6*c^7*d*e - 160*a^7*c^6*e*f + 416
*a^6*b*c^6*d*f - 24*a^3*b^6*c^4*d*e + 218*a^4*b^4*c^5*d*e - 608*a^5*b^2*c^6
*d*e + 28*a^4*b^5*c^4*d*f - 216*a^5*b^3*c^5*d*f - 14*a^5*b^4*c^4*e*f + 96*a
^6*b^2*c^5*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + ((
(1920*a^8*c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 24*a^5*b^6*c^4*d + 120*
```

a^6*b^4*c^5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e - 36*a^6*b^5*c^4*e + 1
92*a^7*b^3*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4*f + 576*a^8*b^2*c^5*f
 - 320*a^8*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
- ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 -
 2688*a^9*b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d
 - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6
*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6
*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d
+ 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2
c^2)))(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b
^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6
- 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) - (((((1920*a^8*c^7*d - 64
0*a^9*c^6*f - 4*a^4*b^8*c^3*d + 24*a^5*b^6*c^4*d + 120*a^6*b^4*c^5*d - 1088
*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e - 36*a^6*b^5*c^4*e + 192*a^7*b^3*c^5*e + 1
6*a^6*b^6*c^3*f - 168*a^7*b^4*c^4*f + 576*a^8*b^2*c^5*f - 320*a^8*b*c^6*e)/
(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6
+ 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(
4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e
- 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^6 - 64*a^9*c^
3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c
+ 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b
^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*b*c^6 + 12
*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*b^
4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)*(4*b
^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 4
8*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8*a^3*(4*a*c - b^2)^(3/2)
*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^
6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e
 - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) + (
(2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 26
88*a^9*b^3*c^5)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*
d + 6*a^2*b*c*e)^2*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d
 - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(32
a^6(4*a*c - b^2)^3*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)
*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*a^3*c^3*d
- 6*b^6*d - 2*a^4*c^2*f + 3*a*b^5*e - 72*a^2*b^2*c^2*d + 42*a*b^4*c*d - 21*
a^2*b^3*c*e + 33*a^3*b*c^2*e + 2*a^3*b^2*c*f))/(8*a^3*c^2*(4*a*c - b^2)^3*(
36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^6*c^2*f
^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a^3*b^2*
c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 288*a^2*
b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f - 12*a^5
*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f)) + (((((((1920*a^8*
c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 24*a^5*b^6*c^4*d + 120*a^6*b^4*c^
5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e - 36*a^6*b^5*c^4*e + 192*a^7*b^3
*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4*f + 576*a^8*b^2*c^5*f - 320*a^8
*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a
^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*
b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*
b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^6 -
 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*
a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*
f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*
b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*
c^5)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b
*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2
*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8*a^3*(4*a*c -
b^2)^(3/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^
6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*

```
e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a
^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c +
192*a^5*b^2*c^2)) + (((80*a^6*b*c^6*e^2 - 1104*a^5*b*c^7*d^2 - 16*a^7*b*c^5
*f^2 + 24*a^2*b^7*c^4*d^2 - 260*a^3*b^5*c^5*d^2 + 932*a^4*b^3*c^6*d^2 + 6*a
^4*b^5*c^4*e^2 - 44*a^5*b^3*c^5*e^2 + 4*a^6*b^3*c^4*f^2 + 480*a^6*c^7*d*e -
 160*a^7*c^6*e*f + 416*a^6*b*c^6*d*f - 24*a^3*b^6*c^4*d*e + 218*a^4*b^4*c^5
*d*e - 608*a^5*b^2*c^6*d*e + 28*a^4*b^5*c^4*d*f - 216*a^5*b^3*c^5*d*f - 14*
a^5*b^4*c^4*e*f + 96*a^6*b^2*c^5*e*f)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c
+ 48*a^8*b^2*c^2) + (((1920*a^8*c^7*d - 640*a^9*c^6*f - 4*a^4*b^8*c^3*d + 2
4*a^5*b^6*c^4*d + 120*a^6*b^4*c^5*d - 1088*a^7*b^2*c^6*d + 2*a^5*b^7*c^3*e
- 36*a^6*b^5*c^4*e + 192*a^7*b^3*c^5*e + 16*a^6*b^6*c^3*f - 168*a^7*b^4*c^4
*f + 576*a^8*b^2*c^5*f - 320*a^8*b*c^6*e)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^
4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^
3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6
*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d
+ 24*a^2*b^4*c*e))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 1
28*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*
c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a
^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f
 - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) + ((2560*a^10*b
*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c
^5)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*
c*e)^3)/(64*a^9*(4*a*c - b^2)^(9/2)*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c +
48*a^8*b^2*c^2)))*(768*b^7*d + 5120*a^4*c^3*e - 384*a*b^6*e + 18432*a^2*b^3
*c^2*d - 8832*a^3*b^2*c^2*e - 6912*a*b^5*c*d - 12544*a^3*b*c^3*d + 3456*a^2
*b^4*c*e - 256*a^3*b^3*c*f + 768*a^4*b*c^2*f))/(1024*a^3*c^2*(4*a*c - b^2)^
(7/2)*(36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^
6*c^2*f^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a
^3*b^2*c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 2
88*a^2*b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f -
 12*a^5*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f)))*(16*a^9*b^
6*(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*
(4*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2)))/(144*a^4*c^6*d
^2 + 4*b^8*c^2*d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^2 + 192*a^2*b^4*c^4*d^
2 - 288*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^4*c^3*e^2 + 36*a^4*b^2
*c^4*e^2 - 96*a^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4*b*c^5*d*e - 48*a^5*b*
c^4*e*f + 48*a^2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e - 16*a^3*b^4*c^3*d*f + 9
6*a^4*b^2*c^4*d*f + 8*a^4*b^3*c^3*e*f) - ((16*a^9*b^6*(4*a*c - b^2)^(9/2) -
 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4*a*c - b^2)^(9/2) + 7
68*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*((8*b^5*c^4*d^3 - 48*a*b^3*c^5*d^3 + 7
2*a^2*b*c^6*d^3 - 36*a^3*c^6*d^2*e - 4*a^5*c^4*e*f^2 - a^3*b^2*c^4*e^3 + 24
*a^4*c^5*d*e*f - 12*a*b^4*c^4*d^2*e - 12*a^3*b*c^5*d*e^2 - 48*a^3*b*c^5*d^2
*f + 8*a^4*b*c^4*d*f^2 + 4*a^4*b*c^4*e^2*f + 48*a^2*b^2*c^5*d^2*e + 6*a^2*b
^3*c^4*d*e^2 + 16*a^2*b^3*c^4*d^2*f - 16*a^3*b^2*c^4*d*e*f)/(a^6*b^4 + 16*a
^8*c^2 - 8*a^7*b^2*c) + (((36*a^5*c^6*d^2 + 4*a^7*c^4*f^2 - 16*a^2*b^6*c^3*
d^2 + 116*a^3*b^4*c^4*d^2 - 216*a^4*b^2*c^5*d^2 - 4*a^4*b^4*c^3*e^2 + 17*a^
5*b^2*c^4*e^2 - 24*a^6*c^5*d*f + 108*a^5*b*c^5*d*e - 36*a^6*b*c^4*e*f + 16*
a^3*b^5*c^3*d*e - 92*a^4*b^3*c^4*d*e - 16*a^4*b^4*c^3*d*f + 72*a^5*b^2*c^4*
d*f + 8*a^5*b^3*c^3*e*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((72*a^5*b
^5*c^3*d - 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b
^4*c^3*e + 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c
^4*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c
^3 + 64*a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2
*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(
2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^
4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3
*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e
))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d
```

+ 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) + (((((72*a^5*b^5*c^3*d - 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8*a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^2*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(32*a^6*(4*a*c - b^2)^3*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(6*a^3*c^3*d - 6*b^6*d - 2*a^4*c^2*f + 3*a*b^5*e - 72*a^2*b^2*c^2*d + 42*a*b^4*c*d - 21*a^2*b^3*c*e + 33*a^3*b*c^2*e + 2*a^3*b^2*c*f))/(8*a^3*c^2*(4*a*c - b^2)^3*(144*a^4*c^6*d^2 + 4*b^8*c^2*d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^2 + 192*a^2*b^4*c^4*d^2 - 288*a^3*b^2*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^4*c^3*e^2 + 36*a^4*b^2*c^4*e^2 - 96*a^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4*b*c^5*d*e - 48*a^5*b*c^4*e*f + 48*a^2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e - 16*a^3*b^4*c^3*d*f + 96*a^4*b^2*c^4*d*f + 8*a^4*b^3*c^3*e*f)*(36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*a^5*c^3*e^2 + 4*a^6*c^2*f^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^4*c^2*d^2 + 1528*a^3*b^2*c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 - 24*a^5*c^3*d*f - 288*a^2*b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f + 2*a^4*b^3*c*e*f - 12*a^5*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2*d*f)) + (((((((72*a^5*b^5*c^3*d - 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(8*a^3*(4*a*c - b^2)^(3/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) - (((36*a^5*c^6*d^2 + 4*a^7*c^4*f^2 - 16*a^2*b^6*c^3*d^2 + 116*a^3*b^4*c^4*d^2 - 216*a^4*b^2*c^5*d^2 - 4*a^4*b^4*c^3*e^2 + 17*a^5*b^2*c^4*e^2 - 24*a^6*c^5*d*f + 108*a^5*b*c^5*d*e - 36*a^6*b*c^4*e*f + 16*a^3*b^5*c^3*d*e - 92*a^4*b^3*c^4*d*e - 16*a^4*b^4*c^3*d*f + 72*a^5*b^2*c^4*d*f + 8*a^5*b^3*c^3*e*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - (((72*a^5*b^5*c^3*d - 8*a^4*b^7*c^2*d - 184*a^6*b^3*c^4*d + 4*a^5*b^6*c^2*e - 36*a^6*b^4*c^3*e + 80*a^7*b^2*c^4*e + 8*a^7*b^3*c^3*f + 96*a^7*b*c^5*d - 32*a^8*b*c^4*f)/(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c) - ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d - 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(a^6*b

```
^4 + 16*a^8*c^2 - 8*a^7*b^2*c)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 19
2*a^5*b^2*c^2)))*(4*b^7*d + 128*a^4*c^3*e - 2*a*b^6*e + 192*a^2*b^3*c^2*d -
 96*a^3*b^2*c^2*e - 48*a*b^5*c*d - 256*a^3*b*c^3*d + 24*a^2*b^4*c*e))/(2*(4
*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*b^4*d + 12*a^
2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e))/(4*a^3*(4*a*c
- b^2)^(3/2)) + ((4*a^7*b^6*c^2 - 32*a^8*b^4*c^3 + 64*a^9*b^2*c^4)*(2*b^4*d
 + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^2*b*c*e)^3)/(64*
a^9*(4*a*c - b^2)^(9/2)*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))*(16*a^9*b^6*
(4*a*c - b^2)^(9/2) - 1024*a^12*c^3*(4*a*c - b^2)^(9/2) - 192*a^10*b^4*c*(4
*a*c - b^2)^(9/2) + 768*a^11*b^2*c^2*(4*a*c - b^2)^(9/2))*(768*b^7*d + 5120
*a^4*c^3*e - 384*a*b^6*e + 18432*a^2*b^3*c^2*d - 8832*a^3*b^2*c^2*e - 6912*
a*b^5*c*d - 12544*a^3*b*c^3*d + 3456*a^2*b^4*c*e - 256*a^3*b^3*c*f + 768*a^
4*b*c^2*f))/(1024*a^3*c^2*(4*a*c - b^2)^(7/2)*(144*a^4*c^6*d^2 + 4*b^8*c^2*
d^2 + 16*a^6*c^4*f^2 - 48*a*b^6*c^3*d^2 + 192*a^2*b^4*c^4*d^2 - 288*a^3*b^2
*c^5*d^2 + a^2*b^6*c^2*e^2 - 12*a^3*b^4*c^3*e^2 + 36*a^4*b^2*c^4*e^2 - 96*a
^5*c^5*d*f - 4*a*b^7*c^2*d*e + 144*a^4*b*c^5*d*e - 48*a^5*b*c^4*e*f + 48*a^
2*b^5*c^3*d*e - 168*a^3*b^3*c^4*d*e - 16*a^3*b^4*c^3*d*f + 96*a^4*b^2*c^4*d
*f + 8*a^4*b^3*c^3*e*f)*(36*a^4*c^4*d^2 - 6*a^2*b^6*e^2 - 24*b^8*d^2 + 400*
a^5*c^3*e^2 + 4*a^6*c^2*f^2 + 72*a^3*b^4*c*e^2 + 24*a*b^7*d*e - 1152*a^2*b^
4*c^2*d^2 + 1528*a^3*b^2*c^3*d^2 - 291*a^4*b^2*c^2*e^2 + 288*a*b^6*c*d^2 -
24*a^5*c^3*d*f - 288*a^2*b^5*c*d*e - 1564*a^4*b*c^3*d*e - 4*a^3*b^4*c*d*f +
 2*a^4*b^3*c*e*f - 12*a^5*b*c^2*e*f + 1158*a^3*b^3*c^2*d*e + 24*a^4*b^2*c^2
*d*f)))*(2*b^4*d + 12*a^2*c^2*d - a*b^3*e - 4*a^3*c*f - 12*a*b^2*c*d + 6*a^
2*b*c*e))/(2*a^3*(4*a*c - b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.67 $\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=329

$$-\frac{\log\left(a+bx^2+cx^4\right)\left(-2abe-a(2cd-af)+3b^2d\right)}{4a^4}+\frac{\log(x)\left(-2abe-a(2cd-af)+3b^2d\right)}{a^4}+\frac{2bd-ae}{2a^3x^2}-\frac{d}{4a^2x^4}+\frac{cx}{}$$

```
[Out] -1/4*d/a^2/x^4+1/2*(-a*e+2*b*d)/a^3/x^2+1/2*(b^4*d-a*b^3*e+3*a^2*b*c*e+2*a^
2*c*(-a*f+c*d)-a*b^2*(-a*f+4*c*d)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*
d))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/2*(3*b^5*d-2*a*b^4*e+12*a^2*b^2
*c*e-12*a^3*c^2*e+6*a^2*b*c*(-a*f+5*c*d)-a*b^3*(-a*f+20*c*d))*arctanh((2*c*
x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^(3/2)+(3*b^2*d-2*a*b*e-a*(-a*f+
2*c*d))*ln(x)/a^4-1/4*(3*b^2*d-2*a*b*e-a*(-a*f+2*c*d))*ln(c*x^4+b*x^2+a)/a^
4
```

Rubi [A] time = 1.16, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1663, 1646, 1628, 634, 618, 206, 628}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(12a^2b^2ce+6a^2bc(5cd-af)-12a^3c^2e-ab^3(20cd-af)-2ab^4e+3b^5d\right)}{2a^4\left(b^2-4ac\right)^{3/2}}+\frac{cx^2\left(2a^2ce-ab^2e-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(4*a^2*x^4) + (2*b*d - a*e)/(2*a^3*x^2) + (b^4*d - a*b^3*e + 3*a^2*b*c*e
 + 2*a^2*c*(c*d - a*f) - a*b^2*(4*c*d - a*f) + c*(b^3*d - a*b^2*e + 2*a^2*c
*e - a*b*(3*c*d - a*f))*x^2)/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((
3*b^5*d - 2*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e + 6*a^2*b*c*(5*c*d - a*
f) - a*b^3*(20*c*d - a*f))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4
*(b^2 - 4*a*c)^(3/2)) + ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[x])/a^4
- ((3*b^2*d - 2*a*b*e - a*(2*c*d - a*f))*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^5\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex+fx^2}{x^3\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2e+2a^2ce-ab\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b^3d-ab^2e+2a^2ce-ab\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

$$= -\frac{d}{4a^2x^4}+\frac{2bd-ae}{2a^3x^2}+\frac{b^4d-ab^3e+3a^2bce+2a^2c(cd-af)-ab^2(4cd-af)+c\left(b\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+c\right.}$$

Mathematica [A] time = 1.22, size = 592, normalized size = 1.80

$$-\frac{\frac{2a\left(2a^2c\left(af-c\left(d+ex^2\right)\right)+b^3\left(ae-cdx^2\right)+ab^2\left(-af+4cd+cex^2\right)-abc\left(3ae+afx^2-3cdx^2\right)+b^4(-d)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\log\left(-\sqrt{b^2-4ac}+b+2cx^2\right)\left(2a^2bc\left(4e\sqrt{b^2-4ac}-3af+1\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((a^2*d)/x^4 + (2*a*(-2*b*d + a*e))/x^2 + (2*a*(-(b^4*d) + b^3*(a*e -
c*d*x^2) + a*b^2*(4*c*d - a*f + c*e*x^2) - a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x
^2) + 2*a^2*c*(a*f - c*(d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 4*(3*b^2*d - 2*a*b*e + a*(-2*c*d + a*f))*Log[x] + ((3*b^5*d + b^4*(3*Sqrt[
b^2 - 4*a*c]*d - 2*a*e) + 2*a^2*b*c*(15*c*d + 4*Sqrt[b^2 - 4*a*c]*e - 3*a*f
) + a*b^3*(-20*c*d - 2*Sqrt[b^2 - 4*a*c]*e + a*f) - 4*a^2*c*(-2*c*Sqrt[b^2
- 4*a*c]*d + 3*a*c*e + a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-14*c*Sqrt[b^2 - 4*a
*c]*d + 12*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*
x^2])/(b^2 - 4*a*c)^(3/2) + ((-3*b^5*d + b^4*(3*Sqrt[b^2 - 4*a*c]*d + 2*a*e
) - a*b^3*(-20*c*d + 2*Sqrt[b^2 - 4*a*c]*e + a*f) + 2*a^2*b*c*(-15*c*d + 4*
Sqrt[b^2 - 4*a*c]*e + 3*a*f) + 4*a^2*c*(2*c*Sqrt[b^2 - 4*a*c]*d + 3*a*c*e -
 a*Sqrt[b^2 - 4*a*c]*f) + a*b^2*(-2*c*(7*Sqrt[b^2 - 4*a*c]*d + 6*a*e) + a*S
qrt[b^2 - 4*a*c]*f))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(3
/2))/a^4
```

fricas [B] time = 16.73, size = 2567, normalized size = 7.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*((3*a*b^5*c - 23*a^2*b^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a
^3*b^2*c^2 + 12*a^4*c^3)*e + (a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 -
 49*a^2*b^4*c + 108*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3
*c + 28*a^4*b*c^2)*e + 2*(a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a
^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*
c^2)*e)*x^2 + (((3*b^5*c - 20*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*
a^2*b^2*c^2 + 6*a^3*c^3)*e + (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 2
0*a*b^4*c + 30*a^2*b^2*c^2)*d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (
a^2*b^4 - 6*a^3*b^2*c)*f)*x^6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d
- 2*(a^2*b^4 - 6*a^3*b^2*c + 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^
2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^
5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4
*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*
(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d -
 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a
^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2 + 64*a
^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e +
 (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 6
4*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*
e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*
c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^
2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^4*c - 8
*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*x^6
 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4), 1/4*(2*((3*a*b^5*c - 23*a^2*b
^3*c^2 + 44*a^3*b*c^3)*d - 2*(a^2*b^4*c - 7*a^3*b^2*c^2 + 12*a^4*c^3)*e + (
```

```
a^3*b^3*c - 4*a^4*b*c^2)*f)*x^6 + ((6*a*b^6 - 49*a^2*b^4*c + 108*a^3*b^2*c^
2 - 32*a^4*c^3)*d - 2*(2*a^2*b^5 - 15*a^3*b^3*c + 28*a^4*b*c^2)*e + 2*(a^3*
b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*f)*x^4 + (3*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*
b*c^2)*d - 2*(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*e)*x^2 + 2*(((3*b^5*c - 2
0*a*b^3*c^2 + 30*a^2*b*c^3)*d - 2*(a*b^4*c - 6*a^2*b^2*c^2 + 6*a^3*c^3)*e +
 (a^2*b^3*c - 6*a^3*b*c^2)*f)*x^8 + ((3*b^6 - 20*a*b^4*c + 30*a^2*b^2*c^2)*
d - 2*(a*b^5 - 6*a^2*b^3*c + 6*a^3*b*c^2)*e + (a^2*b^4 - 6*a^3*b^2*c)*f)*x^
6 + ((3*a*b^5 - 20*a^2*b^3*c + 30*a^3*b*c^2)*d - 2*(a^2*b^4 - 6*a^3*b^2*c +
 6*a^4*c^2)*e + (a^3*b^3 - 6*a^4*b*c)*f)*x^4)*sqrt(-b^2 + 4*a*c)*arctan(-(2
*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (a^3*b^4 - 8*a^4*b^2*c + 16
*a^5*c^2)*d - (((3*b^6*c - 26*a*b^4*c^2 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d -
2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 +
 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)
*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c +
16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c
^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c
 + 16*a^5*c^2)*f)*x^4)*log(c*x^4 + b*x^2 + a) + 4*(((3*b^6*c - 26*a*b^4*c^2
 + 64*a^2*b^2*c^3 - 32*a^3*c^4)*d - 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c
^3)*e + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*f)*x^8 + ((3*b^7 - 26*a*b^
5*c + 64*a^2*b^3*c^2 - 32*a^3*b*c^3)*d - 2*(a*b^6 - 8*a^2*b^4*c + 16*a^3*b^
2*c^2)*e + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*f)*x^6 + ((3*a*b^6 - 26*a
^2*b^4*c + 64*a^3*b^2*c^2 - 32*a^4*c^3)*d - 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a
^4*b*c^2)*e + (a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*f)*x^4)*log(x))/((a^4*b^
4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*x^8 + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c
^2)*x^6 + (a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*x^4)]
```

giac [A] time = 1.89, size = 535, normalized size = 1.63

$$-\frac{\left(3\,b^5 d-20\,ab^3 cd+30\,a^2 bc^2 d+a^2 b^3 f-6\,a^3 bcf-2\,ab^4 e+12\,a^2 b^2 ce-12\,a^3 c^2 e\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^4 b^2-4\,a^5 c\right)\sqrt{-b^2+4\,ac}}+\frac{3\,b^4 cd x}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^5*d - 20*a*b^3*c*d + 30*a^2*b*c^2*d + a^2*b^3*f - 6*a^3*b*c*f - 2
*a*b^4*e + 12*a^2*b^2*c*e - 12*a^3*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 +
4*a*c))/((a^4*b^2 - 4*a^5*c)*sqrt(-b^2 + 4*a*c)) + 1/4*(3*b^4*c*d*x^4 - 14*
a*b^2*c^2*d*x^4 + 8*a^2*c^3*d*x^4 + a^2*b^2*c*f*x^4 - 4*a^3*c^2*f*x^4 - 2*a
*b^3*c*x^4*e + 8*a^2*b*c^2*x^4*e + 3*b^5*d*x^2 - 12*a*b^3*c*d*x^2 + 2*a^2*b
*c^2*d*x^2 + a^2*b^3*f*x^2 - 2*a^3*b*c*f*x^2 - 2*a*b^4*x^2*e + 6*a^2*b^2*c*
x^2*e + 4*a^3*c^2*x^2*e + 5*a*b^4*d - 22*a^2*b^2*c*d + 12*a^3*c^2*d + 3*a^3
*b^2*f - 8*a^4*c*f - 4*a^2*b^3*e + 14*a^3*b*c*e)/((a^4*b^2 - 4*a^5*c)*(c*x^
4 + b*x^2 + a)) - 1/4*(3*b^2*d - 2*a*c*d + a^2*f - 2*a*b*e)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(3*b^2*d - 2*a*c*d + a^2*f - 2*a*b*e)*log(x^2)/a^4 - 1/4*
(9*b^2*d*x^4 - 6*a*c*d*x^4 + 3*a^2*f*x^4 - 6*a*b*x^4*e - 4*a*b*d*x^2 + 2*a^
2*x^2*e + a^2*d)/(a^4*x^4)
```

maple [B] time = 0.03, size = 1078, normalized size = 3.28

$$-\frac{bcf\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}-\frac{c^2 e\,x^2}{\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a}+\frac{b^2 ce\,x^2}{2\left(c\,x^4+b\,x^2+a\right)\left(4ac-b^2\right)a^2}+\frac{3}{2\left(c\,x^4+b\,x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4*d/a^2/x^4+1/a^3/x^2*b*d-2/a^3*ln(x)*b*e-2/a^3*ln(x)*c*d+3/a^4*ln(x)*b^
2*d+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*f+6/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x
```

```
^2+b)/(4*a*c-b^2)^(1/2))*b^2*c*e+15/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))*b*c^2*d-10/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))*b^3*c*d-1/a/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*e-3/2/a/(c
*x^4+b*x^2+a)/(4*a*c-b^2)*b*c*e+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*c*d+2
/a^2/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*b*e-7/2/a^3/(4*a*c-b^2)*c*ln(c*x^4+b*x
^2+a)*b^2*d-3/a/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b*c
*f-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^2*f-1/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*
c^2*d-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b*f+1/2/a^2/(c*x^4+b*x^2+a)*c
/(4*a*c-b^2)*x^2*b^2*e+3/2/a^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^2*b*d-1/2/
a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^2*b^3*d-1/2/a^2/x^2*e+1/a^2*ln(x)*f+1/2
/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^3*e-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*
b^4*d+3/4/a^4/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^4*d+3/2/a^4/(4*a*c-b^2)^(3/2)
*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^5*d-6/a/(4*a*c-b^2)^(3/2)*arctan((
2*c*x^2+b)/(4*a*c-b^2)^(1/2))*c^2*e+1/2/a^2/(4*a*c-b^2)^(3/2)*arctan((2*c*x
^2+b)/(4*a*c-b^2)^(1/2))*b^3*f-1/a^3/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2))*b^4*e-1/a/(4*a*c-b^2)*c*ln(c*x^4+b*x^2+a)*f+1/4/a^2/(4*a*
c-b^2)*ln(c*x^4+b*x^2+a)*b^2*f+2/a^2/(4*a*c-b^2)*c^2*ln(c*x^4+b*x^2+a)*d-1/
2/a^3/(4*a*c-b^2)*ln(c*x^4+b*x^2+a)*b^3*e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^5/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 21.02, size = 15905, normalized size = 48.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^5*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(x)*(3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 - (log((((((((4*b*c^2*(3*
b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e - 17*a*b^3*c*d - 5*a^3*b*c*f +
19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)) - (b*c^2*(a*b + 3*b^2
*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e -
 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*
c - b^2)^3))^(1/2) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/a^4 + (2*c^3*x^2
*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a*b^3*c*d - 10*a^3*b*c
*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^2)))*(a^4*(-(3*b^5*d
+ a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^
2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) + 3*b^2*d + a^2*
f - 2*a*b*e - 2*a*c*d))/(4*a^4) + (c^3*(36*b^8*d^2 + 16*a^2*b^6*e^2 + 4*a^4
*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c*f^2 - 48*a*b^7
*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b^2*c^2*e^2 - 30
9*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^5*c*d*e + 324*a
^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^5*b*c^2*e*f - 8
32*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^2)^2) + (c^4*x^2
*(54*b^7*d^2 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*c^3*d^2 - 164*a^3
*b^3*c*e^2 + 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*b^3*c^2*d^2 - 441*
a*b^5*c*d^2 - 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*c^3*d*e - 24*a^3*b^
4*e*f - 120*a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^3*b^3*c*d*f + 260*a^4*b
*c^2*d*f + 122*a^4*b^2*c*e*f - 1072*a^3*b^2*c^2*d*e))/(a^6*(4*a*c - b^2)^2)
)*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6
```

```
*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2
) + 3*b^2*d + a^2*f - 2*a*b*e - 2*a*c*d))/(4*a^4) - (c^4*(3*b^2*d + a^2*f -
 2*a*b*e - 2*a*c*d)*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a*b*c*d
)^2)/(a^9*(4*a*c - b^2)^2) + (c^5*x^2*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^
2*c*e - 11*a*b*c*d)^3)/(a^9*(4*a*c - b^2)^3))*((((c^3*(36*b^8*d^2 + 16*a^2*
b^6*e^2 + 4*a^4*b^4*f^2 - 36*a^5*c^3*e^2 - 116*a^3*b^4*c*e^2 - 17*a^5*b^2*c
*f^2 - 48*a*b^7*d*e + 778*a^2*b^4*c^2*d^2 - 473*a^3*b^2*c^3*d^2 + 216*a^4*b
^2*c^2*e^2 - 309*a*b^6*c*d^2 + 24*a^2*b^6*d*f - 16*a^3*b^5*e*f + 380*a^2*b^
5*c*d*e + 324*a^4*b*c^3*d*e - 154*a^3*b^4*c*d*f + 92*a^4*b^3*c*e*f - 108*a^
5*b*c^2*e*f - 832*a^3*b^3*c^2*d*e + 230*a^4*b^2*c^2*d*f))/(a^6*(4*a*c - b^2
)^2) - (((b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(a^4*(-(3*b^5*d + a^2*b^3*f
- 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d +
12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) - 3*b^2*d - a^2*f + 2*a*b*e
+ 2*a*c*d))/a^4 + (4*b*c^2*(3*b^5*d + a^2*b^3*f - 6*a^3*c^2*e - 2*a*b^4*e -
 17*a*b^3*c*d - 5*a^3*b*c*f + 19*a^2*b*c^2*d + 10*a^2*b^2*c*e))/(a^3*(4*a*c
 - b^2)) + (2*c^3*x^2*(3*b^5*d + a^2*b^3*f + 60*a^3*c^2*e - 2*a*b^4*e + 4*a
*b^3*c*d - 10*a^3*b*c*f - 70*a^2*b*c^2*d - 4*a^2*b^2*c*e))/(a^3*(4*a*c - b^
2)))*(a^4*(-(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d
- 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(
1/2) - 3*b^2*d - a^2*f + 2*a*b*e + 2*a*c*d))/(4*a^4) + (c^4*x^2*(54*b^7*d^2
 + 24*a^2*b^5*e^2 + 6*a^4*b^3*f^2 - 440*a^3*b*c^3*d^2 - 164*a^3*b^3*c*e^2 +
 276*a^4*b*c^2*e^2 - 72*a*b^6*d*e + 1011*a^2*b^3*c^2*d^2 - 441*a*b^5*c*d^2
- 20*a^5*b*c*f^2 + 36*a^2*b^5*d*f + 240*a^4*c^3*d*e - 24*a^3*b^4*e*f - 120*
a^5*c^2*e*f + 540*a^2*b^4*c*d*e - 207*a^3*b^3*c*d*f + 260*a^4*b*c^2*d*f + 1
22*a^4*b^2*c*e*f - 1072*a^3*b^2*c^2*d*e))/(a^6*(4*a*c - b^2)^2))*(a^4*(-(3*
b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f +
 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2/(a^8*(4*a*c - b^2)^3))^(1/2) - 3*b^2*d
- a^2*f + 2*a*b*e + 2*a*c*d))/(4*a^4) + (c^4*(3*b^2*d + a^2*f - 2*a*b*e - 2
*a*c*d)*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a*b*c*d)^2)/(a^9*(4
*a*c - b^2)^2) - (c^5*x^2*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2*c*e - 11*a
*b*c*d)^3)/(a^9*(4*a*c - b^2)^3)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f -
 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^
3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*
c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a
^6*b^2*c^2)) - (d/(4*a) + (x^2*(2*a*e - 3*b*d))/(4*a^2) + (x^4*(6*b^4*d + 8
*a^2*c^2*d + 2*a^2*b^2*f - 4*a*b^3*e - 4*a^3*c*f - 25*a*b^2*c*d + 14*a^2*b*
c*e))/(4*a^3*(4*a*c - b^2)) + (c*x^6*(3*b^3*d - 2*a*b^2*e + a^2*b*f + 6*a^2
*c*e - 11*a*b*c*d))/(2*a^3*(4*a*c - b^2)))/(a*x^4 + b*x^6 + c*x^8) + (atan(
(x^2*((((((1760*a^7*b*c^8*d^2 - 1104*a^8*b*c^7*e^2 + 80*a^9*b*c^6*f^2 + 54*
a^3*b^9*c^4*d^2 - 657*a^4*b^7*c^5*d^2 + 2775*a^5*b^5*c^6*d^2 - 4484*a^6*b^3
*c^7*d^2 + 24*a^5*b^7*c^4*e^2 - 260*a^6*b^5*c^5*e^2 + 932*a^7*b^3*c^6*e^2 +
 6*a^7*b^5*c^4*f^2 - 44*a^8*b^3*c^5*f^2 - 960*a^8*c^8*d*e + 480*a^9*c^7*e*f
 - 1040*a^8*b*c^7*d*f - 72*a^4*b^8*c^4*d*e + 828*a^5*b^6*c^5*d*e - 3232*a^6
*b^4*c^6*d*e + 4528*a^7*b^2*c^7*d*e + 36*a^5*b^7*c^4*d*f - 351*a^6*b^5*c^5*
d*f + 1088*a^7*b^3*c^6*d*f - 24*a^6*b^6*c^4*e*f + 218*a^7*b^4*c^5*e*f - 608
*a^8*b^2*c^6*e*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)
 - (((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^
5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^4
*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^1
0*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3
- 12*a^10*b^4*c + 48*a^11*b^2*c^2) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 1
84*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8*d + 256*a^4
*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*
a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^
2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^6 - 64*a^12*c^3 -
12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c +
192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f -
 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96
*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b
```

```
^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6
*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*
b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a
*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^
6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)) - (216*a^6*c^8*e^3 + 27*
b^9*c^5*d^3 - 297*a*b^7*c^6*d^3 + 1089*a^2*b^5*c^7*d^3 - 1331*a^3*b^3*c^8*d
^3 - 8*a^3*b^6*c^5*e^3 + 72*a^4*b^4*c^6*e^3 - 216*a^5*b^2*c^7*e^3 + a^6*b^3
*c^5*f^3 - 54*a*b^8*c^5*d^2*e - 1188*a^5*b*c^8*d*e^2 + 108*a^6*b*c^7*e^2*f
+ 558*a^2*b^6*c^6*d^2*e + 36*a^2*b^7*c^5*d*e^2 - 1914*a^3*b^4*c^7*d^2*e - 3
48*a^3*b^5*c^6*d*e^2 + 2178*a^4*b^2*c^8*d^2*e + 1116*a^4*b^3*c^7*d*e^2 + 27
*a^2*b^7*c^5*d^2*f - 198*a^3*b^5*c^6*d^2*f + 363*a^4*b^3*c^7*d^2*f + 9*a^4*
b^5*c^5*d*f^2 - 33*a^5*b^3*c^6*d*f^2 + 12*a^4*b^5*c^5*e^2*f - 72*a^5*b^3*c^
6*e^2*f - 6*a^5*b^4*c^5*e*f^2 + 18*a^6*b^2*c^6*e*f^2 - 36*a^3*b^6*c^5*d*e*f
 + 240*a^4*b^4*c^6*d*e*f - 396*a^5*b^2*c^7*d*e*f)/(a^9*b^6 - 64*a^12*c^3 -
12*a^10*b^4*c + 48*a^11*b^2*c^2) + (((((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d -
 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*
e + 24*a^8*b^6*c^4*e + 120*a^9*b^4*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7*
c^3*f - 36*a^9*b^5*c^4*f + 192*a^10*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^1
1*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + ((25
60*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 268
8*a^12*b^3*c^5)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*
a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^
4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*
c*f))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b
^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f -
 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 1
2*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) + ((2560*a^13*b*c^6 + 12*a^9*b^
9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d
+ a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^
2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^
5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c
^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e -
 24*a^3*b^4*c*f))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^
10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a
^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*
d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3
/2)) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^
5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e
- 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^2*(6*b^8*d
+ 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2
*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*
d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(32*a^8*(4*a*c - b^
2)^3*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 -
 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(9*b^7*d + 3*a^2*b^5*f + 6
*a^4*c^3*e - 6*a*b^6*e + 150*a^2*b^3*c^2*d - 72*a^3*b^2*c^2*e - 69*a*b^5*c*
d - 75*a^3*b*c^3*d + 42*a^2*b^4*c*e - 21*a^3*b^3*c*f + 33*a^4*b*c^2*f))/(8*
a^3*c^2*(4*a*c - b^2)^3*(1600*a^5*c^5*d^2 - 24*a^2*b^8*e^2 - 54*b^10*d^2 -
6*a^4*b^6*f^2 + 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 + 288*a^3*b^6*c*e^2 + 72*a
^5*b^4*c*f^2 + 72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2 + 7200*a^3*b^4*c^3*d^2 -
 5775*a^4*b^2*c^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528*a^5*b^2*c^3*e^2 - 291*a
^6*b^2*c^2*f^2 + 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f + 24*a^3*b^7*e*f - 1600*a
^6*c^4*d*f - 912*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e + 456*a^3*b^6*c*d*f - 2
88*a^4*b^5*c*e*f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^5*c^2*d*e - 6900*a^4*b^3
*c^3*d*e - 2025*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*d*f + 1158*a^5*b^3*c^2*e
*f)) - (((((1760*a^7*b*c^8*d^2 - 1104*a^8*b*c^7*e^2 + 80*a^9*b*c^6*f^2 + 54
*a^3*b^9*c^4*d^2 - 657*a^4*b^7*c^5*d^2 + 2775*a^5*b^5*c^6*d^2 - 4484*a^6*b^
3*c^7*d^2 + 24*a^5*b^7*c^4*e^2 - 260*a^6*b^5*c^5*e^2 + 932*a^7*b^3*c^6*e^2
+ 6*a^7*b^5*c^4*f^2 - 44*a^8*b^3*c^5*f^2 - 960*a^8*c^8*d*e + 480*a^9*c^7*e*
f - 1040*a^8*b*c^7*d*f - 72*a^4*b^8*c^4*d*e + 828*a^5*b^6*c^5*d*e - 3232*a^
```

6*b^4*c^6*d*e + 4528*a^7*b^2*c^7*d*e + 36*a^5*b^7*c^4*d*f - 351*a^6*b^5*c^5
*d*f + 1088*a^7*b^3*c^6*d*f - 24*a^6*b^6*c^4*e*f + 218*a^7*b^4*c^5*e*f - 60
8*a^8*b^2*c^6*e*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2
) - (((1920*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c
^5*d + 1248*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^
4*c^5*e - 1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^
10*b^3*c^5*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3
 - 12*a^10*b^4*c + 48*a^11*b^2*c^2) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 -
184*a^10*b^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8*d + 256*a^
4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576
*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a
^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^6 - 64*a^12*c^3 -
 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c +
 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f
- 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 9
6*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*
b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(
3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f
 + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) - (((((192
0*a^11*c^7*e + 6*a^6*b^9*c^3*d - 40*a^7*b^7*c^4*d - 108*a^8*b^5*c^5*d + 124
8*a^9*b^3*c^6*d - 4*a^7*b^8*c^3*e + 24*a^8*b^6*c^4*e + 120*a^9*b^4*c^5*e -
1088*a^10*b^2*c^6*e + 2*a^8*b^7*c^3*f - 36*a^9*b^5*c^4*f + 192*a^10*b^3*c^5
*f - 2240*a^10*b*c^7*d - 320*a^11*b*c^6*f)/(a^9*b^6 - 64*a^12*c^3 - 12*a^10
*b^4*c + 48*a^11*b^2*c^2) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b
^7*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(6*b^8*d + 256*a^4*c^4*d +
2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c
^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e
 + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b
^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b
^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d -
6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2))
 + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7*c^3 + 1056*a^11*b^5*c^
4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20
*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^8*d + 256*
a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 5
76*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48
*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(8*a^4*(4*a*c - b^2)^(3/2
)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^4*c + 48*a^11*b^2*c^2)*(4*a^4*b^6 - 25
6*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*
a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3
*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e +
 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^
4*c + 192*a^6*b^2*c^2)) + ((2560*a^13*b*c^6 + 12*a^9*b^9*c^2 - 184*a^10*b^7
*c^3 + 1056*a^11*b^5*c^4 - 2688*a^12*b^3*c^5)*(3*b^5*d + a^2*b^3*f - 12*a^3
*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b
^2*c*e)^3)/(64*a^12*(4*a*c - b^2)^(9/2)*(a^9*b^6 - 64*a^12*c^3 - 12*a^10*b^
4*c + 48*a^11*b^2*c^2)))*(4608*b^8*d + 40960*a^4*c^4*d + 1536*a^2*b^6*f - 2
0480*a^5*c^3*f - 3072*a*b^7*e + 138240*a^2*b^4*c^2*d - 145920*a^3*b^2*c^3*d
 - 73728*a^3*b^3*c^2*e + 35328*a^4*b^2*c^2*f - 44544*a*b^6*c*d + 27648*a^2*
b^5*c*e + 50176*a^4*b*c^3*e - 13824*a^3*b^4*c*f))/(4096*a^3*c^2*(4*a*c - b^
2)^(7/2)*(1600*a^5*c^5*d^2 - 24*a^2*b^8*e^2 - 54*b^10*d^2 - 6*a^4*b^6*f^2 +
 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 + 288*a^3*b^6*c*e^2 + 72*a^5*b^4*c*f^2 +
72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2 + 7200*a^3*b^4*c^3*d^2 - 5775*a^4*b^2*c
^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528*a^5*b^2*c^3*e^2 - 291*a^6*b^2*c^2*f^2
+ 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f + 24*a^3*b^7*e*f - 1600*a^6*c^4*d*f - 91
2*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e + 456*a^3*b^6*c*d*f - 288*a^4*b^5*c*e*
f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^5*c^2*d*e - 6900*a^4*b^3*c^3*d*e - 2025
*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*d*f + 1158*a^5*b^3*c^2*e*f))))*(16*a^12*
b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*

```
c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2)))/(144*a^6*c^6
*e^2 + 9*b^10*c^2*d^2 - 120*a*b^8*c^3*d^2 + 580*a^2*b^6*c^4*d^2 - 1200*a^3*
b^4*c^5*d^2 + 900*a^4*b^2*c^6*d^2 + 4*a^2*b^8*c^2*e^2 - 48*a^3*b^6*c^3*e^2
+ 192*a^4*b^4*c^4*e^2 - 288*a^5*b^2*c^5*e^2 + a^4*b^6*c^2*f^2 - 12*a^5*b^4*
c^3*f^2 + 36*a^6*b^2*c^4*f^2 - 12*a*b^9*c^2*d*e - 720*a^5*b*c^6*d*e + 144*a
^6*b*c^5*e*f + 152*a^2*b^7*c^3*d*e - 672*a^3*b^5*c^4*d*e + 1200*a^4*b^3*c^5
*d*e + 6*a^2*b^8*c^2*d*f - 76*a^3*b^6*c^3*d*f + 300*a^4*b^4*c^4*d*f - 360*a
^5*b^2*c^5*d*f - 4*a^3*b^7*c^2*e*f + 48*a^4*b^5*c^3*e*f - 168*a^5*b^3*c^4*e
*f) - ((16*a^12*b^6*(4*a*c - b^2)^(9/2) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2)
 - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2) + 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2
))*((27*b^8*c^4*d^3 - 216*a*b^6*c^5*d^3 - 72*a^5*b*c^6*e^3 - 72*a^5*c^7*d*e
^2 + 36*a^6*c^6*e^2*f + 495*a^2*b^4*c^6*d^3 - 242*a^3*b^2*c^7*d^3 - 8*a^3*b
^5*c^4*e^3 + 48*a^4*b^3*c^5*e^3 + a^6*b^2*c^4*f^3 - 54*a*b^7*c^4*d^2*e + 26
4*a^4*b*c^7*d^2*e + 12*a^6*b*c^5*e*f^2 + 396*a^2*b^5*c^5*d^2*e + 36*a^2*b^6
*c^4*d*e^2 - 798*a^3*b^3*c^6*d^2*e - 240*a^3*b^4*c^5*d*e^2 + 420*a^4*b^2*c^
6*d*e^2 + 27*a^2*b^6*c^4*d^2*f - 144*a^3*b^4*c^5*d^2*f + 165*a^4*b^2*c^6*d^
2*f + 9*a^4*b^4*c^4*d*f^2 - 24*a^5*b^2*c^5*d*f^2 + 12*a^4*b^4*c^4*e^2*f - 4
8*a^5*b^2*c^5*e^2*f - 6*a^5*b^3*c^4*e*f^2 - 156*a^5*b*c^6*d*e*f - 36*a^3*b^
5*c^4*d*e*f + 168*a^4*b^3*c^5*d*e*f)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)
 + (((36*a^8*c^6*e^2 - 36*a^3*b^8*c^3*d^2 + 309*a^4*b^6*c^4*d^2 - 778*a^5*b
^4*c^5*d^2 + 473*a^6*b^2*c^6*d^2 - 16*a^5*b^6*c^3*e^2 + 116*a^6*b^4*c^4*e^2
 - 216*a^7*b^2*c^5*e^2 - 4*a^7*b^4*c^3*f^2 + 17*a^8*b^2*c^4*f^2 - 324*a^7*b
*c^6*d*e + 108*a^8*b*c^5*e*f + 48*a^4*b^7*c^3*d*e - 380*a^5*b^5*c^4*d*e + 8
32*a^6*b^3*c^5*d*e - 24*a^5*b^6*c^3*d*f + 154*a^6*b^4*c^4*d*f - 230*a^7*b^2
*c^5*d*f + 16*a^6*b^5*c^3*e*f - 92*a^7*b^3*c^4*e*f)/(a^9*b^4 + 16*a^11*c^2
- 8*a^10*b^2*c) + (((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d + 348*a^8*b^4*c^4
*d - 304*a^9*b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*e - 184*a^9*b^3*c
^4*e + 4*a^8*b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^4*f + 96*a^10*b*c
^5*e)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b
^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*
c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2
*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 2
4*a^3*b^4*c*f))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*
a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^
2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d
 - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 2
56*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*
c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3
*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e
+ 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a
^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2))
 - (((((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d + 348*a^8*b^4*c^4*d - 304*a^9*
b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*e - 184*a^9*b^3*c^4*e + 4*a^8*
b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^4*f + 96*a^10*b*c^5*e)/(a^9*b^
4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a
^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b
^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b
^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f
))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*
a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^
4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4
*(4*a*c - b^2)^(3/2)) + ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^
4)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b
*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^
6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 1
92*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a
^4*b*c^3*e - 24*a^3*b^4*c*f))/(8*a^4*(4*a*c - b^2)^(3/2)*(a^9*b^4 + 16*a^11
*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*
c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a
```

^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) -
((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(3*b^5*d + a^2*b^3*f
- 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d +
12*a^2*b^2*c*e)^2*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f -
4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*
a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^
4*c*f))/(32*a^8*(4*a*c - b^2)^3*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a
^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(9*b^7*d + 3*a^2*b
^5*f + 6*a^4*c^3*e - 6*a*b^6*e + 150*a^2*b^3*c^2*d - 72*a^3*b^2*c^2*e - 69*
a*b^5*c*d - 75*a^3*b*c^3*d + 42*a^2*b^4*c*e - 21*a^3*b^3*c*f + 33*a^4*b*c^2
*f))/(8*a^3*c^2*(4*a*c - b^2)^3*(144*a^6*c^6*e^2 + 9*b^10*c^2*d^2 - 120*a*b
^8*c^3*d^2 + 580*a^2*b^6*c^4*d^2 - 1200*a^3*b^4*c^5*d^2 + 900*a^4*b^2*c^6*d
^2 + 4*a^2*b^8*c^2*e^2 - 48*a^3*b^6*c^3*e^2 + 192*a^4*b^4*c^4*e^2 - 288*a^5
*b^2*c^5*e^2 + a^4*b^6*c^2*f^2 - 12*a^5*b^4*c^3*f^2 + 36*a^6*b^2*c^4*f^2 -
12*a*b^9*c^2*d*e - 720*a^5*b*c^6*d*e + 144*a^6*b*c^5*e*f + 152*a^2*b^7*c^3*
d*e - 672*a^3*b^5*c^4*d*e + 1200*a^4*b^3*c^5*d*e + 6*a^2*b^8*c^2*d*f - 76*a
^3*b^6*c^3*d*f + 300*a^4*b^4*c^4*d*f - 360*a^5*b^2*c^5*d*f - 4*a^3*b^7*c^2*
e*f + 48*a^4*b^5*c^3*e*f - 168*a^5*b^3*c^4*e*f)*(1600*a^5*c^5*d^2 - 24*a^2*
b^8*e^2 - 54*b^10*d^2 - 6*a^4*b^6*f^2 + 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 +
288*a^3*b^6*c*e^2 + 72*a^5*b^4*c*f^2 + 72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2
+ 7200*a^3*b^4*c^3*d^2 - 5775*a^4*b^2*c^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528
*a^5*b^2*c^3*e^2 - 291*a^6*b^2*c^2*f^2 + 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f +
24*a^3*b^7*e*f - 1600*a^6*c^4*d*f - 912*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e
+ 456*a^3*b^6*c*d*f - 288*a^4*b^5*c*e*f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^
5*c^2*d*e - 6900*a^4*b^3*c^3*d*e - 2025*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*
d*f + 1158*a^5*b^3*c^2*e*f)) + (((((((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d
+ 348*a^8*b^4*c^4*d - 304*a^9*b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*
e - 184*a^9*b^3*c^4*e + 4*a^8*b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^
4*f + 96*a^10*b*c^5*e)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^
6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2
*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d
- 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 25
6*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*
(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2
*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c
^2*d + 12*a^2*b^2*c*e))/(4*a^4*(4*a*c - b^2)^(3/2)) + ((4*a^10*b^6*c^2 - 32
*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*
b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)*(6*b^
8*d + 256*a^4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4
*c^2*d - 576*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^
6*c*d + 48*a^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(8*a^4*(4*a*c -
b^2)^(3/2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*a^4*b^6 - 256*a^7*c^3
- 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^6*f
- 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 192*
a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a^4*
b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192
*a^6*b^2*c^2)) + (((36*a^8*c^6*e^2 - 36*a^3*b^8*c^3*d^2 + 309*a^4*b^6*c^4*d
^2 - 778*a^5*b^4*c^5*d^2 + 473*a^6*b^2*c^6*d^2 - 16*a^5*b^6*c^3*e^2 + 116*a
^6*b^4*c^4*e^2 - 216*a^7*b^2*c^5*e^2 - 4*a^7*b^4*c^3*f^2 + 17*a^8*b^2*c^4*f
^2 - 324*a^7*b*c^6*d*e + 108*a^8*b*c^5*e*f + 48*a^4*b^7*c^3*d*e - 380*a^5*b
^5*c^4*d*e + 832*a^6*b^3*c^5*d*e - 24*a^5*b^6*c^3*d*f + 154*a^6*b^4*c^4*d*f
- 230*a^7*b^2*c^5*d*f + 16*a^6*b^5*c^3*e*f - 92*a^7*b^3*c^4*e*f)/(a^9*b^4
+ 16*a^11*c^2 - 8*a^10*b^2*c) + (((12*a^6*b^8*c^2*d - 116*a^7*b^6*c^3*d + 3
48*a^8*b^4*c^4*d - 304*a^9*b^2*c^5*d - 8*a^7*b^7*c^2*e + 72*a^8*b^5*c^3*e -
184*a^9*b^3*c^4*e + 4*a^8*b^6*c^2*f - 36*a^9*b^4*c^3*f + 80*a^10*b^2*c^4*f
+ 96*a^10*b*c^5*e)/(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c) + ((4*a^10*b^6*c
^2 - 32*a^11*b^4*c^3 + 64*a^12*b^2*c^4)*(6*b^8*d + 256*a^4*c^4*d + 2*a^2*b^
6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576*a^3*b^2*c^3*d - 1
92*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a^2*b^5*c*e + 256*a

```
^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)*(4*
a^4*b^6 - 256*a^7*c^3 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(6*b^8*d + 256*a^
4*c^4*d + 2*a^2*b^6*f - 128*a^5*c^3*f - 4*a*b^7*e + 336*a^2*b^4*c^2*d - 576
*a^3*b^2*c^3*d - 192*a^3*b^3*c^2*e + 96*a^4*b^2*c^2*f - 76*a*b^6*c*d + 48*a
^2*b^5*c*e + 256*a^4*b*c^3*e - 24*a^3*b^4*c*f))/(2*(4*a^4*b^6 - 256*a^7*c^3
 - 48*a^5*b^4*c + 192*a^6*b^2*c^2)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e -
2*a*b^4*e - 20*a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/
(4*a^4*(4*a*c - b^2)^(3/2)) - ((4*a^10*b^6*c^2 - 32*a^11*b^4*c^3 + 64*a^12*
b^2*c^4)*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*a*b^3*c*d - 6
*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e)^3)/(64*a^12*(4*a*c - b^2)^(9/
2)*(a^9*b^4 + 16*a^11*c^2 - 8*a^10*b^2*c)))*(16*a^12*b^6*(4*a*c - b^2)^(9/2
) - 1024*a^15*c^3*(4*a*c - b^2)^(9/2) - 192*a^13*b^4*c*(4*a*c - b^2)^(9/2)
+ 768*a^14*b^2*c^2*(4*a*c - b^2)^(9/2))*(4608*b^8*d + 40960*a^4*c^4*d + 153
6*a^2*b^6*f - 20480*a^5*c^3*f - 3072*a*b^7*e + 138240*a^2*b^4*c^2*d - 14592
0*a^3*b^2*c^3*d - 73728*a^3*b^3*c^2*e + 35328*a^4*b^2*c^2*f - 44544*a*b^6*c
*d + 27648*a^2*b^5*c*e + 50176*a^4*b*c^3*e - 13824*a^3*b^4*c*f))/(4096*a^3*
c^2*(4*a*c - b^2)^(7/2)*(144*a^6*c^6*e^2 + 9*b^10*c^2*d^2 - 120*a*b^8*c^3*d
^2 + 580*a^2*b^6*c^4*d^2 - 1200*a^3*b^4*c^5*d^2 + 900*a^4*b^2*c^6*d^2 + 4*a
^2*b^8*c^2*e^2 - 48*a^3*b^6*c^3*e^2 + 192*a^4*b^4*c^4*e^2 - 288*a^5*b^2*c^5
*e^2 + a^4*b^6*c^2*f^2 - 12*a^5*b^4*c^3*f^2 + 36*a^6*b^2*c^4*f^2 - 12*a*b^9
*c^2*d*e - 720*a^5*b*c^6*d*e + 144*a^6*b*c^5*e*f + 152*a^2*b^7*c^3*d*e - 67
2*a^3*b^5*c^4*d*e + 1200*a^4*b^3*c^5*d*e + 6*a^2*b^8*c^2*d*f - 76*a^3*b^6*c
^3*d*f + 300*a^4*b^4*c^4*d*f - 360*a^5*b^2*c^5*d*f - 4*a^3*b^7*c^2*e*f + 48
*a^4*b^5*c^3*e*f - 168*a^5*b^3*c^4*e*f)*(1600*a^5*c^5*d^2 - 24*a^2*b^8*e^2
- 54*b^10*d^2 - 6*a^4*b^6*f^2 + 36*a^6*c^4*e^2 + 400*a^7*c^3*f^2 + 288*a^3*
b^6*c*e^2 + 72*a^5*b^4*c*f^2 + 72*a*b^9*d*e - 3480*a^2*b^6*c^2*d^2 + 7200*a
^3*b^4*c^3*d^2 - 5775*a^4*b^2*c^4*d^2 - 1152*a^4*b^4*c^2*e^2 + 1528*a^5*b^2
*c^3*e^2 - 291*a^6*b^2*c^2*f^2 + 720*a*b^8*c*d^2 - 36*a^2*b^8*d*f + 24*a^3*
b^7*e*f - 1600*a^6*c^4*d*f - 912*a^2*b^7*c*d*e + 3020*a^5*b*c^4*d*e + 456*a
^3*b^6*c*d*f - 288*a^4*b^5*c*e*f - 1564*a^6*b*c^3*e*f + 4032*a^3*b^5*c^2*d*
e - 6900*a^4*b^3*c^3*d*e - 2025*a^4*b^4*c^2*d*f + 3510*a^5*b^2*c^3*d*f + 11
58*a^5*b^3*c^2*e*f)))*(3*b^5*d + a^2*b^3*f - 12*a^3*c^2*e - 2*a*b^4*e - 20*
a*b^3*c*d - 6*a^3*b*c*f + 30*a^2*b*c^2*d + 12*a^2*b^2*c*e))/(2*a^4*(4*a*c -
 b^2)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**5/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.68 $\int \frac{x^6\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=550

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-b^3c(cd-34af)-19ab^2c^2e+4abc^2(2cd-13af)-5b^5f+3b^4ce}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(\cdots\right.}{2\sqrt{2}\,c^{7/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] (-2*b*f+c*e)*x/c^3+1/3*f*x^3/c^2+1/2*x*(a*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*
a*f+c*d))+(b^3*c*e-3*a*b*c^2*e-b^4*f-b^2*c*(-4*a*f+c*d)+2*a*c^2*(-a*f+c*d))
*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c*(-24*a*f+c*d)+
2*a*c^2*(-7*a*f+3*c*d)+(-3*b^4*c*e+19*a*b^2*c^2*e-20*a^2*c^3*e+5*b^5*f+b^3*
c*(-34*a*f+c*d)-4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*
a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*e-13*a*b*c^2*e-5*b^4*f-b^2*c*(-24*a*
f+c*d)+2*a*c^2*(-7*a*f+3*c*d)+(3*b^4*c*e-19*a*b^2*c^2*e+20*a^2*c^3*e-5*b^5*
f-b^3*c*(-34*a*f+c*d)+4*a*b*c^2*(-13*a*f+2*c*d))/(-4*a*c+b^2)^(1/2))/c^(7/2
)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 13.23, antiderivative size = 550, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1668, 1676, 1166, 205}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{20a^2c^3e-19ab^2c^2e-b^3c(cd-34af)+4abc^2(2cd-13af)+3b^4ce-5b^5f}{\sqrt{b^2-4ac}}-b^2c(cd-24af)-13abc^2e+2ac^2(\cdots\right.}{2\sqrt{2}\,c^{7/2}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*e - 2*b*f)*x)/c^3 + (f*x^3)/(3*c^2) + (x*(a*(b^2*c*e - 2*a*c^2*e - b^3*
f - b*c*(c*d - 3*a*f)) + (b^3*c*e - 3*a*b*c^2*e - b^4*f - b^2*c*(c*d - 4*a*
f) + 2*a*c^2*(c*d - a*f))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
 ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c
*d - 7*a*f) - (3*b^4*c*e - 19*a*b^2*c^2*e + 20*a^2*c^3*e - 5*b^5*f - b^3*c*
(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a
*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*c*e - 13*a*b*c^2*e - 5*b^4*f - b
^2*c*(c*d - 24*a*f) + 2*a*c^2*(3*c*d - 7*a*f) + (3*b^4*c*e - 19*a*b^2*c^2*e
 + 20*a^2*c^3*e - 5*b^5*f - b^3*c*(c*d - 34*a*f) + 4*a*b*c^2*(2*c*d - 13*a*
f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c
]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^6\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}dx = \frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2a\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f-b^2c(cd-4af)+2a\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx\right.}$$

$$= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx\right.}$$

$$= \frac{(ce-2bf)x}{c^3}+\frac{fx^3}{3c^2}+\frac{x\left(a\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)+\left(b^3ce-3abc^2e-b^4f\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx\right.}$$

Mathematica [A] time = 2.13, size = 648, normalized size = 1.18

$$-\frac{6\sqrt{c}\,x\left(a^2c\left(2c\left(e+fx^2\right)-3bf\right)+a\left(b^3f-b^2c\left(e+4fx^2\right)+bc^2\left(d+3ex^2\right)-2c^3dx^2\right)+b^2x^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(abc^2\left(13e\sqrt{b^2-4ac}-52\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*e - 2*b*f)*x + 4*c^(3/2)*f*x^3 - (6*Sqrt[c]*x*(b^2*(c^2*d -
b*c*e + b^2*f)*x^2 + a^2*c*(-3*b*f + 2*c*(e + f*x^2)) + a*(b^3*f - 2*c^3*d*
```

```
x^2 + b*c^2*(d + 3*e*x^2) - b^2*c*(e + 4*f*x^2))))/((b^2 - 4*a*c)*(a + b*x^
2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*f + a*b*c^2*(8*c*d + 13*Sqrt[b^2 - 4*a*c]*
e - 52*a*f) - b^3*c*(c*d + 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + b^4*(3*c*e + 5
*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt[b^2 - 4*a*c]*d - 19*a*c*e - 24*a*Sqrt
[b^2 - 4*a*c]*f) + 2*a*c^2*(-3*c*Sqrt[b^2 - 4*a*c]*d + 10*a*c*e + 7*a*Sqrt[
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*f + b^
3*c*(c*d - 3*Sqrt[b^2 - 4*a*c]*e - 34*a*f) + a*b*c^2*(-8*c*d + 13*Sqrt[b^2
- 4*a*c]*e + 52*a*f) + b^4*(-3*c*e + 5*Sqrt[b^2 - 4*a*c]*f) + b^2*c*(c*Sqrt
[b^2 - 4*a*c]*d + 19*a*c*e - 24*a*Sqrt[b^2 - 4*a*c]*f) - 2*a*c^2*(3*c*Sqrt[
b^2 - 4*a*c]*d + 10*a*c*e - 7*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2
- 4*a*c]]))/(12*c^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 9.04, size = 8957, normalized size = 16.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*c^2*d*x^3 - 2*a*c^3*d*x^3 + b^4*f*x^3 - 4*a*b^2*c*f*x^3 + 2*a^2*c
^2*f*x^3 - b^3*c*x^3*e + 3*a*b*c^2*x^3*e + a*b*c^2*d*x + a*b^3*f*x - 3*a^2*
b*c*f*x - a*b^2*c*x*e + 2*a^2*c^2*x*e)/((b^2*c^3 - 4*a*c^4)*(c*x^4 + b*x^2
+ a)) + 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c
^3 - 4*a*c^4)^2*d + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*c^4 - 112*a^3*
c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 + 44*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 110*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 48*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 56*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 48*(
b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2*c^3 - 4*a*c^4)^2*f
- (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
```

*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2
*c^3 - 4*a*c^4)^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 2*a*b^5*c^7 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^2*c^8 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 + 16*a^2*b^3*
c^8 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^9 - 32*a^3*b*c^9 +
2*(b^2 - 4*a*c)*a*b^3*c^7 - 8*(b^2 - 4*a*c)*a^2*b*c^8)*d*abs(b^2*c^3 - 4*a*
c^4) + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^6*c^5 - 10*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b^3*c^6 + 78*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*
c^6 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 118*a^2*b^5*c^6
 - 304*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^7 - 464*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b*c^8 + 608*a^4*b*c^8 + 10*(b^2 - 4*a*c)*a*b^5*c^5 - 78*(b^2
- 4*a*c)*a^2*b^3*c^6 + 152*(b^2 - 4*a*c)*a^3*b*c^7)*f*abs(b^2*c^3 - 4*a*c^4
) - 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^5*c^6 - 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 +
 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 68*a^2*b^4*c^7 - 160
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^8 - 256*a^3*b^2*c^8 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*c^9 + 320*a^4*c^9 + 6*(b^2 - 4*a*c)*a*b^4*c^6 - 44*(b^2 - 4*a*c)*a^2*b^2
c^7 + 80(b^2 - 4*a*c)*a^3*c^8)*abs(b^2*c^3 - 4*a*c^4)*e - (2*b^8*c^10 - 3
2*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^3*b^2*c^13 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^4*c^11 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^12 - 2*(b^2 - 4*a*c)*b^6*c^10 + 24*(b^2 - 4*
a*c)*a*b^4*c^11 - 64*(b^2 - 4*a*c)*a^2*b^2*c^12)*d - (10*b^10*c^8 - 148*a*b
^8*c^9 + 808*a^2*b^6*c^10 - 1920*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*
c)*b^8*c^8 + 108*(b^2 - 4*a*c)*a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 +
 416*(b^2 - 4*a*c)*a^3*b^2*c^11)*f + (6*b^9*c^9 - 86*a*b^7*c^10 + 440*a^2*b
^5*c^11 - 928*a^3*b^3*c^12 + 640*a^4*b*c^13 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^7 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c

+ sqrt(b^2 - 4*a*c)*c)*b^8*c^8 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6*(b^2 - 4*a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^2*b^3*c^11 + 160*(b^2 - 4*a*c)*a^3*b*c^12)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*a*b*c^4 + sqrt((b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 - 4*a*c^5)))/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*c^8 + 48*a^3*b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*c^10 - 8*a^2*b^2*c^10 + 16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/16*((2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2*c^3 - 4*a*c^4)^2*d + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2*c^3 - 4*a*c^4)^2*f - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*e + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 2*a*b^5*c^7 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 - 16*a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^9 + 32*a^3*b*c^9 - 2*(b^2 - 4*a*c)*a*b^3*c^7 + 8*(b^2 - 4*a*c)*a^2*b*c^8)*d*abs(b^2*c^3 - 4*a*c^4) + 2*(5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 + 10*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^6 + 78*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 118*a^2*b^5*c^6 - 304*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 464*a^3*b^3*c^7 + 76

```
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 608*a^4*b*c^8 - 10*(b^
2 - 4*a*c)*a*b^5*c^5 + 78*(b^2 - 4*a*c)*a^2*b^3*c^6 - 152*(b^2 - 4*a*c)*a^3
*b*c^7)*f*abs(b^2*c^3 - 4*a*c^4) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^6*c^5 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 6*a*b^6*c^6 + 128*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^4*c^7 - 68*a^2*b^4*c^7 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*c^8 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + 256*a^3*b^2*c^8 + 40*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^9 - 320*a^4*c^9 - 6*(b^2 - 4*a*c)*a*b^4
*c^6 + 44*(b^2 - 4*a*c)*a^2*b^2*c^7 - 80*(b^2 - 4*a*c)*a^3*c^8)*abs(b^2*c^3
 - 4*a*c^4)*e - (2*b^8*c^10 - 32*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^3*b^
2*c^13 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8
+ 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 80*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 - 24*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^11 - 32*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^12 - 2*(b^2 -
 4*a*c)*b^6*c^10 + 24*(b^2 - 4*a*c)*a*b^4*c^11 - 64*(b^2 - 4*a*c)*a^2*b^2*c
^12)*d - (10*b^10*c^8 - 148*a*b^8*c^9 + 808*a^2*b^6*c^10 - 1920*a^3*b^4*c^1
1 + 1664*a^4*b^2*c^12 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*b^10*c^6 + 74*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^8*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^9*c^7 - 404*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^6*c^8 - 108*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b
^8*c^8 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^4*c^9 + 376*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^9 + 54*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^6*c^9 - 832*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^10 - 416*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^3*c^10 - 188*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^4*c^10 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^2*c^11 - 10*(b^2 - 4*a*c)*b^8*c^8 + 108*(b^2 - 4*a*c)*a*b^6*c^9 - 3
76*(b^2 - 4*a*c)*a^2*b^4*c^10 + 416*(b^2 - 4*a*c)*a^3*b^2*c^11)*f + (6*b^9*
c^9 - 86*a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a^4*b*c^13
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^7 + 43*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 + 6*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 - 220*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 - 62*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^9 + 464*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 96*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 80*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6*(b^2 - 4*
a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^2*b^3*c^11
 + 160*(b^2 - 4*a*c)*a^3*b*c^12)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*
a*b*c^4 - sqrt((b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4
 - 4*a*c^5)))/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*
c^8 + 48*a^3*b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*
```

```
c^10 - 8*a^2*b^2*c^10 + 16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/3*(
c^4*f*x^3 - 6*b*c^3*f*x + 3*c^4*x*e)/c^6
```

maple [B] time = 0.06, size = 2558, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)

[Out] 1/3*f*x^3/c^2-2/c^3*b*f*x+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a^2*f-1/2/c^2
/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*e+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^
3*b^2*d+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*e+1/2/c^3/(c*x^4+b*x^2+a)/(4*
a*c-b^2)*x^3*b^4*f+3/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d-3/2/(4*a*c-b^2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*a*d+13/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e+6/c
^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*f-6/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*a*b^2*f-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*
d-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1
/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*a*b*d+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*a*b*d-5/4/c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b^5*f+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*e-1
/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-5/4/c^3/(4*
a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5*f+3/4/c^2/(4*a*c-b^2)/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*e-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(
1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e+17/2/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*a*b^3*f-13/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a^2*b*f+17/2/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a
*b^3*f-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*e-
13/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*b*f+1/c^2*e*x
-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*d+1/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2
)*x*b^3*f-2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b^2*f+3/2/c/(c*x^4+b*x^2+
a)/(4*a*c-b^2)*x^3*a*b*e-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*b*f-1/2/
c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*e+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^
2)*x*b*d+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e+5/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*e-3/4/c^2/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^3*e+1/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d-7/2
```

```
/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*f-5/4/c^3/(4*a*c-b^2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^4*f+3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e-1/4/c/(4*a*c-b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b^2*d+7/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*f+5/4/c^
3/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(((b^2*c^2 - 2*a*c^3)*d - (b^3*c - 3*a*b*c^2)*e + (b^4 - 4*a*b^2*c + 2
*a^2*c^2)*f)*x^3 + (a*b*c^2*d - (a*b^2*c - 2*a^2*c^2)*e + (a*b^3 - 3*a^2*b*
c)*f)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*x^4 + (b^3*c^3 - 4*a*
b*c^4)*x^2) + 1/2*integrate((a*b*c^2*d + ((b^2*c^2 - 6*a*c^3)*d - (3*b^3*c
- 13*a*b*c^2)*e + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*f)*x^2 - (3*a*b^2*c - 1
0*a^2*c^2)*e + (5*a*b^3 - 19*a^2*b*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 -
 4*a*c^4) + 1/3*(c*f*x^3 + 3*(c*e - 2*b*f)*x)/c^3
```

mupad [B] time = 4.10, size = 33799, normalized size = 61.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] x*(e/c^2 - (2*b*f)/c^3) + ((x^3*(b^4*f + b^2*c^2*d + 2*a^2*c^2*f - 2*a*c^3*
d - b^3*c*e + 3*a*b*c^2*e - 4*a*b^2*c*f))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2
*e + a*b^3*f + a*b*c^2*d - a*b^2*c*e - 3*a^2*b*c*f))/(2*(4*a*c - b^2)))/(a*
c^3 + c^4*x^4 + b*c^3*x^2) - atan(((((10240*a^5*c^9*e + 192*a^2*b^5*c^7*d -
 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2
*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f - 16
*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 19456*a
^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x
*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2
)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7
*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^
4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^
5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3
*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6
*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*
e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c
^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c -
 b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c
*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e
 - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 1
4784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a
^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2
) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f
+ 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c
```

```
 - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d
*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^
9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*
b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^
7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2
 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*
d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6
*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 15
04*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^
3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*
a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*
a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9
)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360
*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a
*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e
*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c -
 b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^
5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*
e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f -
69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3
*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e
*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e
*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c -
b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*
(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186
*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24
*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 614
4*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^
2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*
a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 7
18*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4
*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c
^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394*a*b
^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f
 - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*
(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^
13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*
a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 +
 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^
6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2
- 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 2
5*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2
 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*
c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f
^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d
*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*
b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*
(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*
a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*
b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*
c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) -
 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3
*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6
*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^
```

2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(
1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^1
2*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*
c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i - (((10240*a^5*c^9*e + 192*a^2*b^5*c^7
*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4
*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f
- 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 194
56*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))
+ (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b
*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 384
0*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^
4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767
*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040
*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^
3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^
11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a
*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b
^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6
*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f
- 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f +
42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*
e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4
*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c
^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2)
+ 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^
8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*
(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2
*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*
c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880
*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2
- 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 1065
6*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*
c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6
366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219
744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^
2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 1
5360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 1
52*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c
^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^
8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^
8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*
f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3
*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c
^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c
^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) -
186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7
- 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 -
6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^
6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 -

```
114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2
 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794
*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b
^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394
*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4
*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))
/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 +
9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3
840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e
^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^
7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*
e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2
 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4
*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(
4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13
*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c
^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 72
4*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*
f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1
548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*
a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*
b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/
2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132
*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280
*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*
a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^
9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 +
 b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*
b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((10240*a^5*c^9*e + 192*a^2*b^5
*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752
*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^
7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f -
 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^
7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-
(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a
^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 +
 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 3024
0*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^
9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 3
5767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 21
5040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^1
2*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*
a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-
(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165
*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6
*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4
*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*
f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)
^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*
c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a
*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*
a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^
2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1
```

/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(1
6*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13
*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^
5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 2
6880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*
d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 -
10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*
a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 -
 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c
- b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2
 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f
 + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^
12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^
2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^
2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^
6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6
*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b
^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b
^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*
c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*
c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^
11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^
4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^
2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4
*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 +
2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e +
10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f +
 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4
*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e
f))/(2(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^
2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2
 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c
^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^
2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*
c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7
*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2
)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7
*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2
*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*
b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^
13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f
 + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*
c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
 - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30
720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*
a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)
^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 3
9132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 16
1280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c
^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) -
 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b
^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^
13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*
a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((10240*a^5*c^9*e + 192*a^2*b^
5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 1075

```
2*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c
^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f
- 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c
^7)) + (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*
a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2
+ 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 302
40*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)
^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 -
35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 2
15040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2
*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^
12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258
*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(
-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 16
5*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^
6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^
4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d
*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2
)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6
*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f
*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*
a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4
*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a
^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(
1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^1
3*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a
^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 +
26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6
*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 -
 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 + 25
*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2
- 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^2*(-(4*a*c
 - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^
2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*
f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b
^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*f^2*(
-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a
^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b
^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c
^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) -
6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*
b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*
b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2
*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1
/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12
*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c
^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a
^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d
^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^
4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 +
 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e +
 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f
+ 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^
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4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*
e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d
^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^
2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*
c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a
^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9
*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^
7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a*c - b^
2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^
7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3*c^3*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a
*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b
^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f
+ 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2
*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 3
0720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352
*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f +
39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 1
61280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*
c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2)
- 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4*a*c -
b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840
*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (216*a^4*c^6*d^3 + 225*a^4*b^6
*f^3 - 2744*a^7*c^3*f^3 - 1300*a^5*b*c^4*e^3 - 2060*a^5*b^4*c*f^3 + 125*a^2
*b^8*d*f^2 + 600*a^5*c^5*d*e^2 - 175*a^3*b^7*e*f^2 - 1512*a^5*c^5*d^2*f + 3
528*a^6*c^4*d*f^2 - 1400*a^6*c^4*e^2*f + 5*a^2*b^4*c^4*d^3 - 66*a^3*b^2*c^5
*d^3 - 63*a^3*b^5*c^2*e^3 + 573*a^4*b^3*c^3*e^3 + 5334*a^6*b^2*c^2*f^3 - 92
4*a^4*b*c^5*d^2*e - 1350*a^3*b^6*c*d*f^2 + 210*a^3*b^6*c*e^2*f + 1485*a^4*b
^5*c*e*f^2 - 364*a^6*b*c^3*e*f^2 - 30*a^2*b^5*c^3*d^2*e + 45*a^2*b^6*c^2*d*
e^2 + 339*a^3*b^3*c^4*d^2*e - 402*a^3*b^4*c^3*d*e^2 + 762*a^4*b^2*c^4*d*e^2
+ 50*a^2*b^6*c^2*d^2*f - 600*a^3*b^4*c^3*d^2*f + 2002*a^4*b^2*c^4*d^2*f +
4835*a^4*b^4*c^2*d*f^2 - 6598*a^5*b^2*c^3*d*f^2 - 1927*a^4*b^4*c^2*e^2*f +
4722*a^5*b^2*c^3*e^2*f - 3061*a^5*b^3*c^2*e*f^2 - 150*a^2*b^7*c*d*e*f + 231
2*a^5*b*c^4*d*e*f + 1480*a^3*b^5*c^2*d*e*f - 4122*a^4*b^3*c^3*d*e*f)/(4*(64
*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))))*(-(25*b^15*f^2 + b^1
1*c^4*d^2 + 9*b^13*c^2*e^2 + 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9
*c^5*d^2 - 3840*a^5*b*c^9*d^2 - 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*
a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f
+ 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*
a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5
*b^3*c^7*e^2 + 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*d^2*(-(4*a
*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928
*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 - 49*a^3
*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f
+ 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*
c^8*d*f + 724*a*b^12*c^2*e*f - 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 246*
a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*
d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f
+ 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f + 42*a^2*c^4*d*f*(-(4*a*c -
b^2)^9)^(1/2) - 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3
*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*
e*f - 161280*a^6*b^2*c^7*e*f + 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 51
*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)
^(1/2) - 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*e*f*(-(4

```
*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*2i - atan(((((10240*a^5*c
^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3
*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f
+ 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e
- 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c
^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 -
 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^
2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b
*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504
*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*
b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^
2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^
2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^
5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^
(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a
^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b
^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f
 + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b
^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*
d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e
+ 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69
120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d
*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f
 - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f
 - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-
(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a
^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a
*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*
a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a
^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^1
1*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9
*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*
a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f
+ 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*
a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5
*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a
*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928
*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3
*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
- 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f
 + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*
c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*
a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*
d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f
+ 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c -
 b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3
*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*
e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51
*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)
^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4
*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2 - 72*a^
3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2
 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2
```

+ 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*
a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f
- 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 1
48*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5
*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f +
3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15
*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2)
- 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1
/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b
^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d
^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 -
44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^
2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^
2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a
^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43
520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1
/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^
4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^
7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*
(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^
2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^
5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c
- b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^
2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^
3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i - (((10240*a
^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224
*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^
6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^
5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b
^4*c^6 - 48*a^2*b^2*c^7)) + (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e
^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^
9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a
^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 -
1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*
a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^
4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 636
6*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 21974
4*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 153
60*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152
*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2
*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*
c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*
d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f
- 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c
^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4
*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7
*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c
- b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*
f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 1
86*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 -
24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6
144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 7

68*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((10240*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^

5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^
8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12
*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c
^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*
b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 268
80*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^
2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10
656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^
2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) +
6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 2
19744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 -
15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f +
152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12
*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4
*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*
b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*
c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*
d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b
^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8
*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2
*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^
3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^
7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11
- 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10
+ 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f
^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) -
27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^1
4*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2
+ 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 4
4800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^
2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2
+ 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2
+ 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7
*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 4352
0*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2
) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c
- b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*
b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*
c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*
b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*
b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c -
b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/
(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*
b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*f^2
- 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^8
*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^2
*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^2
- 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4*
c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^6
*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^2
*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c^

3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((10240*a^5*c^9*e + 192*a^2*b^5*c^7*d - 768*a^3*b^3*c^8*d - 736*a^2*b^6*c^6*e + 4224*a^3*b^4*c^7*e - 10752*a^4*b^2*c^8*e + 1264*a^2*b^7*c^5*f - 7488*a^3*b^5*c^6*f + 19712*a^4*b^3*c^7*f - 16*a*b^7*c^6*d + 1024*a^4*b*c^9*d + 48*a*b^8*c^5*e - 80*a*b^9*c^4*f - 19456*a^5*b*c^8*f)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*b^3*c^8*d^

```
2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*c^6*e^2 -
44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^4*d
^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b^9*c^3*f^2
 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b^3*c^6*f^2
 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a*c - b^2)^
9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e + 35840*a^
7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3*d*f + 435
20*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e - 22400*a^4
*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 14784*a^3*b^7
*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2*c^4*d*f*(
-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2
*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f + 201600*a^5
*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c - b^2)^9)^(
1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e*(-(4*a*c
 - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a*b^3*c^2
*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3
*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*f^
2 - 72*a^3*c^7*d^2 + 200*a^4*c^6*e^2 + b^6*c^4*d^2 - 392*a^5*c^5*f^2 + 9*b^
8*c^2*e^2 - 16*a*b^4*c^5*d^2 - 114*a*b^6*c^3*e^2 - 30*b^9*c*e*f + 74*a^2*b^
2*c^6*d^2 + 481*a^2*b^4*c^4*e^2 - 718*a^3*b^2*c^5*e^2 + 1676*a^2*b^6*c^2*f^
2 - 3536*a^3*b^4*c^3*f^2 + 2794*a^4*b^2*c^4*f^2 - 340*a*b^8*c*f^2 + 336*a^4
*c^6*d*f - 6*b^7*c^3*d*e + 10*b^8*c^2*d*f + 86*a*b^5*c^4*d*e + 472*a^3*b*c^
6*d*e - 148*a*b^6*c^3*d*f + 394*a*b^7*c^2*e*f - 1768*a^4*b*c^5*e*f - 374*a^
2*b^3*c^5*d*e + 698*a^2*b^4*c^4*d*f - 1132*a^3*b^2*c^5*d*f - 1804*a^2*b^5*c
^3*e*f + 3266*a^3*b^3*c^4*e*f))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(
-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*b*c^7*f
^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 3840*a^4*
b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a^4*b^5*
c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
 b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 35767*a^3*b
^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 215040*a^6*b
^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2*(-(4*a
*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c^3*d*e
+ 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b^11*c^3
*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*a*c - b
^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*f
^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^6*d*e -
 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*f - 147
84*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f - 42*a^2
*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
- 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5*e*f +
201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(4*a*c -
 b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^4*d*e
*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 184
*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c - b^2)
^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9
- 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (216
*a^4*c^6*d^3 + 225*a^4*b^6*f^3 - 2744*a^7*c^3*f^3 - 1300*a^5*b*c^4*e^3 - 20
60*a^5*b^4*c*f^3 + 125*a^2*b^8*d*f^2 + 600*a^5*c^5*d*e^2 - 175*a^3*b^7*e*f^
2 - 1512*a^5*c^5*d^2*f + 3528*a^6*c^4*d*f^2 - 1400*a^6*c^4*e^2*f + 5*a^2*b^
4*c^4*d^3 - 66*a^3*b^2*c^5*d^3 - 63*a^3*b^5*c^2*e^3 + 573*a^4*b^3*c^3*e^3 +
 5334*a^6*b^2*c^2*f^3 - 924*a^4*b*c^5*d^2*e - 1350*a^3*b^6*c*d*f^2 + 210*a^
3*b^6*c*e^2*f + 1485*a^4*b^5*c*e*f^2 - 364*a^6*b*c^3*e*f^2 - 30*a^2*b^5*c^3
*d^2*e + 45*a^2*b^6*c^2*d*e^2 + 339*a^3*b^3*c^4*d^2*e - 402*a^3*b^4*c^3*d*e
```

```
^2 + 762*a^4*b^2*c^4*d*e^2 + 50*a^2*b^6*c^2*d^2*f - 600*a^3*b^4*c^3*d^2*f +
 2002*a^4*b^2*c^4*d^2*f + 4835*a^4*b^4*c^2*d*f^2 - 6598*a^5*b^2*c^3*d*f^2 -
 1927*a^4*b^4*c^2*e^2*f + 4722*a^5*b^2*c^3*e^2*f - 3061*a^5*b^3*c^2*e*f^2 -
 150*a^2*b^7*c*d*e*f + 2312*a^5*b*c^4*d*e*f + 1480*a^3*b^5*c^2*d*e*f - 4122
*a^4*b^3*c^3*d*e*f)/(4*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^
7))))*(-(25*b^15*f^2 + b^11*c^4*d^2 + 9*b^13*c^2*e^2 - 25*b^6*f^2*(-(4*a*c
- b^2)^9)^(1/2) - 27*a*b^9*c^5*d^2 - 3840*a^5*b*c^9*d^2 + 9*a*c^5*d^2*(-(4*
a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*e^2 + 26880*a^6*b*c^8*e^2 - 80640*a^7*
b*c^7*f^2 - 30*b^14*c*e*f + 288*a^2*b^7*c^6*d^2 - 1504*a^3*b^5*c^7*d^2 + 38
40*a^4*b^3*c^8*d^2 + 2077*a^2*b^9*c^4*e^2 - 10656*a^3*b^7*c^5*e^2 + 30240*a
^4*b^5*c^6*e^2 - 44800*a^5*b^3*c^7*e^2 - 25*a^2*c^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) - b^2*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*f^2 - 3576
7*a^3*b^9*c^3*f^2 + 116928*a^4*b^7*c^4*f^2 - 219744*a^5*b^5*c^5*f^2 + 21504
0*a^6*b^3*c^6*f^2 + 49*a^3*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^4*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*f^2 - 15360*a^6*c^9*d*e - 6*b^12*c
^3*d*e + 35840*a^7*c^8*e*f + 10*b^13*c^2*d*f + 152*a*b^10*c^4*d*e - 258*a*b
^11*c^3*d*f + 43520*a^6*b*c^8*d*f + 724*a*b^12*c^2*e*f + 30*b^5*c*e*f*(-(4*
a*c - b^2)^9)^(1/2) - 246*a^2*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*
b^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^2*b^8*c^5*d*e + 8064*a^3*b^6*c^
6*d*e - 22400*a^4*b^4*c^7*d*e + 30720*a^5*b^2*c^8*d*e + 2706*a^2*b^9*c^4*d*
f - 14784*a^3*b^7*c^5*d*f + 44352*a^4*b^5*c^6*d*f - 69120*a^5*b^3*c^7*d*f -
 42*a^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 6*b^3*c^3*d*e*(-(4*a*c - b^2)^9)
^(1/2) - 7278*a^2*b^10*c^3*e*f + 39132*a^3*b^8*c^4*e*f - 119616*a^4*b^6*c^5
*e*f + 201600*a^5*b^4*c^6*e*f - 161280*a^6*b^2*c^7*e*f - 10*b^4*c^2*d*f*(-(
4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*
c^4*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a*b^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2
) - 184*a*b^3*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2) + 186*a^2*b*c^3*e*f*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b
^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)
*2i + (f*x^3)/(3*c^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.69 $\int \frac{x^4\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=436

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+c}{}\right.}{}$$

```
[Out] f*x/c^2+1/2*x*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)-(b^2*c*e-2*a*c^2*e-b^3*f-b*
c*(-3*a*f+c*d))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*
c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-6*a*c^2*e-3*b^3*f+b*c*(13*a*
f+c*d)+(-b^3*c*e+8*a*b*c^2*e+3*b^4*f-4*a*c^2*(-5*a*f+c*d)-b^2*c*(19*a*f+c*d
))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^
(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*c*e-6
*a*c^2*e-3*b^3*f+b*c*(13*a*f+c*d)+(b^3*c*e-8*a*b*c^2*e-3*b^4*f+4*a*c^2*(-5*
a*f+c*d)+b^2*c*(19*a*f+c*d))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 5.54, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1668, 1676, 1166, 205}

$$\frac{x\left(a\left(-2acf+b^2f-bce+2c^2d\right)-x^2\left(-bc(cd-3af)-2ac^2e+b^2ce+b^3(-f)\right)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{b^2c(19af+c}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (f*x)/c^2 + (x*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) - (b^2*c*e - 2*a*c^2*
e - b^3*f - b*c*(c*d - 3*a*f))*x^2))/(2*c^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + ((b^2*c*e - 6*a*c^2*e - 3*b^3*f + b*c*(c*d + 13*a*f) - (b^3*c*e - 8*a
*b*c^2*e - 3*b^4*f + 4*a*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2
 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt
[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*c*e - 6*a*c^
2*e - 3*b^3*f + b*c*(c*d + 13*a*f) + (b^3*c*e - 8*a*b*c^2*e - 3*b^4*f + 4*a
*c^2*(c*d - 5*a*f) + b^2*c*(c*d + 19*a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c
)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^4\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \int$$

$$= \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \int\Bigl(-$$

$$= \frac{fx}{c^2} + \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} -$$

$$= \frac{fx}{c^2} + \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} +$$

$$= \frac{fx}{c^2} + \frac{x\left(a\left(2c^2d-bce+b^2f-2acf\right)-\left(b^2ce-2ac^2e-b^3f-bc(cd-3af)\right)x^2\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} +$$

Mathematica [A] time = 1.54, size = 511, normalized size = 1.17

$$\frac{2\sqrt{c}\,x\left(-2a^2cf+a\left(b^2f-bc\left(e+3fx^2\right)+2c^2\left(d+ex^2\right)\right)+bx^2\left(b^2f-bce+c^2d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2ac^2\left(3e\sqrt{b^2-4ac}-10af+2cd\right)+b^2c\left(-e\sqrt{b^2-4a\right.\right.}{\left(b^2-4ac\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*f*x + (2*Sqrt[c]*x*(-2*a^2*c*f + b*(c^2*d - b*c*e + b^2*f)*x^2 +
 a*(b^2*f + 2*c^2*(d + e*x^2) - b*c*(e + 3*f*x^2))))/((b^2 - 4*a*c)*(a + b*
x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*f + 2*a*c^2*(2*c*d + 3*Sqrt[b^2 - 4*a*c]*e
 - 10*a*f) + b^2*c*(c*d - Sqrt[b^2 - 4*a*c]*e + 19*a*f) + b^3*(c*e + 3*Sqrt
[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*d + 8*a*c*e + 13*a*Sqrt[b^2 - 4
*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
```

```
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*f + 2*a*c^2*(-2
*c*d + 3*Sqrt[b^2 - 4*a*c]*e + 10*a*f) - b^2*c*(c*d + Sqrt[b^2 - 4*a*c]*e +
 19*a*f) + b^3*(-(c*e) + 3*Sqrt[b^2 - 4*a*c]*f) - b*c*(c*Sqrt[b^2 - 4*a*c]*
d - 8*a*c*e + 13*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
4*c^(5/2))
```

fricas [B] time = 17.36, size = 12597, normalized size = 28.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(b^2*c - 4*a*c^2)*f*x^5 + 2*(b*c^2*d - (b^2*c - 2*a*c^2)*e + (3*b^3
- 11*a*b*c)*f)*x^3 + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)
*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^
4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*
c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*
((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 15
0*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^
7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e
^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*
e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4
*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)
*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 +
 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^
4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a
^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)
*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*
a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c
^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)
)*log(((3*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^
4*c^4 - 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*
d*e^3 - (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1
971*a^3*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6
*c + 3213*a^2*b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1
323*a^2*b^5*c + 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2
- 117*a*b^4*c^3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^
5*c^2 + 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c
^2 + 1672*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^
5*c^3 - 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3
 + 692*a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^
4)*e^3)*f)*x + 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 +
3*(b^5*c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*
c^6 + 16*a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^
3*b*c^6)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3
 + 11360*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4
*c^4 + 392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^
2*b^5*c^3 - 2536*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*
a*b^4*c^5 - 32*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c
^5 + 16*a^3*b*c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a
^3*b^2*c^5 + 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*
c^9 - 64*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^
3*b^2*c^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 -
 960*a^3*b^3*c^8 + 1024*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^
2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*
b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550
*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*
b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 5
```

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50*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9
*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 +
484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(
3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*
e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b
^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^
2 + 2*(b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 +
60*a^2*b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)
*f^2 - 2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4
*c^2 + 150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a
^2*b^2*c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c
^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*
a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3
+ 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125
*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)
*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51
*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c
^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4
*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*
c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*
a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64
*a^3*c^8))) - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 +
(b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 -
 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^
2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5
*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b
^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*
(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (
81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f
^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (2
7*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*
b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*
a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)
*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e
+ 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c
^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 6
4*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((
(3*b^2*c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 -
 28*a*b^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 -
 (5*a*b^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3
*b^4*c + 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 32
13*a^2*b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2
*b^5*c + 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a
*b^4*c^3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 +
 1461*a^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 16
72*a^3*b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 -
 123*a*b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*
a^2*b^2*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)
*f)*x - 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*
c^5 - 8*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 1
6*a^3*c^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6
)*e^3 + (27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 1136
0*a^4*b^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 +
392*a^3*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c
^3 - 2536*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c
^5 - 32*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16
*a^3*b*c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*
c^5 + 160*a^4*c^6)*e^2)*f + (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 6
```

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4*a^3*b*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^3*b^2*c
^9 + 768*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^
3*b^3*c^8 + 1024*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 -
 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5
 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^
2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4
 + 125*a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*
b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^
3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2
*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c
^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (
3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11
 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(
b^4*c^3 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*
b*c^4)*e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 -
2*((3*b^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 +
150*a^2*b^2*c^3 - 120*a^3*c^4)*e)*f + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*
c^7 - 64*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2
*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6
)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a
^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^
5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3
 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*
c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75
*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^
6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 4
9*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2
*c^12 - 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^
8))) + sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^
2 - 4*a*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^
2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*
b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 -
13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3
- 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8
)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^
5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8
- 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*
((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c
 - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4
 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c
^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f
^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*
b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 19
8*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c
^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3*b^2*
c^6 + 4*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 - 28*a*b
^2*c^5)*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 - (5*a*b
^4*c^3 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3*b^4*c
+ 5625*a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*
b^4*c^2 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c
+ 2727*a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^
3 - 150*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 + 1461*a
^2*b^3*c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 1672*a^3*
b^2*c^3)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 - 123*a*
b^3*c^4 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2
*c^4)*d*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x +
 1/2*sqrt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8
*a*b^3*c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3*c
^7)*d*e^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6)*e^3 +
```

```
(27*b^10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b
^2*c^4 - 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3
*b^2*c^5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 25
36*a^3*b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32
*a^2*b^2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*
c^6)*d*e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*c^5 + 1
60*a^4*c^6)*e^2)*f - (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b
*c^10)*d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^3*b^2*c^9 + 76
8*a^4*c^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c
^8 + 1024*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^
7)*d^2*e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a
^2*c^6)*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 +
 625*a^4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*
a^3*c^5)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*
e)*f^3 + 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*
a*b^3*c^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^
4 - 75*a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*
a*b*c^6)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c
^3 - 49*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3
 - 6*a*b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*
e^2 + (9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b
^5*c^2 - 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2
*b^2*c^3 - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 6
4*a^3*c^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 +
4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 +
 (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)
*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d +
(27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((
9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 6
5*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^
5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*
e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3
*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 -
 64*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) -
sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a
*b*c^3)*x^2)*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*b^2*c^4 -
 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (9*b^7 - 1
05*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2 - 13*a*b^
3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^3 - 120*a
^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt(
(c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*c^5 - 9*a
*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 - 4*((27*b^
6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7*c - 351*
a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c^4 + 3*a*
b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b*c^5)*d*e
 + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)*f^2 - 4*
((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(3*b^4*c^4
 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 + 198*a^2*b
*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/
(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log(((3*b^2*c^6 + 4
*a*c^7)*d^4 + (9*b^3*c^5 - 20*a*b*c^6)*d^3*e + 3*(3*b^4*c^4 - 28*a*b^2*c^5)
*d^2*e^2 + (3*b^5*c^3 - 65*a*b^3*c^4 + 324*a^2*b*c^5)*d*e^3 - (5*a*b^4*c^3
 - 81*a^2*b^2*c^4 + 324*a^3*c^5)*e^4 - (189*a^2*b^6 - 1971*a^3*b^4*c + 5625*
a^4*b^2*c^2 - 2500*a^5*c^3)*f^4 - ((81*b^8 - 945*a*b^6*c + 3213*a^2*b^4*c^2
 - 3000*a^3*b^2*c^3 + 2000*a^4*c^4)*d - (135*a*b^7 - 1323*a^2*b^5*c + 2727*
a^3*b^3*c^2 + 2500*a^4*b*c^3)*e)*f^3 + 3*((27*b^6*c^2 - 117*a*b^4*c^3 - 150
*a^2*b^2*c^4 + 200*a^3*c^5)*d^2 + (27*b^7*c - 405*a*b^5*c^2 + 1461*a^2*b^3*
```

```
c^3 - 500*a^3*b*c^4)*d*e - (45*a*b^6*c - 558*a^2*b^4*c^2 + 1672*a^3*b^2*c^3
)*e^2)*f^2 - ((27*b^4*c^4 + 80*a^2*c^6)*d^3 + 3*(18*b^5*c^3 - 123*a*b^3*c^4
 - 100*a^2*b*c^5)*d^2*e + 3*(9*b^6*c^2 - 165*a*b^4*c^3 + 692*a^2*b^2*c^4)*d
*e^2 - (45*a*b^5*c^2 - 647*a^2*b^3*c^3 + 2268*a^3*b*c^4)*e^3)*f)*x - 1/2*sq
rt(1/2)*(2*(b^4*c^6 - 8*a*b^2*c^7 + 16*a^2*c^8)*d^3 + 3*(b^5*c^5 - 8*a*b^3*
c^6 + 16*a^2*b*c^7)*d^2*e - 18*(a*b^4*c^5 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*d*e
^2 - (b^7*c^3 - 17*a*b^5*c^4 + 88*a^2*b^3*c^5 - 144*a^3*b*c^6)*e^3 + (27*b^
10 - 459*a*b^8*c + 2961*a^2*b^6*c^2 - 8818*a^3*b^4*c^3 + 11360*a^4*b^2*c^4
- 4000*a^5*c^5)*f^3 - 3*(2*(12*a*b^6*c^3 - 121*a^2*b^4*c^4 + 392*a^3*b^2*c^
5 - 400*a^4*c^6)*d + (9*b^9*c - 153*a*b^7*c^2 + 947*a^2*b^5*c^3 - 2536*a^3*
b^3*c^4 + 2480*a^4*b*c^5)*e)*f^2 - 3*((3*b^6*c^4 - 14*a*b^4*c^5 - 32*a^2*b^
2*c^6 + 160*a^3*c^7)*d^2 - 26*(a*b^5*c^4 - 8*a^2*b^3*c^5 + 16*a^3*b*c^6)*d*
e - 3*(b^8*c^2 - 17*a*b^6*c^3 + 98*a^2*b^4*c^4 - 224*a^3*b^2*c^5 + 160*a^4*
c^6)*e^2)*f - (4*(b^7*c^7 - 12*a*b^5*c^8 + 48*a^2*b^3*c^9 - 64*a^3*b*c^10)*
d + (b^8*c^6 - 24*a*b^6*c^7 + 192*a^2*b^4*c^8 - 640*a^3*b^2*c^9 + 768*a^4*c
^10)*e - (3*b^9*c^5 - 52*a*b^7*c^6 + 336*a^2*b^5*c^7 - 960*a^3*b^3*c^8 + 10
24*a^4*b*c^9)*f)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*
e^2 + 4*(b^3*c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)
*e^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)*f^4 - 4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5
)*d + (27*b^7*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3
+ 6*((9*b^4*c^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c
^4 - 65*a^2*b*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*
a^3*c^5)*e^2)*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6
)*d^2*e + 3*(3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49
*a*b^3*c^4 + 198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*
c^12 - 64*a^3*c^13)))*sqrt(-((b^3*c^4 + 12*a*b*c^5)*d^2 + 2*(b^4*c^3 - 6*a*
b^2*c^4 - 24*a^2*c^5)*d*e + (b^5*c^2 - 15*a*b^3*c^3 + 60*a^2*b*c^4)*e^2 + (
9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*f^2 - 2*((3*b^5*c^2
- 13*a*b^3*c^3 - 12*a^2*b*c^4)*d + (3*b^6*c - 40*a*b^4*c^2 + 150*a^2*b^2*c^
3 - 120*a^3*c^4)*e)*f - (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c
^8)*sqrt((c^8*d^4 + 4*b*c^7*d^3*e + 6*(b^2*c^6 - 3*a*c^7)*d^2*e^2 + 4*(b^3*
c^5 - 9*a*b*c^6)*d*e^3 + (b^4*c^4 - 18*a*b^2*c^5 + 81*a^2*c^6)*e^4 + (81*b^
8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*f^4 -
4*((27*b^6*c^2 - 108*a*b^4*c^3 - 180*a^2*b^2*c^4 + 125*a^3*c^5)*d + (27*b^7
*c - 351*a*b^5*c^2 + 1197*a^2*b^3*c^3 - 550*a^3*b*c^4)*e)*f^3 + 6*((9*b^4*c
^4 + 3*a*b^2*c^5 + 25*a^2*c^6)*d^2 + 2*(9*b^5*c^3 - 51*a*b^3*c^4 - 65*a^2*b
*c^5)*d*e + (9*b^6*c^2 - 132*a*b^4*c^3 + 484*a^2*b^2*c^4 - 75*a^3*c^5)*e^2)
*f^2 - 4*((3*b^2*c^6 + 5*a*c^7)*d^3 + 3*(3*b^3*c^5 - 4*a*b*c^6)*d^2*e + 3*(
3*b^4*c^4 - 22*a*b^2*c^5 - 15*a^2*c^6)*d*e^2 + (3*b^5*c^3 - 49*a*b^3*c^4 +
198*a^2*b*c^5)*e^3)*f)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3
*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + 2*(2*a*
c^2*d - a*b*c*e + (3*a*b^2 - 10*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*
c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2)
```

giac [B] time = 8.25, size = 7496, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] f*x/c^2 + 1/2*(b*c^2*d*x^3 + b^3*f*x^3 - 3*a*b*c*f*x^3 - b^2*c*x^3*e + 2*a*
c^2*x^3*e + 2*a*c^2*d*x + a*b^2*f*x - 2*a^2*c*f*x - a*b*c*x*e)/((c*x^4 + b*
x^2 + a)*(b^2*c^2 - 4*a*c^3)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4*a*
c^3)^2*d - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 -
```

```
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(
b^2*c^2 - 4*a*c^3)^2*f + (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^
4)*(b^2*c^2 - 4*a*c^3)^2*e - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 16*a^2*b
^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^8 + 32*a^3*c^8 - 2
*(b^2 - 4*a*c)*a*b^2*c^6 + 8*(b^2 - 4*a*c)*a^2*c^7)*d*abs(-b^2*c^2 + 4*a*c^
3) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5
+ 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 16
0*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^
2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)*f*abs(-b^2*c^2 + 4*a*c^3) + 2*(sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5
+ 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2
- 4*a*c)*a^2*b*c^6)*abs(-b^2*c^2 + 4*a*c^3)*e - (2*b^7*c^8 - 8*a*b^5*c^9 -
32*a^2*b^3*c^10 + 128*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^7*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^6*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*b^5*c^8 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b*c^9 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*d + (6*b^9
*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 -
3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^
```

2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*
c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2
 - 4*a*c)*a^3*b*c^9)*f - (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*
a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*
c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 -
 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 -
 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 -
4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)
*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*
c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4
)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^
3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)
*abs(-b^2*c^2 + 4*a*c^3)*abs(c)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c^2 - 4
*a*c^3)^2*d - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3
)*(b^2*c^2 - 4*a*c^3)^2*f + (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a
c^4)(b^2*c^2 - 4*a*c^3)^2*e + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^4*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*a*b^4*c^6 + 16*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 + 16*a^
2*b^2*c^7 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^8 - 32*a^3*c^8
+ 2*(b^2 - 4*a*c)*a*b^2*c^6 - 8*(b^2 - 4*a*c)*a^2*c^7)*d*abs(-b^2*c^2 + 4*a
c^3) + 2(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^5*c^4 - 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
^5 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 -
 160*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^2*c^6 - 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*c^7 + 320*a^4*c^7 + 6*(b^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2
*b^2*c^5 + 80*(b^2 - 4*a*c)*a^3*c^6)*f*abs(-b^2*c^2 + 4*a*c^3) - 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2

```
- 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^5 - 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^7 - 32*a^3*b*c^7 + 2*(b^2 - 4*a*c)*a*b^3*c^5 - 8*(b
^2 - 4*a*c)*a^2*b*c^6)*abs(-b^2*c^2 + 4*a*c^3)*e - (2*b^7*c^8 - 8*a*b^5*c^9
 - 32*a^2*b^3*c^10 + 128*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^6*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^3*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^5*c^8 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^9 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^2*c^9 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*d + (6*
b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b
^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(
b^2 - 4*a*c)*a^3*b*c^9)*f - (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 2
56*a^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^
6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^
7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2
 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c
^9)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 - sqrt((b^3*c^2 - 4*a
*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*
c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2
*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c
^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c))
```

maple [B] time = 0.05, size = 1977, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e+13/4/c/(4*a*
c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*f-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*
```

```
b*f+f*x/c^2+1/2/c/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b*e+3/2/c/(c*x^4+b*x^2+a)
/(4*a*c-b^2)*x^3*a*b*f-1/2/c^2/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^2*f-3/4/c^
2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*f+1/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2*e+3/4/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*f-1/4/c/(4*a*c-b^2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b^2*e-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)*b^2*d-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+5
/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a
rctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*f+3/4/c^2/(4*a*c-
b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*f-1/4/c/(4*a*c-b^2)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*a*b*e-1/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*d-1/(c*x^4+b*x^2
+a)/(4*a*c-b^2)*x^3*a*e-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b*d-c/(4*a*c-b^
2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*d+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*f+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*a*b*e-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
a*d-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*f-19/
4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*f-1/2/c^2/(c*x^
4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*f+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^2*
e+1/c/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*f-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*a*e+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+3/2/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*e-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+5/(4*a*c-b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bc^2d-\left(b^2c-2\,ac^2\right)e+\left(b^3-3\,abc\right)f\right)x^3+\left(2\,ac^2d-abce+\left(ab^2-2\,a^2c\right)f\right)x}{2\left(ab^2c^2-4\,a^2c^3+\left(b^2c^3-4\,ac^4\right)x^4+\left(b^3c^2-4\,abc^3\right)x^2\right)}+\frac{fx}{c^2}+\frac{-\int\frac{2\,ac^2d-abce-\left(bc^2d+\left(b^2c-6\,ac^2\right)e\right.}{cx^4}}{2\left(b^2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d - (b^2*c - 2*a*c^2)*e + (b^3 - 3*a*b*c)*f)*x^3 + (2*a*c^2*d -
 a*b*c*e + (a*b^2 - 2*a^2*c)*f)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*
c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + f*x/c^2 + 1/2*integrate(-(2*a*c^2*d
 - a*b*c*e - (b*c^2*d + (b^2*c - 6*a*c^2)*e - (3*b^3 - 13*a*b*c)*f)*x^2 + (
3*a*b^2 - 10*a^2*c)*f)/(c*x^4 + b*x^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

mupad [B] time = 2.65, size = 25862, normalized size = 59.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (f*x)/c^2 - atan(((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d +
 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c
^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7
*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^
2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*
d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 38
40*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a
^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^
(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*
c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^
4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2
)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*
b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*
a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8
*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6
*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c
^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b
^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5
+ b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4
*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) -
9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c -
 b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 -
512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4
*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5
*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
- b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*
e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e
- 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 19
2*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^
7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e
*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f +
 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1
280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8
*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6
*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^
4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^
3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*
d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c
^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((
768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c
^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2
 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*
c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2
*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c
^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*
f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^
6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5
```

*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3

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*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^
6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^
2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b
^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*
b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) -
2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b
^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d
*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 80
64*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*
c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3
*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((63*a^3*b^5*f
^3 - 216*a^4*c^4*e^3 + 3*a*b^3*c^4*d^3 + 4*a^2*b*c^5*d^3 - 573*a^4*b^3*c*f^
3 + 1300*a^5*b*c^2*f^3 - 24*a^3*c^5*d^2*e - 45*a^2*b^6*e*f^2 - 600*a^5*c^3*
e*f^2 - 5*a^2*b^4*c^2*e^3 + 66*a^3*b^2*c^3*e^3 + 27*a*b^7*d*f^2 + 240*a^4*c
^4*d*e*f + 6*a*b^4*c^3*d^2*e + 3*a*b^5*c^2*d*e^2 + 204*a^3*b*c^4*d*e^2 - 18
*a*b^5*c^2*d^2*f - 279*a^2*b^5*c*d*f^2 + 12*a^3*b*c^4*d^2*f - 420*a^4*b*c^3
*d*f^2 + 30*a^2*b^5*c*e^2*f + 402*a^3*b^4*c*e*f^2 + 924*a^4*b*c^3*e^2*f - 4
2*a^2*b^2*c^4*d^2*e - 51*a^2*b^3*c^3*d*e^2 + 81*a^2*b^3*c^3*d^2*f + 801*a^3
*b^3*c^2*d*f^2 - 339*a^3*b^3*c^2*e^2*f - 762*a^4*b^2*c^2*e*f^2 - 18*a*b^6*c
*d*e*f + 246*a^2*b^4*c^2*d*e*f - 804*a^3*b^2*c^3*d*e*f)/(4*(64*a^3*c^6 - b^
6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*a^5*c^7*f - 2048*a^4*c^8
*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b
^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 3
2*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*
a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^8*d^
2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f
^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^
7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12
*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1
504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a
^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2
*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*
a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^
11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*
a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) -
 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2
*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 153
6*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^
3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5
*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*
a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*
b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5
*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b
^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^
9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(
4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2
+ 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f
 + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^
3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7
*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(
-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11
*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2
*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*
d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a
*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*
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b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*
d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f -
 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^
2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c
^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c
^10)))^(1/2) - (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2
 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b
^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 -
 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^
3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3
*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4
*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c
- b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^1
3*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)
^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a
^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*
c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*
f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2
*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2
*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*
a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2)
- 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(
-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2
*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4
*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f -
8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^
2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a
^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c
^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^
5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 1075
2*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a
*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x
*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^1
1*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*
e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6
*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*
a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^
9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c
^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b
^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360
*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*
a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^
3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c
^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f
+ 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c -
b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11
 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4
*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^
3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768
*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*
e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 +
3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6
*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^
7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*
f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2
 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)
```

^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c
^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*
c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^
2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 5
1*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*
c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f
 + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 2240
0*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2) + (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^
5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*
a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 71
8*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^
6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^
5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(
2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2
 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^
2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*
b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*
e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 -
 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c -
b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3
072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*
a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a
*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*
e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*
e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548
*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5
*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4
*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/
2)))*((768*a^4*b*c^8*d^2 - b^9*c^4*d^2 - c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) -
 b^11*c^2*e^2 - 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*f^2 + 27*a*b^9*
c^3*e^2 + 3840*a^5*b*c^7*e^2 + 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880
*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 -
288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^
2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b
^3*c^5*f^2 - 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 1
5360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1
536*a^5*b*c^7*d*f + 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2*b*c^3*d*e*(-(
4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f + 6*b^3*c*e*f*(-(4*a*c - b^2)^9)
^(1/2) + 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 12
8*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b
^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*
e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f + 6*b^2*c^2*d*f*(-(4*a*
c - b^2)^9)^(1/2) - 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*
c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840
*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*2i - ((x^3*(b^3*f + 2*a*c^2*e + b
*c^2*d - b^2*c*e - 3*a*b*c*f))/(2*(4*a*c - b^2)) + (x*(2*a*c^2*d + a*b^2*f
 - 2*a^2*c*f - a*b*c*e))/(2*(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) -
atan(((((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^
2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*
a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 102
4*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 4
8*a^2*b^2*c^5)) - (x*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b

^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8
*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^
9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^
3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c
^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f
^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*
c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*
b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a
*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) +
2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-
(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*
b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*
d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8
064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2
*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^
3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192
*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 -
8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^
2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 +
27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/
2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*
c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2
- 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 4
4800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c
^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c
^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c
^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c
- b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^
5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f -
1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^
3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d
f(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*
(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*
c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8*f^2 + 8*a^2
*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2
*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*
a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f +
2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b
^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374
*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4
*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*
b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*
b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2
+ 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 106
56*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2
*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) +
213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f +
6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f -
10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/
2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c
f^2(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e +
1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^
4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4
*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) +
44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 2
4*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144

```
*a^5*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b
^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736
*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d
- 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*
c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/
2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)
^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^
3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^
2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^
5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2
 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 -
 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 3
6*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4
*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^
2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^
2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*
d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 15
48*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a
^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-
(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240
*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(
1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(1
6*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b
^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2)
 + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*
c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2
- 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 302
40*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*
a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^
8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c -
 b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f
- 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^
(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e +
576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2
*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2
*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b
^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) +
 (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4
*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*
a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c*f
^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e - 8*
a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f + 4
*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2
*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^1
1*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b
^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26
880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2
 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077
*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^
5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e
+ 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f
+ 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)
^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e +
```

128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*1i)/((63*a^3*b^5*f^3 - 216*a^4*c^4*e^3 + 3*a*b^3*c^4*d^3 + 4*a^2*b*c^5*d^3 - 573*a^4*b^3*c*f^3 + 1300*a^5*b*c^2*f^3 - 24*a^3*c^5*d^2*e - 45*a^2*b^6*e*f^2 - 600*a^5*c^3*e*f^2 - 5*a^2*b^4*c^2*e^3 + 66*a^3*b^2*c^3*e^3 + 27*a*b^7*d*f^2 + 240*a^4*c^4*d*e*f + 6*a*b^4*c^3*d^2*e + 3*a*b^5*c^2*d*e^2 + 204*a^3*b*c^4*d*e^2 - 18*a*b^5*c^2*d^2*f - 279*a^2*b^5*c*d*f^2 + 12*a^3*b*c^4*d^2*f - 420*a^4*b*c^3*d*f^2 + 30*a^2*b^5*c*e^2*f + 402*a^3*b^4*c*e*f^2 + 924*a^4*b*c^3*e^2*f - 42*a^2*b^2*c^4*d^2*e - 51*a^2*b^3*c^3*d*e^2 + 81*a^2*b^3*c^3*d^2*f + 801*a^3*b^3*c^2*d*f^2 - 339*a^3*b^3*c^2*e^2*f - 762*a^4*b^2*c^2*e*f^2 - 18*a*b^6*c*d*e*f + 246*a^2*b^4*c^2*d*e*f - 804*a^3*b^2*c^3*d*e*f)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (((10240*a^5*c^7*f - 2048*a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 768*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) - (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^3*f^2 - 114*a*b^6*c

*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f - 14*a*b^3*c^4*d*e -
8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f + 472*a^3*b*c^4*e*f +
4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b
^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b
^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a
*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) -
26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d
^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 20
77*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*
a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*
e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*
f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*
e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^
2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e
+ 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*
a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6
*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 +
3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2) + (((10240*a^5*c^7*f - 2048*
a^4*c^8*d - 384*a^2*b^4*c^6*d + 1536*a^3*b^2*c^7*d + 192*a^2*b^5*c^5*e - 76
8*a^3*b^3*c^6*e - 736*a^2*b^6*c^4*f + 4224*a^3*b^4*c^5*f - 10752*a^4*b^2*c^
6*f + 32*a*b^6*c^5*d - 16*a*b^7*c^4*e + 1024*a^4*b*c^7*e + 48*a*b^8*c^3*f)/
(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^4*d^2*(
-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f
^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a
^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 +
6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*
e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 +
10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*
a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f
+ 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*
f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^
2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*
e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512
*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*
b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5
- 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6
144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 76
8*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*
c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(
4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c
^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^1
2*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 +
1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*
a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^
2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213
*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b
^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10
*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
- 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^
2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 15
36*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b
^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^
5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44

```
*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a
*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^
5*b^2*c^10)))^(1/2) + (x*(9*b^8*f^2 + 8*a^2*c^6*d^2 - 72*a^3*c^5*e^2 + b^4*
c^4*d^2 + 200*a^4*c^4*f^2 + b^6*c^2*e^2 + 2*a*b^2*c^5*d^2 - 16*a*b^4*c^3*e^
2 - 6*b^7*c*e*f + 74*a^2*b^2*c^4*e^2 + 481*a^2*b^4*c^2*f^2 - 718*a^3*b^2*c^
3*f^2 - 114*a*b^6*c*f^2 - 80*a^3*c^5*d*f + 2*b^5*c^3*d*e - 6*b^6*c^2*d*f -
14*a*b^3*c^4*d*e - 8*a^2*b*c^5*d*e + 32*a*b^4*c^3*d*f + 86*a*b^5*c^2*e*f +
472*a^3*b*c^4*e*f + 4*a^2*b^2*c^4*d*f - 374*a^2*b^3*c^3*e*f))/(2*(16*a^2*c^
5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d
^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a
^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 3840*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2
- 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a
^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b
^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 + 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2
) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*
d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*
e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*
c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) -
192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*
b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f + 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3
*e*f - 8064*a^3*b^6*c^4*e*f + 22400*a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f
 - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^
9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 -
 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)))*((c^4*d
^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^4*d^2 - 9*b^13*f^2 - b^11*c^2*e^2 + 9*b
^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^8*d^2 + 27*a*b^9*c^3*e^2 + 38
40*a^5*b*c^7*e^2 - 9*a*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*f
^2 + 6*b^12*c*e*f + 96*a^2*b^5*c^6*d^2 - 512*a^3*b^3*c^7*d^2 - 288*a^2*b^7*
c^4*e^2 + 1504*a^3*b^5*c^5*e^2 - 3840*a^4*b^3*c^6*e^2 - 2077*a^2*b^9*c^2*f^
2 + 10656*a^3*b^7*c^3*f^2 - 30240*a^4*b^5*c^4*f^2 + 44800*a^5*b^3*c^5*f^2 +
 25*a^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^2*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 213*a*b^11*c*f^2 - 3072*a^5*c^8*d*e - 2*b^10*c^3*d*e + 15360*a^6*c^7
*e*f + 6*b^11*c^2*d*f + 36*a*b^8*c^4*d*e - 98*a*b^9*c^3*d*f + 1536*a^5*b*c^
7*d*f - 10*a*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^3*d*e*(-(4*a*c - b^2)
^9)^(1/2) - 152*a*b^10*c^2*e*f - 6*b^3*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 51*
a*b^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^5*d*e + 128*a^3*b^4*c^
6*d*e + 1536*a^4*b^2*c^7*d*e + 576*a^2*b^7*c^4*d*f - 1344*a^3*b^5*c^5*d*f +
 512*a^4*b^3*c^6*d*f + 1548*a^2*b^8*c^3*e*f - 8064*a^3*b^6*c^4*e*f + 22400*
a^4*b^4*c^5*e*f - 30720*a^5*b^2*c^6*e*f - 6*b^2*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 44*a*b*c^2*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*
c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9
 - 6144*a^5*b^2*c^10)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.70 $\int \frac{x^2(d+ex^2+fx^4)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=362

$$-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^3f+b^2ce}{c\sqrt{b^2-4ac}}+6af\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*x*(b*c*d-2*a*c*e+a*b*f+(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)*x^2)/c/(-4*a*c+b
^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*(2*c*d-b*e+6*a*f-b^2*f/c+(b^2*c*e+4*a*c^2*e+b^3*f-4*b*c*(2*a*f+c*d))/c/
(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-b*e+6*
a*f-b^2*f/c+(-b^2*c*e-4*a*c^2*e-b^3*f+4*b*c*(2*a*f+c*d))/c/(-4*a*c+b^2)^(1/
2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.50, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1668, 1166, 205}

$$-\frac{x\left(x^2\left(-2acf+b^2f-bce+2c^2d\right)+abf-2ace+bcd\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{-4bc(2af+cd)+4ac^2e+b^2ce+b^3f}{c\sqrt{b^2-4ac}}+6af\right.}{2\sqrt{2}\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -(x*(b*c*d - 2*a*c*e + a*b*f + (2*c^2*d - b*c*e + b^2*f - 2*a*c*f)*x^2))/(2
*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - ((2*c*d - b*e + 6*a*f - (b^2*f)/c +
 (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c
]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((2*c*d - b*e + 6*a*f - (b^2
*f)/c - (b^2*c*e + 4*a*c^2*e + b^3*f - 4*b*c*(c*d + 2*a*f))/(c*Sqrt[b^2 - 4
*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]
*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
```

```
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2+fx^4\right)}{\left(a+bx^2+cx^4\right)^2}dx = -\frac{x\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-\frac{a(bcd-2ace+abf)}{c}+a\left(2cd-be+6af-\right.}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right)}$$

$$= -\frac{x\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}-\frac{b^2ce}{}\right.}{}$$

$$= -\frac{x\left(bcd-2ace+abf+\left(2c^2d-bce+b^2f-2acf\right)x^2\right)}{2c\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\left(2cd-be+6af-\frac{b^2f}{c}+\frac{b^2ce}{}\right.}{2\sqrt{2}\sqrt{c}\left(b^2\right.}$$

Mathematica [A] time = 1.10, size = 414, normalized size = 1.14

$$\frac{-\frac{2\sqrt{c}x\left(abf-2ac\left(e+fx^2\right)+b^2fx^2+bc\left(d-ex^2\right)+2c^2dx^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(bc\left(e\sqrt{b^2-4ac}+8af+4cd\right)-2c\left(cd\sqrt{b^2-4ac}+3af\sqrt{b^2-4ac}+2ace\right)+b\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}}{4c^{3/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*Sqrt[c]*x*(a*b*f + 2*c^2*d*x^2 + b^2*f*x^2 + b*c*(d - e*x^2) - 2*a*c*(
e + f*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*f) + b*c
*(4*c*d + Sqrt[b^2 - 4*a*c]*e + 8*a*f) + b^2*(-(c*e) + Sqrt[b^2 - 4*a*c]*f)
 - 2*c*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*f + b*c*(-4*c*d + Sqrt[b^2 - 4*a*c
]*e - 8*a*f) + b^2*(c*e + Sqrt[b^2 - 4*a*c]*f) - 2*c*(c*Sqrt[b^2 - 4*a*c]*d
 - 2*a*c*e + 3*a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
c^(3/2))
```

fricas [B] time = 8.44, size = 8951, normalized size = 24.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a
*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-((b^3*c^3
+ 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b
*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 2
8*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (a*b^6*c^3 -
```

```
 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*
e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b
^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*
d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 +
 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*
b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a
^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a*c^6)*d^4 -
 (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b
^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^2)*f^4 + ((
a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*c + 324*a^4
*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5
*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^
2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2
*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*
a^3*b*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6
)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^5*c^3 - 8*a
^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*
c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 - 144*a^5*b*c^3)*f^3 -
((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^4*b*c^4)*d + 18*(a^3*b
^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4
 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a
^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2)*f - ((a*b
^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(a^2*b^7*c^4
- 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e - (a^2*b^8*c^3 - 24*a^3
*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^6*c^7)*f)*sqrt((c^6*d^
4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f
^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 -
2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3
*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c
^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))*s
qrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3
*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((
3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f
 + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4
 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^
4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2
*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*
c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^
3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a
*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) - sqrt(1/2)*((b^
2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(
-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2
 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*
b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f + (
a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2
*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 -
4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12
*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)
*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^
3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6
*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^2*c^5 + 4*a
*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b*c^4)*d*e^3
 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c + 324*a^5*c^
2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 - 65*a^3*b^3*
c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24*a^3*c^4)*d
^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*c - 28*a^3*
b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 + 9*(a*b^3*c
^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2 + (9*a^2*b^
3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*b^3*c^5 + 1
```

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6*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^2*e - (a*b^
5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8*a^3*b^2*c^
4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 - 144*a^5*b*
c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^4*b*c^4)*d
+ 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b^5*c^3 - 8*
a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 - 32*a^3*b^2
*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4*b*c^4)*e^2
)*f - ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^8)*d - 4*(a
^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e - (a^2*b^8*c
^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^6*c^7)*f)*s
qrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81
*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2
)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*
c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^
2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a
^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d
*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)
*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^
3*c^3)*e)*f + (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sq
rt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*
a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)
*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c
^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2
 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^
5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))) + sqr
t(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)
*x^2)*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2*c^4)*d*e +
 (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3*b*c^2)*f^2
 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^
3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6)*sqrt((
c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*c + 81*a^4*
c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3*b*c^2)*e)*
f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^2*b^2*c^2 -
 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^4*d*e^2 + a
^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8 - 64*a^5*c^
9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))*log(((3*b^
2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3 + 12*a^2*b
*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*a^4*b^2*c +
 324*a^5*c^2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3*a^2*b^5 -
65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b^2*c^3 - 24
*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e + (3*a^2*b^4*
c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2*c^5)*d^3 +
 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*c^3)*d*e^2
+ (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^5*c^4 - 8*a*
b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*a^3*c^6)*d^
2*e - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*b^4*c^3 - 8
*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^4*b^3*c^2 -
 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c^3 - 240*a^
4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^2 + (7*(a*b
^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^2*b^4*c^3 -
 32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c^3 + 16*a^4
*b*c^4)*e^2)*f + ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 - 256*a^5*c^
8)*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5*b*c^7)*e -
 (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c^6 + 768*a^
6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3
*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c -
9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 -
3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a
^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2
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*c^8 - 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4
*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60
*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*
c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*
a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*
b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9
*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3
*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^
2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*
c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c
^6))) - sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c -
 4*a*b*c^2)*x^2)*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*b^2*c^3 + 4*a^2
*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2*b^3*c + 60*a^3
*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c - 6*a^2*b^2*c^2
- 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*
c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^4 - 18*a^3*b^2*
c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^2*b^3*c - 9*a^3
*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c^4)*d^2 - 3*(a^
2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d^2*e - 3*a^2*c^
4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 + 48*a^4*b^2*c^8
- 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5 - 64*a^4*c^6))
*log(((3*b^2*c^5 + 4*a*c^6)*d^4 - (b^3*c^4 + 12*a*b*c^5)*d^3*e + (a*b^3*c^3
 + 12*a^2*b*c^4)*d*e^3 - (3*a^2*b^2*c^3 + 4*a^3*c^4)*e^4 + (5*a^3*b^4 - 81*
a^4*b^2*c + 324*a^5*c^2)*f^4 + ((a*b^6 - 15*a^2*b^4*c + 432*a^4*c^3)*d - (3
*a^2*b^5 - 65*a^3*b^3*c + 324*a^4*b*c^2)*e)*f^3 - 3*(3*(a*b^4*c^2 - 6*a^2*b
^2*c^3 - 24*a^3*c^4)*d^2 - (a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d*e +
(3*a^2*b^4*c - 28*a^3*b^2*c^2)*e^2)*f^2 - ((b^4*c^3 - 24*a*b^2*c^4 - 48*a^2
*c^5)*d^3 + 9*(a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e - 3*(a*b^4*c^2 + 12*a^2*b^2*
c^3)*d*e^2 + (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((b^5
*c^4 - 8*a*b^3*c^5 + 16*a^2*b*c^6)*d^3 - 2*(a*b^4*c^4 - 8*a^2*b^2*c^5 + 16*
a^3*c^6)*d^2*e - (a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d*e^2 + 2*(a^2*
b^4*c^3 - 8*a^3*b^2*c^4 + 16*a^4*c^5)*e^3 - (a^2*b^7 - 17*a^3*b^5*c + 88*a^
4*b^3*c^2 - 144*a^5*b*c^3)*f^3 - ((a*b^7*c - 23*a^2*b^5*c^2 + 136*a^3*b^3*c
^3 - 240*a^4*b*c^4)*d + 18*(a^3*b^4*c^2 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*e)*f^
2 + (7*(a*b^5*c^3 - 8*a^2*b^3*c^4 + 16*a^3*b*c^5)*d^2 - 2*(a*b^6*c^2 - 2*a^
2*b^4*c^3 - 32*a^3*b^2*c^4 + 96*a^4*c^5)*d*e + 3*(a^2*b^5*c^2 - 8*a^3*b^3*c
^3 + 16*a^4*b*c^4)*e^2)*f + ((a*b^8*c^4 - 8*a^2*b^6*c^5 + 128*a^4*b^2*c^7 -
 256*a^5*c^8)*d - 4*(a^2*b^7*c^4 - 12*a^3*b^5*c^5 + 48*a^4*b^3*c^6 - 64*a^5
*b*c^7)*e - (a^2*b^8*c^3 - 24*a^3*b^6*c^4 + 192*a^4*b^4*c^5 - 640*a^5*b^2*c
^6 + 768*a^6*c^7)*f)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b
^4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a
^2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*
c^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*
d^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 +
 48*a^4*b^2*c^8 - 64*a^5*c^9)))*sqrt(-((b^3*c^3 + 12*a*b*c^4)*d^2 - 4*(3*a*
b^2*c^3 + 4*a^2*c^4)*d*e + (a*b^3*c^2 + 12*a^2*b*c^3)*e^2 + (a*b^5 - 15*a^2
*b^3*c + 60*a^3*b*c^2)*f^2 - 2*((3*a*b^3*c^2 - 28*a^2*b*c^3)*d - (a*b^4*c -
 6*a^2*b^2*c^2 - 24*a^3*c^3)*e)*f - (a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^
2*c^5 - 64*a^4*c^6)*sqrt((c^6*d^4 - 2*a*c^5*d^2*e^2 + a^2*c^4*e^4 + (a^2*b^
4 - 18*a^3*b^2*c + 81*a^4*c^2)*f^4 - 4*(3*(a^2*b^2*c^2 - 9*a^3*c^3)*d - (a^
2*b^3*c - 9*a^3*b*c^2)*e)*f^3 - 2*(12*a^2*b*c^3*d*e + (a*b^2*c^3 - 27*a^2*c
^4)*d^2 - 3*(a^2*b^2*c^2 - 3*a^3*c^3)*e^2)*f^2 + 4*(3*a*c^5*d^3 - a*b*c^4*d
^2*e - 3*a^2*c^4*d*e^2 + a^2*b*c^3*e^3)*f)/(a^2*b^6*c^6 - 12*a^3*b^4*c^7 +
48*a^4*b^2*c^8 - 64*a^5*c^9)))/(a*b^6*c^3 - 12*a^2*b^4*c^4 + 48*a^3*b^2*c^5
 - 64*a^4*c^6))) + 2*(b*c*d - 2*a*c*e + a*b*f)*x)/((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
```

giac [B] time = 6.81, size = 6208, normalized size = 17.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -1/2*(2*c^2*d*x^3 + b^2*f*x^3 - 2*a*c*f*x^3 - b*c*x^3*e + b*c*d*x + a*b*f*x
 - 2*a*c*x*e)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2)) - 1/16*(2*(2*b^2*c^4
- 8*a*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c
^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^4 - 2*(b^2 - 4*a*c)*c^4)
*(b^2*c - 4*a*c^2)^2*d - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^
2*c - 4*a*c^2)^2*f - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*e - 2*(sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b^3*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 2
*b^5*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c^5 + 16*a*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b*c^6 - 32*a^2*b*c^6 + 2*(b^2 - 4*a*c)*b^3*c^4 - 8*(b^2 - 4*a*c)*a*b*c^
5)*d*abs(b^2*c - 4*a*c^2) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^
2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*
c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(b^2*c -
4*a*c^2) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b
^2*c^4 - 8*(b^2 - 4*a*c)*a^2*c^5)*abs(b^2*c - 4*a*c^2)*e - 4*(2*b^6*c^6 - 1
6*a*b^4*c^7 + 32*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*
c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7
 - 2*(b^2 - 4*a*c)*b^4*c^6 + 8*(b^2 - 4*a*c)*a*b^2*c^7)*d + (2*b^8*c^4 - 32
*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
```

- 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*f + (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 + sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) + 1/16*(2*(2*b^2*c^4 - 8*a*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^4 - 2*(b^2 - 4*a*c)*c^4)*(b^2*c - 4*a*c^2)^2*d - (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*f - (2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)^2*e + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 2*b^5*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*a*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + 32*a^2*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*d*abs(b^2*c - 4*a*c^2) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*f*abs(b^2*c - 4*a*c^2) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*abs(b^2*c - 4*a*c^2)*e - 4*(2*b^6*c^6 - 16*a*b^4*c^7 + 32*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt

```
(b^2 - 4*a*c)*c)*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 8*(b^2 - 4*a*c)*a*b^2*c^7
)*d + (2*b^8*c^4 - 32*a*b^6*c^5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*
(b^2 - 4*a*c)*a*b^4*c^5 - 64*(b^2 - 4*a*c)*a^2*b^2*c^6)*f + (2*b^7*c^5 - 8*
a*b^5*c^6 - 32*a^2*b^3*c^7 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^7 - 2*(b^2 - 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*e)
*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 - sqrt((b^3*c - 4*a*b*c^2)^2
- 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))/(b^2*c^2 - 4*a*c^3)))/((a*b
^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*b^2*c^5 + 16*a^2*b^3*c^5 + a
*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^2*c^6 + 16*a^3*c^7)*abs(b^2*
c - 4*a*c^2)*abs(c))
```

maple [B] time = 0.04, size = 1300, normalized size = 3.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2*(2*a*c*f-b^2*f+b*c*e-2*c^2*d)/(4*a*c-b^2)/c*x^3+1/2*(a*b*f-2*a*c*e+b*
c*d)/(4*a*c-b^2)/c*x)/(c*x^4+b*x^2+a)-3/2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*a*f+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+1/4/(4*a*c-b^2)*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*b*e-1/2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+2/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*f-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*b^3*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^2*e+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+3/2/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f-1/4/(4*a*c-b^2)/c*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2
*f-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e+1/2/(4*a*c-b^2)*c*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*d+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
```

```
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*f-1/(4*a*c-b^
2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*e-1/4/(4*a*c-b^2)/c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*c*x)*b^2*e+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*
d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,c^2d-bce+\left(b^2-2\,ac\right)f\right)x^3+\left(bcd-2\,ace+abf\right)x}{2\left(\left(b^2c^2-4\,ac^3\right)x^4+ab^2c-4\,a^2c^2+\left(b^3c-4\,abc^2\right)x^2\right)}-\frac{\int\frac{bcd-2\,ace+abf-\left(2\,c^2d-bce-\left(b^2-6\,ac\right)f\right)x^2}{cx^4+bx^2+a}\,dx}{2\left(b^2c-4\,ac^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*c^2*d - b*c*e + (b^2 - 2*a*c)*f)*x^3 + (b*c*d - 2*a*c*e + a*b*f)*x
)/((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)
 - 1/2*integrate(-(b*c*d - 2*a*c*e + a*b*f - (2*c^2*d - b*c*e - (b^2 - 6*a*
c)*f)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^2*c - 4*a*c^2)
```

mupad [B] time = 6.54, size = 19494, normalized size = 53.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2 + f*x^4))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(2*c^2*d + b^2*f - 2*a*c*f - b*c*e))/(2*c*(4*a*c - b^2)) + (x*(a*b*f
- 2*a*c*e + b*c*d))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((204
8*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a
^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 10
24*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^
6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*a^4*b*c^7*d^2
 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*
e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*
a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*
c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a
*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c
- b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c
^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f -
192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10
*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c
^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^
7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6
+ 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7
*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*
c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^
2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*
c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 +
 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*
b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f +
12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*
a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b
```

^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*
f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*
b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^
12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^
4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e
^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^
5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*
d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f
 + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*
d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c
^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c
f^2(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 +
96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b
^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 1
2*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a
*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^
7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f
 - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b
^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^1
2*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4
*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*1i - (((2048*a^4*c^6*e + 16*b^7*c^3*d + 76
8*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*
f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e
 + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2
 + 48*a^2*b^2*c^3)) + (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a
*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2
f^2(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2
*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96
*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c
^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2
- 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f
+ 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c
^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f -
3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b
^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*
b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(
16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c +
 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-
(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a
*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27
*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b
^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*
f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*
d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b
^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*
f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a
^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a
c - b^2)^9)^(1/2))/(32(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*
a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/
2) - (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^
3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 +
16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*
c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c +
16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(
4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*
b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*

a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) +
96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^
3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f
^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d
*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^
6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f
- 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^
4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a
^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2
)*1i)/((((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c
^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b
^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*
c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((768*
a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2
- a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) +
a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2
- 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*
c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 +
1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c
^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*
d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e -
128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b
^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e
*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c
^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 38
40*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 10
24*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((
768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11
*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/
2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5
*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*
b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^
2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a
^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*
c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*
e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a
^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c
^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6
+ 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2
- 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c
^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f
+ 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*
a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((7
68*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*
f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2
) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*
f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b
^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2
+ 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^
6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c
^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e
- 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^
2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^
5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 +
3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (((2048*a^4*c^6*e + 16*b^7*
c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^

2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a
*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12
*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*
d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e
^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^
(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512
*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^
3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^
9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128
*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*
c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f +
128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e*f*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4
+ 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)
))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(
2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 -
c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c
^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)
^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 -
512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^
5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*
a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*
b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e
*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*
e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*
c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*
c^8)))^(1/2) - (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 +
72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*
c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f -
52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2
*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((768*a^4*b*c^7*d^2 - b^9*c^3*d^2 - c
^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^
6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a*c^2*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 - 9*a^2*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 -
512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5
*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a
*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b
^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*
f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f + 2*a*b*c*e
f(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c
^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c
^8)))^(1/2) + (8*a*c^5*d^3 + b^6*d*f^2 + 5*a^2*b^4*f^3 + 6*b^2*c^4*d^3 + 21
6*a^4*c^2*f^3 - 3*a*b^3*c^2*e^3 - 4*a^2*b*c^3*e^3 - 66*a^3*b^2*c*f^3 + 8*a^
2*c^4*d*e^2 + 72*a^2*c^4*d^2*f + 216*a^3*c^3*d*f^2 - 5*b^3*c^3*d^2*e + b^4*
c^2*d*e^2 + 24*a^3*c^3*e^2*f - 5*b^4*c^2*d^2*f - 3*a*b^5*e*f^2 - 28*a*b*c^4
*d^2*e - 12*a*b^4*c*d*f^2 - 6*a*b^4*c*e^2*f + 18*a*b^2*c^3*d*e^2 + 26*a*b^2
*c^3*d^2*f + 51*a^2*b^3*c*e*f^2 - 204*a^3*b*c^2*e*f^2 + 2*b^5*c*d*e*f + 2*a
^2*b^2*c^2*d*f^2 + 42*a^2*b^2*c^2*e^2*f + 6*a*b^3*c^2*d*e*f - 152*a^2*b*c^3
*d*e*f)/(4*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*((768*a^
4*b*c^7*d^2 - b^9*c^3*d^2 - c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*f^2 -
a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 + a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + a
*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 -
9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^
6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 15

04*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6
*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f - 6*a*c^2*d*
f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 12
8*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8
*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f
 - 2*a*b^10*c*e*f + 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9
 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840
*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*2i - atan(((((2048*a^4*c^6*e + 16*
b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 19
2*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d -
32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4
 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) -
 b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c
^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)
^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 -
 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^
5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*
a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*
b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e
*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*
e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*
c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*
c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5
))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/
2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5
*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e
^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 384
0*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e
+ 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/
2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*
a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c
^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*
b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b
^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*
b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d
^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*
b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d
*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e)
)/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2
) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*
b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^
9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b
^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^
2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840
*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e +
 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a
^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^
3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b
*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^
10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b
^2*c^8)))^(1/2)*1i - (((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d +
 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c
^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f
 - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3

)) + (x*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768
*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b
^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840
*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2
- 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*
b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*
e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*
d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*
b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*
d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536
*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^
4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*
a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a
*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 +
768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 +
3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*
d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*
a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^
7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*
c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*
a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*
c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f +
1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 128
0*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) - (x*(8*a*c^5*
d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e
^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 +
48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^
3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*
b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 +
768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3
840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d
^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a
^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7
*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c
^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a
^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c
^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1
536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
)/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280
*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*1i)/((((2048*a^
4*c^6*e + 16*b^7*c^3*d + 768*a^2*b^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b
^2*c^5*e - 192*a^2*b^5*c^3*f + 768*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a
^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c
- 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*((c^3*d^2*(-(4*a*c - b^
2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2
+ 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a
*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-
(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*
b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*
f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8
*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d
*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*
a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e
*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 -

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 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 -
6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 76
8*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c
 - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*
e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*
a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*
c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a
*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c
 - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c
^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f -
192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10
*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c
^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^
7 - 6144*a^6*b^2*c^8)))^(1/2) + (x*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 -
 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 - b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*
e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e
 + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8
*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e
^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2
*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a
^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c
^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*
b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c -
 b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^
3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 1
92*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*
c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^
3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7
 - 6144*a^6*b^2*c^8)))^(1/2) + (((2048*a^4*c^6*e + 16*b^7*c^3*d + 768*a^2*b
^3*c^5*d + 384*a^2*b^4*c^4*e - 1536*a^3*b^2*c^5*e - 192*a^2*b^5*c^3*f + 768
*a^3*b^3*c^4*f - 192*a*b^5*c^4*d - 1024*a^3*b*c^6*d - 32*a*b^6*c^3*e + 16*a
*b^7*c^2*f - 1024*a^4*b*c^5*f)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a
^2*b^2*c^3)) + (x*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b^11
*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9*c*
f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2*b^
5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e^2
 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1024*
a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3584*
a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d*e
 + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072*a^
4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c^4*
e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*c^5
 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2)*(16*b^7*
c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2
*c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*
b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^
2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^
9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^
2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*
e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 1
024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 3
584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*
d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 307
```

```
2*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*
c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8
*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) - (x
*(8*a*c^5*d^2 - b^6*f^2 - 8*a^2*c^4*e^2 - 10*b^2*c^4*d^2 + 72*a^3*c^3*f^2 -
 b^4*c^2*e^2 - 2*a*b^2*c^3*e^2 - 2*b^5*c*e*f - 74*a^2*b^2*c^2*f^2 + 16*a*b^
4*c*f^2 + 48*a^2*c^4*d*f + 6*b^3*c^3*d*e + 6*b^4*c^2*d*f - 52*a*b^2*c^3*d*f
 + 14*a*b^3*c^2*e*f + 8*a^2*b*c^3*e*f + 8*a*b*c^4*d*e))/(2*(b^4*c + 16*a^2*
c^3 - 8*a*b^2*c^2)))*((c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^3*d^2 - a*b
^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 + 768*a^5*b*c^6*e^2 - a*b^2*f^2
*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^2*b^9
*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^2
*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^5*c^4*e^2 - 512*a^4*b^3*c^5*e
^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^2 - 3840*a^5*b^3*c^4*f^2 - 10
24*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c^3*d*e + 6*a*b^9*c^2*d*f + 35
84*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 128*a^2*b^6*c^4*d
*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f + 960*a^3*b^5*c^4*d*f - 3072
*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^3*b^6*c^3*e*f + 128*a^4*b^4*c
^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f - 2*a*b*c*e*f*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 24*a^2*b^10*c^4 + 240*a^3*b^8*
c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 6144*a^6*b^2*c^8)))^(1/2) + (8*
a*c^5*d^3 + b^6*d*f^2 + 5*a^2*b^4*f^3 + 6*b^2*c^4*d^3 + 216*a^4*c^2*f^3 - 3
*a*b^3*c^2*e^3 - 4*a^2*b*c^3*e^3 - 66*a^3*b^2*c*f^3 + 8*a^2*c^4*d*e^2 + 72*
a^2*c^4*d^2*f + 216*a^3*c^3*d*f^2 - 5*b^3*c^3*d^2*e + b^4*c^2*d*e^2 + 24*a^
3*c^3*e^2*f - 5*b^4*c^2*d^2*f - 3*a*b^5*e*f^2 - 28*a*b*c^4*d^2*e - 12*a*b^4
*c*d*f^2 - 6*a*b^4*c*e^2*f + 18*a*b^2*c^3*d*e^2 + 26*a*b^2*c^3*d^2*f + 51*a
^2*b^3*c*e*f^2 - 204*a^3*b*c^2*e*f^2 + 2*b^5*c*d*e*f + 2*a^2*b^2*c^2*d*f^2
+ 42*a^2*b^2*c^2*e^2*f + 6*a*b^3*c^2*d*e*f - 152*a^2*b*c^3*d*e*f)/(4*(b^6*c
 - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3))))*((c^3*d^2*(-(4*a*c - b^2)
^9)^(1/2) - b^9*c^3*d^2 - a*b^11*f^2 + 768*a^4*b*c^7*d^2 - a*b^9*c^2*e^2 +
768*a^5*b*c^6*e^2 - a*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - a*c^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 27*a^2*b^9*c*f^2 + 3840*a^6*b*c^5*f^2 + 9*a^2*c*f^2*(-(4
*a*c - b^2)^9)^(1/2) + 96*a^2*b^5*c^5*d^2 - 512*a^3*b^3*c^6*d^2 + 96*a^3*b^
5*c^4*e^2 - 512*a^4*b^3*c^5*e^2 - 288*a^3*b^7*c^2*f^2 + 1504*a^4*b^5*c^3*f^
2 - 3840*a^5*b^3*c^4*f^2 - 1024*a^5*c^7*d*e - 3072*a^6*c^6*e*f + 12*a*b^8*c
^3*d*e + 6*a*b^9*c^2*d*f + 3584*a^5*b*c^6*d*f + 6*a*c^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) - 128*a^2*b^6*c^4*d*e + 384*a^3*b^4*c^5*d*e - 128*a^2*b^7*c^3*d*f
 + 960*a^3*b^5*c^4*d*f - 3072*a^4*b^3*c^5*d*f + 36*a^2*b^8*c^2*e*f - 192*a^
3*b^6*c^3*e*f + 128*a^4*b^4*c^4*e*f + 1536*a^5*b^2*c^5*e*f - 2*a*b^10*c*e*f
 - 2*a*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^9 + a*b^12*c^3 - 2
4*a^2*b^10*c^4 + 240*a^3*b^8*c^5 - 1280*a^4*b^6*c^6 + 3840*a^5*b^4*c^7 - 61
44*a^6*b^2*c^8)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.71 $\int \frac{d+ex^2+fx^4}{\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=346

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2a\right.}{2\sqrt{2}\,a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(b^2*d-a*b*e-2*a*(-a*f+c*d)+(a*b*f-2*a*c*e+b*c*d)*x^2)/a/(-4*a*c+b^2)
/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))
*(b*c*d-2*a*c*e+a*b*f+(4*a*b*c*e+b^2*(-a*f+c*d)-4*a*c*(a*f+3*c*d))/(-4*a*c+
b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-2*a*c*e+a*b*
f+(-4*a*b*c*e-b^2*(-a*f+c*d)+4*a*c*(a*f+3*c*d))/(-4*a*c+b^2)^(1/2))/a/(-4*a
*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 1.90, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1678, 1166, 205}

$$\frac{x\left(x^2(abf-2ace+bcd)-abe-2a(cd-af)+b^2d\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2(cd-af)+4abce-4ac(af+3cd)}{\sqrt{b^2-4ac}}+abf-2a\right.}{2\sqrt{2}\,a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(b^2*d - a*b*e - 2*a*(c*d - a*f) + (b*c*d - 2*a*c*e + a*b*f)*x^2))/(2*a*
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b*c*d - 2*a*c*e + a*b*f + (4*a*b*c*e
 + b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*
c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*c*d - 2*a*c*e + a*b*f - (4*a*b*c*e +
b^2*(c*d - a*f) - 4*a*c*(3*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
```

```
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-b^2d-abe+2a(3cd+af)+(-bcd+2ace-ab\ldots}{a+bx^2+cx^4}}{2a\left(b^2-4ac\right)\ldots}$$

$$= \frac{x\left(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf-\frac{4abce+b^2(cd-a\ldots}{\sqrt{b^2\ldots}}\ldots\right.}{4a\left(b^2\ldots\right.}$$

$$= \frac{x\left(b^2d-abe-2a(cd-af)+(bcd-2ace+abf)x^2\right)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(bcd-2ace+abf+\frac{4abce+b^2(cd-a\ldots}{\sqrt{b^2\ldots}}\ldots\right.}{2\sqrt{2}\,a\sqrt{c}\left(b^2-4a\ldots\right.}$$

Mathematica [A] time = 1.08, size = 382, normalized size = 1.10

$$\frac{\frac{2x\left(b\left(-ae+afx^2+cdx^2\right)+2a\left(af-c\left(d+ex^2\right)\right)+b^2d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(b\left(cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ace\right)-2ac\left(e\sqrt{b^2-4ac}+2af+6cd\right)+b^2(cd-a\ldots\right.}{\sqrt{c}\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}\ldots}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(b^2*d + b*(-(a*e) + c*d*x^2 + a*f*x^2) + 2*a*(a*f - c*(d + e*x^2))))
/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(b^2*(c*d - a*f) - 2*a*c*(6
*c*d + Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e +
a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*
c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*
(b^2*(-(c*d) + a*f) + 2*a*c*(6*c*d - Sqrt[b^2 - 4*a*c]*e + 2*a*f) + b*(c*Sq
rt[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]]))/(4*a)
```

fricas [B] time = 8.49, size = 8991, normalized size = 25.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*c*d - 2*a*c*e + a*b*f)*x^3 + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4
 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^
2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2
*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c -
 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b
^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4
 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4
*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*
```

```
(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*
b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^
2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4
*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a
^2*c^5)*d^4 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4
*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*
a^3*b^2*c^2 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a
^4*b^2*c - 48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c -
6*a^3*b^2*c^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b
^6*c - 15*a*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a
^3*b*c^3)*d^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^
4*b*c^2)*e^3)*f)*x + 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3
 - 672*a^3*b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^
3*b^3*c^3 - 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b
^2*c^3 - 32*a^5*c^4)*d*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3
 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c +
160*a^5*b^2*c^2 - 288*a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e
)*f^2 - 2*((4*a^2*b^6*c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d
^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*
b^2*c^2 + 16*a^6*c^3)*e^2)*f - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c
^3 - 448*a^6*b^3*c^4 + 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*
a^7*b^2*c^4 - 256*a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c
^3 - 64*a^8*b*c^4)*f)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^
3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*
a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e +
 a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^
4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c
^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b
*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*
a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^
2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*
a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*
d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2
- 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d
*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e
+ a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b
^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^
5*b^2*c^3 - 64*a^6*c^4))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2
- 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*
b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12
*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c
^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48
*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c
*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2
 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*
d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e
 + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*
b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a
^5*b^2*c^3 - 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4
 - (3*b^5*c^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a
^2*b^2*c^3)*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2
 + 4*a^4*c^3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c -
48*a^5*c^2)*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c
^2 - 24*a^4*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a
*b^4*c^2 + 432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d
^2*e + 3*(a^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^
3)*f)*x - 1/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*
b^2*c^4 + 864*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 -
 112*a^4*b*c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32
```

```
*a^5*c^4)*d*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b
^4 - 8*a^6*b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2
*c^2 - 288*a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*(
(4*a^2*b^6*c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 1
6*a^6*c^3)*e^2)*f - ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^
6*b^3*c^4 + 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4
 - 256*a^8*c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8
*b*c^4)*f)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4
 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*
d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2
+ (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2
 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8
*b^2*c^4 - 64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 +
 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*
e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3
*a^3*b^2*c + 4*a^4*c^2)*e)*f + (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6
*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c
^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*
e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d
*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48
*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 -
 64*a^6*c^4))) + sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c +
 (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2
+ 2*(a*b^4*c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)
*e^2 + (a^3*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(
3*a^3*b^2*c + 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^
3 - 64*a^6*c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^
6*f^4 + (b^4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*
c^3)*d^3*e + 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c
*e^2 + (a^3*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*
d*e^2 + (a^2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 4
8*a^8*b^2*c^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3
- 64*a^6*c^4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4 - (3*b^5*c
^2 - 65*a*b^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a^2*b^2*c^3)
*d^2*e^2 - (9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2 + 4*a^4*c^
3)*e^4 + (3*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)
*d + (a^4*b^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c^2 - 24*a^4
*c^3)*d^2 + (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a*b^4*c^2 +
432*a^3*c^4)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d^2*e + 3*(a
^2*b^4*c + 12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^3)*f)*x + 1
/2*sqrt(1/2)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 8
64*a^4*c^5)*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*
c^4)*d^2*e + 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32*a^5*c^4)*d
*e^2 + (a^3*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b^4 - 8*a^6*
b^2*c + 16*a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2*c^2 - 288*
a^6*c^3)*d + (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*((4*a^2*b^6*
c - 59*a^3*b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3*b^5*c - 8*
a^4*b^3*c^2 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*
e^2)*f + ((a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 +
 512*a^7*b*c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*
c^5)*e - 4*(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*f)*
sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2
 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(
a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*
c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2
*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 -
64*a^9*c^5)))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c
```

```
 - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*
b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c
+ 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c
^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4
*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e +
 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*
b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2
*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^
4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4
))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4
*a^2*b*c)*x^2)*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*
c - 6*a^2*b^2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3
*b^3 + 12*a^4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c
 + 4*a^4*c^2)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*
c^4)*sqrt((4*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^
4*c^2 - 18*a*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e
+ 6*(a^2*b^2*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3
*b^2*c - 27*a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^
2*b^2*c^2 - 9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c
^4 - 64*a^9*c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^
4))*log(((5*b^4*c^3 - 81*a*b^2*c^4 + 324*a^2*c^5)*d^4 - (3*b^5*c^2 - 65*a*b
^3*c^3 + 324*a^2*b*c^4)*d^3*e - 3*(3*a*b^4*c^2 - 28*a^2*b^2*c^3)*d^2*e^2 -
(9*a^2*b^3*c^2 - 20*a^3*b*c^3)*d*e^3 - (3*a^3*b^2*c^2 + 4*a^4*c^3)*e^4 + (3
*a^5*b^2 + 4*a^6*c)*f^4 - ((a^3*b^4 - 24*a^4*b^2*c - 48*a^5*c^2)*d + (a^4*b
^3 + 12*a^5*b*c)*e)*f^3 - 9*((a^2*b^4*c - 6*a^3*b^2*c^2 - 24*a^4*c^3)*d^2 +
 (a^3*b^3*c + 12*a^4*b*c^2)*d*e)*f^2 + ((b^6*c - 15*a*b^4*c^2 + 432*a^3*c^4
)*d^3 + 3*(a*b^5*c + 3*a^2*b^3*c^2 - 108*a^3*b*c^3)*d^2*e + 3*(a^2*b^4*c +
12*a^3*b^2*c^2)*d*e^2 + (a^3*b^3*c + 12*a^4*b*c^2)*e^3)*f)*x - 1/2*sqrt(1/2
)*((b^8*c - 23*a*b^6*c^2 + 190*a^2*b^4*c^3 - 672*a^3*b^2*c^4 + 864*a^4*c^5)
*d^3 + 3*(a*b^7*c - 15*a^2*b^5*c^2 + 72*a^3*b^3*c^3 - 112*a^4*b*c^4)*d^2*e
+ 3*(a^2*b^6*c - 10*a^3*b^4*c^2 + 32*a^4*b^2*c^3 - 32*a^5*c^4)*d*e^2 + (a^3
*b^5*c - 8*a^4*b^3*c^2 + 16*a^5*b*c^3)*e^3 + 2*(a^5*b^4 - 8*a^6*b^2*c + 16*
a^7*c^2)*f^3 - ((a^3*b^6 - 26*a^4*b^4*c + 160*a^5*b^2*c^2 - 288*a^6*c^3)*d
+ (a^4*b^5 - 8*a^5*b^3*c + 16*a^6*b*c^2)*e)*f^2 - 2*((4*a^2*b^6*c - 59*a^3*
b^4*c^2 + 280*a^4*b^2*c^3 - 432*a^5*c^4)*d^2 + 5*(a^3*b^5*c - 8*a^4*b^3*c^2
 + 16*a^5*b*c^3)*d*e + (a^4*b^4*c - 8*a^5*b^2*c^2 + 16*a^6*c^3)*e^2)*f + ((
a^3*b^9*c - 20*a^4*b^7*c^2 + 144*a^5*b^5*c^3 - 448*a^6*b^3*c^4 + 512*a^7*b*
c^5)*d + (a^4*b^8*c - 8*a^5*b^6*c^2 + 128*a^7*b^2*c^4 - 256*a^8*c^5)*e - 4*
(a^5*b^7*c - 12*a^6*b^5*c^2 + 48*a^7*b^3*c^3 - 64*a^8*b*c^4)*f)*sqrt((4*a^3
*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a*b^2
*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2*c^2
 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*a^4*
c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 - 9*a^
3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*c^5)
))*sqrt(-((b^5*c - 15*a*b^3*c^2 + 60*a^2*b*c^3)*d^2 + 2*(a*b^4*c - 6*a^2*b^
2*c^2 - 24*a^3*c^3)*d*e + (a^2*b^3*c + 12*a^3*b*c^2)*e^2 + (a^3*b^3 + 12*a^
4*b*c)*f^2 - 2*((3*a^2*b^3*c - 28*a^3*b*c^2)*d + 2*(3*a^3*b^2*c + 4*a^4*c^2
)*e)*f - (a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4)*sqrt((4
*a^3*b*c^2*d*e^3 + a^4*c^2*e^4 + 12*a^5*c*d*f^3 + a^6*f^4 + (b^4*c^2 - 18*a
*b^2*c^3 + 81*a^2*c^4)*d^4 + 4*(a*b^3*c^2 - 9*a^2*b*c^3)*d^3*e + 6*(a^2*b^2
*c^2 - 3*a^3*c^3)*d^2*e^2 - 2*(2*a^4*b*c*d*e + a^5*c*e^2 + (a^3*b^2*c - 27*
a^4*c^2)*d^2)*f^2 - 12*(2*a^3*b*c^2*d^2*e + a^4*c^2*d*e^2 + (a^2*b^2*c^2 -
9*a^3*c^3)*d^3)*f)/(a^6*b^6*c^2 - 12*a^7*b^4*c^3 + 48*a^8*b^2*c^4 - 64*a^9*
c^5)))/(a^3*b^6*c - 12*a^4*b^4*c^2 + 48*a^5*b^2*c^3 - 64*a^6*c^4))) - 2*(a*
b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*x)/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 -
4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
```

giac [B] time = 6.97, size = 6356, normalized size = 18.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c*d*x^3 + a*b*f*x^3 - 2*a*c*x^3*e + b^2*d*x - 2*a*c*d*x + 2*a^2*f*x
- a*b*x*e)/((c*x^4 + b*x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^3 - 8*a
*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c +
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c
^3)*(a*b^2 - 4*a^2*c)^2*d + (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2
*f - 2*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4*a^2*c)^2*e + 2*(sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 2*a*b^
6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^4*c^3 + 28*a^2*b^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 1
0*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 128*a^3*b^2*c^4 + 2
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 192*a^4*c^5 + 2*(b^2 -
4*a*c)*a*b^4*c^2 - 20*(b^2 - 4*a*c)*a^2*b^2*c^3 + 48*(b^2 - 4*a*c)*a^3*c^4)
*d*abs(a*b^2 - 4*a^2*c) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^
4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^2 - 2*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*a^3*b^4*c^2 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 16*a^
4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 32*a^5*c^4
+ 2*(b^2 - 4*a*c)*a^3*b^2*c^2 - 8*(b^2 - 4*a*c)*a^4*c^3)*f*abs(a*b^2 - 4*a^
2*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^4*c^2 - 2*a^2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 16*a^3*b^3*c^3 - 4*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 32*a^4*b*c^4 + 2*(b^2 -
4*a*c)*a^2*b^3*c^2 - 8*(b^2 - 4*a*c)*a^3*b*c^3)*abs(a*b^2 - 4*a^2*c)*e + (2
*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*
b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 - 96*(b^2 - 4*a*c)*a^4*b*c^5)*d - (2
*a^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5*b^3*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c
```

```
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 3
2*(b^2 - 4*a*c)*a^5*b*c^4)*f + 4*(2*a^3*b^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b
^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*
c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2
 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2
- 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3
- 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(
b^2 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c)*a^4*b^2*c^4)*e)*arctan(2*sqrt(1/
2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*
a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4
*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 -
64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4 + 16*a^5*c^5)*abs(a*b^2 - 4*a^2*c
)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(a*b^2 - 4*a^2*c)^2*d + (2*a*b^3*c^2
- 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4
*a*c)*a*b*c^2)*(a*b^2 - 4*a^2*c)^2*f - 2*(2*a*b^2*c^3 - 8*a^2*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*a*c^3)*(a*b^2 - 4
*a^2*c)^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 14*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^2 + 2*a*b^6*c^2 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^3*b^2*c^3 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 28*a^2*b^4*c^3 -
96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 48*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^2*c^4 + 128*a^3*b^2*c^4 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*c^5 - 192*a^4*c^5 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 20*(b^2 - 4*a*c)*a^2*b
^2*c^3 - 48*(b^2 - 4*a*c)*a^3*c^4)*d*abs(a*b^2 - 4*a^2*c) + 4*(sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^4*b^2*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2
+ 2*a^3*b^4*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^3 + 8*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^3 - 16*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^4*c^4 + 32*a^5*c^4 - 2*(b^2 - 4*a*c)*a^3*b^2*c^2 + 8*(b^2 -
4*a*c)*a^4*c^3)*f*abs(a*b^2 - 4*a^2*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2
- 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 2*a^2*b^5*c^2 + 1
6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 + 8*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c^3 - 16*a^3*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b*c^4 + 32*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^3*c^2 + 8*(b^2 - 4*a*c)*a^3*
b*c^3)*abs(a*b^2 - 4*a^2*c)*e + (2*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 224*a^4*b
^3*c^5 - 384*a^5*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
```

```
c)*c)*a^4*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^5*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b*c^5 - 2*(b^2 - 4*a*c)*a^2*b^5*c^3 + 32*(b^2 - 4*a*c)*a^3*b^3*c^4 -
96*(b^2 - 4*a*c)*a^4*b*c^5)*d - (2*a^3*b^7*c^2 - 8*a^4*b^5*c^3 - 32*a^5*b^3
*c^4 + 128*a^6*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*
b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5
*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c
^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c
^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4
 - 2*(b^2 - 4*a*c)*a^3*b^5*c^2 + 32*(b^2 - 4*a*c)*a^5*b*c^4)*f + 4*(2*a^3*b
^6*c^3 - 16*a^4*b^4*c^4 + 32*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 2*(b^2 - 4*a*c)*a^3*b^4*c^3 + 8*(b^2 - 4*a*c
)*a^4*b^2*c^4)*e)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c - sqrt((a*b^
3 - 4*a^2*b*c)^2 - 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c -
 4*a^2*c^2)))/((a^3*b^6*c - 12*a^4*b^4*c^2 - 2*a^3*b^5*c^2 + 48*a^5*b^2*c^3
 + 16*a^4*b^3*c^3 + a^3*b^4*c^3 - 64*a^6*c^4 - 32*a^5*b*c^4 - 8*a^4*b^2*c^4
 + 16*a^5*c^5)*abs(a*b^2 - 4*a^2*c)*abs(c))
```

maple [B] time = 0.04, size = 1182, normalized size = 3.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] (-1/2/a*(a*b*f-2*a*c*e+b*c*d)/(4*a*c-b^2)*x^3-1/2*(2*a^2*f-a*b*e-2*a*c*d+b^
2*d)/a/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b*f-1/2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e+1/4/a/(4*a*c-b^2)*c*2^(1/2)/
((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*b*d-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+1/(4*a*c-b
^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-3/(4*a*c-b^2)*c^2/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2*d-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f+1/2/(4*
a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-1/4/a/(4*a*c-b^2)*c*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-
a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f-1/4/(4*a*c-b^2)/(-4
```

```
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b*e-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*d+1/4/a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c*d - 2*a*c*e + a*b*f)*x^3 - (a*b*e - 2*a^2*f - (b^2 - 2*a*c)*d)*x)
/((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)
+ 1/2*integrate((a*b*e - 2*a^2*f + (b*c*d - 2*a*c*e + a*b*f)*x^2 + (b^2 - 6
*a*c)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2 - 4*a^2*c)
```

mupad [B] time = 6.55, size = 19589, normalized size = 56.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4
*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^
4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 1
6*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2)) - (x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c
- b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^
2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*
a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*
c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 -
3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f
- 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*
d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 12
8*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^
6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b
^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^
8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(10
24*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2
*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f
^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2
*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(
-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^
4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 +
 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^
3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^
6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*
a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*
c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f
- 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*
(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2
+ 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)
))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2
 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^
```

2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f +
4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b
^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2)
- b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) -
a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b
^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2
+ 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a
^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*
e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(
-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b
^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d
*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384
*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^
6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c
^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b
^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^
3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 102
4*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) +
(x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b
^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2
*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c
d^2(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1
504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b
^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e -
1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*
a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c
^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f +
960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5
*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(
4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^
6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a
^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^
9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(
1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7
*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2
- 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^
6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2
*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 1
92*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b
^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e
*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 -
1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a
^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2
+ a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f
+ 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f
- 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*
c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 384
0*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 76
8*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4
*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96
*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*
f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(
1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^
4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^

3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f -
2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 +
a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^
7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i)/((8*a^3*c^4*e^3 + 5*b^3*c^4*d^3 -
3*a^3*b^3*c*f^3 - 4*a^4*b*c^2*f^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 8
*a^4*c^3*e*f^2 + b^5*c^2*d^2*f + 6*a^2*b^2*c^3*e^3 - 36*a*b*c^5*d^3 + a*b^5
*c*d*f^2 + 48*a^3*c^4*d*e*f + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a
^2*b*c^4*d*e^2 - a*b^3*c^3*d^2*f - 60*a^2*b*c^4*d^2*f - 28*a^3*b*c^3*d*f^2
+ a^2*b^4*c*e*f^2 - 28*a^3*b*c^3*e^2*f - 9*a^2*b^3*c^2*d*f^2 - 5*a^2*b^3*c^
2*e^2*f + 18*a^3*b^2*c^2*e*f^2 - 4*a*b^4*c^2*d*e*f + 52*a^2*b^2*c^3*d*e*f)/
(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c^
6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^
2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3
*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024
*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) -
(x*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^
11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*
b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 15
04*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^
3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1
024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a
*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^
3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f +
960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*
b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6
*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^
2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 -
8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9
)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^
(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1
/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*
c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2
- 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6
*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*
c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 19
2*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^
7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*
f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 -
1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^
2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2
+ a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f +
2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f -
6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c
^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840
*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768
*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*
d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*
a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f
+ 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4
*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3
*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2
*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a
^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7
*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*a^5*c^6*d + 2048*a^6*c^5*f -

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 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c
^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b
^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*
b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((27*a*b^9*c^2*d^2
- a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*
c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*
c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3
840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5
*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^
2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1
2*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^
4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f -
3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10
*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12
*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^
5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^
5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*
b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 +
 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2
+ 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a
*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5
*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2
+ 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^
5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^
9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 12
8*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^
5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*
f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^
7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 384
0*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*
e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a
^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*
a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(
2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 -
 a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c
^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c
*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^
7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5
*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*
a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3
584*a^6*b*c^5*d*f - 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f
 + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^
5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^
4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*
c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^1
0*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^
2*c^6)))^(1/2)))*((27*a*b^9*c^2*d^2 - a^3*b^9*f^2 - a^3*f^2*(-(4*a*c - b^2)
^9)^(1/2) - b^11*c*d^2 + 3840*a^5*b*c^6*d^2 - 9*a*c^2*d^2*(-(4*a*c - b^2)^9
)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 + a^2*c*e^2*(-(4*a*c - b^2)^9)^
(1/2) + b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^
7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^
2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a
^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f - 6*a^
2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e -
192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*
b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*
e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e + 2*a*b*c*d*e*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3
```

- 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*2i - ((x*(b^2*d + 2*a^2*f - a*b*e - 2*a*c*d))/(2*a*(4*a*c - b^2)) + (x^3*(a*b*f - 2*a*c*e + b*c*d))/(2*a*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + atan(((((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i - (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4* d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*1i)/((8*a^3*c^4*e^3 + 5*b^3*c^4*d^3 - 3*a^3*b^3*c*f^3 - 4*a^4*b*c^2*f^3 + 72*a^2*c^5*d^2*e - 3*b^4*c^3*d^2*e + 8*a^4*c^3*e*f^2 + b^5*c^2*d^2*f + 6*a^2*b^2*c^3*e^3 - 36*a*b*c^5*d^3 + a*b^5*c*d*f^2 + 48*a^3*c^4*d*e*f + 18*a*b^2*c^4*d^2*e + 3*a*b^3*c^3*d*e^2 - 60*a^2*b*c^4*d*e^2 - a*b^3*c^3*d^2*f - 60*a^2*b*c^4*d^2*f - 28*a^3*b*c^3*d*f^2 + a^2*b^4*c*e*f^2 - 28*a^3*b*c^3*e^2*f - 9*a^2*b^3*c^2*d*f^2 - 5*a^2*b^3*c^2*e^2*f + 18*a^3*b^2*c^2*e*f^2 - 4*a*b^4*c^2*d*e*f + 52*a^2*b^2*c^3*d*e*f)/(4*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4* b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 + 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e - 8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2) + (((6144*a^5*c^6*d + 2048*a^6*c^5*f - 288*a^2*b^6*c^3*d + 1920*a^3*b^4*c^4*d - 5632*a^4*b^2*c^5*d + 16*a^2*b^7*c^2*e - 192*a^3*b^5*c^3*e + 768*a^4*b^3*c^4*e - 32*a^3*b^6*c^2*f + 384*a^4*b^4*c^3*f - 1536*a^5*b^2*c^4*f + 16*a*b^8*c^2*d - 1024*a^5*b*c^5*e)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2

```
*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^
9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^
3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*
e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072
*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d
*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c -
 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 -
6144*a^8*b^2*c^6)))^(1/2) - (x*(72*a^2*c^5*d^2 - 8*a^3*c^4*e^2 + b^4*c^3*d^
2 + 8*a^4*c^3*f^2 - 14*a*b^2*c^4*d^2 + a^2*b^4*c*f^2 + 10*a^2*b^2*c^3*e^2 +
 2*a^3*b^2*c^2*f^2 + 48*a^3*c^4*d*f + 2*a*b^3*c^3*d*e - 40*a^2*b*c^4*d*e -
8*a^3*b*c^3*e*f + 4*a^2*b^2*c^3*d*f - 6*a^2*b^3*c^2*e*f))/(2*(a^2*b^4 + 16*
a^4*c^2 - 8*a^3*b^2*c)))*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 -
 b^11*c*d^2 + 27*a*b^9*c^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c
- b^2)^9)^(1/2) - a^2*b^9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c -
b^2)^9)^(1/2) - b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 28
8*a^2*b^7*c^3*d^2 + 1504*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^
5*c^3*e^2 - 512*a^5*b^3*c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2
- 3072*a^6*c^6*d*e - 1024*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*
f + 6*a^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^
2*d*e - 192*a^3*b^6*c^3*d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e -
128*a^3*b^7*c^2*d*f + 960*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*
b^6*c^2*e*f + 384*a^5*b^4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c -
 b^2)^9)^(1/2))/(32*(4096*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*
b^8*c^3 - 1280*a^6*b^6*c^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)))
*((a^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*f^2 - b^11*c*d^2 + 27*a*b^9*c
^2*d^2 + 3840*a^5*b*c^6*d^2 + 9*a*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^
9*c*e^2 + 768*a^6*b*c^5*e^2 - a^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c*d^
2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^7*b*c^4*f^2 - 288*a^2*b^7*c^3*d^2 + 1504
*a^3*b^5*c^4*d^2 - 3840*a^4*b^3*c^5*d^2 + 96*a^4*b^5*c^3*e^2 - 512*a^5*b^3*
c^4*e^2 + 96*a^5*b^5*c^2*f^2 - 512*a^6*b^3*c^3*f^2 - 3072*a^6*c^6*d*e - 102
4*a^7*c^5*e*f + 6*a^2*b^9*c*d*f + 3584*a^6*b*c^5*d*f + 6*a^2*c*d*f*(-(4*a*c
 - b^2)^9)^(1/2) + 12*a^3*b^8*c*e*f + 36*a^2*b^8*c^2*d*e - 192*a^3*b^6*c^3*
d*e + 128*a^4*b^4*c^4*d*e + 1536*a^5*b^2*c^5*d*e - 128*a^3*b^7*c^2*d*f + 96
0*a^4*b^5*c^3*d*f - 3072*a^5*b^3*c^4*d*f - 128*a^4*b^6*c^2*e*f + 384*a^5*b^
4*c^3*e*f - 2*a*b^10*c*d*e - 2*a*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^9*c^7 + a^3*b^12*c - 24*a^4*b^10*c^2 + 240*a^5*b^8*c^3 - 1280*a^6*b^6*c
^4 + 3840*a^7*b^4*c^5 - 6144*a^8*b^2*c^6)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.72 $\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2} dx$

Optimal. Leaf size=399

$$-\frac{x\left(a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+cx^2\left(-abe-2a(cd-af)+b^2d\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2}{\cdots}\right.}{2\sqrt{2}a^2\left(b^2\right.}$$

```
[Out] -d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e))+c*(b^2*d-a*b*e-2*a*
(-a*f+c*d))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d)+(
3*b^3*d-a*b^2*e+12*a^2*c*e-4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a
*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(
b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*b^2*d-a*b*e-2*a*(-a*f+5*c*d)+(-3*b^
3*d+a*b^2*e-12*a^2*c*e+4*a*b*(a*f+4*c*d))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b
^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.20, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1669, 1664, 1166, 205}

$$-\frac{x\left(cx^2\left(-abe-2a(cd-af)+b^2d\right)+a\left(\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{12a^2ce-ab^2}{\cdots}\right.}{2\sqrt{2}a^2\left(b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f)) + c*(b
^2*d - a*b*e - 2*a*(c*d - a*f))*x^2))/(2*a^2*(b^2 - 4*a*c)*(a + b*x^2 + c*x
^4)) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) + (3*b^3*d - a*b^2*e +
 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b - S
qrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*b^2*d - a*b*e - 2*a*(5*c*d - a*f) - (3*b^3
*d - a*b^2*e + 12*a^2*c*e - 4*a*b*(4*c*d + a*f))/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a
*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
```

```
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-2(b^2-4 \cdots}{\cdots}}{\cdots}$$

$$= -\frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \left(\frac{2(-b^2 \cdots}{ax \cdots}\right.}{\cdots}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \cdots}{\cdots}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{(c \cdots}{\cdots}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a}-b(3cd+be)+a(2ce+bf)\right)+c\left(b^2d-abe-2a(cd-af)\right)x^2\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\sqrt{\cdots}}{\cdots}$$

Mathematica [A] time = 1.32, size = 444, normalized size = 1.11

$$-\frac{2x\left(b^2\left(cdx^2-ae\right)+ab\left(af-c\left(3d+ex^2\right)\right)+2ac\left(a\left(e+fx^2\right)-cdx^2\right)+b^3d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(ab\left(e\sqrt{b^2-4ac}+4af+16cd\right)-2a\left(-5cd\sqrt{b^2-4ac}+af \cdots\right.\right.}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*d)/x - (2*x*(b^3*d + b^2*(-(a*e) + c*d*x^2) + a*b*(a*f - c*(3*d + e*x^
2)) + 2*a*c*(-(c*d*x^2) + a*(e + f*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*Sqrt[c]*(-3*b^3*d + b^2*(-3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b
*(16*c*d + Sqrt[b^2 - 4*a*c]*e + 4*a*f) - 2*a*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6
*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*Sqrt[c]*(3*b^3*d - b^2*(3*Sqrt[b^2 - 4*a*c]*d + a*e) + a*b*(-16*c*d + Sqrt
[b^2 - 4*a*c]*e - 4*a*f) + 2*a*(5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[
```

```
b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

fricas [B] time = 19.29, size = 13111, normalized size = 32.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - 2*(a^2*b*f + (3
*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 + sqrt(1/2)*((a^2*b^2*c - 4*a
^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*
b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*
a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c +
60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3
*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6
 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 91
8*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*
a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*
b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5
- 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^
2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*
b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*
c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^
3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 -
 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6
 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c
 + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625
*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2
*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672
*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)
*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c
+ 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b
*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a
^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4
*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b
^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5
*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b
^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b
*c^3)*e^3)*f)*x + 1/2*sqrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2
- 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10
 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 -
 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*
a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4
*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8
*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)
*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*
a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4
)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 1
60*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)
*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3
+ 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b
^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128
*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 30
51*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^
2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*
b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c
 + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^
7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6
```

```
*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 -
4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*
a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c
 - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4
*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2
*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2
 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^
3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3
*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^
2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^
2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 119
7*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a
^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*
c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^
6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2
*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^
6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*
a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)
*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*
b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*
c^3))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3
 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 4
20*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c
^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b
*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4
*b^2*c - 24*a^5*c^2)*e)*f + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a
^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3
*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^
2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 -
 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 +
 (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^
7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3
 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*
b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c -
65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^
5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64
*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-(
(189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135
*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(4
5*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2
 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^
3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*
a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a
^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b
^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((8
1*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^
5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4
)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^
3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((27*b^
11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4
 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4
138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64
*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (
a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e
^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^
5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b
^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c
^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 1
66*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5
```

```
*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^2)*f - ((3*a^5*b^10 - 55*a^6*b^8*c +
 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d -
 (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c
^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((
a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625
*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*
c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d
^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18
*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 +
6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d
*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^
4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*
d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*
b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*s
qrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*
b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3
*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 1
3*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81
*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4
 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e +
6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3
*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c +
81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4
 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2
 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 1
25*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*
a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/
(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*
a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*
c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7
 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2
*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*
a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c
 - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 -
12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a
*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b
^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6
 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 4
9*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*
e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2
*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*
e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*
d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22
*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 -
12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c +
48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-((189*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^
2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^
3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^
3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 - 647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*
e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 + 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4
*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^
2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*
c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^
4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*
c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^
2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*
c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^
3)*e^3)*f)*x + 1/2*sqrt(1/2)*((27*b^11 - 486*a*b^9*c + 3330*a^2*b^7*c^2 - 1
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0549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 - 5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 -
177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 80
0*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5
*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^
2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3 - (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*
c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d
- (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2
*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2 - 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d
^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*
a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c + 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^
2)*f + ((3*a^5*b^10 - 55*a^6*b^8*c + 392*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2
176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*
c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^
10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*
a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b
^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4
*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c +
198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b
*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^
2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*(
(27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3
*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 1
5*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c
+ 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^
3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 -
120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 +
 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^
4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c
^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 -
 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a
^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*
b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2
)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b
^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3
*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 -
 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4
*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*
e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2
*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3
))) - sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 +
(a^3*b^2 - 4*a^4*c)*x)*sqrt(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*
a^3*b*c^3)*d^2 - 2*(3*a*b^6 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)
*d*e + (a^2*b^5 - 15*a^3*b^3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)
*f^2 - 2*((3*a^2*b^5 - 13*a^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^
2*c - 24*a^5*c^2)*e)*f - (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*
c^3)*sqrt((a^8*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^
2*c^3 + 625*a^4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 -
 550*a^4*b*c^3)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75
*a^5*c^3)*d^2*e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a
^4*b^4 - 18*a^5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c
)*d)*f^3 + 6*((9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4
*a^6*b*c)*d*e + (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4
*c - 180*a^4*b^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*
a^5*b*c^2)*d^2*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b
^3 - 9*a^6*b*c)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^
13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log(-((18
9*b^6*c^3 - 1971*a*b^4*c^4 + 5625*a^2*b^2*c^5 - 2500*a^3*c^6)*d^4 - (135*b^
7*c^2 - 1323*a*b^5*c^3 + 2727*a^2*b^3*c^4 + 2500*a^3*b*c^5)*d^3*e + 3*(45*a
*b^6*c^2 - 558*a^2*b^4*c^3 + 1672*a^3*b^2*c^4)*d^2*e^2 - (45*a^2*b^5*c^2 -
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647*a^3*b^3*c^3 + 2268*a^4*b*c^4)*d*e^3 + (5*a^3*b^4*c^2 - 81*a^4*b^2*c^3 +
 324*a^5*c^4)*e^4 - (3*a^6*b^2*c + 4*a^7*c^2)*f^4 + ((27*a^4*b^4*c + 80*a^6
*c^3)*d - (9*a^5*b^3*c - 20*a^6*b*c^2)*e)*f^3 - 3*((27*a^2*b^6*c - 117*a^3*
b^4*c^2 - 150*a^4*b^2*c^3 + 200*a^5*c^4)*d^2 - (18*a^3*b^5*c - 123*a^4*b^3*
c^2 - 100*a^5*b*c^3)*d*e + (3*a^4*b^4*c - 28*a^5*b^2*c^2)*e^2)*f^2 + ((81*b
^8*c - 945*a*b^6*c^2 + 3213*a^2*b^4*c^3 - 3000*a^3*b^2*c^4 + 2000*a^4*c^5)*
d^3 - 3*(27*a*b^7*c - 405*a^2*b^5*c^2 + 1461*a^3*b^3*c^3 - 500*a^4*b*c^4)*d
^2*e + 3*(9*a^2*b^6*c - 165*a^3*b^4*c^2 + 692*a^4*b^2*c^3)*d*e^2 - (3*a^3*b
^5*c - 65*a^4*b^3*c^2 + 324*a^5*b*c^3)*e^3)*f)*x - 1/2*sqrt(1/2)*((27*b^11
- 486*a*b^9*c + 3330*a^2*b^7*c^2 - 10549*a^3*b^5*c^3 + 14408*a^4*b^3*c^4 -
5200*a^5*b*c^5)*d^3 - 3*(9*a*b^10 - 177*a^2*b^8*c + 1285*a^3*b^6*c^2 - 4138
*a^4*b^4*c^3 + 5216*a^5*b^2*c^4 - 800*a^6*c^5)*d^2*e + 3*(3*a^2*b^9 - 64*a^
3*b^7*c + 495*a^4*b^5*c^2 - 1656*a^5*b^3*c^3 + 2032*a^6*b*c^4)*d*e^2 - (a^3
*b^8 - 23*a^4*b^6*c + 190*a^5*b^4*c^2 - 672*a^6*b^2*c^3 + 864*a^7*c^4)*e^3
- (a^6*b^5 - 8*a^7*b^3*c + 16*a^8*b*c^2)*f^3 + 3*((3*a^4*b^7 - 25*a^5*b^5*c
 + 56*a^6*b^3*c^2 - 16*a^7*b*c^3)*d - (a^5*b^6 - 10*a^6*b^4*c + 32*a^7*b^2*
c^2 - 32*a^8*c^3)*e)*f^2 - 3*((9*a^2*b^9 - 105*a^3*b^7*c + 373*a^4*b^5*c^2
- 248*a^5*b^3*c^3 - 560*a^6*b*c^4)*d^2 - 2*(3*a^3*b^8 - 40*a^4*b^6*c + 166*
a^5*b^4*c^2 - 176*a^6*b^2*c^3 - 160*a^7*c^4)*d*e + (a^4*b^7 - 15*a^5*b^5*c
+ 72*a^6*b^3*c^2 - 112*a^7*b*c^3)*e^2)*f + ((3*a^5*b^10 - 55*a^6*b^8*c + 39
2*a^7*b^6*c^2 - 1344*a^8*b^4*c^3 + 2176*a^9*b^2*c^4 - 1280*a^10*c^5)*d - (a
^6*b^9 - 20*a^7*b^7*c + 144*a^8*b^5*c^2 - 448*a^9*b^3*c^3 + 512*a^10*b*c^4)
*e - (a^7*b^8 - 8*a^8*b^6*c + 128*a^10*b^2*c^3 - 256*a^11*c^4)*f)*sqrt((a^8
*f^4 + (81*b^8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^
4*c^4)*d^4 - 4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3
)*d^3*e + 6*(9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*
e^2 - 4*(3*a^3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^
5*b^2*c + 81*a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*(
(9*a^4*b^4 + 3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e
+ (a^6*b^2 - 3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b
^2*c^2 + 125*a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2
*e + 3*(3*a^4*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c
)*e^3)*f)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt
(-((9*b^7 - 105*a*b^5*c + 385*a^2*b^3*c^2 - 420*a^3*b*c^3)*d^2 - 2*(3*a*b^6
 - 40*a^2*b^4*c + 150*a^3*b^2*c^2 - 120*a^4*c^3)*d*e + (a^2*b^5 - 15*a^3*b^
3*c + 60*a^4*b*c^2)*e^2 + (a^4*b^3 + 12*a^5*b*c)*f^2 - 2*((3*a^2*b^5 - 13*a
^3*b^3*c - 12*a^4*b*c^2)*d - (a^3*b^4 - 6*a^4*b^2*c - 24*a^5*c^2)*e)*f - (a
^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((a^8*f^4 + (81*b^
8 - 918*a*b^6*c + 3051*a^2*b^4*c^2 - 2550*a^3*b^2*c^3 + 625*a^4*c^4)*d^4 -
4*(27*a*b^7 - 351*a^2*b^5*c + 1197*a^3*b^3*c^2 - 550*a^4*b*c^3)*d^3*e + 6*(
9*a^2*b^6 - 132*a^3*b^4*c + 484*a^4*b^2*c^2 - 75*a^5*c^3)*d^2*e^2 - 4*(3*a^
3*b^5 - 49*a^4*b^3*c + 198*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 18*a^5*b^2*c + 81*
a^6*c^2)*e^4 + 4*(a^7*b*e - (3*a^6*b^2 + 5*a^7*c)*d)*f^3 + 6*((9*a^4*b^4 +
3*a^5*b^2*c + 25*a^6*c^2)*d^2 - 2*(3*a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 -
3*a^7*c)*e^2)*f^2 - 4*((27*a^2*b^6 - 108*a^3*b^4*c - 180*a^4*b^2*c^2 + 125*
a^5*c^3)*d^3 - 3*(9*a^3*b^5 - 51*a^4*b^3*c - 65*a^5*b*c^2)*d^2*e + 3*(3*a^4
*b^4 - 22*a^5*b^2*c - 15*a^6*c^2)*d*e^2 - (a^5*b^3 - 9*a^6*b*c)*e^3)*f)/(a^
10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6
*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))) - 4*(a*b^2 - 4*a^2*c)*d)/((a^2*b^2*
c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)
```

giac [B] time = 7.09, size = 7182, normalized size = 18.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")

[Out] -1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a*b*c*x^4*e + 3*b^3*
d*x^2 - 11*a*b*c*d*x^2 + a^2*b*f*x^2 - a*b^2*x^2*e + 2*a^2*c*x^2*e + 2*a*b^
```

```
2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*b^4
*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b
^2 - 4*a*c)*b^2*c^2 + 20*(b^2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*d + 2*(
2*a^2*b^2*c^2 - 8*a^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2
- 2*(b^2 - 4*a*c)*a^2*c^2)*(a^2*b^2 - 4*a^3*c)^2*f - (2*a*b^3*c^2 - 8*a^2*b
*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b
*c^2)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*
c^2 + 74*a^3*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^
3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a
^2*b^5*c - 50*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*d*ab
s(a^2*b^2 - 4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5 -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c - 2*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*a^4*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^6*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b
^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 16*a^5*b^3*c
^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 32*a^6*b*c^3 + 2
*(b^2 - 4*a*c)*a^4*b^3*c - 8*(b^2 - 4*a*c)*a^5*b*c^2)*f*abs(a^2*b^2 - 4*a^3
*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 28*a^4*b^4*c^2 - 96*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^3 - 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^4
 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a*c)*a^4*b^2*c^2 +
 48*(b^2 - 4*a*c)*a^5*c^3)*abs(a^2*b^2 - 4*a^3*c)*e + (6*a^4*b^8*c^2 - 80*a
^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(
b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*d - 4*(2*a^6*b^6*
c^2 - 16*a^7*b^4*c^3 + 32*a^8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^6*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
```

```
t(b^2 - 4*a*c)*c)*a^7*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^6*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^8*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^7*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^6*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^7*b^2*c^3 - 2*(b^2 - 4*a*c)*a^6*b^4*c^2 + 8*(b^2 - 4*a*c)*a^7*b^2
*c^3)*f - (2*a^5*b^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c
)*a^5*b^5*c^2 + 32*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*
e)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b
*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*
c^2)))/((a^5*b^6 - 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3
*c^2 + a^5*b^4*c^2 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4
)*abs(a^2*b^2 - 4*a^3*c)*abs(c)) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^
2 - 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*d + 2*(2*a^2*b^2*c^2 - 8*a^3*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c + 2*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*c^2)*(
a^2*b^2 - 4*a^3*c)^2*f - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*
e - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^7 - 37*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^6*c + 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 3*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 74*a^3*b^5*c^2 - 208*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^3*b^3*c^3 + 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b*c^4 - 416*a^5*b*c^4 - 6*(b^2 - 4*a*c)*a^2*b^5*c + 50*(b^2 - 4*a*c)*
a^3*b^3*c^2 - 104*(b^2 - 4*a*c)*a^4*b*c^3)*d*abs(a^2*b^2 - 4*a^3*c) + 2*(sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^5*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^
4*c + 2*a^4*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^2 +
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 + sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 16*a^5*b^3*c^2 - 4*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 32*a^6*b*c^3 - 2*(b^2 - 4*a*c)*a^4*b^3*c + 8
*(b^2 - 4*a*c)*a^5*b*c^2)*f*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*a^3*b^6*
```

```
c + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^2 + 20*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^3*b^4*c^2 - 28*a^4*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 10*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 128*a^5*b^2*c^3 + 24*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^4 - 192*a^6*c^4 - 2*(b^2 - 4*
a*c)*a^3*b^4*c + 20*(b^2 - 4*a*c)*a^4*b^2*c^2 - 48*(b^2 - 4*a*c)*a^5*c^3)*a
bs(a^2*b^2 - 4*a^3*c)*e + (6*a^4*b^8*c^2 - 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4
 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4
*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 + 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 1
28*(b^2 - 4*a*c)*a^6*b^2*c^4)*d - 4*(2*a^6*b^6*c^2 - 16*a^7*b^4*c^3 + 32*a^
8*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b
^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c
+ 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c - 1
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^2 - 8
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 - 2*(b^2
 - 4*a*c)*a^6*b^4*c^2 + 8*(b^2 - 4*a*c)*a^7*b^2*c^3)*f - (2*a^5*b^7*c^2 - 4
0*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32*(b^2 - 4*
a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*e)*arctan(2*sqrt(1/2)*x/sqrt
((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c
)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 - 12*a^6*b^
4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2 - 64*a^8*
c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*a^3*c)*abs
(c))
```

maple [B] time = 0.05, size = 1575, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+4/a*c^2/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-3/4/a^2*c/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
```

```
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b^2*e-d/a^2/x+1/4/a*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/a
/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*d-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*
b^2*e+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^3*d-1/2*c/(4*a*c-b^2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*f+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f-1/4/a*c/(4*a*c-b^2)
*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b*e+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b*f+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f-3/4/a^2*c/(
4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c*x)*b^2*d+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*e+1/2/(c*x^4+b*x^2+a)/(4*a*c-b
^2)*x*b*f+c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*f+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*
a*c-b^2)*x^3*b^2*d+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-5/2/a*c^2/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*e-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
e-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*e-3/2/a/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*x*b*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(abce - 2\,a^2cf - \left(3\,b^2c - 10\,ac^2\right)d\right)x^4 - \left(a^2bf + \left(3\,b^3 - 11\,abc\right)d - \left(ab^2 - 2\,a^2c\right)e\right)x^2 - 2\,\left(ab^2 - 4\,a^2c\right)d}{2\,\left(\left(a^2b^2c - 4\,a^3c^2\right)x^5 + \left(a^2b^3 - 4\,a^3bc\right)x^3 + \left(a^3b^2 - 4\,a^4c\right)x\right)} - \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*f + (3*b^3
 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 - 4*a^2*c)*d)/((a^2*b^
2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) -
 1/2*integrate(-(a^2*b*f + (a*b*c*e - 2*a^2*c*f - (3*b^2*c - 10*a*c^2)*d)*x
^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 - 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a
^2*b^2 - 4*a^3*c)
```

mupad [B] time = 6.86, size = 28164, normalized size = 70.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((x^2*(3*b^3*d - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*c
 - b^2)) - d/a + (c*x^4*(3*b^2*d + 2*a^2*f - a*b*e - 10*a*c*d))/(2*a^2*(4*a
*c - b^2)))/(a*x + b*x^3 + c*x^5) - atan(((x*(204800*a^12*c^9*d^2 - 73728*a
^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*
d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*
```

d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 +
4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 +
160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 1638
4*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^1
3*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^
5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3
*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f
+ 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 92
16*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 - a^2*b^
11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 +
9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e -
2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 4480
0*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840
*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*
d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*
f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*
c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*
c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c
- b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*
b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*
a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d
e(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c
+ 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5
)))^(1/2)*(x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^
9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6
*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/
2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^
7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b
^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f
^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^
6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*
b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36
*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2
*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e
+ 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*
b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e
*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a
^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^
3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256
*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*
c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*a^15*c^8*e + 192
*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*
b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^
2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e -
327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*
a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b
^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f))*((27*a^3*b^9*c*e^2 -
a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(
-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5
*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*
d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2
+ 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2
d^2(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2
- 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b

^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*
c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^
3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 1
0*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-
(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 2240
0*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9
)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f
- 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2
*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b
^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b
^2*c^5)))^(1/2)*1i + (x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^
14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c
^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2
*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^
2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2
- 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 -
81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*
b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b
^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c
^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7
*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f -
32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-
(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2
6880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2
+ 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a
^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96
*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f
+ 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^
5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 15
48*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a
^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d
*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128
*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)
^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1
280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(393216*a^1
5*c^8*e + x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9
)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*
d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7
*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^
7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^
2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6
*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(
1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b
*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*
a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*
d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e
+ 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b
^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*
f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*
a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c
^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 467

2*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f))*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/((x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^

```
2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f
+ 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*
d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c -
 b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^
4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2)
+ 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^
5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5))
)^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 6144
0*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b
^3*c^7) - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47
360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 126156
8*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^
8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c
^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40
960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^1
5*b*c^7*f))*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9
)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*
d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7
*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^
7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^
2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6
*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(
1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b
*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*
a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*
d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e
+ 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b
^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*
f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) - (x*(204800*a^12*c^9*d^2
- 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*
b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10
*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*
c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c
^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f
^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e +
40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a
^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8
*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^
4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4
*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((27*a^3*b^9*c*e^2
 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^
2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*
c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^
12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d
^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*
c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e
^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*
a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a
^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98
*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2
*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 2
```

2400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2
)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d
*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*
a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^
6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^1
0*b^2*c^5)))^(1/2)*(393216*a^15*c^8*e + x*((27*a^3*b^9*c*e^2 - a^2*b^11*e^2
 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 - a^4*f^2*(-(4*a*c - b^
2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*a^7*b*c^5*e^2 + 9*a^3
*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a
^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*
b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^2*c^2*d^2*(-(4*a*
c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b
^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 +
6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f + 6*
a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f
+ 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) + 10*a^3*c*d*f*
(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2*c*d^2*(-(4*a*c - b^2
)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^
4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 57
6*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^
6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f - 44*a^2*b*c*d*e*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*
a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1
/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^
12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c
^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256
000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^
9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^
6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*
f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 819
20*a^14*b^3*c^6*f - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f))*((27*a^3*b^9
*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f^2 -
a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 + 3840*
a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 +
6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5
*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 2
5*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5
*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2
+ 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f -
3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*
e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^
(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*a*b^2
*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d
*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(4*a*c
 - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3
*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f
 - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 -
 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 61
44*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9*d^3 - 1024*a^13*c^6*f^3 + 4608*a
^11*b*c^7*e^3 + 46080*a^11*c^8*d*e^2 - 76800*a^11*c^8*d^2*f + 15360*a^12*c^
7*d*f^2 - 9216*a^12*c^7*e^2*f + 504*a^6*b^8*c^5*d^3 - 8112*a^7*b^6*c^6*d^3
+ 48704*a^8*b^4*c^7*d^3 - 129280*a^9*b^2*c^8*d^3 - 40*a^8*b^7*c^4*e^3 + 608
*a^9*b^5*c^5*e^3 - 2944*a^10*b^3*c^6*e^3 - 48*a^10*b^6*c^3*f^3 + 320*a^11*b
^4*c^4*f^3 - 256*a^12*b^2*c^5*f^3 - 84480*a^10*b*c^8*d^2*e + 7680*a^12*b*c^
6*e*f^2 - 360*a^6*b^9*c^4*d^2*e + 5736*a^7*b^7*c^5*d^2*e + 240*a^7*b^8*c^4*
d*e^2 - 33888*a^8*b^5*c^6*d^2*e - 3792*a^8*b^6*c^5*d*e^2 + 87936*a^9*b^3*c^
7*d^2*e + 21696*a^9*b^4*c^6*d*e^2 - 52992*a^10*b^2*c^7*d*e^2 + 216*a^6*b^10
*c^3*d^2*f - 3744*a^7*b^8*c^4*d^2*f + 25200*a^8*b^6*c^5*d^2*f + 72*a^8*b^8*
c^3*d*f^2 - 81984*a^9*b^4*c^6*d^2*f - 1296*a^9*b^6*c^4*d*f^2 + 128256*a^10*

b^2*c^7*d^2*f + 7872*a^10*b^4*c^5*d*f^2 - 19200*a^11*b^2*c^6*d*f^2 + 24*a^8
*b^8*c^3*e^2*f - 336*a^9*b^6*c^4*e^2*f - 24*a^9*b^7*c^3*e*f^2 + 960*a^10*b^
4*c^5*e^2*f + 672*a^10*b^5*c^4*e*f^2 + 2304*a^11*b^2*c^6*e^2*f - 4224*a^11*
b^3*c^5*e*f^2 - 21504*a^11*b*c^7*d*e*f - 144*a^7*b^9*c^3*d*e*f + 2256*a^8*b
^7*c^4*d*e*f - 12480*a^9*b^5*c^5*d*e*f + 28416*a^10*b^3*c^6*d*e*f))*((27*a^
3*b^9*c*e^2 - a^2*b^11*e^2 - 9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*f
^2 - a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 - 9*b^13*d^2 +
3840*a^7*b*c^5*e^2 + 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f
^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^
4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2
) - 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^
5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3
*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e
*f - 3072*a^8*c^5*e*f + 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10
*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f - 2*a^3*b*e*f*(-(4*a*c - b^2
)^9)^(1/2) + 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f + 51*
a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*
c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e + 6*a^2*b^2*d*f*(-(
4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^
6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^
4*e*f - 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*
c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
 - 6144*a^10*b^2*c^5)))^(1/2)*2i - atan(((x*(204800*a^12*c^9*d^2 - 73728*a^
13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d
^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d
^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 +
4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 1
60*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384
*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13
*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5
*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*
d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f
+ 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 921
6*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((9*b^4*d^2*(-(4*a*c - b^2)^
9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2
)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 -
9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2
077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800
*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-
(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*
a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d
^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f
 - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c
*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c
*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c
- b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b
^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2)
 + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a
^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*
e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c +
 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)
))^(1/2)*(x*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^
2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 +
27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7
*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^
7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^
2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6

*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(
1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b
*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*
a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*
d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e
- 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b
^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*
f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*
a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c
^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*a^15*c^8*e + 192*
a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b
^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2
*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e -
327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a
^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^
3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f))*((9*b^4*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c
 - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*
e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d
*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 +
 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*
d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 -
 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^
11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c
^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3
*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10
*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400
*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f -
 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*
b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^
10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^
2*c^5)))^(1/2)*1i + (x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^1
4*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^
5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*
c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2
 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 -
 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 8
1920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b
^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^
5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^
4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*
d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f -
32768*a^12*b^3*c^6*e*f) + ((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e
^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^
6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 +
 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^
2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*
a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f +
 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a
*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5
*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^
9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 154
8*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^

6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*
f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*
a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 12
80*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(393216*a^15
*c^8*e + x*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2
 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 2
7*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*
c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7
*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2
 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*
d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1
/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*
e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a
^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d
*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e -
 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^
5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f
 + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a
^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^
5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672
*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^1
2*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c
^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e
 - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240
*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 851968*a^14
*b*c^8*d - 65536*a^15*b*c^7*f))*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*
b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26
880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2
*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^
2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9
)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2
 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11
*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7
*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c -
 b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
 + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30
720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*
c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f
+ 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^
2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*1i)/((
x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b
^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^
6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10
*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6
*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^
2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 2
37568*a^12*b*c^8*d*e + 40960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^
8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^
11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6
*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*
e*f - 1024*a^10*b^7*c^4*e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*

```
f) + ((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4
*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*
b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768
*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^
2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c -
b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e
^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512
*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e -
2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) -
152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-
(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8
*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8
064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2
*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*
d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 153
6*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12
+ 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840
*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*((9*b^4*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*
a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 207
7*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a
^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4
*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^
6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2
 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f -
 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d
*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d
*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4
*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) +
 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5
*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 2
40*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))
^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440
*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^
3*c^7) - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 473
60*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568
*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8
*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^
7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 409
60*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 851968*a^14*b*c^8*d + 65536*a^15
*b*c^7*f))*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2
 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 2
7*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*
c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7
*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2
 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*
d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1
/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*
e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a
^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d
*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e -
 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^
5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f
 + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5
```

```
*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3
+ 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2) - (x*(204800*a^12*c^9*d^2 -
 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b
^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*
b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c
^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^
7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^
2 - 16384*a^13*b^2*c^6*f^2 - 81920*a^13*c^8*d*f + 237568*a^12*b*c^8*d*e + 4
0960*a^13*b*c^7*e*f - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^
9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*
b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4
*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 32*a^9*b^9*c^3*e*f - 1024*a^10*b^7*c^4*
e*f + 9216*a^11*b^5*c^5*e*f - 32768*a^12*b^3*c^6*e*f) + ((9*b^4*d^2*(-(4*a*
c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c
^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^1
2*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^
2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c
^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^
2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a
*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^
8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*
a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) -
 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*
(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22
400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*
f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a
^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6
*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10
*b^2*c^5)))^(1/2)*(393216*a^15*c^8*e + x*((9*b^4*d^2*(-(4*a*c - b^2)^9)^(1/
2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2*(-(4*a*c - b^2)^9)^(
1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a^7*b*c^5*e^2 - 9*a^3*
c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6*a*b^12*d*e - 2077*a^
2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*c^4*d^2 + 44800*a^5*b
^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^2*c^2*d^2*(-(4*a*c
 - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*c^3*e^2 - 3840*a^6*b^
3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 + 213*a*b^11*c*d^2 + 6
*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3072*a^8*c^5*e*f - 6*a
*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e - 98*a^3*b^9*c*d*f +
 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(1/2) - 10*a^3*c*d*f*(
-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*c*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*e + 22400*a^5*b^4*c^4
*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 576
*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*c^4*d*f - 192*a^5*b^6
*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f + 44*a^2*b*c*d*e*(-(4
*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a
^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/
2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^1
2*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^
7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 2560
00*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9
*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6
*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f
 - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 8192
0*a^14*b^3*c^6*f - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f))*((9*b^4*d^2*(
-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^4*f^2
*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3840*a
^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^2 + 6
```

```
*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4*b^5*
c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 25
*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5*b^5*
c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*f^2 +
 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*f - 3
072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c*d*e
 - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)^9)^(
1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a*b^2*
c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c^3*d*
e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6*b^3*
c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4*e*f
+ 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 -
24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 614
4*a^10*b^2*c^5)))^(1/2) + 128000*a^10*c^9*d^3 - 1024*a^13*c^6*f^3 + 4608*a^
11*b*c^7*e^3 + 46080*a^11*c^8*d*e^2 - 76800*a^11*c^8*d^2*f + 15360*a^12*c^7
*d*f^2 - 9216*a^12*c^7*e^2*f + 504*a^6*b^8*c^5*d^3 - 8112*a^7*b^6*c^6*d^3 +
 48704*a^8*b^4*c^7*d^3 - 129280*a^9*b^2*c^8*d^3 - 40*a^8*b^7*c^4*e^3 + 608*
a^9*b^5*c^5*e^3 - 2944*a^10*b^3*c^6*e^3 - 48*a^10*b^6*c^3*f^3 + 320*a^11*b^
4*c^4*f^3 - 256*a^12*b^2*c^5*f^3 - 84480*a^10*b*c^8*d^2*e + 7680*a^12*b*c^6
*e*f^2 - 360*a^6*b^9*c^4*d^2*e + 5736*a^7*b^7*c^5*d^2*e + 240*a^7*b^8*c^4*d
*e^2 - 33888*a^8*b^5*c^6*d^2*e - 3792*a^8*b^6*c^5*d*e^2 + 87936*a^9*b^3*c^7
*d^2*e + 21696*a^9*b^4*c^6*d*e^2 - 52992*a^10*b^2*c^7*d*e^2 + 216*a^6*b^10*
c^3*d^2*f - 3744*a^7*b^8*c^4*d^2*f + 25200*a^8*b^6*c^5*d^2*f + 72*a^8*b^8*c
^3*d*f^2 - 81984*a^9*b^4*c^6*d^2*f - 1296*a^9*b^6*c^4*d*f^2 + 128256*a^10*b
^2*c^7*d^2*f + 7872*a^10*b^4*c^5*d*f^2 - 19200*a^11*b^2*c^6*d*f^2 + 24*a^8*
b^8*c^3*e^2*f - 336*a^9*b^6*c^4*e^2*f - 24*a^9*b^7*c^3*e*f^2 + 960*a^10*b^4
*c^5*e^2*f + 672*a^10*b^5*c^4*e*f^2 + 2304*a^11*b^2*c^6*e^2*f - 4224*a^11*b
^3*c^5*e*f^2 - 21504*a^11*b*c^7*d*e*f - 144*a^7*b^9*c^3*d*e*f + 2256*a^8*b^
7*c^4*d*e*f - 12480*a^9*b^5*c^5*d*e*f + 28416*a^10*b^3*c^6*d*e*f))*((9*b^4*
d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^11*e^2 - 9*b^13*d^2 - a^4*b^9*f^2 + a^
4*f^2*(-(4*a*c - b^2)^9)^(1/2) - 26880*a^6*b*c^6*d^2 + 27*a^3*b^9*c*e^2 + 3
840*a^7*b*c^5*e^2 - 9*a^3*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^8*b*c^4*f^
2 + 6*a*b^12*d*e - 2077*a^2*b^9*c^2*d^2 + 10656*a^3*b^7*c^3*d^2 - 30240*a^4
*b^5*c^4*d^2 + 44800*a^5*b^3*c^5*d^2 + a^2*b^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 25*a^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^4*b^7*c^2*e^2 + 1504*a^5
*b^5*c^3*e^2 - 3840*a^6*b^3*c^4*e^2 + 96*a^6*b^5*c^2*f^2 - 512*a^7*b^3*c^3*
f^2 + 213*a*b^11*c*d^2 + 6*a^2*b^11*d*f + 15360*a^7*c^6*d*e - 2*a^3*b^10*e*
f - 3072*a^8*c^5*e*f - 6*a*b^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*
c*d*e - 98*a^3*b^9*c*d*f + 1536*a^7*b*c^5*d*f + 2*a^3*b*e*f*(-(4*a*c - b^2)
^9)^(1/2) - 10*a^3*c*d*f*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c*e*f - 51*a
*b^2*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 1548*a^3*b^8*c^2*d*e - 8064*a^4*b^6*c
^3*d*e + 22400*a^5*b^4*c^4*d*e - 30720*a^6*b^2*c^5*d*e - 6*a^2*b^2*d*f*(-(4
*a*c - b^2)^9)^(1/2) + 576*a^4*b^7*c^2*d*f - 1344*a^5*b^5*c^3*d*f + 512*a^6
*b^3*c^4*d*f - 192*a^5*b^6*c^2*e*f + 128*a^6*b^4*c^3*e*f + 1536*a^7*b^2*c^4
*e*f + 44*a^2*b*c*d*e*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c
^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4
- 6144*a^10*b^2*c^5)))^(1/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.73 $\int \frac{d+ex^2+fx^4}{x^4(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=575

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^2
*(b*e+4*c*d)/a+b^2*f-2*a*c*f)+c*(b^3*d-a*b^2*e+2*a^2*c*e-a*b*(-a*f+3*c*d))*
x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d+b^3*(-3*a*e+5*d*(-4*a*c+b^2)^(1/2))-
a*b^2*(29*c*d-a*f+3*e*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f+5*e*(-4*a*c
+b^2)^(1/2))-a*b*(-16*a*c*e+19*(-4*a*c+b^2)^(1/2)*c*d-(-4*a*c+b^2)^(1/2)*a*
f))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^4*d-b^3*(3*a*e
+5*d*(-4*a*c+b^2)^(1/2))+2*a^2*c*(14*c*d-6*a*f-5*e*(-4*a*c+b^2)^(1/2))-a*b^
2*(29*c*d-a*f-3*e*(-4*a*c+b^2)^(1/2))+a*b*(16*a*c*e+19*(-4*a*c+b^2)^(1/2)*c
*d-(-4*a*c+b^2)^(1/2)*a*f))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a*c+b^2)^
(1/2))^(1/2)
```

Rubi [A] time = 9.91, antiderivative size = 575, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1669, 1664, 1166, 205}

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-2acf+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2ce-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\tan^{-1}\left(\frac{\cdots}{\sqrt{b\cdots}}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3
*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - 2*a*c*f) + c*(b^3*d - a*b^2*e + 2*
a^2*c*e - a*b*(3*c*d - a*f))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + (Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*c
*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) - a*b^2*(29*c*d + 3*Sqrt[b^2 - 4*a*c]*e
 - a*f) - a*b*(19*c*Sqrt[b^2 - 4*a*c]*d - 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f)
)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(
b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(5*b^4*d - b^3*(
5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + 2*a^2*c*(14*c*d - 5*Sqrt[b^2 - 4*a*c]*e -
6*a*f) - a*b^2*(29*c*d - 3*Sqrt[b^2 - 4*a*c]*e - a*f) + a*b*(19*c*Sqrt[b^2
- 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/
Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + S
qrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-ab^2e+2a^2ce-ab(3cd\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-ab^2e+2a^2ce-ab(3cd\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-2acf\right)+c\left(b^3d-\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.80, size = 548, normalized size = 0.95

$$\frac{\frac{6x\left(2a^2c\left(c\left(d+ex^2\right)-af\right)+b^3\left(cdx^2-ae\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+abc\left(3ae+afx^2-3cdx^2\right)+b^4d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2a^2c\left(5e\sqrt{b^2-4ac}-6af+14c\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b*c*(3*a*e - 3*c*d*x^2 + a*f*x^2) + 2*a^2*c*(-(a*f) + c*(d + e*x^2)) +
a*b^2*(a*f - c*(4*d + e*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sq
rt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3*a*e) + 2*a^2*c*(14*
c*d + 5*Sqrt[b^2 - 4*a*c]*e - 6*a*f) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]
*e + a*f) + a*b*(-19*c*Sqrt[b^2 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]
*f))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(-5*b^4*d + b^3*(
5*Sqrt[b^2 - 4*a*c]*d + 3*a*e) - a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a
*f) + 2*a^2*c*(-14*c*d + 5*Sqrt[b^2 - 4*a*c]*e + 6*a*f) + a*b*(-19*c*Sqrt[b
^2 - 4*a*c]*d - 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4
*a*c]]))/(12*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [B] time = 8.59, size = 8660, normalized size = 15.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 + a^2*b*c*f*x^3 - a*b^2*c*x^3*e + 2*a^2*
c^2*x^3*e + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x + a^2*b^2*f*x - 2*a^3*c
*f*x - a*b^3*x*e + 3*a^2*b*c*x*e)/((a^3*b^2 - 4*a^4*c)*(c*x^4 + b*x^2 + a))
 + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*
(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 -
4*a^4*c)^2*f - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*
c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
```

*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 22
4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4
*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*d*abs
(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6 -
14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
6*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 28*a^6*b^
4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^6*b^2*c^3 - 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^7*c^4 + 192*a^8*c^4 + 2*(b^2 - 4*a*c)*a^5*b^4*c - 20*(b^2 - 4*a*
c)*a^6*b^2*c^2 + 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c) - 2*(3*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^6*c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*
c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3
*c^3 - 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c
^4 + 416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c
^2 + 104*(b^2 - 4*a*c)*a^6*b*c^3)*abs(a^3*b^2 - 4*a^4*c)*e + (10*a^6*b^9*c^
2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 -
5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b
^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*
a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d + (2*a^8*b^7*c^2 - 40*a^9*b^5*
c^3 + 224*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(b^2 - 4*a*c)
*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f - (6*a^7*b^8*c^2 - 80*a^8*b^6
*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +

```
sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2
- 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e)*arctan(2*sqrt(1/2)
*x/sqrt((a^3*b^3 - 4*a^4*b*c + sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 -
4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12
*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 -
64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^
4*c)*abs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*
c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)
*(a^3*b^2 - 4*a^4*c)^2*f - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20
*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e - 2*(5*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*a^3*b^8
*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*
b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 572*a^5*
b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*(b^2
- 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*c)*a^
6*c^4)*d*abs(a^3*b^2 - 4*a^4*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^5*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c + 2*a^5*b^6*c + 64*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2
 - 28*a^6*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 + 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^7*c^4 - 192*a^8*c^4 - 2*(b^2 - 4*a*c)*a^5*b^4*c + 20
*(b^2 - 4*a*c)*a^6*b^2*c^2 - 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^
4*c) + 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*b^6*c + 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
```

```
*c)*a^6*b^3*c^2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 +
3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 - 74*a^5*b^5*c^2 - 20
8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^3*c^3 + 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^6*b*c^4 - 416*a^7*b*c^4 - 6*(b^2 - 4*a*c)*a^4*b^5*c + 50*(b^2 - 4*a*
c)*a^5*b^3*c^2 - 104*(b^2 - 4*a*c)*a^6*b*c^3)*abs(a^3*b^2 - 4*a^4*c)*e + (1
0*a^6*b^9*c^2 - 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*
a^10*b*c^6 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
6*b^9 + 69*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^
7*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*
c - 340*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c
^2 - 98*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c
^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^
2 + 688*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c
^3 + 288*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*
c^3 + 49*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*
c^3 - 448*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*
c^4 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2
*c^4 - 144*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^
3*c^4 + 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b
*c^5 - 10*(b^2 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b
^2 - 4*a*c)*a^8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d + (2*a^8*b^7*c^2 -
 40*a^9*b^5*c^3 + 224*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(
b^2 - 4*a*c)*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f - (6*a^7*b^8*c^2
- 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^
2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e)*arctan
(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4
*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((
a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^
7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^
3*b^2 - 4*a^4*c)*abs(c)) + 1/3*(6*b*d*x^2 - 3*a*x^2*e - a*d)/(a^3*x^3)
```

maple [B] time = 0.06, size = 2180, normalized size = 3.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/3*d/a^2/x^3+2/a^3/x*b*d-5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d+7
/a*c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/4/a*c/(4*a*
c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b^2*e-1/4/a*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f+3/4/a^2*c/(4*a*c-b^2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*c*x)*b^2*e+19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+7/a*
c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d-19/4/a^2*c^2/(4*a*c
-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b^3*d-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^
2*d+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+4/a*c^2
/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-3/4/a^2*c/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*e+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*b^3*e+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*b^2*f+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^4*d+5/4/a^3*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+
b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)
*b^4*d-29/4/a^2*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d
-1/2/a/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b*f+1/2/a^2/(c*x^4+b*x^2+a)*c/(4*a
*c-b^2)*x^3*b^2*e+3/2/a^2/(c*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*b*d-3/2/a/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*x*b*c*e+2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*c*
d-1/2/a^3/(c*x^4+b*x^2+a)*c/(4*a*c-b^2)*x^3*b^3*d+5/2/a*c^2/(4*a*c-b^2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*e-5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-3*c^2/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f-3*c^2/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*c*f-1/a/(c
*x^4+b*x^2+a)*c^2/(4*a*c-b^2)*x^3*e-1/2/a^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b
^4*d-1/2/a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^2*f-1/a/(c*x^4+b*x^2+a)/(4*a*c-b
^2)*x*c^2*d+1/2/a^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b^3*e-1/a^2/x*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(a^2*b*c*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*
x^6 + ((15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d - 3*(3*a*b^3 - 11*a^2*b*c)*e +
```

```
3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*b*c)*d - 3*(a^2*b^2 - 4*
a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^
3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) + 1/2*integrate(((a^2*b*c
*f + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 -
24*a*b^2*c + 14*a^2*c^2)*d - (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)
*f)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

mupad [B] time = 7.37, size = 36097, normalized size = 62.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 +
 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 -
488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d
^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*
e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^
7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f
^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^
2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f -
480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e +
 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*
d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d
*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^
8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e
*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 +
9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^
2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d
*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2
 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c
^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*
e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f
^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*
f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^1
2*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 2
46*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^
6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f +
 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b
^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e
*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f -
 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)
^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 9
17504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c
- b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 +
 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*
(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^
3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^
6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 3024
0*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^
```

(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 150
4*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d
*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*
(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*
a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e +
 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e -
161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4
*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^
3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*
c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7
*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2
*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^1
0*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b
^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^
3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 157286
4*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b
^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b
^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^
3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e
+ 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 1792
0*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a
^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*
b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213
*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f
^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c
^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^
5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 4
9*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b
^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*
b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^
2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*
f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^
11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a
^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a
^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d
*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^
4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^
3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*
b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(
1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13
*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*
c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + (x*(204800*a^17*c^9*e^2 - 401408*a^16
*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*
d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*
c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^
12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*
b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b
^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4
*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^
9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4
*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5
*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^1

0*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*
b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^
9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*
b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)
^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880
*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*
c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*
c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c
- b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*
b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*
b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 3
5840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a
*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*
c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2)
 - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132
*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280
*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c
^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*
d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c
^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^
2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f
*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c +
240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5
)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2 + 9*a^2
*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*
b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3
840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e +
6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 21
9744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^
2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 +
 a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(
1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 -
615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 1
5360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d
*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^
2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4
*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(
4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 4435
2*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9
)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f +
8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a
^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(
1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b
^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 +
 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a
^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^
5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 793
6*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000
*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^1
3*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^
7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2
*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f -

327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f + 851968*a^19*b*c^8*e))*(-(25
*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11
*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*
a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) -
30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4
*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 20
77*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*
a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(
4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a
^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6
*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) +
724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^
10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 1
19616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 1
0*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b
^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*
(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a
^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b
^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^
3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 12
80*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/((x*(2
04800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^
14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12
*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 187187
2*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112
*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458
752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a
^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*
a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^
13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^1
3*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a
^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704
*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*
a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520
*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*
e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*
d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^
9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a
^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a
^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10
656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b
^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) +
288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*
b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a
^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 2
58*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*
c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
- 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e +
201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*
b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a
^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2
*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) +

184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a

```
^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*
a^12*b^2*c^5)))^(1/2) - (x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73
728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^
11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401
280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3
264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 +
365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 -
416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 696
32*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568
*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*
a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 212
1728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39
488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f -
720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22
528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) +
 (-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^
4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2
 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(
1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 1169
28*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^
2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 -
44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 +
3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d
*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(
1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*
a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c
*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d
*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d
*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784
*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^
3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720
*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e
*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) -
186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^
2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^
2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917
504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25
*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 21
3*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*
f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*
c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c
^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*
b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(
-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6
*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d
^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e
*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b
^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*
(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*
a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*
a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9
)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*
d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a
^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c
^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(
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-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2
*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^
(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^1
3*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4
*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 -
 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^1
9*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^
3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*
d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e -
 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 7782
40*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15
*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^
4*c^6*f + 557056*a^19*b^2*c^7*f + 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*
a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a
^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2
- 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e
 + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 -
 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c -
 b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2
*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^
2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2
 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f
- 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*
c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246
*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)
^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*
c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*
(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 4
4352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2
)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f
 + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 5
1*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9
)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-
(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^
7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^
3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - 128000*a^15*c^9*e^3 +
476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f^3 - 250880*a^14*c^10*d^2*e - 460
80*a^16*c^8*e*f^2 + 1800*a^9*b^9*c^6*d^3 - 29080*a^10*b^7*c^7*d^3 + 176032*
a^11*b^5*c^8*d^3 - 473216*a^12*b^3*c^9*d^3 - 504*a^11*b^8*c^5*e^3 + 8112*a^
12*b^6*c^6*e^3 - 48704*a^13*b^4*c^7*e^3 + 129280*a^14*b^2*c^8*e^3 + 40*a^13
*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^3 + 2944*a^15*b^3*c^6*f^3 + 215040*a^15*c
^9*d*e*f + 442880*a^14*b*c^9*d*e^2 - 433664*a^14*b*c^9*d^2*f + 109056*a^15*
b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f - 1400*a^9*b^10*c^5*d^2*e + 21680*a^10
*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 - 121648*a^11*b^6*c^7*d^2*e - 2717
6*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8*d^2*e + 164448*a^12*b^5*c^7*d*e^
2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^13*b^3*c^8*d*e^2 + 1000*a^9*b^11*c
^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 124280*a^11*b^7*c^6*d^2*f + 400*a^11*
b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f - 6600*a^12*b^7*c^5*d*f^2 + 69491
2*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6*d*f^2 - 108928*a^14*b^3*c^7*d*f^2
 + 360*a^11*b^9*c^4*e^2*f - 5736*a^12*b^7*c^5*e^2*f - 240*a^12*b^8*c^4*e*f^
2 + 33888*a^13*b^5*c^6*e^2*f + 3792*a^13*b^6*c^5*e*f^2 - 87936*a^14*b^3*c^7
*e^2*f - 21696*a^14*b^4*c^6*e*f^2 + 52992*a^15*b^2*c^7*e*f^2 - 1200*a^10*b^
10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f - 130656*a^12*b^6*c^6*d*e*f + 39436
8*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8*d*e*f))*(-(25*b^15*d^2 + 9*a^2*b
^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*
c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 384
0*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 63
66*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 2197
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44*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - (d/(3*a) + (x^2*(3*a*e - 5*b*d))/(3*a^2) + (x^4*(15*b^4*d + 14*a^2*c^2*d + 3*a^2*b^2*f - 9*a*b^3*e - 6*a^3*c*f - 62*a*b^2*c*d + 33*a^2*b*c*e))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*b^3*d - 3*a*b^2*e + a^2*b*f + 10*a^2*c*e - 19*a*b*c*d))/(2*a^3*(4*a*c - b^2)))/(a*x^3 + b*x^5 + c*x^7) + atan(((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d* f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + (x*(

204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9

)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*
a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3
*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d
+ 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d -
2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 4736
0*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*
a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8
*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^
7*f + 851968*a^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-
(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*
c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5
*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 3
5767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 21
5040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2
+ 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b
^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^
2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2
*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b
^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f +
43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^
2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5
*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2
706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120
*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22
400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*
(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*
a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*
a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144
*a^12*b^2*c^5)))^(1/2)*1i)/((x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2
- 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 9281
6*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 -
2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2
- 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^
2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^
2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 +
69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f - 1236992*a^16*b*c^9*d*e + 23
7568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105
824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e +
2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f
+ 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*
f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3*e*f + 2336*a^13*b^9*c^4*e*f
- 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6*e*f - 253952*a^16*b^3*c^7*e*
f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6
*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 +
116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 -
9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^
(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^
2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^
2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c

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^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^
9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f +
152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b
^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c
^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c
^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 1
4784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^
3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 3
0720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b
*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2
) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^
8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*
(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2
- 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2
- 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*
c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b
^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b
^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*
a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f
^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288
*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13
*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c
^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a
^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7
278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201
600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^
2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*
c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) -
42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b
^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184
*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)
^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096
*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11
*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c
^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040
*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^1
2*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*
c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2
*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e -
778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*
a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^1
8*b^4*c^6*f - 557056*a^19*b^2*c^7*f - 851968*a^19*b*c^8*e))*(-(25*b^15*d^2
+ 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 806
40*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*
f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14
*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d
^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a
*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9
*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^
5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4
*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*
e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b
^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f -
```

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246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c -
b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*
b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*
d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f
 + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c -
 b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2
*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f
 + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^
2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^
6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - (x*(204800*a^17*c^
9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 400*a^9*b^14*c^3*d^2 -
9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2
+ 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^
9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5
*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*
c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f
^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 344064*a^17*c^9*d*f
- 1236992*a^16*b*c^9*d*e + 237568*a^17*b*c^8*e*f - 480*a^10*b^13*c^3*d*e +
11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e
 - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*
d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6
*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f - 96*a^12*b^11*c^3
*e*f + 2336*a^13*b^9*c^4*e*f - 22528*a^14*b^7*c^5*e*f + 107520*a^15*b^5*c^6
*e*f - 253952*a^16*b^3*c^7*e*f) + (-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*
d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3
*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 +
 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d
^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^
2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^
3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c
^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a
*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*
c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 +
10*a^2*b^13*d*f + 35840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f +
30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c
*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*
a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b
^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b
^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/
2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f -
 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^
2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*
f - 22400*a^7*b^4*c^4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a
*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*
c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2)
 + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(
25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^
11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 2
7*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a
^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 - 9*a^2*b^
4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) +
2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 4480
0*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a^4*c^2*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840
```

*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e -
6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2)
+ 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f + 152*a^4*
b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165*a*b^4*c*d^2
*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e -
119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e -
10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5
*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 6*a^3*b^3*e*
f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1548
*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f + 30720*a^8
*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a^4*b*c*e*f*(
-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 186*
a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9
)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 -
1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576
*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 -
327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^
12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*
b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*
b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4
*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e
- 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 10240
0*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f + 851968*a
^19*b*c^8*e))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^
9)^(1/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*
a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c
^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c
^6*d^2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b
^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) -
25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b
^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35
840*a^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*
c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c
^6*d*f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*
a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*
a^7*b^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^
2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d
*f + 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2
)^9)^(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^
4*e*f + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) -
44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2
)^9)^(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*
(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 2
40*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)
))^(1/2) - 128000*a^15*c^9*e^3 + 476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f
^3 - 250880*a^14*c^10*d^2*e - 46080*a^16*c^8*e*f^2 + 1800*a^9*b^9*c^6*d^3 -
29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^3 - 473216*a^12*b^3*c^9*d^3
- 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3 - 48704*a^13*b^4*c^7*e^3 +
129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^3 + 2944
*a^15*b^3*c^6*f^3 + 215040*a^15*c^9*d*e*f + 442880*a^14*b*c^9*d*e^2 - 43366
4*a^14*b*c^9*d^2*f + 109056*a^15*b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f - 140
0*a^9*b^10*c^5*d^2*e + 21680*a^10*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 -
121648*a^11*b^6*c^7*d^2*e - 27176*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8
*d^2*e + 164448*a^12*b^5*c^7*d*e^2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^1
3*b^3*c^8*d*e^2 + 1000*a^9*b^11*c^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 1242
80*a^11*b^7*c^6*d^2*f + 400*a^11*b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f

```
- 6600*a^12*b^7*c^5*d*f^2 + 694912*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6*
d*f^2 - 108928*a^14*b^3*c^7*d*f^2 + 360*a^11*b^9*c^4*e^2*f - 5736*a^12*b^7*
c^5*e^2*f - 240*a^12*b^8*c^4*e*f^2 + 33888*a^13*b^5*c^6*e^2*f + 3792*a^13*b
^6*c^5*e*f^2 - 87936*a^14*b^3*c^7*e^2*f - 21696*a^14*b^4*c^6*e*f^2 + 52992*
a^15*b^2*c^7*e*f^2 - 1200*a^10*b^10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f -
130656*a^12*b^6*c^6*d*e*f + 394368*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8
*d*e*f))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 - 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1
/2) + a^4*b^11*f^2 - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b
*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 + 9*a^5*c*f^2*(-(4*a*c - b
^2)^9)^(1/2) - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^
2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^
2 - 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 49*a^3*c^3*d^2*(-(4*a*c - b^2)
^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^
4*e^2 - 44800*a^7*b^3*c^5*e^2 - a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 25*a
^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^
3*f^2 + 3840*a^8*b^3*c^4*f^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a
^8*c^7*d*e - 6*a^3*b^12*e*f - 15360*a^9*c^6*e*f + 30*a*b^5*d*e*(-(4*a*c - b
^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*
f + 152*a^4*b^10*c*e*f - 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + 165
*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b
^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b
^2*c^6*d*e - 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f
 - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f +
6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) - 1548*a^5*b^8*c^2*e*f + 8064*a^6*b^6*c^3*e*f - 22400*a^7*b^4*c^4*e*f
 + 30720*a^8*b^2*c^5*e*f + 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 44*a
^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^
(1/2) + 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 78*a^3*b^2*c*d*f*(-(4*
a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^
9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1
/2)*2i
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.74 $\int \frac{x^9\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=68

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)+\frac{415x^2+414}{2\left(x^4+3x^2+2\right)}$$

```
[Out] -293/2*x^2+49/2*x^4-9/2*x^6+5/8*x^8+1/2*(415*x^2+414)/(x^4+3*x^2+2)+2*ln(x^
2+1)+392*ln(x^2+2)
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{415x^2+414}{2\left(x^4+3x^2+2\right)}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8 + (414 + 415*x^2)/(2*(2 +
 3*x^2 + x^4)) + 2*Log[1 + x^2] + 392*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4\left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{414+415x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-206-105x+53x^2-27x^3+12x^4-5x^5}{2+3x+x^2}\,dx,x,$$

$$= \frac{414+415x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \left(293-98x+27x^2-5x^3-\frac{4(198+197x)}{2+3x+x^2}\right)dx,$$

$$= -\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}+\frac{414+415x^2}{2\left(2+3x^2+x^4\right)}+2\,\text{Subst}\left(\int \frac{198+197x}{2+3x+x^2}\,dx,$$

$$= -\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}+\frac{414+415x^2}{2\left(2+3x^2+x^4\right)}+2\,\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)$$

$$= -\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}+\frac{414+415x^2}{2\left(2+3x^2+x^4\right)}+2\log\left(1+x^2\right)+392\log\left(2+\right.$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.91

$$\frac{1}{8}\left(5x^8-36x^6+196x^4-1172x^2+16\log\left(x^2+1\right)+3136\log\left(x^2+2\right)+\frac{4\left(415x^2+414\right)}{x^4+3x^2+2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-1172*x^2 + 196*x^4 - 36*x^6 + 5*x^8 + (4*(414 + 415*x^2))/(2 + 3*x^2 + x^
4) + 16*Log[1 + x^2] + 3136*Log[2 + x^2])/8
```

fricas [A] time = 0.78, size = 82, normalized size = 1.21

$$\frac{5x^{12}-21x^{10}+98x^8-656x^6-3124x^4-684x^2+3136\left(x^4+3x^2+2\right)\log\left(x^2+2\right)+16\left(x^4+3x^2+2\right)\log\left(x^2\right.}{8\left(x^4+3x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(5*x^12 - 21*x^10 + 98*x^8 - 656*x^6 - 3124*x^4 - 684*x^2 + 3136*(x^4 +
 3*x^2 + 2)*log(x^2 + 2) + 16*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 1656)/(x^4 +
 3*x^2 + 2)
```

giac [A] time = 0.36, size = 63, normalized size = 0.93

$$\frac{5}{8}x^8-\frac{9}{2}x^6+\frac{49}{2}x^4-\frac{293}{2}x^2-\frac{394x^4+767x^2+374}{2\left(x^4+3x^2+2\right)}+392\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 - 1/2*(394*x^4 + 767*x^2 + 374)/(x
^4 + 3*x^2 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

maple [A] time = 0.02, size = 56, normalized size = 0.82

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)-\frac{1}{2\left(x^2+1\right)}+\frac{208}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/8*x^8-9/2*x^6+49/2*x^4-293/2*x^2+2*ln(x^2+1)-1/2/(x^2+1)+208/(x^2+2)+392*
ln(x^2+2)
```

maxima [A] time = 0.60, size = 58, normalized size = 0.85

$$\frac{5}{8}x^8-\frac{9}{2}x^6+\frac{49}{2}x^4-\frac{293}{2}x^2+\frac{415x^2+414}{2\left(x^4+3x^2+2\right)}+392\log\left(x^2+2\right)+2\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 9/2*x^6 + 49/2*x^4 - 293/2*x^2 + 1/2*(415*x^2 + 414)/(x^4 + 3*x^2
 + 2) + 392*log(x^2 + 2) + 2*log(x^2 + 1)
```

mupad [B] time = 0.06, size = 57, normalized size = 0.84

$$2\ln\left(x^2+1\right)+392\ln\left(x^2+2\right)+\frac{\frac{415x^2}{2}+207}{x^4+3x^2+2}-\frac{293x^2}{2}+\frac{49x^4}{2}-\frac{9x^6}{2}+\frac{5x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 2*log(x^2 + 1) + 392*log(x^2 + 2) + ((415*x^2)/2 + 207)/(3*x^2 + x^4 + 2) -
 (293*x^2)/2 + (49*x^4)/2 - (9*x^6)/2 + (5*x^8)/8
```

sympy [A] time = 0.17, size = 61, normalized size = 0.90

$$\frac{5x^8}{8}-\frac{9x^6}{2}+\frac{49x^4}{2}-\frac{293x^2}{2}+\frac{415x^2+414}{2x^4+6x^2+4}+2\log\left(x^2+1\right)+392\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**8/8 - 9*x**6/2 + 49*x**4/2 - 293*x**2/2 + (415*x**2 + 414)/(2*x**4 + 6
*x**2 + 4) + 2*log(x**2 + 1) + 392*log(x**2 + 2)
```

3.75 $\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=61

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)-\frac{207x^2+206}{2\left(x^4+3x^2+2\right)}$$

```
[Out] 49*x^2-27/4*x^4+5/6*x^6+1/2*(-207*x^2-206)/(x^4+3*x^2+2)-5/2*ln(x^2+1)-144*
ln(x^2+2)
```

Rubi [A] time = 0.12, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{207x^2+206}{2\left(x^4+3x^2+2\right)}-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 - (206 + 207*x^2)/(2*(2 + 3*x^2 + x^4)) - (
5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
```

```
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3\left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{102+53x-27x^2+12x^3-5x^4}{2+3x+x^2}\,dx, x, x^2\right)\\
&= -\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-98+27x-5x^2+\frac{298+293x}{2+3x+x^2}\right)dx, x, x^2\right)\\
&= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{298+293x}{2+3x+x^2}\,dx, x, x^2\right)\\
&= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{5}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)-144\,\text{S}\\
&= 49x^2-\frac{27x^4}{4}+\frac{5x^6}{6}-\frac{206+207x^2}{2\left(2+3x^2+x^4\right)}-\frac{5}{2}\log\left(1+x^2\right)-144\log\left(2+x^2\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.00

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5}{2}\log\left(x^2+1\right)-144\log\left(x^2+2\right)+\frac{-207x^2-206}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 49*x^2 - (27*x^4)/4 + (5*x^6)/6 + (-206 - 207*x^2)/(2*(2 + 3*x^2 + x^4)) -
(5*Log[1 + x^2])/2 - 144*Log[2 + x^2]
```

fricas [A] time = 0.83, size = 77, normalized size = 1.26

$$\frac{10\,x^{10}-51\,x^8+365\,x^6+1602\,x^4-66\,x^2-1728\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right)-30\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right)}{12\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(10*x^10 - 51*x^8 + 365*x^6 + 1602*x^4 - 66*x^2 - 1728*(x^4 + 3*x^2 +
2)*log(x^2 + 2) - 30*(x^4 + 3*x^2 + 2)*log(x^2 + 1) - 1236)/(x^4 + 3*x^2 +
2)
```

giac [A] time = 0.32, size = 58, normalized size = 0.95

$$\frac{5}{6}\,x^6-\frac{27}{4}\,x^4+49\,x^2+\frac{293\,x^4+465\,x^2+174}{4\left(x^4+3\,x^2+2\right)}-144\,\log\left(x^2+2\right)-\frac{5}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 + 1/4*(293*x^4 + 465*x^2 + 174)/(x^4 + 3*x^2 +
2) - 144*log(x^2 + 2) - 5/2*log(x^2 + 1)
```

maple [A] time = 0.02, size = 51, normalized size = 0.84

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2-\frac{5\ln\left(x^2+1\right)}{2}-144\ln\left(x^2+2\right)+\frac{1}{2x^2+2}-\frac{104}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/6*x^6-27/4*x^4+49*x^2-5/2*ln(x^2+1)+1/2/(x^2+1)-104/(x^2+2)-144*ln(x^2+2)
```

maxima [A] time = 0.72, size = 53, normalized size = 0.87

$$\frac{5}{6}\,x^6-\frac{27}{4}\,x^4+49\,x^2-\frac{207\,x^2+206}{2\left(x^4+3\,x^2+2\right)}-144\,\log\left(x^2+2\right)-\frac{5}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 27/4*x^4 + 49*x^2 - 1/2*(207*x^2 + 206)/(x^4 + 3*x^2 + 2) - 144*l
og(x^2 + 2) - 5/2*log(x^2 + 1)
```

mupad [B] time = 0.04, size = 53, normalized size = 0.87

$$49\,x^2-144\,\ln\left(x^2+2\right)-\frac{\frac{207\,x^2}{2}+103}{x^4+3\,x^2+2}-\frac{5\,\ln\left(x^2+1\right)}{2}-\frac{27\,x^4}{4}+\frac{5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 49*x^2 - 144*log(x^2 + 2) - ((207*x^2)/2 + 103)/(3*x^2 + x^4 + 2) - (5*log(
x^2 + 1))/2 - (27*x^4)/4 + (5*x^6)/6
```

sympy [A] time = 0.17, size = 56, normalized size = 0.92

$$\frac{5x^6}{6}-\frac{27x^4}{4}+49x^2+\frac{-207x^2-206}{2x^4+6x^2+4}-\frac{5\log\left(x^2+1\right)}{2}-144\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**6/6 - 27*x**4/4 + 49*x**2 + (-207*x**2 - 206)/(2*x**4 + 6*x**2 + 4) -
5*log(x**2 + 1)/2 - 144*log(x**2 + 2)
```

3.76 $\int \frac{x^5(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

Optimal. Leaf size=54

$$\frac{5x^4}{4} - \frac{27x^2}{2} + 3\log(x^2+1) + 46\log(x^2+2) + \frac{103x^2+102}{2(x^4+3x^2+2)}$$

```
[Out] -27/2*x^2+5/4*x^4+1/2*(103*x^2+102)/(x^4+3*x^2+2)+3*ln(x^2+1)+46*ln(x^2+2)
```

Rubi [A] time = 0.11, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^4}{4} - \frac{27x^2}{2} + \frac{103x^2+102}{2(x^4+3x^2+2)} + 3\log(x^2+1) + 46\log(x^2+2)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
```

```
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{x^2\left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)$$
$$= \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{-50-27x+12x^2-5x^3}{2+3x+x^2}\,dx,x,x^2\right)$$
$$= \frac{102+103x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \left(27-5x-\frac{2(52+49x)}{2+3x+x^2}\right)dx,x,x^2\right)$$
$$= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)}+\operatorname{Subst}\left(\int \frac{52+49x}{2+3x+x^2}\,dx,x,x^2\right)$$
$$= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)}+3\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)+46\operatorname{Subst}\left(\int$$
$$= -\frac{27x^2}{2}+\frac{5x^4}{4}+\frac{102+103x^2}{2\left(2+3x^2+x^4\right)}+3\log\left(1+x^2\right)+46\log\left(2+x^2\right)$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.00

$$\frac{5x^4}{4}-\frac{27x^2}{2}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (-27*x^2)/2 + (5*x^4)/4 + (102 + 103*x^2)/(2*(2 + 3*x^2 + x^4)) + 3*Log[1 +
 x^2] + 46*Log[2 + x^2]
```

fricas [A] time = 0.71, size = 72, normalized size = 1.33

$$\frac{5\,x^8-39\,x^6-152\,x^4+98\,x^2+184\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right)+12\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right)+204}{4\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(5*x^8 - 39*x^6 - 152*x^4 + 98*x^2 + 184*(x^4 + 3*x^2 + 2)*log(x^2 + 2)
 + 12*(x^4 + 3*x^2 + 2)*log(x^2 + 1) + 204)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.37, size = 53, normalized size = 0.98

$$\frac{5}{4}\,x^4-\frac{27}{2}\,x^2-\frac{49\,x^4+44\,x^2-4}{2\left(x^4+3\,x^2+2\right)}+46\,\log\left(x^2+2\right)+3\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 27/2*x^2 - 1/2*(49*x^4 + 44*x^2 - 4)/(x^4 + 3*x^2 + 2) + 46*log(x
^2 + 2) + 3*log(x^2 + 1)
```

maple [A] time = 0.02, size = 46, normalized size = 0.85

$$\frac{5x^4}{4}-\frac{27x^2}{2}+3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)-\frac{1}{2\left(x^2+1\right)}+\frac{52}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/4*x^4-27/2*x^2+3*ln(x^2+1)-1/2/(x^2+1)+52/(x^2+2)+46*ln(x^2+2)
```

maxima [A] time = 1.07, size = 48, normalized size = 0.89

$$\frac{5}{4}x^4-\frac{27}{2}x^2+\frac{103x^2+102}{2\left(x^4+3x^2+2\right)}+46\log\left(x^2+2\right)+3\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 27/2*x^2 + 1/2*(103*x^2 + 102)/(x^4 + 3*x^2 + 2) + 46*log(x^2 + 2
) + 3*log(x^2 + 1)
```

mupad [B] time = 0.90, size = 47, normalized size = 0.87

$$3\ln\left(x^2+1\right)+46\ln\left(x^2+2\right)+\frac{\frac{103x^2}{2}+51}{x^4+3x^2+2}-\frac{27x^2}{2}+\frac{5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 3*log(x^2 + 1) + 46*log(x^2 + 2) + ((103*x^2)/2 + 51)/(3*x^2 + x^4 + 2) - (
27*x^2)/2 + (5*x^4)/4
```

sympy [A] time = 0.17, size = 48, normalized size = 0.89

$$\frac{5x^4}{4}-\frac{27x^2}{2}+\frac{103x^2+102}{2x^4+6x^2+4}+3\log\left(x^2+1\right)+46\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**4/4 - 27*x**2/2 + (103*x**2 + 102)/(2*x**4 + 6*x**2 + 4) + 3*log(x**2
+ 1) + 46*log(x**2 + 2)
```

3.77 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=49

$$\frac{5x^2}{2}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}$$

```
[Out] 5/2*x^2+1/2*(-51*x^2-50)/(x^4+3*x^2+2)-7/2*ln(x^2+1)-10*ln(x^2+2)
```

Rubi [A] time = 0.09, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1663, 1660, 1657, 632, 31}

$$\frac{5x^2}{2}-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (50 + 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*L
og[2 + x^2]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
```

```
(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x\left(4+x+3x^2+5x^3\right)}{\left(2+3x+x^2\right)^2}\,dx,x,x^2\right)\\
&= -\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{24+12x-5x^2}{2+3x+x^2}\,dx,x,x^2\right)\\
&= -\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \left(-5+\frac{34+27x}{2+3x+x^2}\right)dx,x,x^2\right)\\
&= \frac{5x^2}{2}-\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\text{Subst}\left(\int \frac{34+27x}{2+3x+x^2}\,dx,x,x^2\right)\\
&= \frac{5x^2}{2}-\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{7}{2}\text{Subst}\left(\int \frac{1}{1+x}\,dx,x,x^2\right)-10\,\text{Subst}\left(\int \frac{1}{2+x}\,d\right.\\
&= \frac{5x^2}{2}-\frac{50+51x^2}{2\left(2+3x^2+x^4\right)}-\frac{7}{2}\log\left(1+x^2\right)-10\log\left(2+x^2\right)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.00

$$\frac{5x^2}{2}-\frac{7}{2}\log\left(x^2+1\right)-10\log\left(x^2+2\right)+\frac{-51x^2-50}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 + (-50 - 51*x^2)/(2*(2 + 3*x^2 + x^4)) - (7*Log[1 + x^2])/2 - 10*
Log[2 + x^2]
```

fricas [A] time = 0.90, size = 67, normalized size = 1.37

$$\frac{5\,x^6+15\,x^4-41\,x^2-20\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right)-7\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right)-50}{2\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(5*x^6 + 15*x^4 - 41*x^2 - 20*(x^4 + 3*x^2 + 2)*log(x^2 + 2) - 7*(x^4 +
 3*x^2 + 2)*log(x^2 + 1) - 50)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.39, size = 45, normalized size = 0.92

$$\frac{5}{2}\,x^2-\frac{51\,x^2+50}{2\left(x^2+2\right)\left(x^2+1\right)}-10\,\log\left(x^2+2\right)-\frac{7}{2}\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/((x^2 + 2)*(x^2 + 1)) - 10*log(x^2 + 2) - 7/2*l
og(x^2 + 1)
```

maple [A] time = 0.02, size = 41, normalized size = 0.84

$$\frac{5x^2}{2}-\frac{7\ln\left(x^2+1\right)}{2}-10\ln\left(x^2+2\right)+\frac{1}{2x^2+2}-\frac{26}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/2*x^2-7/2*ln(x^2+1)+1/2/(x^2+1)-26/(x^2+2)-10*ln(x^2+2)
```

maxima [A] time = 0.51, size = 43, normalized size = 0.88

$$\frac{5}{2}x^2-\frac{51x^2+50}{2\left(x^4+3x^2+2\right)}-10\log\left(x^2+2\right)-\frac{7}{2}\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 1/2*(51*x^2 + 50)/(x^4 + 3*x^2 + 2) - 10*log(x^2 + 2) - 7/2*log(x
^2 + 1)
```

mupad [B] time = 0.04, size = 43, normalized size = 0.88

$$\frac{5x^2}{2}-10\ln\left(x^2+2\right)-\frac{\frac{51x^2}{2}+25}{x^4+3x^2+2}-\frac{7\ln\left(x^2+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (5*x^2)/2 - 10*log(x^2 + 2) - ((51*x^2)/2 + 25)/(3*x^2 + x^4 + 2) - (7*log(
x^2 + 1))/2
```

sympy [A] time = 0.17, size = 44, normalized size = 0.90

$$\frac{5x^2}{2}+\frac{-51x^2-50}{2x^4+6x^2+4}-\frac{7\log\left(x^2+1\right)}{2}-10\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**2/2 + (-51*x**2 - 50)/(2*x**4 + 6*x**2 + 4) - 7*log(x**2 + 1)/2 - 10*l
og(x**2 + 2)
```

3.78 $\int \frac{x(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=42

$$4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)+\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}$$

```
[Out] 1/2*(25*x^2+24)/(x^4+3*x^2+2)+4*ln(x^2+1)-3/2*ln(x^2+2)
```

Rubi [A] time = 0.05, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1663, 1660, 632, 31}

$$\frac{25x^2+24}{2\left(x^4+3x^2+2\right)}+4\log\left(x^2+1\right)-\frac{3}{2}\log\left(x^2+2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 632

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-13-5x}{2+3x+x^2}\,dx, x, x^2\right)$$
$$= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)} - \frac{3}{2}\text{Subst}\left(\int \frac{1}{2+x}\,dx, x, x^2\right) + 4\,\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, x^2\right)$$
$$= \frac{24+25x^2}{2\left(2+3x^2+x^4\right)} + 4\log\left(1+x^2\right) - \frac{3}{2}\log\left(2+x^2\right)$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$4\log\left(x^2+1\right) - \frac{3}{2}\log\left(x^2+2\right) + \frac{25x^2+24}{2\left(x^4+3x^2+2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] (24 + 25*x^2)/(2*(2 + 3*x^2 + x^4)) + 4*Log[1 + x^2] - (3*Log[2 + x^2])/2
```

fricas [A] time = 1.05, size = 57, normalized size = 1.36

$$\frac{25\,x^2 - 3\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right) + 8\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right) + 24}{2\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/2*(25*x^2 - 3*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 8*(x^4 + 3*x^2 + 2)*log(x^
2 + 1) + 24)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.35, size = 40, normalized size = 0.95

$$\frac{25\,x^2+24}{2\left(x^2+2\right)\left(x^2+1\right)} - \frac{3}{2}\,\log\left(x^2+2\right) + 4\,\log\left(x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] 1/2*(25*x^2 + 24)/((x^2 + 2)*(x^2 + 1)) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

maple [A] time = 0.02, size = 36, normalized size = 0.86

$$4\ln\left(x^2+1\right) - \frac{3\ln\left(x^2+2\right)}{2} - \frac{1}{2\left(x^2+1\right)} + \frac{13}{x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
[Out] 4*ln(x^2+1)-1/2/(x^2+1)+13/(x^2+2)-3/2*ln(x^2+2)
```

maxima [A] time = 0.52, size = 38, normalized size = 0.90

$$\frac{25\,x^{2}+24}{2\left(x^{4}+3\,x^{2}+2\right)}-\frac{3}{2}\,\log\left(x^{2}+2\right)+4\,\log\left(x^{2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
[Out] 1/2*(25*x^2 + 24)/(x^4 + 3*x^2 + 2) - 3/2*log(x^2 + 2) + 4*log(x^2 + 1)
```

mupad [B] time = 0.05, size = 37, normalized size = 0.88

$$4\,\ln\left(x^{2}+1\right)-\frac{3\,\ln\left(x^{2}+2\right)}{2}+\frac{\frac{25\,x^{2}}{2}+12}{x^{4}+3\,x^{2}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
[Out] 4*log(x^2 + 1) - (3*log(x^2 + 2))/2 + ((25*x^2)/2 + 12)/(3*x^2 + x^4 + 2)
```

sympy [A] time = 0.17, size = 36, normalized size = 0.86

$$\frac{25x^{2}+24}{2x^{4}+6x^{2}+4}+4\log\left(x^{2}+1\right)-\frac{3\log\left(x^{2}+2\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
[Out] (25*x**2 + 24)/(2*x**4 + 6*x**2 + 4) + 4*log(x**2 + 1) - 3*log(x**2 + 2)/2
```

3.79 $\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=44

$$-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}+\log(x)$$

```
[Out] 1/2*(-12*x^2-11)/(x^4+3*x^2+2)+ln(x)-9/2*ln(x^2+1)+4*ln(x^2+2)
```

Rubi [A] time = 0.08, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1663, 1646, 800}

$$-\frac{12x^2+11}{2\left(x^4+3x^2+2\right)}-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -(11 + 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\operatorname{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \frac{-2+7x}{x\left(2+3x+x^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)} - \frac{1}{2}\operatorname{Subst}\left(\int \left(-\frac{1}{x}+\frac{9}{1+x}-\frac{8}{2+x}\right)dx, x, x^2\right)$$
$$= -\frac{11+12x^2}{2\left(2+3x^2+x^4\right)} + \log(x) - \frac{9}{2}\log\left(1+x^2\right) + 4\log\left(2+x^2\right)$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$-\frac{9}{2}\log\left(x^2+1\right)+4\log\left(x^2+2\right)+\frac{-12x^2-11}{2\left(x^4+3x^2+2\right)}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-11 - 12*x^2)/(2*(2 + 3*x^2 + x^4)) + Log[x] - (9*Log[1 + x^2])/2 + 4*Log[
2 + x^2]
```

fricas [A] time = 0.90, size = 71, normalized size = 1.61

$$-\frac{12\,x^2-8\left(x^4+3\,x^2+2\right)\log\left(x^2+2\right)+9\left(x^4+3\,x^2+2\right)\log\left(x^2+1\right)-2\left(x^4+3\,x^2+2\right)\log(x)+11}{2\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(12*x^2 - 8*(x^4 + 3*x^2 + 2)*log(x^2 + 2) + 9*(x^4 + 3*x^2 + 2)*log(x
^2 + 1) - 2*(x^4 + 3*x^2 + 2)*log(x) + 11)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.38, size = 47, normalized size = 1.07

$$\frac{x^4-21\,x^2-20}{4\left(x^4+3\,x^2+2\right)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(x^4 - 21*x^2 - 20)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 +
1) + 1/2*log(x^2)
```

maple [A] time = 0.02, size = 38, normalized size = 0.86

$$\ln(x)-\frac{9\ln\left(x^2+1\right)}{2}+4\ln\left(x^2+2\right)+\frac{1}{2x^2+2}-\frac{13}{2\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x)
```

```
[Out] ln(x)-9/2*ln(x^2+1)+1/2/(x^2+1)-13/2/(x^2+2)+4*ln(x^2+2)
```

maxima [A] time = 0.72, size = 44, normalized size = 1.00

$$-\frac{12\,x^2+11}{2\,\left(x^4+3\,x^2+2\right)}+4\,\log\left(x^2+2\right)-\frac{9}{2}\,\log\left(x^2+1\right)+\frac{1}{2}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(12*x^2 + 11)/(x^4 + 3*x^2 + 2) + 4*log(x^2 + 2) - 9/2*log(x^2 + 1) +
1/2*log(x^2)
```

mupad [B] time = 0.04, size = 40, normalized size = 0.91

$$4\,\ln\left(x^2+2\right)-\frac{9\,\ln\left(x^2+1\right)}{2}+\ln(x)-\frac{6\,x^2+\frac{11}{2}}{x^4+3\,x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] 4*log(x^2 + 2) - (9*log(x^2 + 1))/2 + log(x) - (6*x^2 + 11/2)/(3*x^2 + x^4
+ 2)
```

sympy [A] time = 0.18, size = 41, normalized size = 0.93

$$\frac{-12x^2-11}{2x^4+6x^2+4}+\log\left(x\right)-\frac{9\log\left(x^2+1\right)}{2}+4\log\left(x^2+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+3*x**2+2)**2,x)
```

```
[Out] (-12*x**2 - 11)/(2*x**4 + 6*x**2 + 4) + log(x) - 9*log(x**2 + 1)/2 + 4*log(
x**2 + 2)
```

3.80 $\int \frac{4+x^2+3x^4+5x^6}{x^3(2+3x^2+x^4)^2} dx$

Optimal. Leaf size=55

$$-\frac{1}{2x^2}+5\log\left(x^2+1\right)-\frac{29}{8}\log\left(x^2+2\right)+\frac{11x^2+9}{4\left(x^4+3x^2+2\right)}-\frac{11\log(x)}{4}$$

```
[Out] -1/2/x^2+1/4*(11*x^2+9)/(x^4+3*x^2+2)-11/4*ln(x)+5*ln(x^2+1)-29/8*ln(x^2+2)
```

Rubi [A] time = 0.10, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1663, 1646, 1628}

$$\frac{11x^2+9}{4\left(x^4+3x^2+2\right)}-\frac{1}{2x^2}+5\log\left(x^2+1\right)-\frac{29}{8}\log\left(x^2+2\right)-\frac{11\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(2*x^2) + (9 + 11*x^2)/(4*(2 + 3*x^2 + x^4)) - (11*Log[x])/4 + 5*Log[1 +
 x^2] - (29*Log[2 + x^2])/8
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)$$
$$= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{11x^2}{2}}{x^2\left(2+3x+x^2\right)}\,dx, x, x^2\right)$$
$$= \frac{9+11x^2}{4\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^2}+\frac{11}{4x}-\frac{10}{1+x}+\frac{29}{4(2+x)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{2x^2} + \frac{9+11x^2}{4\left(2+3x^2+x^4\right)} - \frac{11\log(x)}{4} + 5\log\left(1+x^2\right) - \frac{29}{8}\log\left(2+x^2\right)$$

Mathematica [A] time = 0.03, size = 50, normalized size = 0.91

$$\frac{1}{8}\left(-\frac{4}{x^2}+40\log\left(x^2+1\right)-29\log\left(x^2+2\right)+\frac{22x^2+18}{x^4+3x^2+2}-22\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-4/x^2 + (18 + 22*x^2)/(2 + 3*x^2 + x^4) - 22*Log[x] + 40*Log[1 + x^2] - 2
9*Log[2 + x^2])/8
```

fricas [A] time = 0.98, size = 92, normalized size = 1.67

$$\frac{18\,x^4+6\,x^2-29\,\left(x^6+3\,x^4+2\,x^2\right)\log\left(x^2+2\right)+40\,\left(x^6+3\,x^4+2\,x^2\right)\log\left(x^2+1\right)-22\,\left(x^6+3\,x^4+2\,x^2\right)\log(x}{8\,\left(x^6+3\,x^4+2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(18*x^4 + 6*x^2 - 29*(x^6 + 3*x^4 + 2*x^2)*log(x^2 + 2) + 40*(x^6 + 3*x
^4 + 2*x^2)*log(x^2 + 1) - 22*(x^6 + 3*x^4 + 2*x^2)*log(x) - 8)/(x^6 + 3*x^
4 + 2*x^2)
```

giac [A] time = 0.37, size = 53, normalized size = 0.96

$$\frac{9\,x^4+3\,x^2-4}{4\,\left(x^6+3\,x^4+2\,x^2\right)} - \frac{29}{8}\,\log\left(x^2+2\right) + 5\,\log\left(x^2+1\right) - \frac{11}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x
^2 + 1) - 11/8*log(x^2)
```

maple [A] time = 0.02, size = 45, normalized size = 0.82

$$-\frac{11\ln(x)}{4} + 5\ln\left(x^2+1\right) - \frac{29\ln\left(x^2+2\right)}{8} - \frac{1}{2x^2} - \frac{1}{2\left(x^2+1\right)} + \frac{13}{4\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/2/x^2-11/4*ln(x)+5*ln(x^2+1)-1/2/(x^2+1)+13/4/(x^2+2)-29/8*ln(x^2+2)
```

maxima [A] time = 0.79, size = 53, normalized size = 0.96

$$\frac{9\,x^4+3\,x^2-4}{4\,\left(x^6+3\,x^4+2\,x^2\right)}-\frac{29}{8}\,\log\left(x^2+2\right)+5\,\log\left(x^2+1\right)-\frac{11}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^4 + 3*x^2 - 4)/(x^6 + 3*x^4 + 2*x^2) - 29/8*log(x^2 + 2) + 5*log(x
^2 + 1) - 11/8*log(x^2)
```

mupad [B] time = 0.04, size = 50, normalized size = 0.91

$$5\,\ln\left(x^2+1\right)-\frac{29\,\ln\left(x^2+2\right)}{8}-\frac{11\,\ln(x)}{4}+\frac{\frac{9\,x^4}{4}+\frac{3\,x^2}{4}-1}{x^6+3\,x^4+2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] 5*log(x^2 + 1) - (29*log(x^2 + 2))/8 - (11*log(x))/4 + ((3*x^2)/4 + (9*x^4)
/4 - 1)/(2*x^2 + 3*x^4 + x^6)
```

sympy [A] time = 0.20, size = 51, normalized size = 0.93

$$\frac{9x^4+3x^2-4}{4x^6+12x^4+8x^2}-\frac{11\log{(x)}}{4}+5\log{\left(x^2+1\right)}-\frac{29\log{\left(x^2+2\right)}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+3*x**2+2)**2,x)
```

```
[Out] (9*x**4 + 3*x**2 - 4)/(4*x**6 + 12*x**4 + 8*x**2) - 11*log(x)/4 + 5*log(x**
2 + 1) - 29*log(x**2 + 2)/8
```

3.81 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=64

$$-\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right)-\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{23\log(x)}{4}$$

```
[Out] -1/4/x^4+11/8/x^2+1/8*(-9*x^2-5)/(x^4+3*x^2+2)+23/4*ln(x)-11/2*ln(x^2+1)+21
/8*ln(x^2+2)
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1663, 1646, 1628}

$$-\frac{9x^2+5}{8\left(x^4+3x^2+2\right)}+\frac{11}{8x^2}-\frac{1}{4x^4}-\frac{11}{2}\log\left(x^2+1\right)+\frac{21}{8}\log\left(x^2+2\right)+\frac{23\log(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(4*x^4) + 11/(8*x^2) - (5 + 9*x^2)/(8*(2 + 3*x^2 + x^4)) + (23*Log[x])/4
 - (11*Log[1 + x^2])/2 + (21*Log[2 + x^2])/8
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(2+3x^2+x^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3\left(2+3x+x^2\right)^2}\,dx, x, x^2\right)$$
$$= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \frac{-2+\frac{5x}{2}-\frac{17x^2}{4}+\frac{9x^3}{4}}{x^3\left(2+3x+x^2\right)}\,dx, x, x^2\right)$$
$$= -\frac{5+9x^2}{8\left(2+3x^2+x^4\right)} - \frac{1}{2}\text{Subst}\left(\int \left(-\frac{1}{x^3}+\frac{11}{4x^2}-\frac{23}{4x}+\frac{11}{1+x}-\frac{21}{4(2+x)}\right)dx, x, x^2\right)$$
$$= -\frac{1}{4x^4}+\frac{11}{8x^2}-\frac{5+9x^2}{8\left(2+3x^2+x^4\right)}+\frac{23\log(x)}{4}-\frac{11}{2}\log\left(1+x^2\right)+\frac{21}{8}\log\left(2+x^2\right)$$

Mathematica [A] time = 0.03, size = 56, normalized size = 0.88

$$\frac{1}{8}\left(-\frac{2}{x^4}+\frac{11}{x^2}-44\log\left(x^2+1\right)+21\log\left(x^2+2\right)-\frac{9x^2+5}{x^4+3x^2+2}+46\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-2/x^4 + 11/x^2 - (5 + 9*x^2)/(2 + 3*x^2 + x^4) + 46*Log[x] - 44*Log[1 + x
^2] + 21*Log[2 + x^2])/8
```

fricas [A] time = 1.05, size = 97, normalized size = 1.52

$$\frac{2\,x^6+26\,x^4+16\,x^2+21\left(x^8+3\,x^6+2\,x^4\right)\log\left(x^2+2\right)-44\left(x^8+3\,x^6+2\,x^4\right)\log\left(x^2+1\right)+46\left(x^8+3\,x^6+\right.}{8\left(x^8+3\,x^6+2\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*x^6 + 26*x^4 + 16*x^2 + 21*(x^8 + 3*x^6 + 2*x^4)*log(x^2 + 2) - 44*(
x^8 + 3*x^6 + 2*x^4)*log(x^2 + 1) + 46*(x^8 + 3*x^6 + 2*x^4)*log(x) - 4)/(x
^8 + 3*x^6 + 2*x^4)
```

giac [A] time = 0.34, size = 66, normalized size = 1.03

$$\frac{23\,x^4+51\,x^2+36}{16\left(x^4+3\,x^2+2\right)}-\frac{69\,x^4-22\,x^2+4}{16\,x^4}+\frac{21}{8}\log\left(x^2+2\right)-\frac{11}{2}\log\left(x^2+1\right)+\frac{23}{8}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(23*x^4 + 51*x^2 + 36)/(x^4 + 3*x^2 + 2) - 1/16*(69*x^4 - 22*x^2 + 4)/
x^4 + 21/8*log(x^2 + 2) - 11/2*log(x^2 + 1) + 23/8*log(x^2)
```

maple [A] time = 0.02, size = 50, normalized size = 0.78

$$\frac{23\ln(x)}{4}-\frac{11\ln\left(x^2+1\right)}{2}+\frac{21\ln\left(x^2+2\right)}{8}+\frac{11}{8x^2}-\frac{1}{4x^4}+\frac{1}{2x^2+2}-\frac{13}{8\left(x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/4/x^4+11/8/x^2+23/4*ln(x)-11/2*ln(x^2+1)+1/2/(x^2+1)-13/8/(x^2+2)+21/8*l
n(x^2+2)
```

maxima [A] time = 0.68, size = 56, normalized size = 0.88

$$\frac{x^6 + 13\,x^4 + 8\,x^2 - 2}{4\left(x^8 + 3\,x^6 + 2\,x^4\right)} + \frac{21}{8}\,\log\left(x^2 + 2\right) - \frac{11}{2}\,\log\left(x^2 + 1\right) + \frac{23}{8}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(x^6 + 13*x^4 + 8*x^2 - 2)/(x^8 + 3*x^6 + 2*x^4) + 21/8*log(x^2 + 2) -
11/2*log(x^2 + 1) + 23/8*log(x^2)
```

mupad [B] time = 0.92, size = 55, normalized size = 0.86

$$\frac{21\,\ln\left(x^2+2\right)}{8} - \frac{11\,\ln\left(x^2+1\right)}{2} + \frac{23\,\ln(x)}{4} + \frac{\frac{x^6}{4} + \frac{13\,x^4}{4} + 2\,x^2 - \frac{1}{2}}{x^8 + 3\,x^6 + 2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (21*log(x^2 + 2))/8 - (11*log(x^2 + 1))/2 + (23*log(x))/4 + (2*x^2 + (13*x^
4)/4 + x^6/4 - 1/2)/(2*x^4 + 3*x^6 + x^8)
```

sympy [A] time = 0.21, size = 56, normalized size = 0.88

$$\frac{23\log{(x)}}{4} - \frac{11\log{\left(x^2+1\right)}}{2} + \frac{21\log{\left(x^2+2\right)}}{8} + \frac{x^6 + 13x^4 + 8x^2 - 2}{4x^8 + 12x^6 + 8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+3*x**2+2)**2,x)
```

```
[Out] 23*log(x)/4 - 11*log(x**2 + 1)/2 + 21*log(x**2 + 2)/8 + (x**6 + 13*x**4 + 8
*x**2 - 2)/(4*x**8 + 12*x**6 + 8*x**4)
```

3.82 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=70

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)x}{2(x^4+3x^2+2)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -293*x+98/3*x^3-27/5*x^5+5/7*x^7-1/2*x*(207*x^2+206)/(x^4+3*x^2+2)+9/2*arct
an(x)+340*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-\frac{(207x^2+206)x}{2(x^4+3x^2+2)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 - (x*(206 + 207*x^2))/(2*(2 +
3*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= -\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-412-6x^2+212x^4-108x^6+48x^8-20x^{10}}{2+3x^2+x^4}\,dx \\
&= -\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(1172-392x^2+108x^4-20x^6-\frac{2\left(1378+1369x^2\right)}{2+3x^2+x^4}\right) \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{1}{2}\int \frac{1378+1369x^2}{2+3x^2+x^4}\,dx \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{9}{2}\int \frac{1}{1+x^2}\,dx+680\int \frac{1}{2+} \\
&= -293x+\frac{98x^3}{3}-\frac{27x^5}{5}+\frac{5x^7}{7}-\frac{x\left(206+207x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\Big(
\end{aligned}
$$

Mathematica [A] time = 0.04, size = 71, normalized size = 1.01

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}+\frac{-207x^3-206x}{2\left(x^4+3x^2+2\right)}-293x+\frac{9}{2}\tan^{-1}(x)+340\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -293*x + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7 + (-206*x - 207*x^3)/(2*(2 + 3
*x^2 + x^4)) + (9*ArcTan[x])/2 + 340*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

fricas [A] time = 1.09, size = 79, normalized size = 1.13

$$\frac{150\,x^{11}-684\,x^9+3758\,x^7-43218\,x^5-192605\,x^3+71400\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+945\left(x^4+3\,x^2\right.}{210\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/210*(150*x^11 - 684*x^9 + 3758*x^7 - 43218*x^5 - 192605*x^3 + 71400*sqrt(
2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) + 945*(x^4 + 3*x^2 + 2)*arctan(x
) - 144690*x)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.33, size = 58, normalized size = 0.83

$$\frac{5}{7}\,x^7-\frac{27}{5}\,x^5+\frac{98}{3}\,x^3+340\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-293\,x-\frac{207\,x^3+206\,x}{2\left(x^4+3\,x^2+2\right)}+\frac{9}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.80

$$\frac{5x^7}{7}-\frac{27x^5}{5}+\frac{98x^3}{3}-293x+\frac{x}{2x^2+2}-\frac{104x}{x^2+2}+\frac{9\arctan(x)}{2}+340\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/7*x^7-27/5*x^5+98/3*x^3-293*x+1/2*x/(x^2+1)+9/2*arctan(x)-104*x/(x^2+2)+3
40*arctan(1/2*2^(1/2)*x)*2^(1/2)
```

maxima [A] time = 1.64, size = 58, normalized size = 0.83

$$\frac{5}{7}x^7 - \frac{27}{5}x^5 + \frac{98}{3}x^3 + 340\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 293x - \frac{207x^3 + 206x}{2\left(x^4 + 3x^2 + 2\right)} + \frac{9}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 27/5*x^5 + 98/3*x^3 + 340*sqrt(2)*arctan(1/2*sqrt(2)*x) - 293*x -
 1/2*(207*x^3 + 206*x)/(x^4 + 3*x^2 + 2) + 9/2*arctan(x)
```

mupad [B] time = 0.95, size = 58, normalized size = 0.83

$$\frac{9\,\mathrm{atan}(x)}{2} - 293x + 340\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{207x^3}{2} + 103x}{x^4 + 3x^2 + 2} + \frac{98x^3}{3} - \frac{27x^5}{5} + \frac{5x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (9*atan(x))/2 - 293*x + 340*2^(1/2)*atan((2^(1/2)*x)/2) - (103*x + (207*x^3
)/2)/(3*x^2 + x^4 + 2) + (98*x^3)/3 - (27*x^5)/5 + (5*x^7)/7
```

sympy [A] time = 0.21, size = 68, normalized size = 0.97

$$\frac{5x^7}{7} - \frac{27x^5}{5} + \frac{98x^3}{3} - 293x + \frac{-207x^3 - 206x}{2x^4 + 6x^2 + 4} + \frac{9\,\mathrm{atan}\,(x)}{2} + 340\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**7/7 - 27*x**5/5 + 98*x**3/3 - 293*x + (-207*x**3 - 206*x)/(2*x**4 + 6*
x**2 + 4) + 9*atan(x)/2 + 340*sqrt(2)*atan(sqrt(2)*x/2)
```

3.83 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2}\,dx$

Optimal. Leaf size=57

$$x^5-9x^3+\frac{(103x^2+102)\,x}{2\,(x^4+3x^2+2)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 98*x-9*x^3+x^5+1/2*x*(103*x^2+102)/(x^4+3*x^2+2)-11/2*arctan(x)-118*arctan(
1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$x^5-9x^3+\frac{(103x^2+102)\,x}{2\,(x^4+3x^2+2)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 98*x - 9*x^3 + x^5 + (x*(102 + 103*x^2))/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan
[x])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{204+6x^2-108x^4+48x^6-20x^8}{2+3x^2+x^4}\,dx\\ &= \frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-392+108x^2-20x^4+\frac{2\left(494+483x^2\right)}{2+3x^2+x^4}\right)dx\\ &= 98x-9x^3+x^5+\frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\int \frac{494+483x^2}{2+3x^2+x^4}\,dx\\ &= 98x-9x^3+x^5+\frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{11}{2}\int \frac{1}{1+x^2}\,dx-236\int \frac{1}{2+x^2}\,dx\\ &= 98x-9x^3+x^5+\frac{x\left(102+103x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 58, normalized size = 1.02

$$x^5-9x^3+\frac{103x^3+102x}{2\left(x^4+3x^2+2\right)}+98x-\frac{11}{2}\tan^{-1}(x)-118\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] 98*x - 9*x^3 + x^5 + (102*x + 103*x^3)/(2*(2 + 3*x^2 + x^4)) - (11*ArcTan[x
])/2 - 118*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

fricas [A] time = 0.87, size = 74, normalized size = 1.30

$$\frac{2\,x^9-12\,x^7+146\,x^5+655\,x^3-236\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-11\left(x^4+3\,x^2+2\right)\arctan(x)+494\,x}{2\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/2*(2*x^9 - 12*x^7 + 146*x^5 + 655*x^3 - 236*sqrt(2)*(x^4 + 3*x^2 + 2)*arc
tan(1/2*sqrt(2)*x) - 11*(x^4 + 3*x^2 + 2)*arctan(x) + 494*x)/(x^4 + 3*x^2 +
 2)
```

giac [A] time = 0.31, size = 51, normalized size = 0.89

$$x^5-9\,x^3-118\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+98\,x+\frac{103\,x^3+102\,x}{2\left(x^4+3\,x^2+2\right)}-\frac{11}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

maple [A] time = 0.01, size = 49, normalized size = 0.86

$$x^5-9x^3+98x-\frac{x}{2\left(x^2+1\right)}+\frac{52x}{x^2+2}-\frac{11\arctan(x)}{2}-118\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] x^5-9*x^3+98*x-1/2/(x^2+1)*x-11/2*arctan(x)+52/(x^2+2)*x-118*2^(1/2)*arctan
(1/2*2^(1/2)*x)
```

maxima [A] time = 1.62, size = 51, normalized size = 0.89

$$x^5 - 9\,x^3 - 118\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) + 98\,x + \frac{103\,x^3 + 102\,x}{2\left(x^4 + 3\,x^2 + 2\right)} - \frac{11}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 - 118*sqrt(2)*arctan(1/2*sqrt(2)*x) + 98*x + 1/2*(103*x^3 + 102
*x)/(x^4 + 3*x^2 + 2) - 11/2*arctan(x)
```

mupad [B] time = 0.05, size = 50, normalized size = 0.88

$$98\,x - \frac{11\,\mathrm{atan}(x)}{2} - 118\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) + \frac{\frac{103\,x^3}{2} + 51\,x}{x^4 + 3\,x^2 + 2} - 9\,x^3 + x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 98*x - (11*atan(x))/2 - 118*2^(1/2)*atan((2^(1/2)*x)/2) + (51*x + (103*x^3)
/2)/(3*x^2 + x^4 + 2) - 9*x^3 + x^5
```

sympy [A] time = 0.21, size = 54, normalized size = 0.95

$$x^5 - 9x^3 + 98x + \frac{103x^3 + 102x}{2x^4 + 6x^2 + 4} - \frac{11\,\mathrm{atan}\,(x)}{2} - 118\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] x**5 - 9*x**3 + 98*x + (103*x**3 + 102*x)/(2*x**4 + 6*x**2 + 4) - 11*atan(x
)/2 - 118*sqrt(2)*atan(sqrt(2)*x/2)
```

3.84 $\int \frac{x^4(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^2} dx$

Optimal. Leaf size=56

$$\frac{5x^3}{3} - \frac{(51x^2+50)\,x}{2(x^4+3x^2+2)} - 27x + \frac{13}{2}\tan^{-1}(x) + 33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] -27*x+5/3*x^3-1/2*x*(51*x^2+50)/(x^4+3*x^2+2)+13/2*arctan(x)+33*arctan(1/2*
x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$\frac{5x^3}{3} - \frac{(51x^2+50)\,x}{2(x^4+3x^2+2)} - 27x + \frac{13}{2}\tan^{-1}(x) + 33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -27*x + (5*x^3)/3 - (x*(50 + 51*x^2))/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x]
)/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx=-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{-100-6x^2+48x^4-20x^6}{2+3x^2+x^4}\,dx$$
$$=-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(108-20x^2-\frac{2\left(158+145x^2\right)}{2+3x^2+x^4}\right)dx$$
$$=-27x+\frac{5x^3}{3}-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{1}{2}\int \frac{158+145x^2}{2+3x^2+x^4}\,dx$$
$$=-27x+\frac{5x^3}{3}-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{13}{2}\int \frac{1}{1+x^2}\,dx+66\int \frac{1}{2+x^2}\,dx$$
$$=-27x+\frac{5x^3}{3}-\frac{x\left(50+51x^2\right)}{2\left(2+3x^2+x^4\right)}+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Mathematica [A] time = 0.04, size = 57, normalized size = 1.02

$$\frac{5x^3}{3}+\frac{-51x^3-50x}{2\left(x^4+3x^2+2\right)}-27x+\frac{13}{2}\tan^{-1}(x)+33\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
[Out] -27*x + (5*x^3)/3 + (-50*x - 51*x^3)/(2*(2 + 3*x^2 + x^4)) + (13*ArcTan[x])
/2 + 33*Sqrt[2]*ArcTan[x/Sqrt[2]]
```

fricas [A] time = 0.91, size = 69, normalized size = 1.23

$$\frac{10\,x^7-132\,x^5-619\,x^3+198\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+39\left(x^4+3\,x^2+2\right)\arctan(x)-474\,x}{6\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/6*(10*x^7 - 132*x^5 - 619*x^3 + 198*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*
sqrt(2)*x) + 39*(x^4 + 3*x^2 + 2)*arctan(x) - 474*x)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.31, size = 48, normalized size = 0.86

$$\frac{5}{3}\,x^3+33\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-27\,x-\frac{51\,x^3+50\,x}{2\left(x^4+3\,x^2+2\right)}+\frac{13}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

maple [A] time = 0.01, size = 46, normalized size = 0.82

$$\frac{5x^3}{3}-27x+\frac{x}{2x^2+2}-\frac{26x}{x^2+2}+\frac{13\arctan(x)}{2}+33\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5/3*x^3-27*x+1/2/(x^2+1)*x+13/2*arctan(x)-26/(x^2+2)*x+33*2^(1/2)*arctan(1/
2*2^(1/2)*x)
```

maxima [A] time = 1.64, size = 48, normalized size = 0.86

$$\frac{5}{3}x^3 + 33\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 27x - \frac{51x^3 + 50x}{2\left(x^4 + 3x^2 + 2\right)} + \frac{13}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 33*sqrt(2)*arctan(1/2*sqrt(2)*x) - 27*x - 1/2*(51*x^3 + 50*x)/(x^
4 + 3*x^2 + 2) + 13/2*arctan(x)
```

mupad [B] time = 0.92, size = 48, normalized size = 0.86

$$\frac{13\,\mathrm{atan}(x)}{2} - 27x + 33\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right) - \frac{\frac{51x^3}{2} + 25x}{x^4 + 3x^2 + 2} + \frac{5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] (13*atan(x))/2 - 27*x + 33*2^(1/2)*atan((2^(1/2)*x)/2) - (25*x + (51*x^3)/2
)/(3*x^2 + x^4 + 2) + (5*x^3)/3
```

sympy [A] time = 0.21, size = 54, normalized size = 0.96

$$\frac{5x^3}{3} - 27x + \frac{-51x^3 - 50x}{2x^4 + 6x^2 + 4} + \frac{13\,\mathrm{atan}(x)}{2} + 33\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x**3/3 - 27*x + (-51*x**3 - 50*x)/(2*x**4 + 6*x**2 + 4) + 13*atan(x)/2 +
33*sqrt(2)*atan(sqrt(2)*x/2)
```

3.85 $\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2} dx$

Optimal. Leaf size=49

$$\frac{\left(25x^2+24\right)x}{2\left(x^4+3x^2+2\right)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] 5*x+1/2*x*(25*x^2+24)/(x^4+3*x^2+2)-15/2*arctan(x)-7/2*arctan(1/2*x*2^(1/2)
)*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1676, 1166, 203}

$$\frac{\left(25x^2+24\right)x}{2\left(x^4+3x^2+2\right)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (x*(24 + 25*x^2))/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTa
n[x/Sqrt[2]])/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^2}\,dx &= \frac{x\left(24+25x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \frac{48-2x^2-20x^4}{2+3x^2+x^4}\,dx \\
&= \frac{x\left(24+25x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{4}\int \left(-20+\frac{2\left(44+29x^2\right)}{2+3x^2+x^4}\right)dx \\
&= 5x+\frac{x\left(24+25x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{1}{2}\int \frac{44+29x^2}{2+3x^2+x^4}\,dx \\
&= 5x+\frac{x\left(24+25x^2\right)}{2\left(2+3x^2+x^4\right)}-7\int \frac{1}{2+x^2}\,dx-\frac{15}{2}\int \frac{1}{1+x^2}\,dx \\
&= 5x+\frac{x\left(24+25x^2\right)}{2\left(2+3x^2+x^4\right)}-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.02

$$\frac{25x^3+24x}{2\left(x^4+3x^2+2\right)}+5x-\frac{15}{2}\tan^{-1}(x)-\frac{7\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (24*x + 25*x^3)/(2*(2 + 3*x^2 + x^4)) - (15*ArcTan[x])/2 - (7*ArcTan[
x/Sqrt[2]])/Sqrt[2]
```

fricas [A] time = 0.91, size = 64, normalized size = 1.31

$$\frac{10\,x^5+55\,x^3-7\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-15\left(x^4+3\,x^2+2\right)\arctan(x)+44\,x}{2\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(10*x^5 + 55*x^3 - 7*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) -
15*(x^4 + 3*x^2 + 2)*arctan(x) + 44*x)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.34, size = 43, normalized size = 0.88

$$-\frac{7}{2}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+5\,x+\frac{25\,x^3+24\,x}{2\left(x^4+3\,x^2+2\right)}-\frac{15}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

maple [A] time = 0.01, size = 41, normalized size = 0.84

$$5x-\frac{x}{2\left(x^2+1\right)}+\frac{13x}{x^2+2}-\frac{15\arctan(x)}{2}-\frac{7\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

```
[Out] 5*x-1/2/(x^2+1)*x-15/2*arctan(x)+13/(x^2+2)*x-7/2*2^(1/2)*arctan(1/2*2^(1/2
)*x)
```

maxima [A] time = 1.63, size = 43, normalized size = 0.88

$$-\frac{7}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5x+\frac{25x^3+24x}{2\left(x^4+3x^2+2\right)}-\frac{15}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -7/2*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x + 1/2*(25*x^3 + 24*x)/(x^4 + 3*x^2
 + 2) - 15/2*arctan(x)
```

mupad [B] time = 0.07, size = 42, normalized size = 0.86

$$5x-\frac{15\operatorname{atan}(x)}{2}-\frac{7\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}+\frac{\frac{25x^3}{2}+12x}{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^2,x)
```

```
[Out] 5*x - (15*atan(x))/2 - (7*2^(1/2)*atan((2^(1/2)*x)/2))/2 + (12*x + (25*x^3)
/2)/(3*x^2 + x^4 + 2)
```

sympy [A] time = 0.21, size = 48, normalized size = 0.98

$$5x+\frac{25x^3+24x}{2x^4+6x^2+4}-\frac{15\operatorname{atan}(x)}{2}-\frac{7\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

```
[Out] 5*x + (25*x**3 + 24*x)/(2*x**4 + 6*x**2 + 4) - 15*atan(x)/2 - 7*sqrt(2)*ata
n(sqrt(2)*x/2)/2
```

3.86 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=48

$$-\frac{x\left(12x^2+11\right)}{2\left(x^4+3x^2+2\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] -1/2*x*(12*x^2+11)/(x^4+3*x^2+2)+17/2*arctan(x)-19/4*arctan(1/2*x*2^(1/2))*
2^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1678, 1166, 203}

$$-\frac{x\left(12x^2+11\right)}{2\left(x^4+3x^2+2\right)}+\frac{17}{2}\tan^{-1}(x)-\frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] -(x*(11 + 12*x^2))/(2*(2 + 3*x^2 + x^4)) + (17*ArcTan[x])/2 - (19*ArcTan[x/
Sqrt[2]])/(2*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^2}\,dx = -\frac{x\left(11+12x^2\right)}{2\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-30+4x^2}{2+3x^2+x^4}\,dx$$
$$= -\frac{x\left(11+12x^2\right)}{2\left(2+3x^2+x^4\right)} + \frac{17}{2}\int \frac{1}{1+x^2}\,dx - \frac{19}{2}\int \frac{1}{2+x^2}\,dx$$
$$= -\frac{x\left(11+12x^2\right)}{2\left(2+3x^2+x^4\right)} + \frac{17}{2}\tan^{-1}(x) - \frac{19\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 0.96

$$\frac{1}{4}\left(-\frac{2x\left(12x^2+11\right)}{x^4+3x^2+2} + 34\tan^{-1}(x) - 19\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^2,x]
```

```
[Out] ((-2*x*(11 + 12*x^2))/(2 + 3*x^2 + x^4) + 34*ArcTan[x] - 19*Sqrt[2]*ArcTan[
x/Sqrt[2]])/4
```

fricas [A] time = 1.08, size = 59, normalized size = 1.23

$$-\frac{24\,x^3 + 19\,\sqrt{2}\left(x^4+3\,x^2+2\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) - 34\left(x^4+3\,x^2+2\right)\arctan(x) + 22\,x}{4\left(x^4+3\,x^2+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(24*x^3 + 19*sqrt(2)*(x^4 + 3*x^2 + 2)*arctan(1/2*sqrt(2)*x) - 34*(x^4
 + 3*x^2 + 2)*arctan(x) + 22*x)/(x^4 + 3*x^2 + 2)
```

giac [A] time = 0.34, size = 40, normalized size = 0.83

$$-\frac{19}{4}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) - \frac{12\,x^3+11\,x}{2\left(x^4+3\,x^2+2\right)} + \frac{17}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2)
 + 17/2*arctan(x)
```

maple [A] time = 0.01, size = 38, normalized size = 0.79

$$\frac{x}{2x^2+2} - \frac{13x}{2\left(x^2+2\right)} + \frac{17\arctan(x)}{2} - \frac{19\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x)
```

[Out] 1/2/(x^2+1)*x+17/2*arctan(x)-13/2/(x^2+2)*x-19/4*2^(1/2)*arctan(1/2*2^(1/2)*x)

maxima [A] time = 1.53, size = 40, normalized size = 0.83

$$-\frac{19}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{12x^3+11x}{2\left(x^4+3x^2+2\right)}+\frac{17}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

[Out] -19/4*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/2*(12*x^3 + 11*x)/(x^4 + 3*x^2 + 2) + 17/2*arctan(x)

mupad [B] time = 0.07, size = 40, normalized size = 0.83

$$\frac{17\,\mathrm{atan}(x)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}-\frac{6x^3+\frac{11x}{2}}{x^4+3x^2+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^2,x)
```

[Out] (17*atan(x))/2 - (19*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((11*x)/2 + 6*x^3)/(3*x^2 + x^4 + 2)

sympy [A] time = 0.20, size = 46, normalized size = 0.96

$$\frac{-12x^3-11x}{2x^4+6x^2+4}+\frac{17\,\mathrm{atan}\,(x)}{2}-\frac{19\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**2,x)
```

[Out] (-12*x**3 - 11*x)/(2*x**4 + 6*x**2 + 4) + 17*atan(x)/2 - 19*sqrt(2)*atan(sqrt(2)*x/2)/4

3.87 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=53

$$\frac{x\left(11x^2+9\right)}{4\left(x^4+3x^2+2\right)}-\frac{1}{x}-\frac{19}{2}\tan^{-1}(x)+\frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/x+1/4*x*(11*x^2+9)/(x^4+3*x^2+2)-19/2*arctan(x)+45/8*arctan(1/2*x*2^(1/2
))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$\frac{x\left(11x^2+9\right)}{4\left(x^4+3x^2+2\right)}-\frac{1}{x}-\frac{19}{2}\tan^{-1}(x)+\frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -x^(-1) + (x*(9 + 11*x^2))/(4*(2 + 3*x^2 + x^4)) - (19*ArcTan[x])/2 + (45*A
rcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^2}\,dx = \frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+19x^2-11x^4}{x^2\left(2+3x^2+x^4\right)}\,dx$$
$$= \frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^2}+\frac{38}{1+x^2}-\frac{45}{2+x^2}\right)dx$$
$$= -\frac{1}{x}+\frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)} - \frac{19}{2}\int \frac{1}{1+x^2}\,dx + \frac{45}{4}\int \frac{1}{2+x^2}\,dx$$
$$= -\frac{1}{x}+\frac{x\left(9+11x^2\right)}{4\left(2+3x^2+x^4\right)} - \frac{19}{2}\tan^{-1}(x) + \frac{45\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 51, normalized size = 0.96

$$\frac{1}{8}\left(\frac{2x\left(11x^2+9\right)}{x^4+3x^2+2}-\frac{8}{x}-76\tan^{-1}(x)+45\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^2),x]
[Out] (-8/x + (2*x*(9 + 11*x^2))/(2 + 3*x^2 + x^4) - 76*ArcTan[x] + 45*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/8
```

fricas [A] time = 0.92, size = 68, normalized size = 1.28

$$\frac{14\,x^4+45\,\sqrt{2}\left(x^5+3\,x^3+2\,x\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-6\,x^2-76\left(x^5+3\,x^3+2\,x\right)\arctan(x)-16}{8\left(x^5+3\,x^3+2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="fricas")
[Out] 1/8*(14*x^4 + 45*sqrt(2)*(x^5 + 3*x^3 + 2*x)*arctan(1/2*sqrt(2)*x) - 6*x^2
- 76*(x^5 + 3*x^3 + 2*x)*arctan(x) - 16)/(x^5 + 3*x^3 + 2*x)
```

giac [A] time = 0.38, size = 45, normalized size = 0.85

$$\frac{45}{8}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+\frac{7\,x^4-3\,x^2-8}{4\left(x^5+3\,x^3+2\,x\right)}-\frac{19}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="giac")
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

maple [A] time = 0.01, size = 43, normalized size = 0.81

$$-\frac{x}{2\left(x^2+1\right)}+\frac{13x}{4\left(x^2+2\right)}-\frac{19\arctan(x)}{2}+\frac{45\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{8}-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/x-1/2/(x^2+1)*x-19/2*arctan(x)+13/4/(x^2+2)*x+45/8*2^(1/2)*arctan(1/2*2^
(1/2)*x)
```

maxima [A] time = 1.55, size = 45, normalized size = 0.85

$$\frac{45}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{7x^4-3x^2-8}{4\left(x^5+3x^3+2x\right)}-\frac{19}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 45/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/4*(7*x^4 - 3*x^2 - 8)/(x^5 + 3*x^3 +
 2*x) - 19/2*arctan(x)
```

mupad [B] time = 0.07, size = 45, normalized size = 0.85

$$\frac{45\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}-\frac{19\operatorname{atan}(x)}{2}-\frac{-\frac{7x^4}{4}+\frac{3x^2}{4}+2}{x^5+3x^3+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (45*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (19*atan(x))/2 - ((3*x^2)/4 - (7*x^4)/
4 + 2)/(2*x + 3*x^3 + x^5)
```

sympy [A] time = 0.22, size = 49, normalized size = 0.92

$$\frac{7x^4-3x^2-8}{4x^5+12x^3+8x}-\frac{19\operatorname{atan}(x)}{2}+\frac{45\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**2,x)
```

```
[Out] (7*x**4 - 3*x**2 - 8)/(4*x**5 + 12*x**3 + 8*x) - 19*atan(x)/2 + 45*sqrt(2)*
atan(sqrt(2)*x/2)/8
```

3.88 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=62

$$-\frac{1}{3x^3}-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}+\frac{11}{4x}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] -1/3/x^3+11/4/x-1/8*x*(9*x^2+5)/(x^4+3*x^2+2)+21/2*arctan(x)-71/16*arctan(1
/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$-\frac{x\left(9x^2+5\right)}{8\left(x^4+3x^2+2\right)}-\frac{1}{3x^3}+\frac{11}{4x}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(3*x^3) + 11/(4*x) - (x*(5 + 9*x^2))/(8*(2 + 3*x^2 + x^4)) + (21*ArcTan[
x])/2 - (71*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^2}\,dx = -\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+10x^2-\frac{39x^4}{2}+\frac{9x^6}{2}}{x^4\left(2+3x^2+x^4\right)}\,dx$$

$$= -\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^4}+\frac{11}{x^2}-\frac{42}{1+x^2}+\frac{71}{2\left(2+x^2\right)}\right)dx$$

$$= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{71}{8}\int \frac{1}{2+x^2}\,dx+\frac{21}{2}\int \frac{1}{1+x^2}\,dx$$

$$= -\frac{1}{3x^3}+\frac{11}{4x}-\frac{x\left(5+9x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{21}{2}\tan^{-1}(x)-\frac{71\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.90

$$\frac{1}{48}\left(-\frac{16}{x^3}-\frac{6x\left(9x^2+5\right)}{x^4+3x^2+2}+\frac{132}{x}+504\tan^{-1}(x)-213\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-16/x^3 + 132/x - (6*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4) + 504*ArcTan[x] - 21
3*Sqrt[2]*ArcTan[x/Sqrt[2]])/48
```

fricas [A] time = 0.91, size = 79, normalized size = 1.27

$$\frac{78\,x^6+350\,x^4-213\,\sqrt{2}\left(x^7+3\,x^5+2\,x^3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+216\,x^2+504\left(x^7+3\,x^5+2\,x^3\right)\arctan(x)-32}{48\left(x^7+3\,x^5+2\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(78*x^6 + 350*x^4 - 213*sqrt(2)*(x^7 + 3*x^5 + 2*x^3)*arctan(1/2*sqrt(
2)*x) + 216*x^2 + 504*(x^7 + 3*x^5 + 2*x^3)*arctan(x) - 32)/(x^7 + 3*x^5 +
2*x^3)
```

giac [A] time = 0.39, size = 52, normalized size = 0.84

$$-\frac{71}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-\frac{9\,x^3+5\,x}{8\left(x^4+3\,x^2+2\right)}+\frac{33\,x^2-4}{12\,x^3}+\frac{21}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(9*x^3 + 5*x)/(x^4 + 3*x^2 + 2)
+ 1/12*(33*x^2 - 4)/x^3 + 21/2*arctan(x)
```

maple [A] time = 0.02, size = 48, normalized size = 0.77

$$\frac{x}{2x^2+2}-\frac{13x}{8\left(x^2+2\right)}+\frac{21\arctan(x)}{2}-\frac{71\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{16}+\frac{11}{4x}-\frac{1}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/3/x^3+11/4/x+1/2/(x^2+1)*x+21/2*arctan(x)-13/8/(x^2+2)*x-71/16*2^(1/2)*a
rctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.60, size = 52, normalized size = 0.84

$$-\frac{71}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+\frac{39x^6+175x^4+108x^2-16}{24\left(x^7+3x^5+2x^3\right)}+\frac{21}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -71/16*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/24*(39*x^6 + 175*x^4 + 108*x^2 - 1
6)/(x^7 + 3*x^5 + 2*x^3) + 21/2*arctan(x)
```

mupad [B] time = 0.92, size = 51, normalized size = 0.82

$$\frac{21\,\mathrm{atan}(x)}{2}-\frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16}+\frac{\frac{13x^6}{8}+\frac{175x^4}{24}+\frac{9x^2}{2}-\frac{2}{3}}{x^7+3x^5+2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (21*atan(x))/2 - (71*2^(1/2)*atan((2^(1/2)*x)/2))/16 + ((9*x^2)/2 + (175*x^
4)/24 + (13*x^6)/8 - 2/3)/(2*x^3 + 3*x^5 + x^7)
```

sympy [A] time = 0.24, size = 56, normalized size = 0.90

$$\frac{21\,\mathrm{atan}\,(x)}{2}-\frac{71\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}+\frac{39x^6+175x^4+108x^2-16}{24x^7+72x^5+48x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**2,x)
```

```
[Out] 21*atan(x)/2 - 71*sqrt(2)*atan(sqrt(2)*x/2)/16 + (39*x**6 + 175*x**4 + 108*
x**2 - 16)/(24*x**7 + 72*x**5 + 48*x**3)
```

3.89 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2} \, dx$

Optimal. Leaf size=69

$$-\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}-\frac{23}{4x}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/5/x^5+11/12/x^3-23/4/x-1/16*x*(-5*x^2+3)/(x^4+3*x^2+2)-23/2*arctan(x)+97
/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$-\frac{x\left(3-5x^2\right)}{16\left(x^4+3x^2+2\right)}+\frac{11}{12x^3}-\frac{1}{5x^5}-\frac{23}{4x}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(5*x^5) + 11/(12*x^3) - 23/(4*x) - (x*(3 - 5*x^2))/(16*(2 + 3*x^2 + x^4)
) - (23*ArcTan[x])/2 + (97*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^2}\,dx = -\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{39x^6}{4}-\frac{5x^8}{4}}{x^6\left(2+3x^2+x^4\right)}\,dx$$

$$= -\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^6}+\frac{11}{x^4}-\frac{23}{x^2}+\frac{46}{1+x^2}-\frac{97}{4\left(2+x^2\right)}\right)dx$$

$$= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)}+\frac{97}{16}\int \frac{1}{2+x^2}\,dx-\frac{23}{2}\int \frac{1}{1+x^2}\,dx$$

$$= -\frac{1}{5x^5}+\frac{11}{12x^3}-\frac{23}{4x}-\frac{x\left(3-5x^2\right)}{16\left(2+3x^2+x^4\right)}-\frac{23}{2}\tan^{-1}(x)+\frac{97\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 0.88

$$\frac{1}{480}\left(-\frac{96}{x^5}+\frac{440}{x^3}+\frac{30x\left(5x^2-3\right)}{x^4+3x^2+2}-\frac{2760}{x}-5520\tan^{-1}(x)+1455\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^5 + 440/x^3 - 2760/x + (30*x*(-3 + 5*x^2))/(2 + 3*x^2 + x^4) - 5520*
ArcTan[x] + 1455*Sqrt[2]*ArcTan[x/Sqrt[2]])/480
```

fricas [A] time = 0.87, size = 84, normalized size = 1.22

$$-\frac{2610\,x^8+7930\,x^6+4296\,x^4-1455\,\sqrt{2}\left(x^9+3\,x^7+2\,x^5\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-592\,x^2+5520\left(x^9+3\,x^7+2\,x^5\right.}{480\left(x^9+3\,x^7+2\,x^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] -1/480*(2610*x^8 + 7930*x^6 + 4296*x^4 - 1455*sqrt(2)*(x^9 + 3*x^7 + 2*x^5)
*arctan(1/2*sqrt(2)*x) - 592*x^2 + 5520*(x^9 + 3*x^7 + 2*x^5)*arctan(x) + 1
92)/(x^9 + 3*x^7 + 2*x^5)
```

giac [A] time = 0.34, size = 57, normalized size = 0.83

$$\frac{97}{32}\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+\frac{5\,x^3-3\,x}{16\left(x^4+3\,x^2+2\right)}-\frac{345\,x^4-55\,x^2+12}{60\,x^5}-\frac{23}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(5*x^3 - 3*x)/(x^4 + 3*x^2 + 2)
- 1/60*(345*x^4 - 55*x^2 + 12)/x^5 - 23/2*arctan(x)
```

maple [A] time = 0.02, size = 53, normalized size = 0.77

$$-\frac{x}{2\left(x^2+1\right)}+\frac{13x}{16\left(x^2+2\right)}-\frac{23\arctan(x)}{2}+\frac{97\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{23}{4x}+\frac{11}{12x^3}-\frac{1}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/5/x^5+11/12/x^3-23/4/x-1/2/(x^2+1)*x-23/2*arctan(x)+13/16/(x^2+2)*x+97/3
2*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.76, size = 57, normalized size = 0.83

$$\frac{97}{32}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)-\frac{1305x^8+3965x^6+2148x^4-296x^2+96}{240\left(x^9+3x^7+2x^5\right)}-\frac{23}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] 97/32*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/240*(1305*x^8 + 3965*x^6 + 2148*x^4
 - 296*x^2 + 96)/(x^9 + 3*x^7 + 2*x^5) - 23/2*arctan(x)
```

mupad [B] time = 0.92, size = 57, normalized size = 0.83

$$\frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{23\,\mathrm{atan}(x)}{2}-\frac{\frac{87x^8}{16}+\frac{793x^6}{48}+\frac{179x^4}{20}-\frac{37x^2}{30}+\frac{2}{5}}{x^9+3x^7+2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (97*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (23*atan(x))/2 - ((179*x^4)/20 - (37*
x^2)/30 + (793*x^6)/48 + (87*x^8)/16 + 2/5)/(2*x^5 + 3*x^7 + x^9)
```

sympy [A] time = 0.25, size = 61, normalized size = 0.88

$$-\frac{23\,\mathrm{atan}(x)}{2}+\frac{97\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{32}+\frac{-1305x^8-3965x^6-2148x^4+296x^2-96}{240x^9+720x^7+480x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**2,x)
```

```
[Out] -23*atan(x)/2 + 97*sqrt(2)*atan(sqrt(2)*x/2)/32 + (-1305*x**8 - 3965*x**6 -
 2148*x**4 + 296*x**2 - 96)/(240*x**9 + 720*x**7 + 480*x**5)
```

3.90 $\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=76

$$-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/7/x^7+11/20/x^5-23/12/x^3+137/16/x+1/32*x*(3*x^2+19)/(x^4+3*x^2+2)+25/2*
arctan(x)-123/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$\frac{x\left(3x^2+19\right)}{32\left(x^4+3x^2+2\right)}-\frac{23}{12x^3}+\frac{11}{20x^5}-\frac{1}{7x^7}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/(7*x^7) + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (x*(19 + 3*x^2))/(32*
(2 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2]
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^8\left(2+3x^2+x^4\right)^2}\,dx = \frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \frac{-8+10x^2-17x^4+\frac{21x^6}{2}-\frac{39x^8}{8}-\frac{3x^{10}}{8}}{x^8\left(2+3x^2+x^4\right)}\,dx$$
$$= \frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)} - \frac{1}{4}\int \left(-\frac{4}{x^8}+\frac{11}{x^6}-\frac{23}{x^4}+\frac{137}{4x^2}-\frac{50}{1+x^2}+\frac{123}{8\left(2+x^2\right)}\right)dx$$
$$= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)}-\frac{123}{32}\int \frac{1}{2+x^2}\,dx+\frac{25}{2}\int \frac{1}{1+x^2}$$
$$= -\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{137}{16x}+\frac{x\left(19+3x^2\right)}{32\left(2+3x^2+x^4\right)}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.01

$$-\frac{1}{7x^7}+\frac{11}{20x^5}-\frac{23}{12x^3}+\frac{3x^3+19x}{32\left(x^4+3x^2+2\right)}+\frac{137}{16x}+\frac{25}{2}\tan^{-1}(x)-\frac{123\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^8*(2 + 3*x^2 + x^4)^2),x]
```

```
[Out] -1/7*1/x^7 + 11/(20*x^5) - 23/(12*x^3) + 137/(16*x) + (19*x + 3*x^3)/(32*(2
 + 3*x^2 + x^4)) + (25*ArcTan[x])/2 - (123*ArcTan[x/Sqrt[2]])/(32*Sqrt[2])
```

fricas [A] time = 0.88, size = 89, normalized size = 1.17

$$\frac{58170\,x^{10}+163730\,x^8+80136\,x^6-15632\,x^4-12915\,\sqrt{2}\left(x^{11}+3\,x^9+2\,x^7\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+4512\,x^2+84000}{6720\left(x^{11}+3\,x^9+2\,x^7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(58170*x^10 + 163730*x^8 + 80136*x^6 - 15632*x^4 - 12915*sqrt(2)*(x^
11 + 3*x^9 + 2*x^7)*arctan(1/2*sqrt(2)*x) + 4512*x^2 + 84000*(x^11 + 3*x^9
+ 2*x^7)*arctan(x) - 1920)/(x^11 + 3*x^9 + 2*x^7)
```

giac [A] time = 0.45, size = 62, normalized size = 0.82

$$-\frac{123}{64}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+\frac{3\,x^3+19\,x}{32\left(x^4+3\,x^2+2\right)}+\frac{14385\,x^6-3220\,x^4+924\,x^2-240}{1680\,x^7}+\frac{25}{2}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="giac")
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/32*(3*x^3 + 19*x)/(x^4 + 3*x^2 +
2) + 1/1680*(14385*x^6 - 3220*x^4 + 924*x^2 - 240)/x^7 + 25/2*arctan(x)
```

maple [A] time = 0.02, size = 58, normalized size = 0.76

$$\frac{x}{2x^2+2}-\frac{13x}{32\left(x^2+2\right)}+\frac{25\arctan(x)}{2}-\frac{123\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{64}+\frac{137}{16x}-\frac{23}{12x^3}+\frac{11}{20x^5}-\frac{1}{7x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x)
```

```
[Out] -1/7/x^7+11/20/x^5-23/12/x^3+137/16/x+1/2/(x^2+1)*x+25/2*arctan(x)-13/32/(x
^2+2)*x-123/64*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.53, size = 62, normalized size = 0.82

$$-\frac{123}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{29085\,x^{10}+81865\,x^{8}+40068\,x^{6}-7816\,x^{4}+2256\,x^{2}-960}{3360\left(x^{11}+3\,x^{9}+2\,x^{7}\right)}+\frac{25}{2}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^8/(x^4+3*x^2+2)^2,x, algorithm="maxima")
```

```
[Out] -123/64*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/3360*(29085*x^10 + 81865*x^8 + 40
068*x^6 - 7816*x^4 + 2256*x^2 - 960)/(x^11 + 3*x^9 + 2*x^7) + 25/2*arctan(x
)
```

mupad [B] time = 0.07, size = 61, normalized size = 0.80

$$\frac{25\,\mathrm{atan}(x)}{2}-\frac{123\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}+\frac{\frac{277\,x^{10}}{32}+\frac{2339\,x^{8}}{96}+\frac{477\,x^{6}}{40}-\frac{977\,x^{4}}{420}+\frac{47\,x^{2}}{70}-\frac{2}{7}}{x^{11}+3\,x^{9}+2\,x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^8*(3*x^2 + x^4 + 2)^2),x)
```

```
[Out] (25*atan(x))/2 - (123*2^(1/2)*atan((2^(1/2)*x)/2))/64 + ((47*x^2)/70 - (977
*x^4)/420 + (477*x^6)/40 + (2339*x^8)/96 + (277*x^10)/32 - 2/7)/(2*x^7 + 3*
x^9 + x^11)
```

sympy [A] time = 0.28, size = 66, normalized size = 0.87

$$\frac{25\,\mathrm{atan}\,(x)}{2}-\frac{123\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}+\frac{29085x^{10}+81865x^{8}+40068x^{6}-7816x^{4}+2256x^{2}-960}{3360x^{11}+10080x^{9}+6720x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**8/(x**4+3*x**2+2)**2,x)
```

```
[Out] 25*atan(x)/2 - 123*sqrt(2)*atan(sqrt(2)*x/2)/64 + (29085*x**10 + 81865*x**8
 + 40068*x**6 - 7816*x**4 + 2256*x**2 - 960)/(3360*x**11 + 10080*x**9 + 672
0*x**7)
```

3.91 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} \, dx$

Optimal. Leaf size=81

$$x^5-14x^3+\frac{(1669x^2+824)\,x}{8\,(x^4+3x^2+2)}+\frac{(415x^2+414)\,x}{4\,(x^4+3x^2+2)^2}+214x+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

```
[Out] 214*x-14*x^3+x^5+1/4*x*(415*x^2+414)/(x^4+3*x^2+2)^2+1/8*x*(1669*x^2+824)/(
x^4+3*x^2+2)+477/8*arctan(x)-351*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1668, 1678, 1676, 1166, 203}

$$x^5-14x^3+\frac{(1669x^2+824)\,x}{8\,(x^4+3x^2+2)}+\frac{(415x^2+414)\,x}{4\,(x^4+3x^2+2)^2}+214x+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 214*x - 14*x^3 + x^5 + (x*(414 + 415*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(82
4 + 1669*x^2))/(8*(2 + 3*x^2 + x^4)) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcT
an[x/Sqrt[2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^{10}\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx &= \frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{828-2478x^2-840x^4+424x^6-216x^8+96x^{10}-}{\left(2+3x^2+x^4\right)^2} \\
&= \frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{-4952-2700x^2+3136x^4-}{2+3x^2+x^4} \\
&= \frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \left(6848-1344x^2+160x^4-\frac{3}{}\right. \\
&= 214x-14x^3+x^5+\frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{9}{8}\int \frac{518+571}{2+3x^2+} \\
&= 214x-14x^3+x^5+\frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{477}{8}\int \frac{1}{1+x^2} \\
&= 214x-14x^3+x^5+\frac{x\left(414+415x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(824+1669x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{477}{8}\tan^{-1}(x)-
\end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.88

$$\frac{x\left(8x^{12}-64x^{10}+1144x^8+10581x^6+26775x^4+26736x^2+9324\right)}{8\left(x^4+3x^2+2\right)^2}+\frac{477}{8}\tan^{-1}(x)-351\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(9324 + 26736*x^2 + 26775*x^4 + 10581*x^6 + 1144*x^8 - 64*x^10 + 8*x^12)
)/(8*(2 + 3*x^2 + x^4)^2) + (477*ArcTan[x])/8 - 351*Sqrt[2]*ArcTan[x/Sqrt[2
]]
```

fricas [A] time = 0.88, size = 114, normalized size = 1.41

$$\frac{8\,x^{13}-64\,x^{11}+1144\,x^9+10581\,x^7+26775\,x^5+26736\,x^3-2808\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\right.}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*x^13 - 64*x^11 + 1144*x^9 + 10581*x^7 + 26775*x^5 + 26736*x^3 - 2808
*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 477*(x
^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(x) + 9324*x)/(x^8 + 6*x^6 + 13*x^4
 + 12*x^2 + 4)
```

giac [A] time = 0.32, size = 61, normalized size = 0.75

$$x^5-14\,x^3-351\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+214\,x+\frac{1669\,x^7+5831\,x^5+6640\,x^3+2476\,x}{8\,\left(x^4+3\,x^2+2\right)^2}+\frac{477}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^4 + 3*x^2 + 2)^2 + 477/8*arctan(x)
```

maple [A] time = 0.01, size = 64, normalized size = 0.79

$$x^5-14x^3+214x+\frac{477\arctan(x)}{8}-351\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)+\frac{-\frac{11}{8}x^3-\frac{13}{8}x}{\left(x^2+1\right)^2}-\frac{16\left(-\frac{105}{8}x^3-\frac{79}{4}x\right)}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] x^5-14*x^3+214*x+(-11/8*x^3-13/8*x)/(x^2+1)^2+477/8*arctan(x)-16*(-105/8*x^
3-79/4*x)/(x^2+2)^2-351*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.58, size = 71, normalized size = 0.88

$$x^5-14\,x^3-351\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+214\,x+\frac{1669\,x^7+5831\,x^5+6640\,x^3+2476\,x}{8\,\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}+\frac{477}{8}\,\arctan\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 14*x^3 - 351*sqrt(2)*arctan(1/2*sqrt(2)*x) + 214*x + 1/8*(1669*x^7 +
5831*x^5 + 6640*x^3 + 2476*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 477/8*a
rctan(x)
```

mupad [B] time = 0.06, size = 70, normalized size = 0.86

$$214\,x+\frac{477\,\mathrm{atan}\left(x\right)}{8}-351\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)+\frac{\frac{1669\,x^7}{8}+\frac{5831\,x^5}{8}+830\,x^3+\frac{619\,x}{2}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}-14\,x^3+x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] 214*x + (477*atan(x))/8 - 351*2^(1/2)*atan((2^(1/2)*x)/2) + ((619*x)/2 + 83
0*x^3 + (5831*x^5)/8 + (1669*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4) -
14*x^3 + x^5
```

sympy [A] time = 0.26, size = 75, normalized size = 0.93

$$x^5-14x^3+214x+\frac{1669x^7+5831x^5+6640x^3+2476x}{8x^8+48x^6+104x^4+96x^2+32}+\frac{477\,\mathrm{atan}\,(x)}{8}-351\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] x**5 - 14*x**3 + 214*x + (1669*x**7 + 5831*x**5 + 6640*x**3 + 2476*x)/(8*x*
*8 + 48*x**6 + 104*x**4 + 96*x**2 + 32) + 477*atan(x)/8 - 351*sqrt(2)*atan(
sqrt(2)*x/2)
```

3.92 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3} dx$

Optimal. Leaf size=80

$$\frac{5x^3}{3}+\frac{(24-409x^2)\,x}{8\,(x^4+3x^2+2)}-\frac{(207x^2+206)\,x}{4\,(x^4+3x^2+2)^2}-42x-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] -42*x+5/3*x^3-1/4*x*(207*x^2+206)/(x^4+3*x^2+2)^2+1/8*x*(-409*x^2+24)/(x^4+
3*x^2+2)-449/8*arctan(x)+219/2*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1668, 1678, 1676, 1166, 203}

$$\frac{5x^3}{3}+\frac{(24-409x^2)\,x}{8\,(x^4+3x^2+2)}-\frac{(207x^2+206)\,x}{4\,(x^4+3x^2+2)^2}-42x-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -42*x + (5*x^3)/3 - (x*(206 + 207*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(24 -
409*x^2))/(8*(2 + 3*x^2 + x^4)) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]
])/Sqrt[2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
```

```
2] && Expon[Pq, x^2] > 1
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^8(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx = -\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}-\frac{1}{8}\int \frac{-412+1230x^2+424x^4-216x^6+96x^8-40x^{10}}{(2+3x^2+x^4)^2}\,d$$

$$= -\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int \frac{728+1500x^2-864x^4+160}{2+3x^2+x^4}$$

$$= -\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int\left(-1344+160x^2+\frac{4(854+1}{2+3x^2}\right.$$

$$= -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)}+\frac{1}{8}\int \frac{854+1303x^2}{2+3x^2+x^4}\,dx$$

$$= -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)}-\frac{449}{8}\int \frac{1}{1+x^2}\,dx+21$$

$$= -42x+\frac{5x^3}{3}-\frac{x(206+207x^2)}{4(2+3x^2+x^4)^2}+\frac{x(24-409x^2)}{8(2+3x^2+x^4)}-\frac{449}{8}\tan^{-1}(x)+\frac{219\,\text{ta}}{}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.82

$$\frac{x(40x^{10}-768x^8-6755x^6-16233x^4-15416x^2-5124)}{24(x^4+3x^2+2)^2}-\frac{449}{8}\tan^{-1}(x)+\frac{219\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(-5124 - 15416*x^2 - 16233*x^4 - 6755*x^6 - 768*x^8 + 40*x^10))/(24*(2 +
 3*x^2 + x^4)^2) - (449*ArcTan[x])/8 + (219*ArcTan[x/Sqrt[2]])/Sqrt[2]
```

fricas [A] time = 0.86, size = 109, normalized size = 1.36

$$\frac{40\,x^{11}-768\,x^9-6755\,x^7-16233\,x^5-15416\,x^3+2628\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-}{24\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(40*x^11 - 768*x^9 - 6755*x^7 - 16233*x^5 - 15416*x^3 + 2628*sqrt(2)*(
x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1347*(x^8 + 6*x^
6 + 13*x^4 + 12*x^2 + 4)*arctan(x) - 5124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2
 + 4)
```

giac [A] time = 0.41, size = 58, normalized size = 0.72

$$\frac{5}{3}x^3 + \frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8\left(x^4 + 3x^2 + 2\right)^2} - \frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^4 + 3*x^2 + 2)^2 - 449/8*arctan(x)
```

maple [A] time = 0.01, size = 62, normalized size = 0.78

$$\frac{5x^3}{3} - 42x - \frac{449\arctan(x)}{8} + \frac{219\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{-\frac{15}{8}x^3 - \frac{17}{8}x}{\left(x^2+1\right)^2} + \frac{-53x^3 - 54x}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 5/3*x^3-42*x-(-15/8*x^3-17/8*x)/(x^2+1)^2-449/8*arctan(x)+16*(-53/16*x^3-27
/8*x)/(x^2+2)^2+219/2*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.91, size = 68, normalized size = 0.85

$$\frac{5}{3}x^3 + \frac{219}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right) - 42x - \frac{409x^7 + 1203x^5 + 1160x^3 + 364x}{8\left(x^8 + 6x^6 + 13x^4 + 12x^2 + 4\right)} - \frac{449}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 + 219/2*sqrt(2)*arctan(1/2*sqrt(2)*x) - 42*x - 1/8*(409*x^7 + 1203*
x^5 + 1160*x^3 + 364*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 449/8*arctan(
x)
```

mupad [B] time = 0.05, size = 68, normalized size = 0.85

$$\frac{219\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2} - \frac{449\,\mathrm{atan}(x)}{8} - 42x - \frac{\frac{409x^7}{8} + \frac{1203x^5}{8} + 145x^3 + \frac{91x}{2}}{x^8 + 6x^6 + 13x^4 + 12x^2 + 4} + \frac{5x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (219*2^(1/2)*atan((2^(1/2)*x)/2))/2 - (449*atan(x))/8 - 42*x - ((91*x)/2 +
145*x^3 + (1203*x^5)/8 + (409*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4) +
 (5*x^3)/3
```

sympy [A] time = 0.26, size = 76, normalized size = 0.95

$$\frac{5x^3}{3} - 42x + \frac{-409x^7 - 1203x^5 - 1160x^3 - 364x}{8x^8 + 48x^6 + 104x^4 + 96x^2 + 32} - \frac{449\,\mathrm{atan}(x)}{8} + \frac{219\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x**3/3 - 42*x + (-409*x**7 - 1203*x**5 - 1160*x**3 - 364*x)/(8*x**8 + 48*
x**6 + 104*x**4 + 96*x**2 + 32) - 449*atan(x)/8 + 219*sqrt(2)*atan(sqrt(2)*
x/2)/2
```

3.93 $\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\left(15x^2+244\right)x}{8\left(x^4+3x^2+2\right)}+\frac{\left(103x^2+102\right)x}{4\left(x^4+3x^2+2\right)^2}+5x+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

```
[Out] 5*x+1/4*x*(103*x^2+102)/(x^4+3*x^2+2)^2-1/8*x*(15*x^2+244)/(x^4+3*x^2+2)+41
3/8*arctan(x)-191/4*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {1668, 1678, 1676, 1166, 203}

$$-\frac{\left(15x^2+244\right)x}{8\left(x^4+3x^2+2\right)}+\frac{\left(103x^2+102\right)x}{4\left(x^4+3x^2+2\right)^2}+5x+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] 5*x + (x*(102 + 103*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(244 + 15*x^2))/(8*(
2 + 3*x^2 + x^4)) + (413*ArcTan[x])/8 - (191*ArcTan[x/Sqrt[2]])/(2*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^6(4+x^2+3x^4+5x^6)}{(2+3x^2+x^4)^3}\,dx &= \frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{1}{8}\int \frac{204-606x^2-216x^4+96x^6-40x^8}{(2+3x^2+x^4)^2}\,dx\\
&= \frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int \frac{568-924x^2+160x^4}{2+3x^2+x^4}\,dx\\
&= \frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)}+\frac{1}{32}\int \left(160+\frac{4(62-351x^2)}{2+3x^2+x^4}\right)dx\\
&= 5x+\frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)}+\frac{1}{8}\int \frac{62-351x^2}{2+3x^2+x^4}\,dx\\
&= 5x+\frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)}+\frac{413}{8}\int \frac{1}{1+x^2}\,dx-\frac{191}{2}\int \frac{1}{2+x
\end{aligned}$$

$$= 5x+\frac{x(102+103x^2)}{4(2+3x^2+x^4)^2}-\frac{x(244+15x^2)}{8(2+3x^2+x^4)}+\frac{413}{8}\tan^{-1}(x)-\frac{191\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 0.80

$$\frac{1}{8}\left(\frac{x(40x^8+225x^6+231x^4-76x^2-124)}{(x^4+3x^2+2)^2}+413\tan^{-1}(x)-382\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(-124 - 76*x^2 + 231*x^4 + 225*x^6 + 40*x^8))/(2 + 3*x^2 + x^4)^2 + 413
*ArcTan[x] - 382*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

fricas [A] time = 0.64, size = 104, normalized size = 1.39

$$\frac{40\,x^9+225\,x^7+231\,x^5-76\,x^3-382\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+413\left(x^8+6\,x^6+13\right.}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(40*x^9 + 225*x^7 + 231*x^5 - 76*x^3 - 382*sqrt(2)*(x^8 + 6*x^6 + 13*x^
4 + 12*x^2 + 4)*arctan(1/2*sqrt(2)*x) + 413*(x^8 + 6*x^6 + 13*x^4 + 12*x^2
+ 4)*arctan(x) - 124*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

giac [A] time = 0.32, size = 53, normalized size = 0.71

$$-\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5x-\frac{15x^7+289x^5+556x^3+284x}{8\left(x^4+3x^2+2\right)^2}+\frac{413}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^4 + 3*x^2 + 2)^2 + 413/8*arctan(x)
```

maple [A] time = 0.01, size = 56, normalized size = 0.75

$$5x+\frac{413\arctan(x)}{8}-\frac{191\sqrt{2}\arctan\left(\frac{\sqrt{2}x}{2}\right)}{4}+\frac{-\frac{19}{8}x^3-\frac{21}{8}x}{\left(x^2+1\right)^2}-\frac{16\left(-\frac{1}{32}x^3+\frac{25}{16}x\right)}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] 5*x+(-19/8*x^3-21/8*x)/(x^2+1)^2+413/8*arctan(x)-16*(-1/32*x^3+25/16*x)/(x^
2+2)^2-191/4*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.85, size = 63, normalized size = 0.84

$$-\frac{191}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}x\right)+5x-\frac{15x^7+289x^5+556x^3+284x}{8\left(x^8+6x^6+13x^4+12x^2+4\right)}+\frac{413}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -191/4*sqrt(2)*arctan(1/2*sqrt(2)*x) + 5*x - 1/8*(15*x^7 + 289*x^5 + 556*x^
3 + 284*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 413/8*arctan(x)
```

mupad [B] time = 0.93, size = 63, normalized size = 0.84

$$5x+\frac{413\,\mathrm{atan}(x)}{8}-\frac{191\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}-\frac{\frac{15x^7}{8}+\frac{289x^5}{8}+\frac{139x^3}{2}+\frac{71x}{2}}{x^8+6x^6+13x^4+12x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] 5*x + (413*atan(x))/8 - (191*2^(1/2)*atan((2^(1/2)*x)/2))/4 - ((71*x)/2 + (
139*x^3)/2 + (289*x^5)/8 + (15*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

sympy [A] time = 0.26, size = 70, normalized size = 0.93

$$5x+\frac{-15x^7-289x^5-556x^3-284x}{8x^8+48x^6+104x^4+96x^2+32}+\frac{413\,\mathrm{atan}\,(x)}{8}-\frac{191\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] 5*x + (-15*x**7 - 289*x**5 - 556*x**3 - 284*x)/(8*x**8 + 48*x**6 + 104*x**4
 + 96*x**2 + 32) + 413*atan(x)/8 - 191*sqrt(2)*atan(sqrt(2)*x/2)/4
```

3.94 $\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=72

$$-\frac{x\left(51x^2+50\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(125x^2+254\right)}{8\left(x^4+3x^2+2\right)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] -1/4*x*(51*x^2+50)/(x^4+3*x^2+2)^2+1/8*x*(125*x^2+254)/(x^4+3*x^2+2)-369/8*
arctan(x)+267/8*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1678, 1166, 203}

$$-\frac{x\left(51x^2+50\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(125x^2+254\right)}{8\left(x^4+3x^2+2\right)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -(x*(50 + 51*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(254 + 125*x^2))/(8*(2 + 3*
x^2 + x^4)) - (369*ArcTan[x])/8 + (267*ArcTan[x/Sqrt[2]])/(4*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
```

```
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx &= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-100+294x^2+96x^4-40x^6}{\left(2+3x^2+x^4\right)^2}\,dx\\
&= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(254+125x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{-816+660x^2}{2+3x^2+x^4}\,dx\\
&= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(254+125x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{369}{8}\int \frac{1}{1+x^2}\,dx+\frac{267}{4}\int \frac{1}{2+x^2}\,dx\\
&= -\frac{x\left(50+51x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(254+125x^2\right)}{8\left(2+3x^2+x^4\right)}-\frac{369}{8}\tan^{-1}(x)+\frac{267\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{4\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 0.76

$$\frac{1}{8}\left(\frac{x\left(125x^6+629x^4+910x^2+408\right)}{\left(x^4+3x^2+2\right)^2}-369\tan^{-1}(x)+267\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((x*(408 + 910*x^2 + 629*x^4 + 125*x^6))/(2 + 3*x^2 + x^4)^2 - 369*ArcTan[x
] + 267*Sqrt[2]*ArcTan[x/Sqrt[2]])/8
```

fricas [A] time = 0.85, size = 99, normalized size = 1.38

$$\frac{125\,x^7+629\,x^5+910\,x^3+267\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-369\left(x^8+6\,x^6+13\,x^4+12\,x\right.}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 267*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 12*x
^2 + 4)*arctan(1/2*sqrt(2)*x) - 369*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)*arc
tan(x) + 408*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

giac [A] time = 0.34, size = 50, normalized size = 0.69

$$\frac{267}{8}\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+\frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\left(x^4+3\,x^2+2\right)^2}-\frac{369}{8}\,\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^4 + 3*x^2 + 2)^2 - 369/8*arctan(x)
```

maple [A] time = 0.01, size = 54, normalized size = 0.75

$$-\frac{369\arctan(x)}{8}+\frac{267\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{8}-\frac{-\frac{23}{8}x^3-\frac{25}{8}x}{\left(x^2+1\right)^2}+\frac{\frac{51}{4}x^3+\frac{77}{2}x}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] -(-23/8*x^3-25/8*x)/(x^2+1)^2-369/8*arctan(x)+2*(51/8*x^3+77/4*x)/(x^2+2)^2
+267/8*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.53, size = 60, normalized size = 0.83

$$\frac{267}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{125\,x^7+629\,x^5+910\,x^3+408\,x}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}-\frac{369}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 267/8*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/8*(125*x^7 + 629*x^5 + 910*x^3 + 40
8*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 369/8*arctan(x)
```

mupad [B] time = 0.93, size = 59, normalized size = 0.82

$$\frac{267\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{8}-\frac{369\,\mathrm{atan}(x)}{8}+\frac{\frac{125\,x^7}{8}+\frac{629\,x^5}{8}+\frac{455\,x^3}{4}+51\,x}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (267*2^(1/2)*atan((2^(1/2)*x)/2))/8 - (369*atan(x))/8 + (51*x + (455*x^3)/4
 + (629*x^5)/8 + (125*x^7)/8)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

sympy [A] time = 0.26, size = 65, normalized size = 0.90

$$\frac{125x^7+629x^5+910x^3+408x}{8x^8+48x^6+104x^4+96x^2+32}-\frac{369\,\mathrm{atan}\,(x)}{8}+\frac{267\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (125*x**7 + 629*x**5 + 910*x**3 + 408*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96*
x**2 + 32) - 369*atan(x)/8 + 267*sqrt(2)*atan(sqrt(2)*x/2)/8
```

3.95 $\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=72

$$\frac{x\left(25x^2+24\right)}{4\left(x^4+3x^2+2\right)^2}-\frac{x\left(130x^2+211\right)}{8\left(x^4+3x^2+2\right)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/4*x*(25*x^2+24)/(x^4+3*x^2+2)^2-1/8*x*(130*x^2+211)/(x^4+3*x^2+2)+317/8*a
rctan(x)-447/16*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1668, 1678, 1166, 203}

$$\frac{x\left(25x^2+24\right)}{4\left(x^4+3x^2+2\right)^2}-\frac{x\left(130x^2+211\right)}{8\left(x^4+3x^2+2\right)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] (x*(24 + 25*x^2))/(4*(2 + 3*x^2 + x^4)^2) - (x*(211 + 130*x^2))/(8*(2 + 3*x
^2 + x^4)) + (317*ArcTan[x])/8 - (447*ArcTan[x/Sqrt[2]])/(8*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
```

```
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(2+3x^2+x^4\right)^3}\,dx &= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{48-154x^2-40x^4}{\left(2+3x^2+x^4\right)^2}\,dx \\
&= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(211+130x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{748-520x^2}{2+3x^2+x^4}\,dx \\
&= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(211+130x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{317}{8}\int \frac{1}{1+x^2}\,dx-\frac{447}{8}\int \frac{1}{2+x^2}\,dx \\
&= \frac{x\left(24+25x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{x\left(211+130x^2\right)}{8\left(2+3x^2+x^4\right)}+\frac{317}{8}\tan^{-1}(x)-\frac{447\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.78

$$\frac{1}{16}\left(-\frac{2x\left(130x^6+601x^4+843x^2+374\right)}{\left(x^4+3x^2+2\right)^2}+634\tan^{-1}(x)-447\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((-2*x*(374 + 843*x^2 + 601*x^4 + 130*x^6))/(2 + 3*x^2 + x^4)^2 + 634*ArcTa
n[x] - 447*Sqrt[2]*ArcTan[x/Sqrt[2]])/16
```

fricas [A] time = 0.69, size = 99, normalized size = 1.38

$$-\frac{260\,x^7+1202\,x^5+1686\,x^3+447\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-634\left(x^8+6\,x^6+13\,x\right.}{16\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(260*x^7 + 1202*x^5 + 1686*x^3 + 447*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 +
12*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 634*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
*arctan(x) + 748*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

giac [A] time = 0.38, size = 50, normalized size = 0.69

$$-\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\left(x^4+3\,x^2+2\right)^2}+\frac{317}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
374*x)/(x^4 + 3*x^2 + 2)^2 + 317/8*arctan(x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.74

$$\frac{317\arctan(x)}{8}-\frac{447\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{16}+\frac{-\frac{27}{8}x^3-\frac{29}{8}x}{\left(x^2+1\right)^2}-\frac{\frac{103}{8}x^3+\frac{129}{4}x}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] (-27/8*x^3-29/8*x)/(x^2+1)^2+317/8*arctan(x)-(103/8*x^3+129/4*x)/(x^2+2)^2-
447/16*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.74, size = 60, normalized size = 0.83

$$-\frac{447}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{130\,x^7+601\,x^5+843\,x^3+374\,x}{8\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}+\frac{317}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -447/16*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/8*(130*x^7 + 601*x^5 + 843*x^3 +
374*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) + 317/8*arctan(x)
```

mupad [B] time = 0.07, size = 60, normalized size = 0.83

$$\frac{317\,\mathrm{atan}(x)}{8}-\frac{447\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{16}-\frac{\frac{65\,x^7}{4}+\frac{601\,x^5}{8}+\frac{843\,x^3}{8}+\frac{187\,x}{4}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (317*atan(x))/8 - (447*2^(1/2)*atan((2^(1/2)*x)/2))/16 - ((187*x)/4 + (843*
x^3)/8 + (601*x^5)/8 + (65*x^7)/4)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

sympy [A] time = 0.25, size = 66, normalized size = 0.92

$$\frac{-130x^7-601x^5-843x^3-374x}{8x^8+48x^6+104x^4+96x^2+32}+\frac{317\,\mathrm{atan}\,(x)}{8}-\frac{447\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (-130*x**7 - 601*x**5 - 843*x**3 - 374*x)/(8*x**8 + 48*x**6 + 104*x**4 + 96
*x**2 + 32) + 317*atan(x)/8 - 447*sqrt(2)*atan(sqrt(2)*x/2)/16
```

3.96 $\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=72

$$-\frac{x\left(12x^2+11\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(217x^2+335\right)}{16\left(x^4+3x^2+2\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

```
[Out] -1/4*x*(12*x^2+11)/(x^4+3*x^2+2)^2+1/16*x*(217*x^2+335)/(x^4+3*x^2+2)-257/8
*arctan(x)+731/32*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1678, 1178, 1166, 203}

$$-\frac{x\left(12x^2+11\right)}{4\left(x^4+3x^2+2\right)^2}+\frac{x\left(217x^2+335\right)}{16\left(x^4+3x^2+2\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] -(x*(11 + 12*x^2))/(4*(2 + 3*x^2 + x^4)^2) + (x*(335 + 217*x^2))/(16*(2 + 3
*x^2 + x^4)) - (257*ArcTan[x])/8 + (731*ArcTan[x/Sqrt[2]])/(16*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
```

```
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{\left(2+3x^2+x^4\right)^3}\,dx &= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}-\frac{1}{8}\int \frac{-38+80x^2}{\left(2+3x^2+x^4\right)^2}\,dx \\
&= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(335+217x^2\right)}{16\left(2+3x^2+x^4\right)}+\frac{1}{32}\int \frac{-594+434x^2}{2+3x^2+x^4}\,dx \\
&= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(335+217x^2\right)}{16\left(2+3x^2+x^4\right)}-\frac{257}{8}\int \frac{1}{1+x^2}\,dx+\frac{731}{16}\int \frac{1}{2+x^2}\,dx \\
&= -\frac{x\left(11+12x^2\right)}{4\left(2+3x^2+x^4\right)^2}+\frac{x\left(335+217x^2\right)}{16\left(2+3x^2+x^4\right)}-\frac{257}{8}\tan^{-1}(x)+\frac{731\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{16\sqrt{2}}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 56, normalized size = 0.78

$$
\frac{1}{32}\left(\frac{2x\left(217x^6+986x^4+1391x^2+626\right)}{\left(x^4+3x^2+2\right)^2}-1028\tan^{-1}(x)+731\sqrt{2}\,\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(2 + 3*x^2 + x^4)^3,x]
```

```
[Out] ((2*x*(626 + 1391*x^2 + 986*x^4 + 217*x^6))/(2 + 3*x^2 + x^4)^2 - 1028*ArcT
an[x] + 731*Sqrt[2]*ArcTan[x/Sqrt[2]])/32
```

fricas [A] time = 0.90, size = 99, normalized size = 1.38

$$
\frac{434\,x^7+1972\,x^5+2782\,x^3+731\,\sqrt{2}\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)-1028\left(x^8+6\,x^6+13\,x^4+\right.}{32\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(434*x^7 + 1972*x^5 + 2782*x^3 + 731*sqrt(2)*(x^8 + 6*x^6 + 13*x^4 + 1
2*x^2 + 4)*arctan(1/2*sqrt(2)*x) - 1028*(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
*arctan(x) + 1252*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4)
```

giac [A] time = 0.33, size = 50, normalized size = 0.69

$$
\frac{731}{32}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right)+\frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\left(x^4+3\,x^2+2\right)^2}-\frac{257}{8}\,\arctan(x)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^4 + 3*x^2 + 2)^2 - 257/8*arctan(x)
```

maple [A] time = 0.01, size = 53, normalized size = 0.74

$$-\frac{257\arctan(x)}{8}+\frac{731\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{-\frac{31}{8}x^3-\frac{33}{8}x}{\left(x^2+1\right)^2}+\frac{\frac{155}{16}x^3+\frac{181}{8}x}{\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x)
```

```
[Out] -(-31/8*x^3-33/8*x)/(x^2+1)^2-257/8*arctan(x)+(155/16*x^3+181/8*x)/(x^2+2)^
2+731/32*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.67, size = 60, normalized size = 0.83

$$\frac{731}{32}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{217\,x^7+986\,x^5+1391\,x^3+626\,x}{16\left(x^8+6\,x^6+13\,x^4+12\,x^2+4\right)}-\frac{257}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 731/32*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/16*(217*x^7 + 986*x^5 + 1391*x^3 +
 626*x)/(x^8 + 6*x^6 + 13*x^4 + 12*x^2 + 4) - 257/8*arctan(x)
```

mupad [B] time = 0.07, size = 59, normalized size = 0.82

$$\frac{731\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}-\frac{257\,\mathrm{atan}(x)}{8}+\frac{\frac{217\,x^7}{16}+\frac{493\,x^5}{8}+\frac{1391\,x^3}{16}+\frac{313\,x}{8}}{x^8+6\,x^6+13\,x^4+12\,x^2+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(3*x^2 + x^4 + 2)^3,x)
```

```
[Out] (731*2^(1/2)*atan((2^(1/2)*x)/2))/32 - (257*atan(x))/8 + ((313*x)/8 + (1391
*x^3)/16 + (493*x^5)/8 + (217*x^7)/16)/(12*x^2 + 13*x^4 + 6*x^6 + x^8 + 4)
```

sympy [A] time = 0.25, size = 65, normalized size = 0.90

$$\frac{217x^7+986x^5+1391x^3+626x}{16x^8+96x^6+208x^4+192x^2+64}-\frac{257\,\mathrm{atan}\,(x)}{8}+\frac{731\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+3*x**2+2)**3,x)
```

```
[Out] (217*x**7 + 986*x**5 + 1391*x**3 + 626*x)/(16*x**8 + 96*x**6 + 208*x**4 + 1
92*x**2 + 64) - 257*atan(x)/8 + 731*sqrt(2)*atan(sqrt(2)*x/2)/32
```

3.97 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=79

$$\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}-\frac{x\left(347x^2+547\right)}{32\left(x^4+3x^2+2\right)}-\frac{1}{2x}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

```
[Out] -1/2/x+1/8*x*(11*x^2+9)/(x^4+3*x^2+2)^2-1/32*x*(347*x^2+547)/(x^4+3*x^2+2)+
189/8*arctan(x)-1119/64*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$\frac{x\left(11x^2+9\right)}{8\left(x^4+3x^2+2\right)^2}-\frac{x\left(347x^2+547\right)}{32\left(x^4+3x^2+2\right)}-\frac{1}{2x}+\frac{189}{8}\tan^{-1}(x)-\frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(2*x) + (x*(9 + 11*x^2))/(8*(2 + 3*x^2 + x^4)^2) - (x*(547 + 347*x^2))/(
32*(2 + 3*x^2 + x^4)) + (189*ArcTan[x])/8 - (1119*ArcTan[x/Sqrt[2]])/(32*Sq
rt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(2+3x^2+x^4\right)^3}\,dx = \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+29x^2-55x^4}{x^2\left(2+3x^2+x^4\right)^2}\,dx$$

$$= \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2} - \frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32+441x^2-347x^4}{x^2\left(2+3x^2+x^4\right)}\,dx$$

$$= \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2} - \frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^2}+\frac{756}{1+x^2}-\frac{1119}{2+x^2}\right)dx$$

$$= -\frac{1}{2x} + \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2} - \frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)} + \frac{189}{8}\int \frac{1}{1+x^2}\,dx - \frac{1119}{32}\int \frac{1}{2+x}$$

$$= -\frac{1}{2x} + \frac{x\left(9+11x^2\right)}{8\left(2+3x^2+x^4\right)^2} - \frac{x\left(547+347x^2\right)}{32\left(2+3x^2+x^4\right)} + \frac{189}{8}\tan^{-1}(x) - \frac{1119\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{32\sqrt{2}}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.80

$$\frac{1}{64}\left(-\frac{2\left(363x^8+1684x^6+2499x^4+1250x^2+64\right)}{x\left(x^4+3x^2+2\right)^2}+1512\tan^{-1}(x)-1119\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(64 + 1250*x^2 + 2499*x^4 + 1684*x^6 + 363*x^8))/(x*(2 + 3*x^2 + x^4)^
2) + 1512*ArcTan[x] - 1119*Sqrt[2]*ArcTan[x/Sqrt[2]])/64
```

fricas [A] time = 0.86, size = 108, normalized size = 1.37

$$-\frac{726\,x^8+3368\,x^6+4998\,x^4+1119\,\sqrt{2}\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+2500\,x^2-1512\,(x}{64\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/64*(726*x^8 + 3368*x^6 + 4998*x^4 + 1119*sqrt(2)*(x^9 + 6*x^7 + 13*x^5 +
 12*x^3 + 4*x)*arctan(1/2*sqrt(2)*x) + 2500*x^2 - 1512*(x^9 + 6*x^7 + 13*x^
5 + 12*x^3 + 4*x)*arctan(x) + 128)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x)
```

giac [A] time = 0.34, size = 55, normalized size = 0.70

$$-\frac{1119}{64}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{347\,x^7+1588\,x^5+2291\,x^3+1058\,x}{32\left(x^4+3\,x^2+2\right)^2}-\frac{1}{2\,x}+\frac{189}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(347*x^7 + 1588*x^5 + 2291*x^
3 + 1058*x)/(x^4 + 3*x^2 + 2)^2 - 1/2/x + 189/8*arctan(x)
```

maple [A] time = 0.01, size = 58, normalized size = 0.73

$$\frac{189\arctan(x)}{8}-\frac{1119\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{64}-\frac{1}{2x}+\frac{-\frac{35}{8}x^3-\frac{37}{8}x}{\left(x^2+1\right)^2}-\frac{\frac{207}{16}x^3+\frac{233}{8}x}{2\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/2/x+(-35/8*x^3-37/8*x)/(x^2+1)^2+189/8*arctan(x)-1/2*(207/16*x^3+233/8*x
)/(x^2+2)^2-1119/64*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.80, size = 65, normalized size = 0.82

$$-\frac{1119}{64}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{363\,x^8+1684\,x^6+2499\,x^4+1250\,x^2+64}{32\left(x^9+6\,x^7+13\,x^5+12\,x^3+4\,x\right)}+\frac{189}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -1119/64*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/32*(363*x^8 + 1684*x^6 + 2499*x^
4 + 1250*x^2 + 64)/(x^9 + 6*x^7 + 13*x^5 + 12*x^3 + 4*x) + 189/8*arctan(x)
```

mupad [B] time = 0.92, size = 65, normalized size = 0.82

$$\frac{189\,\mathrm{atan}(x)}{8}-\frac{1119\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{64}-\frac{\frac{363\,x^8}{32}+\frac{421\,x^6}{8}+\frac{2499\,x^4}{32}+\frac{625\,x^2}{16}+2}{x^9+6\,x^7+13\,x^5+12\,x^3+4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] (189*atan(x))/8 - (1119*2^(1/2)*atan((2^(1/2)*x)/2))/64 - ((625*x^2)/16 + (
2499*x^4)/32 + (421*x^6)/8 + (363*x^8)/32 + 2)/(4*x + 12*x^3 + 13*x^5 + 6*x
^7 + x^9)
```

sympy [A] time = 0.28, size = 71, normalized size = 0.90

$$\frac{-363x^8-1684x^6-2499x^4-1250x^2-64}{32x^9+192x^7+416x^5+384x^3+128x}+\frac{189\,\mathrm{atan}\,(x)}{8}-\frac{1119\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+3*x**2+2)**3,x)
```

```
[Out] (-363*x**8 - 1684*x**6 - 2499*x**4 - 1250*x**2 - 64)/(32*x**9 + 192*x**7 +
416*x**5 + 384*x**3 + 128*x) + 189*atan(x)/8 - 1119*sqrt(2)*atan(sqrt(2)*x/
2)/64
```

3.98 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=86

$$-\frac{1}{6x^3}-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}+\frac{x\left(571x^2+951\right)}{64\left(x^4+3x^2+2\right)}+\frac{17}{8x}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

```
[Out] -1/6/x^3+17/8/x-1/16*x*(9*x^2+5)/(x^4+3*x^2+2)^2+1/64*x*(571*x^2+951)/(x^4+
3*x^2+2)-113/8*arctan(x)+1611/128*arctan(1/2*x*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1669, 1664, 203}

$$-\frac{x\left(9x^2+5\right)}{16\left(x^4+3x^2+2\right)^2}+\frac{x\left(571x^2+951\right)}{64\left(x^4+3x^2+2\right)}-\frac{1}{6x^3}+\frac{17}{8x}-\frac{113}{8}\tan^{-1}(x)+\frac{1611\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{64\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(6*x^3) + 17/(8*x) - (x*(5 + 9*x^2))/(16*(2 + 3*x^2 + x^4)^2) + (x*(951
+ 571*x^2))/(64*(2 + 3*x^2 + x^4)) - (113*ArcTan[x])/8 + (1611*ArcTan[x/Sqr
t[2]])/(64*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(2+3x^2+x^4\right)^3}\,dx = -\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+20x^2-\frac{73x^4}{2}+\frac{45x^6}{2}}{x^4\left(2+3x^2+x^4\right)^2}\,dx$$

$$= -\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32-88x^2-\frac{573x^4}{2}+\frac{571x^6}{2}}{x^4\left(2+3x^2+x^4\right)}\,dx$$

$$= -\frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^4} - \frac{68}{x^2} - \frac{452}{1+x^2} + \frac{1611}{2\left(2+x^2\right)}\right) d$$

$$= -\frac{1}{6x^3} + \frac{17}{8x} - \frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} - \frac{113}{8}\int \frac{1}{1+x^2}\,dx + \frac{1611}{64}\int$$

$$= -\frac{1}{6x^3} + \frac{17}{8x} - \frac{x\left(5+9x^2\right)}{16\left(2+3x^2+x^4\right)^2} + \frac{x\left(951+571x^2\right)}{64\left(2+3x^2+x^4\right)} - \frac{113}{8}\tan^{-1}(x) + \frac{1611\tan^{-1}\left(\right.}{64\sqrt{2}}$$

Mathematica [A] time = 0.06, size = 78, normalized size = 0.91

$$\frac{1}{384}\left(-\frac{64}{x^3} - \frac{24x\left(9x^2+5\right)}{\left(x^4+3x^2+2\right)^2} + \frac{6x\left(571x^2+951\right)}{x^4+3x^2+2} + \frac{816}{x} - 5424\tan^{-1}(x) + 4833\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] (-64/x^3 + 816/x - (24*x*(5 + 9*x^2))/(2 + 3*x^2 + x^4)^2 + (6*x*(951 + 571
*x^2))/(2 + 3*x^2 + x^4) - 5424*ArcTan[x] + 4833*Sqrt[2]*ArcTan[x/Sqrt[2]])
/384
```

fricas [A] time = 1.01, size = 119, normalized size = 1.38

$$\frac{4242\,x^{10} + 20816\,x^8 + 33978\,x^6 + 20252\,x^4 + 4833\,\sqrt{2}\left(x^{11} + 6\,x^9 + 13\,x^7 + 12\,x^5 + 4\,x^3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\,x\right) + 249}{384\left(x^{11} + 6\,x^9 + 13\,x^7 + 12\,x^5 + 4\,x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(4242*x^10 + 20816*x^8 + 33978*x^6 + 20252*x^4 + 4833*sqrt(2)*(x^11 +
 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(1/2*sqrt(2)*x) + 2496*x^2 - 5424*(
x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^3)*arctan(x) - 256)/(x^11 + 6*x^9 + 13
*x^7 + 12*x^5 + 4*x^3)
```

giac [A] time = 0.36, size = 62, normalized size = 0.72

$$\frac{1611}{128}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right) + \frac{571\,x^7 + 2664\,x^5 + 3959\,x^3 + 1882\,x}{64\left(x^4 + 3\,x^2 + 2\right)^2} + \frac{51\,x^2 - 4}{24\,x^3} - \frac{113}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/64*(571*x^7 + 2664*x^5 + 3959*x^
3 + 1882*x)/(x^4 + 3*x^2 + 2)^2 + 1/24*(51*x^2 - 4)/x^3 - 113/8*arctan(x)
```

maple [A] time = 0.02, size = 64, normalized size = 0.74

$$-\frac{113\arctan(x)}{8}+\frac{1611\sqrt{2}\ \arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{128}+\frac{17}{8x}-\frac{1}{6x^3}-\frac{-\frac{39}{8}x^3-\frac{41}{8}x}{\left(x^2+1\right)^2}+\frac{\frac{259}{8}x^3+\frac{285}{4}x}{8\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/6/x^3+17/8/x-(-39/8*x^3-41/8*x)/(x^2+1)^2-113/8*arctan(x)+1/8*(259/8*x^3
+285/4*x)/(x^2+2)^2+1611/128*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.53, size = 72, normalized size = 0.84

$$\frac{1611}{128}\sqrt{2}\ \arctan\left(\frac{1}{2}\sqrt{2}\,x\right)+\frac{2121\,x^{10}+10408\,x^8+16989\,x^6+10126\,x^4+1248\,x^2-128}{192\left(x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3\right)}-\frac{113}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] 1611/128*sqrt(2)*arctan(1/2*sqrt(2)*x) + 1/192*(2121*x^10 + 10408*x^8 + 169
89*x^6 + 10126*x^4 + 1248*x^2 - 128)/(x^11 + 6*x^9 + 13*x^7 + 12*x^5 + 4*x^
3) - 113/8*arctan(x)
```

mupad [B] time = 0.92, size = 71, normalized size = 0.83

$$\frac{\frac{707\,x^{10}}{64}+\frac{1301\,x^8}{24}+\frac{5663\,x^6}{64}+\frac{5063\,x^4}{96}+\frac{13\,x^2}{2}-\frac{2}{3}}{x^{11}+6\,x^9+13\,x^7+12\,x^5+4\,x^3}-\frac{113\,\mathrm{atan}(x)}{8}+\frac{1611\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] ((13*x^2)/2 + (5063*x^4)/96 + (5663*x^6)/64 + (1301*x^8)/24 + (707*x^10)/64
 - 2/3)/(4*x^3 + 12*x^5 + 13*x^7 + 6*x^9 + x^11) - (113*atan(x))/8 + (1611*
2^(1/2)*atan((2^(1/2)*x)/2))/128
```

sympy [A] time = 0.29, size = 76, normalized size = 0.88

$$-\frac{113\,\mathrm{atan}\,(x)}{8}+\frac{1611\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}x}{2}\right)}{128}+\frac{2121x^{10}+10408x^8+16989x^6+10126x^4+1248x^2-128}{192x^{11}+1152x^9+2496x^7+2304x^5+768x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+3*x**2+2)**3,x)
```

```
[Out] -113*atan(x)/8 + 1611*sqrt(2)*atan(sqrt(2)*x/2)/128 + (2121*x**10 + 10408*x
**8 + 16989*x**6 + 10126*x**4 + 1248*x**2 - 128)/(192*x**11 + 1152*x**9 + 2
496*x**7 + 2304*x**5 + 768*x**3)
```

3.99 $\int \frac{4+x^2+3x^4+5x^6}{x^6(2+3x^2+x^4)^3}\,dx$

Optimal. Leaf size=93

$$-\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{x(3-5x^2)}{32(x^4+3x^2+2)^2}-\frac{x(999x^2+1771)}{128(x^4+3x^2+2)}-\frac{93}{16x}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

```
[Out] -1/10/x^5+17/24/x^3-93/16/x-1/32*x*(-5*x^2+3)/(x^4+3*x^2+2)^2-1/128*x*(999*
x^2+1771)/(x^4+3*x^2+2)+29/8*arctan(x)-2207/256*arctan(1/2*x*2^(1/2))*2^(1/
2)
```

Rubi [A] time = 0.13, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1669, 1664, 203}

$$-\frac{x(3-5x^2)}{32(x^4+3x^2+2)^2}-\frac{x(999x^2+1771)}{128(x^4+3x^2+2)}+\frac{17}{24x^3}-\frac{1}{10x^5}-\frac{93}{16x}+\frac{29}{8}\tan^{-1}(x)-\frac{2207\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] -1/(10*x^5) + 17/(24*x^3) - 93/(16*x) - (x*(3 - 5*x^2))/(32*(2 + 3*x^2 + x^
4)^2) - (x*(1771 + 999*x^2))/(128*(2 + 3*x^2 + x^4)) + (29*ArcTan[x])/8 - (
2207*ArcTan[x/Sqrt[2]])/(128*Sqrt[2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(2+3x^2+x^4\right)^3}dx = -\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2} - \frac{1}{8}\int \frac{-16+20x^2-34x^4+\frac{81x^6}{4}-\frac{25x^8}{4}}{x^6\left(2+3x^2+x^4\right)^2}dx$$

$$= -\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2} - \frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \frac{32-88x^2+184x^4+\frac{681x^6}{4}-\frac{9}{}}{x^6\left(2+3x^2+x^4\right)}$$

$$= -\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2} - \frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)} + \frac{1}{32}\int \left(\frac{16}{x^6}-\frac{68}{x^4}+\frac{186}{x^2}+\frac{116}{1+x^2}-\frac{}{4}\right.$$

$$= -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2}-\frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)}+\frac{29}{8}\int \frac{1}{1+x^2}d$$

$$= -\frac{1}{10x^5}+\frac{17}{24x^3}-\frac{93}{16x}-\frac{x\left(3-5x^2\right)}{32\left(2+3x^2+x^4\right)^2}-\frac{x\left(1771+999x^2\right)}{128\left(2+3x^2+x^4\right)}+\frac{29}{8}\tan^{-1}(x)-$$

Mathematica [**A**] time = 0.08, size = 73, normalized size = 0.78

$$\frac{-\frac{2\left(26145x^{12}+137120x^{10}+246477x^8+170702x^6+30816x^4-3136x^2+768\right)}{x^5\left(x^4+3x^2+2\right)^2}+13920\tan^{-1}(x)-33105\sqrt{2}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)}{3840}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(2 + 3*x^2 + x^4)^3),x]
```

```
[Out] ((-2*(768 - 3136*x^2 + 30816*x^4 + 170702*x^6 + 246477*x^8 + 137120*x^10 +
26145*x^12))/(x^5*(2 + 3*x^2 + x^4)^2) + 13920*ArcTan[x] - 33105*Sqrt[2]*Ar
cTan[x/Sqrt[2]])/3840
```

fricas [**A**] time = 1.22, size = 124, normalized size = 1.33

$$-\frac{52290\,x^{12}+274240\,x^{10}+492954\,x^8+341404\,x^6+61632\,x^4+33105\,\sqrt{2}\left(x^{13}+6\,x^{11}+13\,x^9+12\,x^7+4\,x^5\right.}{3840\left(x^{13}+6\,x^{11}+13\,x^9+12\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="fricas")
```

```
[Out] -1/3840*(52290*x^12 + 274240*x^10 + 492954*x^8 + 341404*x^6 + 61632*x^4 + 3
3105*sqrt(2)*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(1/2*sqrt(2)*x
) - 6272*x^2 - 13920*(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)*arctan(x) +
1536)/(x^13 + 6*x^11 + 13*x^9 + 12*x^7 + 4*x^5)
```

giac [**A**] time = 0.36, size = 67, normalized size = 0.72

$$-\frac{2207}{256}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{999\,x^7+4768\,x^5+7291\,x^3+3554\,x}{128\left(x^4+3\,x^2+2\right)^2}-\frac{1395\,x^4-170\,x^2+24}{240\,x^5}+\frac{29}{8}\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="giac")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/128*(999*x^7 + 4768*x^5 + 7291*
x^3 + 3554*x)/(x^4 + 3*x^2 + 2)^2 - 1/240*(1395*x^4 - 170*x^2 + 24)/x^5 + 2
9/8*arctan(x)
```

maple [A] time = 0.02, size = 68, normalized size = 0.73

$$\frac{29\arctan(x)}{8}-\frac{2207\sqrt{2}\,\arctan\left(\frac{\sqrt{2}\,x}{2}\right)}{256}-\frac{93}{16x}+\frac{17}{24x^3}-\frac{1}{10x^5}+\frac{-\frac{43}{8}x^3-\frac{45}{8}x}{\left(x^2+1\right)^2}-\frac{\frac{311}{8}x^3+\frac{337}{4}x}{16\left(x^2+2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x)
```

```
[Out] -1/10/x^5+17/24/x^3-93/16/x+(-43/8*x^3-45/8*x)/(x^2+1)^2+29/8*arctan(x)-1/1
6*(311/8*x^3+337/4*x)/(x^2+2)^2-2207/256*2^(1/2)*arctan(1/2*2^(1/2)*x)
```

maxima [A] time = 1.74, size = 77, normalized size = 0.83

$$-\frac{2207}{256}\sqrt{2}\,\arctan\left(\frac{1}{2}\sqrt{2}\,x\right)-\frac{26145\,x^{12}+137120\,x^{10}+246477\,x^{8}+170702\,x^{6}+30816\,x^{4}-3136\,x^{2}+768}{1920\left(x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}\right)}+\frac{29}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+3*x^2+2)^3,x, algorithm="maxima")
```

```
[Out] -2207/256*sqrt(2)*arctan(1/2*sqrt(2)*x) - 1/1920*(26145*x^12 + 137120*x^10
+ 246477*x^8 + 170702*x^6 + 30816*x^4 - 3136*x^2 + 768)/(x^13 + 6*x^11 + 13
*x^9 + 12*x^7 + 4*x^5) + 29/8*arctan(x)
```

mupad [B] time = 0.93, size = 77, normalized size = 0.83

$$\frac{29\,\mathrm{atan}(x)}{8}-\frac{2207\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{256}-\frac{\frac{1743\,x^{12}}{128}+\frac{857\,x^{10}}{12}+\frac{82159\,x^{8}}{640}+\frac{85351\,x^{6}}{960}+\frac{321\,x^{4}}{20}-\frac{49\,x^{2}}{30}+\frac{2}{5}}{x^{13}+6\,x^{11}+13\,x^{9}+12\,x^{7}+4\,x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(3*x^2 + x^4 + 2)^3),x)
```

```
[Out] (29*atan(x))/8 - (2207*2^(1/2)*atan((2^(1/2)*x)/2))/256 - ((321*x^4)/20 - (
49*x^2)/30 + (85351*x^6)/960 + (82159*x^8)/640 + (857*x^10)/12 + (1743*x^12
)/128 + 2/5)/(4*x^5 + 12*x^7 + 13*x^9 + 6*x^11 + x^13)
```

sympy [A] time = 0.31, size = 82, normalized size = 0.88

$$\frac{29\,\mathrm{atan}\,(x)}{8}-\frac{2207\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x}{2}\right)}{256}+\frac{-26145x^{12}-137120x^{10}-246477x^{8}-170702x^{6}-30816x^{4}+3136x^{2}-768}{1920x^{13}+11520x^{11}+24960x^{9}+23040x^{7}+7680x^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+3*x**2+2)**3,x)
```

```
[Out] 29*atan(x)/8 - 2207*sqrt(2)*atan(sqrt(2)*x/2)/256 + (-26145*x**12 - 137120*
x**10 - 246477*x**8 - 170702*x**6 - 30816*x**4 + 3136*x**2 - 768)/(1920*x**
13 + 11520*x**11 + 24960*x**9 + 23040*x**7 + 7680*x**5)
```

3.100 $\int \frac{x^9\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=86

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{201\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{25\left(7x^2+15\right)}{8\left(x^4+2x^2+3\right)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)$$

```
[Out] 19*x^2+19/4*x^4-17/6*x^6+5/8*x^8-25/8*(7*x^2+15)/(x^4+2*x^2+3)-183/4*ln(x^4
+2*x^2+3)+201/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2-\frac{25\left(7x^2+15\right)}{8\left(x^4+2x^2+3\right)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)+\frac{201\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x^2 + (19*x^4)/4 - (17*x^6)/6 + (5*x^8)/8 - (25*(15 + 7*x^2))/(8*(3 + 2*
x^2 + x^4)) + (201*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) - (183*Log[3 + 2*
x^2 + x^4])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}dx, x, x^2\right)$$

$$= -\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{-150-400x+200x^2-56x^4+40x^5}{3+2x+x^2}dx, x, x^2\right)$$

$$= -\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \left(304+152x-136x^2+40x^3-\frac{6(177+244x)}{3+2x+x^2}\right)\right.$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{3}{8}\text{Subst}\left(\int \frac{177+244x}{3+2x+x^2}dx, x,\right.$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{201}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}dx,\right.$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{183}{4}\log\left(3+2x^2+x^4\right)-\frac{201}{4}\text{S}$$

$$= 19x^2+\frac{19x^4}{4}-\frac{17x^6}{6}+\frac{5x^8}{8}-\frac{25\left(15+7x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{201\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{183}{4}\log\left(3\right.$$

Mathematica [A] time = 0.04, size = 78, normalized size = 0.91

$$\frac{1}{48}\left(30x^8-136x^6+228x^4+912x^2+603\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)-\frac{150\left(7x^2+15\right)}{x^4+2x^2+3}-2196\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (912*x^2 + 228*x^4 - 136*x^6 + 30*x^8 - (150*(15 + 7*x^2))/(3 + 2*x^2 + x^4
) + 603*Sqrt[2]*ArcTan[(1 + x^2)/Sqrt[2]] - 2196*Log[3 + 2*x^2 + x^4])/48
```

fricas [A] time = 1.05, size = 95, normalized size = 1.10

$$\frac{30\,x^{12}-76\,x^{10}+46\,x^{8}+960\,x^{6}+2508\,x^{4}+603\,\sqrt{2}\left(x^{4}+2\,x^{2}+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)+1686\,x^{2}-2196\left(x^{4}\right.}{48\left(x^{4}+2\,x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(30*x^12 - 76*x^10 + 46*x^8 + 960*x^6 + 2508*x^4 + 603*sqrt(2)*(x^4 +
2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1686*x^2 - 2196*(x^4 + 2*x^2 + 3
)*log(x^4 + 2*x^2 + 3) - 2250)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.02, size = 76, normalized size = 0.88

$$\frac{5}{8}x^8-\frac{17}{6}x^6+\frac{19}{4}x^4+19x^2+\frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{366x^4+557x^2+723}{8\left(x^4+2x^2+3\right)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) + 1/8*(366*x^4 + 557*x^2 + 723)/(x^4 + 2*x^2 + 3) - 183/4*log(x^
4 + 2*x^2 + 3)
```

maple [A] time = 0.01, size = 74, normalized size = 0.86

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{201\sqrt{2}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16}-\frac{183\ln\left(x^4+2x^2+3\right)}{4}-\frac{\frac{175x^2}{4}+\frac{375}{4}}{2\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/8*x^8-17/6*x^6+19/4*x^4+19*x^2-1/2*(175/4*x^2+375/4)/(x^4+2*x^2+3)-183/4*
ln(x^4+2*x^2+3)+201/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 1.33, size = 71, normalized size = 0.83

$$\frac{5}{8}x^8-\frac{17}{6}x^6+\frac{19}{4}x^4+19x^2+\frac{201}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{25\left(7x^2+15\right)}{8\left(x^4+2x^2+3\right)}-\frac{183}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/8*x^8 - 17/6*x^6 + 19/4*x^4 + 19*x^2 + 201/16*sqrt(2)*arctan(1/2*sqrt(2)*
(x^2 + 1)) - 25/8*(7*x^2 + 15)/(x^4 + 2*x^2 + 3) - 183/4*log(x^4 + 2*x^2 +
3)
```

mupad [B] time = 0.90, size = 75, normalized size = 0.87

$$\frac{201\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}-\frac{\frac{175\,x^2}{8}+\frac{375}{8}}{x^4+2\,x^2+3}-\frac{183\,\ln\left(x^4+2\,x^2+3\right)}{4}+19\,x^2+\frac{19\,x^4}{4}-\frac{17\,x^6}{6}+\frac{5\,x^8}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (201*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - ((175*x^2)/8 + 375/8)/
(2*x^2 + x^4 + 3) - (183*log(2*x^2 + x^4 + 3))/4 + 19*x^2 + (19*x^4)/4 - (1
7*x^6)/6 + (5*x^8)/8
```

sympy [A] time = 0.18, size = 87, normalized size = 1.01

$$\frac{5x^8}{8}-\frac{17x^6}{6}+\frac{19x^4}{4}+19x^2+\frac{-175x^2-375}{8x^4+16x^2+24}-\frac{183\log\left(x^4+2x^2+3\right)}{4}+\frac{201\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**8/8 - 17*x**6/6 + 19*x**4/4 + 19*x**2 + (-175*x**2 - 375)/(8*x**4 + 16
*x**2 + 24) - 183*log(x**4 + 2*x**2 + 3)/4 + 201*sqrt(2)*atan(sqrt(2)*x**2/
2 + sqrt(2)/2)/16
```

3.101 $\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=81

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}-\frac{455\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(5x^2+3\right)}{8\left(x^4+2x^2+3\right)}+\frac{19}{2}\log\left(x^4+2x^2+3\right)$$

```
[Out] 19/2*x^2-17/4*x^4+5/6*x^6+25/8*(5*x^2+3)/(x^4+2*x^2+3)+19/2*ln(x^4+2*x^2+3)
-455/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{25\left(5x^2+3\right)}{8\left(x^4+2x^2+3\right)}+\frac{19}{2}\log\left(x^4+2x^2+3\right)-\frac{455\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (19*x^2)/2 - (17*x^4)/4 + (5*x^6)/6 + (25*(3 + 5*x^2))/(8*(3 + 2*x^2 + x^4)
) - (455*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])
/2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{-150+200x-56x^3+40x^4}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \left(152-136x+40x^2-\frac{2(303-152x)}{3+2x+x^2}\right)dx,x,x^2\right.\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{8}\text{Subst}\left(\int \frac{303-152x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{2}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)-\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{2}\log\left(3+2x^2+x^4\right)+\frac{455}{4}\text{Subst}\left(\int\right.\\
&= \frac{19x^2}{2}-\frac{17x^4}{4}+\frac{5x^6}{6}+\frac{25\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{455\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{2}\log\left(3+2x^2+\right.
\end{aligned}$$

Mathematica [A] time = 0.03, size = 73, normalized size = 0.90

$$\frac{1}{48}\left(40x^6-204x^4+456x^2-1365\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)+\frac{150\left(5x^2+3\right)}{x^4+2x^2+3}+456\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (456*x^2 - 204*x^4 + 40*x^6 + (150*(3 + 5*x^2))/(3 + 2*x^2 + x^4) - 1365*Sq
rt[2]*ArcTan[(1 + x^2)/Sqrt[2]] + 456*Log[3 + 2*x^2 + x^4])/48
```

fricas [A] time = 1.01, size = 90, normalized size = 1.11

$$\frac{40\,x^{10}-124\,x^{8}+168\,x^{6}+300\,x^{4}-1365\,\sqrt{2}\left(x^{4}+2\,x^{2}+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)+2118\,x^{2}+456\left(x^{4}+2\,x^{2}+\right.}{48\left(x^{4}+2\,x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(40*x^10 - 124*x^8 + 168*x^6 + 300*x^4 - 1365*sqrt(2)*(x^4 + 2*x^2 + 3
)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 2118*x^2 + 456*(x^4 + 2*x^2 + 3)*log(x^4
+ 2*x^2 + 3) + 450)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.17, size = 71, normalized size = 0.88

$$\frac{5}{6}\,x^6-\frac{17}{4}\,x^4+\frac{19}{2}\,x^2-\frac{455}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{76\,x^4+27\,x^2+153}{8\left(x^4+2\,x^2+3\right)}+\frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) - 1/8*(76*x^4 + 27*x^2 + 153)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 +
3)
```

maple [A] time = 0.01, size = 69, normalized size = 0.85

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}-\frac{455\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}+\frac{19\ln\left(x^4+2x^2+3\right)}{2}+\frac{\frac{125x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/6*x^6-17/4*x^4+19/2*x^2+1/2*(125/4*x^2+75/4)/(x^4+2*x^2+3)+19/2*ln(x^4+2*
x^2+3)-455/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 1.82, size = 66, normalized size = 0.81

$$\frac{5}{6}\,x^6-\frac{17}{4}\,x^4+\frac{19}{2}\,x^2-\frac{455}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(5\,x^2+3\right)}{8\left(x^4+2\,x^2+3\right)}+\frac{19}{2}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/6*x^6 - 17/4*x^4 + 19/2*x^2 - 455/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)
) + 25/8*(5*x^2 + 3)/(x^4 + 2*x^2 + 3) + 19/2*log(x^4 + 2*x^2 + 3)
```

mupad [B] time = 0.05, size = 69, normalized size = 0.85

$$\frac{19\,\ln\left(x^4+2\,x^2+3\right)}{2}+\frac{\frac{125\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3}-\frac{455\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}+\frac{19\,x^2}{2}-\frac{17\,x^4}{4}+\frac{5\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (19*log(2*x^2 + x^4 + 3))/2 + ((125*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (455
*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 + (19*x^2)/2 - (17*x^4)/4 +
(5*x^6)/6
```

sympy [A] time = 0.18, size = 80, normalized size = 0.99

$$\frac{5x^6}{6}-\frac{17x^4}{4}+\frac{19x^2}{2}+\frac{125x^2+75}{8x^4+16x^2+24}+\frac{19\log\left(x^4+2x^2+3\right)}{2}-\frac{455\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**6/6 - 17*x**4/4 + 19*x**2/2 + (125*x**2 + 75)/(8*x**4 + 16*x**2 + 24)
+ 19*log(x**4 + 2*x**2 + 3)/2 - 455*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
)/16
```

3.102 $\int \frac{x^5\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=74

$$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{203\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(3-x^2\right)}{8\left(x^4+2x^2+3\right)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)$$

```
[Out] -17/2*x^2+5/4*x^4+25/8*(-x^2+3)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+203/16*a
rctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^4}{4}-\frac{17x^2}{2}+\frac{25\left(3-x^2\right)}{8\left(x^4+2x^2+3\right)}+\frac{19}{4}\log\left(x^4+2x^2+3\right)+\frac{203\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-17*x^2)/2 + (5*x^4)/4 + (25*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (203*ArcTa
n[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2]) + (19*Log[3 + 2*x^2 + x^4])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{x^2\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\,\text{Subst}\left(\int \frac{150-56x^2+40x^3}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\,\text{Subst}\left(\int \left(-136+40x+\frac{2(279+76x)}{3+2x+x^2}\right)dx,x,x^2\right)\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{8}\,\text{Subst}\left(\int \frac{279+76x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{4}\,\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)+\frac{203}{8}\,\text{S}\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{19}{4}\log\left(3+2x^2+x^4\right)-\frac{203}{4}\,\text{Subst}\left(\int \frac{1}{-8-}\right.\\
&= -\frac{17x^2}{2}+\frac{5x^4}{4}+\frac{25\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{203\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{19}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 66, normalized size = 0.89

$$\frac{1}{16}\left(20x^4-136x^2+203\sqrt{2}\,\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)-\frac{50\left(x^2-3\right)}{x^4+2x^2+3}+76\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (-136*x^2 + 20*x^4 - (50*(-3 + x^2))/(3 + 2*x^2 + x^4) + 203*Sqrt[2]*ArcTan
[(1 + x^2)/Sqrt[2]] + 76*Log[3 + 2*x^2 + x^4])/16
```

fricas [A] time = 1.06, size = 85, normalized size = 1.15

$$\frac{20\,x^8-96\,x^6-212\,x^4+203\,\sqrt{2}\left(x^4+2\,x^2+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-458\,x^2+76\left(x^4+2\,x^2+3\right)\log\left(x^4+2.\right.}{16\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(20*x^8 - 96*x^6 - 212*x^4 + 203*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*
sqrt(2)*(x^2 + 1)) - 458*x^2 + 76*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) +
150)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.09, size = 66, normalized size = 0.89

$$\frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{38x^4+101x^2+39}{8\left(x^4+2x^2+3\right)} + \frac{19}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/8*(38
*x^4 + 101*x^2 + 39)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

maple [A] time = 0.01, size = 64, normalized size = 0.86

$$\frac{5x^4}{4} - \frac{17x^2}{2} + \frac{203\sqrt{2}\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{16} + \frac{19\ln\left(x^4+2x^2+3\right)}{4} + \frac{-\frac{25x^2}{4}+\frac{75}{4}}{2x^4+4x^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/4*x^4-17/2*x^2+1/2*(-25/4*x^2+75/4)/(x^4+2*x^2+3)+19/4*ln(x^4+2*x^2+3)+20
3/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 1.59, size = 59, normalized size = 0.80

$$\frac{5}{4}x^4 - \frac{17}{2}x^2 + \frac{203}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right) - \frac{25\left(x^2-3\right)}{8\left(x^4+2x^2+3\right)} + \frac{19}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/4*x^4 - 17/2*x^2 + 203/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x
^2 - 3)/(x^4 + 2*x^2 + 3) + 19/4*log(x^4 + 2*x^2 + 3)
```

mupad [B] time = 0.05, size = 65, normalized size = 0.88

$$\frac{19\ln\left(x^4+2x^2+3\right)}{4} - \frac{\frac{25x^2}{8}-\frac{75}{8}}{x^4+2x^2+3} + \frac{203\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17x^2}{2} + \frac{5x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (19*log(2*x^2 + x^4 + 3))/4 - ((25*x^2)/8 - 75/8)/(2*x^2 + x^4 + 3) + (203*
2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16 - (17*x^2)/2 + (5*x^4)/4
```

sympy [A] time = 0.18, size = 73, normalized size = 0.99

$$\frac{5x^4}{4} - \frac{17x^2}{2} + \frac{75-25x^2}{8x^4+16x^2+24} + \frac{19\log\left(x^4+2x^2+3\right)}{4} + \frac{203\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**4/4 - 17*x**2/2 + (75 - 25*x**2)/(8*x**4 + 16*x**2 + 24) + 19*log(x**4
 + 2*x**2 + 3)/4 + 203*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

3.103 $\int \frac{x^3(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=65

$$\frac{5x^2}{2} - \frac{17\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4}\log(x^4+2x^2+3)$$

```
[Out] 5/2*x^2-25/8*(x^2+3)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*arctan(1/2*(x
^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1660, 1657, 634, 618, 204, 628}

$$\frac{5x^2}{2} - \frac{25(x^2+3)}{8(x^4+2x^2+3)} - \frac{17}{4}\log(x^4+2x^2+3) - \frac{17\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5*x^2)/2 - (25*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (17*ArcTan[(1 + x^2)/Sqr
t[2]])/(8*Sqrt[2]) - (17*Log[3 + 2*x^2 + x^4])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{x\left(4+x+3x^2+5x^3\right)}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\
&= -\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \frac{-50-56x+40x^2}{3+2x+x^2}\,dx, x, x^2\right)\\
&= -\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\operatorname{Subst}\left(\int \left(40-\frac{34(5+4x)}{3+2x+x^2}\right)dx, x, x^2\right)\\
&= \frac{5x^2}{2}-\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{17}{8}\operatorname{Subst}\left(\int \frac{5+4x}{3+2x+x^2}\,dx, x, x^2\right)\\
&= \frac{5x^2}{2}-\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{17}{8}\operatorname{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right)-\frac{17}{4}\operatorname{Subst}\left(\int \frac{}{3}\right.\\
&= \frac{5x^2}{2}-\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{17}{4}\log\left(3+2x^2+x^4\right)+\frac{17}{4}\operatorname{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\right.\\
&= \frac{5x^2}{2}-\frac{25\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{17\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}-\frac{17}{4}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 0.94

$$\frac{1}{16}\left(40x^2-17\sqrt{2}\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)-\frac{50\left(x^2+3\right)}{x^4+2x^2+3}-68\log\left(x^4+2x^2+3\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (40*x^2 - (50*(3 + x^2))/(3 + 2*x^2 + x^4) - 17*Sqrt[2]*ArcTan[(1 + x^2)/Sq
rt[2]] - 68*Log[3 + 2*x^2 + x^4])/16
```

fricas [A] time = 0.97, size = 80, normalized size = 1.23

$$\frac{40\,x^6+80\,x^4-17\,\sqrt{2}\left(x^4+2\,x^2+3\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+70\,x^2-68\left(x^4+2\,x^2+3\right)\log\left(x^4+2\,x^2+3\right)-15}{16\left(x^4+2\,x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*x^6 + 80*x^4 - 17*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^
2 + 1)) + 70*x^2 - 68*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 150)/(x^4 +
2*x^2 + 3)
```

giac [A] time = 1.18, size = 54, normalized size = 0.83

$$\frac{5}{2}\,x^2 - \frac{17}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right) - \frac{25\left(x^2+3\right)}{8\left(x^4+2\,x^2+3\right)} - \frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

maple [A] time = 0.01, size = 59, normalized size = 0.91

$$\frac{5x^2}{2} - \frac{17\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16} - \frac{17\ln\left(x^4+2x^2+3\right)}{4} - \frac{\frac{25x^2}{4}+\frac{75}{4}}{2\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/2*x^2-1/2*(25/4*x^2+75/4)/(x^4+2*x^2+3)-17/4*ln(x^4+2*x^2+3)-17/16*2^(1/2
)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 1.72, size = 54, normalized size = 0.83

$$\frac{5}{2}\,x^2 - \frac{17}{16}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right) - \frac{25\left(x^2+3\right)}{8\left(x^4+2\,x^2+3\right)} - \frac{17}{4}\,\log\left(x^4+2\,x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/2*x^2 - 17/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/8*(x^2 + 3)/(x^4
 + 2*x^2 + 3) - 17/4*log(x^4 + 2*x^2 + 3)
```

mupad [B] time = 0.92, size = 60, normalized size = 0.92

$$\frac{5\,x^2}{2} - \frac{\frac{25\,x^2}{8}+\frac{75}{8}}{x^4+2\,x^2+3} - \frac{17\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16} - \frac{17\,\ln\left(x^4+2\,x^2+3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (5*x^2)/2 - ((25*x^2)/8 + 75/8)/(2*x^2 + x^4 + 3) - (17*2^(1/2)*atan(2^(1/2
)/2 + (2^(1/2)*x^2)/2))/16 - (17*log(2*x^2 + x^4 + 3))/4
```

sympy [A] time = 0.18, size = 68, normalized size = 1.05

$$\frac{5x^2}{2} + \frac{-25x^2-75}{8x^4+16x^2+24} - \frac{17\log\left(x^4+2x^2+3\right)}{4} - \frac{17\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**2/2 + (-25*x**2 - 75)/(8*x**4 + 16*x**2 + 24) - 17*log(x**4 + 2*x**2 +
 3)/4 - 17*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

3.104 $\int \frac{x(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=58

$$-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

```
[Out] 25/8*(x^2+1)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*arctan(1/2*(x^2+1)*2^(
1/2))*2^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {1663, 1660, 634, 618, 204, 628}

$$\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
```

```
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{-6+40x}{3+2x+x^2}\,dx, x, x^2\right)\\ &= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{5}{4}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx, x, x^2\right)-\frac{23}{8}\text{Subst}\left(\int \frac{1}{3+2x+x\right.\\ &= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{5}{4}\log\left(3+2x^2+x^4\right)+\frac{23}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right.\\ &= \frac{25\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{23\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{5}{4}\log\left(3+2x^2+x^4\right)\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.00

$$-\frac{23\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) - (23*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sq
rt[2]) + (5*Log[3 + 2*x^2 + x^4])/4
```

fricas [A] time = 0.79, size = 70, normalized size = 1.21

$$-\frac{23\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-50x^2-20\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right)-50}{16\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(23*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 50*x^2
- 20*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) - 50)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.13, size = 49, normalized size = 0.84

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

maple [A] time = 0.01, size = 54, normalized size = 0.93

$$-\frac{23\sqrt{2}\,\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}+\frac{5\ln\left(x^4+2x^2+3\right)}{4}+\frac{\frac{25x^2}{4}+\frac{25}{4}}{2x^4+4x^2+6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 1/2*(25/4*x^2+25/4)/(x^4+2*x^2+3)+5/4*ln(x^4+2*x^2+3)-23/16*2^(1/2)*arctan(
1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 1.41, size = 49, normalized size = 0.84

$$-\frac{23}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{25\left(x^2+1\right)}{8\left(x^4+2x^2+3\right)}+\frac{5}{4}\log\left(x^4+2x^2+3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -23/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 25/8*(x^2 + 1)/(x^4 + 2*x^2
+ 3) + 5/4*log(x^4 + 2*x^2 + 3)
```

mupad [B] time = 0.05, size = 69, normalized size = 1.19

$$\frac{5\ln\left(x^4+2x^2+3\right)}{4}+\frac{25x^2}{8\left(x^4+2x^2+3\right)}+\frac{25}{8\left(x^4+2x^2+3\right)}-\frac{23\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (5*log(2*x^2 + x^4 + 3))/4 + (25*x^2)/(8*(2*x^2 + x^4 + 3)) + 25/(8*(2*x^2
+ x^4 + 3)) - (23*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/16
```

sympy [A] time = 0.18, size = 60, normalized size = 1.03

$$\frac{25x^2+25}{8x^4+16x^2+24}+\frac{5\log\left(x^4+2x^2+3\right)}{4}-\frac{23\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (25*x**2 + 25)/(8*x**4 + 16*x**2 + 24) + 5*log(x**4 + 2*x**2 + 3)/4 - 23*sq
rt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/16
```

3.105 $\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{89\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{25\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{4\log(x)}{9}$$

```
[Out] 25/24*(-x^2+1)/(x^4+2*x^2+3)+4/9*ln(x)-1/9*ln(x^4+2*x^2+3)+89/144*arctan(1/
2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 800, 634, 618, 204, 628}

$$\frac{25\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{89\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{4\log(x)}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (25*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) + (89*ArcTan[(1 + x^2)/Sqrt[2]])/(72*
Sqrt[2]) + (4*Log[x])/9 - Log[3 + 2*x^2 + x^4]/9
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 800

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a
+ b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{4+x^2+3x^4+5x^6}{x\left(3+2x^2+x^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}+\frac{70x}{3}}{x\left(3+2x+x^2\right)}\,dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int\left(\frac{32}{9x}-\frac{2(-73+16x)}{9\left(3+2x+x^2\right)}\right)dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{4\log(x)}{9}-\frac{1}{72}\text{Subst}\left(\int \frac{-73+16x}{3+2x+x^2}\,dx,x,x^2\right)\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{4\log(x)}{9}-\frac{1}{9}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)+\frac{89}{72}\text{Subst}\left(\int\right.\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{4\log(x)}{9}-\frac{1}{9}\log\left(3+2x^2+x^4\right)-\frac{89}{36}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx,\right.\\
&= \frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{89\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{72\sqrt{2}}+\frac{4\log(x)}{9}-\frac{1}{9}\log\left(3+2x^2+x^4\right)
\end{aligned}$$

Mathematica [C] time = 0.06, size = 93, normalized size = 1.41

$$\frac{1}{288}\left(-\sqrt{2}\left(16\sqrt{2}+89i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(-16\sqrt{2}+89i\right)\log\left(x^2+i\sqrt{2}+1\right)-\frac{300\left(x^2-1\right)}{x^4+2x^2+3}+128\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((-300*(-1 + x^2))/(3 + 2*x^2 + x^4) + 128*Log[x] - Sqrt[2]*(89*I + 16*Sqrt
[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(89*I - 16*Sqrt[2])*Log[1 + I*Sqrt[
2] + x^2])/288
```

fricas [A] time = 0.78, size = 84, normalized size = 1.27

$$\frac{89\sqrt{2}\left(x^4+2x^2+3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-150x^2-16\left(x^4+2x^2+3\right)\log\left(x^4+2x^2+3\right)+64\left(x^4+2x^2+3\right)}{144\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(89*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 150*x^2
 - 16*(x^4 + 2*x^2 + 3)*log(x^4 + 2*x^2 + 3) + 64*(x^4 + 2*x^2 + 3)*log(x)
+ 150)/(x^4 + 2*x^2 + 3)
```

giac [A] time = 1.10, size = 62, normalized size = 0.94

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)+\frac{8x^4-59x^2+99}{72\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{2}{9}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(8*x^4 - 59*x^2 + 99)/(
x^4 + 2*x^2 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

maple [A] time = 0.01, size = 58, normalized size = 0.88

$$\frac{89\sqrt{2}\ \arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{144}+\frac{4\ln(x)}{9}-\frac{\ln\left(x^4+2x^2+3\right)}{9}-\frac{\frac{75x^2}{4}-\frac{75}{4}}{18\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x)
```

```
[Out] 4/9*ln(x)-1/18*(75/4*x^2-75/4)/(x^4+2*x^2+3)-1/9*ln(x^4+2*x^2+3)+89/144*2^(
1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 1.68, size = 55, normalized size = 0.83

$$\frac{89}{144}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^2+1\right)\right)-\frac{25\left(x^2-1\right)}{24\left(x^4+2x^2+3\right)}-\frac{1}{9}\log\left(x^4+2x^2+3\right)+\frac{2}{9}\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 89/144*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 25/24*(x^2 - 1)/(x^4 + 2*x^2
 + 3) - 1/9*log(x^4 + 2*x^2 + 3) + 2/9*log(x^2)
```

mupad [B] time = 0.91, size = 59, normalized size = 0.89

$$\frac{4\ \ln(x)}{9}-\frac{\ln\left(x^4+2x^2+3\right)}{9}-\frac{\frac{25x^2}{24}-\frac{25}{24}}{x^4+2x^2+3}+\frac{89\sqrt{2}\ \mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (4*log(x))/9 - log(2*x^2 + x^4 + 3)/9 - ((25*x^2)/24 - 25/24)/(2*x^2 + x^4
+ 3) + (89*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/144
```

sympy [A] time = 0.20, size = 65, normalized size = 0.98

$$\frac{25-25x^2}{24x^4+48x^2+72}+\frac{4\log(x)}{9}-\frac{\log\left(x^4+2x^2+3\right)}{9}+\frac{89\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{144}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x/(x**4+2*x**2+3)**2,x)
```

```
[Out] (25 - 25*x**2)/(24*x**4 + 48*x**2 + 72) + 4*log(x)/9 - log(x**4 + 2*x**2 +
3)/9 + 89*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2)/144
```

3.106 $\int \frac{4+x^2+3x^4+5x^6}{x^3(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{2}{9x^2}-\frac{71\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{25\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{13\log(x)}{27}$$

```
[Out] -2/9/x^2-25/72*(x^2+5)/(x^4+2*x^2+3)-13/27*ln(x)+13/108*ln(x^4+2*x^2+3)-71/
432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$-\frac{25\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{2}{9x^2}+\frac{13}{108}\log\left(x^4+2x^2+3\right)-\frac{71\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(9*x^2) - (25*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - (71*ArcTan[(1 + x^2)/S
qrt[2]])/(216*Sqrt[2]) - (13*Log[x])/27 + (13*Log[3 + 2*x^2 + x^4])/108
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^2\left(3+2x+x^2\right)^2}\,dx,x,x^2\right)$$

$$= -\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}-\frac{50x^2}{9}}{x^2\left(3+2x+x^2\right)}\,dx,x,x^2\right)$$

$$= -\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{16}\text{Subst}\left(\int \left(\frac{32}{9x^2}-\frac{104}{27x}+\frac{2(-19+52x)}{27\left(3+2x+x^2\right)}\right)dx,x,x^2\right)$$

$$= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{1}{216}\text{Subst}\left(\int \frac{-19+52x}{3+2x+x^2}\,dx,x,x^2\right)$$

$$= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{13}{108}\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,x,x^2\right)-\ldots$$

$$= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\right)+\frac{71}{108}\text{Subst}\left(\int \ldots\right.$$

$$= -\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{71\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}-\frac{13\log(x)}{27}+\frac{13}{108}\log\left(3+2x^2+x^4\ldots\right.$$

Mathematica [C] time = 0.05, size = 97, normalized size = 1.37

$$\frac{1}{864}\left(-\frac{192}{x^2}+\sqrt{2}\left(52\sqrt{2}+71i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(52\sqrt{2}-71i\right)\log\left(x^2+i\sqrt{2}+1\right)-\frac{300\left(x^2+5\right)}{x^4+2x^2+3}-\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^3*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-192/x^2 - (300*(5 + x^2))/(3 + 2*x^2 + x^4) - 416*Log[x] + Sqrt[2]*(71*I
+ 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(-71*I + 52*Sqrt[2])*Log[1
 + I*Sqrt[2] + x^2])/864
```

fricas [A] time = 0.58, size = 105, normalized size = 1.48

$$-\frac{246\,x^{4}+71\,\sqrt{2}\left(x^{6}+2\,x^{4}+3\,x^{2}\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)+942\,x^{2}-52\left(x^{6}+2\,x^{4}+3\,x^{2}\right)\log\left(x^{4}+2\,x^{2}+3\right)+}{432\left(x^{6}+2\,x^{4}+3\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/432*(246*x^4 + 71*sqrt(2)*(x^6 + 2*x^4 + 3*x^2)*arctan(1/2*sqrt(2)*(x^2
+ 1)) + 942*x^2 - 52*(x^6 + 2*x^4 + 3*x^2)*log(x^4 + 2*x^2 + 3) + 208*(x^6
+ 2*x^4 + 3*x^2)*log(x) + 288)/(x^6 + 2*x^4 + 3*x^2)
```

giac [A] time = 1.07, size = 66, normalized size = 0.93

$$-\frac{71}{432}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)-\frac{41\,x^{4}+157\,x^{2}+48}{72\left(x^{6}+2\,x^{4}+3\,x^{2}\right)}+\frac{13}{108}\,\log\left(x^{4}+2\,x^{2}+3\right)-\frac{13}{54}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

maple [A] time = 0.01, size = 63, normalized size = 0.89

$$-\frac{71\sqrt{2}\,\arctan\left(\frac{\left(2x^{2}+2\right)\sqrt{2}}{4}\right)}{432}-\frac{13\ln(x)}{27}+\frac{13\ln\left(x^{4}+2x^{2}+3\right)}{108}-\frac{2}{9x^{2}}+\frac{-\frac{75x^{2}}{4}-\frac{375}{4}}{54x^{4}+108x^{2}+162}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x)
```

```
[Out] -2/9/x^2-13/27*ln(x)+1/54*(-75/4*x^2-375/4)/(x^4+2*x^2+3)+13/108*ln(x^4+2*x
^2+3)-71/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 1.43, size = 66, normalized size = 0.93

$$-\frac{71}{432}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^{2}+1\right)\right)-\frac{41\,x^{4}+157\,x^{2}+48}{72\left(x^{6}+2\,x^{4}+3\,x^{2}\right)}+\frac{13}{108}\,\log\left(x^{4}+2\,x^{2}+3\right)-\frac{13}{54}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^3/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -71/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/72*(41*x^4 + 157*x^2 + 48
)/(x^6 + 2*x^4 + 3*x^2) + 13/108*log(x^4 + 2*x^2 + 3) - 13/54*log(x^2)
```

mupad [B] time = 0.06, size = 68, normalized size = 0.96

$$\frac{13\,\ln\left(x^{4}+2\,x^{2}+3\right)}{108}-\frac{13\,\ln(x)}{27}-\frac{\frac{41\,x^{4}}{72}+\frac{157\,x^{2}}{72}+\frac{2}{3}}{x^{6}+2\,x^{4}+3\,x^{2}}-\frac{71\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^3*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (13*log(2*x^2 + x^4 + 3))/108 - (13*log(x))/27 - ((157*x^2)/72 + (41*x^4)/7
2 + 2/3)/(3*x^2 + 2*x^4 + x^6) - (71*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)
/2))/432
```

sympy [A] time = 0.21, size = 76, normalized size = 1.07

$$\frac{-41x^4-157x^2-48}{72x^6+144x^4+216x^2}-\frac{13\log(x)}{27}+\frac{13\log\left(x^4+2x^2+3\right)}{108}-\frac{71\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**3/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-41*x**4 - 157*x**2 - 48)/(72*x**6 + 144*x**4 + 216*x**2) - 13*log(x)/27 +
 13*log(x**4 + 2*x**2 + 3)/108 - 71*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt(2)/2
)/432
```

3.107 $\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=80

$$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{125\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{25\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{13\log(x)}{27}$$

```
[Out] -1/9/x^4+13/54/x^2+25/216*(5*x^2+7)/(x^4+2*x^2+3)+13/27*ln(x)-13/108*ln(x^4
+2*x^2+3)+125/432*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$\frac{25\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}+\frac{13}{54x^2}-\frac{1}{9x^4}-\frac{13}{108}\log\left(x^4+2x^2+3\right)+\frac{125\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -1/(9*x^4) + 13/(54*x^2) + (25*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) + (125*
ArcTan[(1 + x^2)/Sqrt[2]])/(216*Sqrt[2]) + (13*Log[x])/27 - (13*Log[3 + 2*x
^2 + x^4])/108
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^3\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{250x^3}{27}}{x^3\left(3+2x+x^2\right)}\,dx, x, x^2\right)$$

$$= \frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int \left(\frac{32}{9x^3}-\frac{104}{27x^2}+\frac{104}{27x}-\frac{2(-73+52x)}{27\left(3+2x+x^2\right)}\right)dx, x$$

$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}-\frac{1}{216}\,\text{Subst}\left(\int \frac{-73+52x}{3+2x+x^2}\,dx,\right.$$

$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}-\frac{13}{108}\,\text{Subst}\left(\int \frac{2+2x}{3+2x+x^2}\,dx,\right.$$

$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3+2x^2+x^4\right)-\frac{125}{108}\,\text{S}$$

$$= -\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{125\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{216\sqrt{2}}+\frac{13\log(x)}{27}-\frac{13}{108}\log\left(3\right.$$

Mathematica [C] time = 0.06, size = 105, normalized size = 1.31

$$\frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}-\sqrt{2}\left(52\sqrt{2}+125i\right)\log\left(x^2-i\sqrt{2}+1\right)+\sqrt{2}\left(-52\sqrt{2}+125i\right)\log\left(x^2+i\sqrt{2}+1\right)+\frac{100(}{x^4+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^5*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-96/x^4 + 208/x^2 + (100*(7 + 5*x^2))/(3 + 2*x^2 + x^4) + 416*Log[x] - Sqr
t[2]*(125*I + 52*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] + Sqrt[2]*(125*I - 52*Sq
rt[2])*Log[1 + I*Sqrt[2] + x^2])/864
```

fricas [A] time = 0.80, size = 110, normalized size = 1.38

$$\frac{354\,x^6+510\,x^4+125\,\sqrt{2}\left(x^8+2\,x^6+3\,x^4\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+216\,x^2-52\left(x^8+2\,x^6+3\,x^4\right)\log\left(x^4+2\,x^{\cdots}\right.}{432\left(x^8+2\,x^6+3\,x^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/432*(354*x^6 + 510*x^4 + 125*sqrt(2)*(x^8 + 2*x^6 + 3*x^4)*arctan(1/2*sqr
t(2)*(x^2 + 1)) + 216*x^2 - 52*(x^8 + 2*x^6 + 3*x^4)*log(x^4 + 2*x^2 + 3) +
 208*(x^8 + 2*x^6 + 3*x^4)*log(x) - 144)/(x^8 + 2*x^6 + 3*x^4)
```

giac [A] time = 1.16, size = 79, normalized size = 0.99

$$\frac{125}{432}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{26\,x^4+177\,x^2+253}{216\left(x^4+2\,x^2+3\right)}-\frac{39\,x^4-26\,x^2+12}{108\,x^4}-\frac{13}{108}\,\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\,\log\left(x^{2}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/216*(26*x^4 + 177*x^2 + 2
53)/(x^4 + 2*x^2 + 3) - 1/108*(39*x^4 - 26*x^2 + 12)/x^4 - 13/108*log(x^4 +
 2*x^2 + 3) + 13/54*log(x^2)
```

maple [A] time = 0.01, size = 68, normalized size = 0.85

$$\frac{125\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{432}+\frac{13\ln(x)}{27}-\frac{13\ln\left(x^4+2x^2+3\right)}{108}+\frac{13}{54x^2}-\frac{1}{9x^4}-\frac{-\frac{125x^2}{4}-\frac{175}{4}}{54\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x)
```

```
[Out] -1/9/x^4+13/54/x^2+13/27*ln(x)-1/54*(-125/4*x^2-175/4)/(x^4+2*x^2+3)-13/108
*ln(x^4+2*x^2+3)+125/432*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

maxima [A] time = 2.38, size = 71, normalized size = 0.89

$$\frac{125}{432}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{59\,x^6+85\,x^4+36\,x^2-24}{72\left(x^8+2\,x^6+3\,x^4\right)}-\frac{13}{108}\,\log\left(x^4+2\,x^2+3\right)+\frac{13}{54}\,\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^5/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 125/432*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/72*(59*x^6 + 85*x^4 + 36*
x^2 - 24)/(x^8 + 2*x^6 + 3*x^4) - 13/108*log(x^4 + 2*x^2 + 3) + 13/54*log(x
^2)
```

mupad [B] time = 0.06, size = 72, normalized size = 0.90

$$\frac{13\,\ln(x)}{27}-\frac{13\,\ln\left(x^4+2\,x^2+3\right)}{108}+\frac{\frac{59\,x^6}{72}+\frac{85\,x^4}{72}+\frac{x^2}{2}-\frac{1}{3}}{x^8+2\,x^6+3\,x^4}+\frac{125\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^5*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (13*log(x))/27 - (13*log(2*x^2 + x^4 + 3))/108 + (x^2/2 + (85*x^4)/72 + (59
*x^6)/72 - 1/3)/(3*x^4 + 2*x^6 + x^8) + (125*2^(1/2)*atan(2^(1/2)/2 + (2^(1
/2)*x^2)/2))/432
```

sympy [A] time = 0.23, size = 80, normalized size = 1.00

$$\frac{13\log(x)}{27} - \frac{13\log\left(x^4+2x^2+3\right)}{108} + \frac{125\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{432} + \frac{59x^6+85x^4+36x^2-24}{72x^8+144x^6+216x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**5/(x**4+2*x**2+3)**2,x)
```

```
[Out] 13*log(x)/27 - 13*log(x**4 + 2*x**2 + 3)/108 + 125*sqrt(2)*atan(sqrt(2)*x**
2/2 + sqrt(2)/2)/432 + (59*x**6 + 85*x**4 + 36*x**2 - 24)/(72*x**8 + 144*x*
*6 + 216*x**4)
```

3.108 $\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}-\frac{1237\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{25\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{61}{972}\log\left(x^4+2x^2+3\right)+\frac{61\log(x)}{243}$$

```
[Out] -2/27/x^6+13/108/x^4-13/54/x^2+25/648*(-7*x^2+1)/(x^4+2*x^2+3)+61/243*ln(x)
-61/972*ln(x^4+2*x^2+3)-1237/3888*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1663, 1646, 1628, 634, 618, 204, 628}

$$\frac{25\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}-\frac{13}{54x^2}+\frac{13}{108x^4}-\frac{2}{27x^6}-\frac{61}{972}\log\left(x^4+2x^2+3\right)-\frac{1237\tan^{-1}\left(\frac{x^2+1}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -2/(27*x^6) + 13/(108*x^4) - 13/(54*x^2) + (25*(1 - 7*x^2))/(648*(3 + 2*x^2
 + x^4)) - (1237*ArcTan[(1 + x^2)/Sqrt[2]])/(1944*Sqrt[2]) + (61*Log[x])/24
3 - (61*Log[3 + 2*x^2 + x^4])/972
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1628

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1646

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p
 + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx = \frac{1}{2}\,\text{Subst}\left(\int \frac{4+x+3x^2+5x^3}{x^4\left(3+2x+x^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int \frac{\frac{32}{3}-\frac{40x}{9}+\frac{200x^2}{27}+\frac{800x^3}{81}-\frac{350x^4}{81}}{x^4\left(3+2x+x^2\right)}\,dx, x, x^2\right)$$

$$= \frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)} + \frac{1}{16}\,\text{Subst}\left(\int\left(\frac{32}{9x^4}-\frac{104}{27x^3}+\frac{104}{27x^2}+\frac{488}{243x}-\frac{2(1481+244}{243\left(3+2x+\right.}\right.\right.$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{61\log(x)}{243}-\frac{\text{Subst}\left(\int\frac{1481+244x}{3+2x+x^2}\,d\right.}{1944}$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{61\log(x)}{243}-\frac{61}{972}\,\text{Subst}\left(\int\frac{2+}{3+2}\right.$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{61\log(x)}{243}-\frac{61}{972}\log\left(3+2x^2+x\right.$$

$$= -\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{1237\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{1944\sqrt{2}}+\frac{61\log(x)}{243}-$$

Mathematica [C] time = 0.07, size = 110, normalized size = 1.26

$$\frac{-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}+\sqrt{2}\left(-244\sqrt{2}+1237i\right)\log\left(x^2-i\sqrt{2}+1\right)-\sqrt{2}\left(244\sqrt{2}+1237i\right)\log\left(x^2+i\sqrt{2}+1\right)-}{7776}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^7*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] (-576/x^6 + 936/x^4 - 1872/x^2 - (300*(-1 + 7*x^2))/(3 + 2*x^2 + x^4) + 195
2*Log[x] + Sqrt[2]*(1237*I - 244*Sqrt[2])*Log[1 - I*Sqrt[2] + x^2] - Sqrt[2
]*(1237*I + 244*Sqrt[2])*Log[1 + I*Sqrt[2] + x^2])/7776
```

fricas [A] time = 0.84, size = 115, normalized size = 1.32

$$-\frac{1986\,x^8+1254\,x^6+2160\,x^4+1237\,\sqrt{2}\left(x^{10}+2\,x^8+3\,x^6\right)\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-828\,x^2+244\left(x^{10}+2\,x^8+\right.}{3888\left(x^{10}+2\,x^8+3\,x^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/3888*(1986*x^8 + 1254*x^6 + 2160*x^4 + 1237*sqrt(2)*(x^10 + 2*x^8 + 3*x^
6)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 828*x^2 + 244*(x^10 + 2*x^8 + 3*x^6)*log
(x^4 + 2*x^2 + 3) - 976*(x^10 + 2*x^8 + 3*x^6)*log(x) + 864)/(x^10 + 2*x^8
+ 3*x^6)
```

giac [A] time = 1.17, size = 84, normalized size = 0.97

$$-\frac{1237}{3888}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)+\frac{122\,x^4-281\,x^2+441}{1944\left(x^4+2\,x^2+3\right)}-\frac{671\,x^6+702\,x^4-351\,x^2+216}{2916\,x^6}-\frac{61}{972}\,\log\left(x^4+2\,x^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/1944*(122*x^4 - 281*x^
2 + 441)/(x^4 + 2*x^2 + 3) - 1/2916*(671*x^6 + 702*x^4 - 351*x^2 + 216)/x^6
 - 61/972*log(x^4 + 2*x^2 + 3) + 61/486*log(x^2)
```

maple [A] time = 0.01, size = 73, normalized size = 0.84

$$-\frac{1237\sqrt{2}\,\arctan\left(\frac{\left(2x^2+2\right)\sqrt{2}}{4}\right)}{3888}+\frac{61\ln(x)}{243}-\frac{61\ln\left(x^4+2x^2+3\right)}{972}-\frac{13}{54x^2}+\frac{13}{108x^4}-\frac{2}{27x^6}-\frac{\frac{525x^2}{4}-\frac{75}{4}}{486\left(x^4+2x^2+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x)
```

```
[Out] -2/27/x^6+13/108/x^4-13/54/x^2+61/243*ln(x)-1/486*(525/4*x^2-75/4)/(x^4+2*x
^2+3)-61/972*ln(x^4+2*x^2+3)-1237/3888*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2)
)
```

maxima [A] time = 2.47, size = 76, normalized size = 0.87

$$-\frac{1237}{3888}\,\sqrt{2}\,\arctan\left(\frac{1}{2}\,\sqrt{2}\left(x^2+1\right)\right)-\frac{331\,x^8+209\,x^6+360\,x^4-138\,x^2+144}{648\left(x^{10}+2\,x^8+3\,x^6\right)}-\frac{61}{972}\,\log\left(x^4+2\,x^2+3\right)+\frac{61}{486}\,\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^7/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1237/3888*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) - 1/648*(331*x^8 + 209*x^6
 + 360*x^4 - 138*x^2 + 144)/(x^10 + 2*x^8 + 3*x^6) - 61/972*log(x^4 + 2*x^2
 + 3) + 61/486*log(x^2)
```

mupad [B] time = 0.07, size = 78, normalized size = 0.90

$$\frac{61\,\ln(x)}{243}-\frac{61\,\ln\left(x^4+2\,x^2+3\right)}{972}-\frac{\frac{331\,x^8}{648}+\frac{209\,x^6}{648}+\frac{5\,x^4}{9}-\frac{23\,x^2}{108}+\frac{2}{9}}{x^{10}+2\,x^8+3\,x^6}-\frac{1237\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^7*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (61*log(x))/243 - (61*log(2*x^2 + x^4 + 3))/972 - ((5*x^4)/9 - (23*x^2)/108
 + (209*x^6)/648 + (331*x^8)/648 + 2/9)/(3*x^6 + 2*x^8 + x^10) - (1237*2^(1
/2)*atan(2^(1/2)/2 + (2^(1/2)*x^2)/2))/3888
```

sympy [A] time = 0.24, size = 85, normalized size = 0.98

$$\frac{61\log(x)}{243} - \frac{61\log\left(x^4+2x^2+3\right)}{972} - \frac{1237\sqrt{2}\operatorname{atan}\left(\frac{\sqrt{2}x^2}{2}+\frac{\sqrt{2}}{2}\right)}{3888} + \frac{-331x^8-209x^6-360x^4+138x^2-144}{648x^{10}+1296x^8+1944x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**7/(x**4+2*x**2+3)**2,x)
```

```
[Out] 61*log(x)/243 - 61*log(x**4 + 2*x**2 + 3)/972 - 1237*sqrt(2)*atan(sqrt(2)*x
**2/2 + sqrt(2)/2)/3888 + (-331*x**8 - 209*x**6 - 360*x**4 + 138*x**2 - 144
)/(648*x**10 + 1296*x**8 + 1944*x**6)
```

3.109 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2} dx$

Optimal. Leaf size=248

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right.}$$

```
[Out] 38*x+19/3*x^3-17/5*x^5+5/7*x^7+25/8*x*(5*x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+3
^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/64*ln(x^2+
3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-525542+1236582*3^(1/2))^(1/2)+1/32*arctan
((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525542+1236582*3^(1/2))^
(1/2)-1/32*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(525542+1
236582*3^(1/2))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+\frac{25\left(5x^2+3\right)x}{8\left(x^4+2x^2+3\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(618291\sqrt{3}-262771\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(61\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) + (Sqrt[(262771 + 618291*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(262771 + 618291*Sqrt[3])/2]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-
262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32 + (Sqrt[(-262771 + 618291*Sqrt[3])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3]
)]*x + x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \frac{-450-1650x^2+1200x^4-336x^8+240x^{10}}{3+2x^2+x^4}\,dx$$

$$= \frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \left(1824+912x^2-816x^4+240x^6-\frac{6\left(987+1339x^2\right.}{3+2x^2+x^4}\right.$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{8}\int \frac{987+1339x^2}{3+2x^2+x^4}\,dx$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{\int \frac{987\sqrt{2\left(-1+\sqrt{3}\right)}-\left(987-1339\sqrt{3}\right)}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}}{16\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\left(1339+329\sqrt{3}\right)\int \frac{}{\sqrt{3}}$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\right.}$$

$$= 38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right.}$$

Mathematica [C] time = 0.17, size = 145, normalized size = 0.58

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+\frac{25\left(5x^2+3\right)x}{8\left(x^4+2x^2+3\right)}+38x-\frac{\left(1339\sqrt{2}+352i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}-\frac{\left(1339\sqrt{2}-352i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 38*x + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7 + (25*x*(3 + 5*x^2))/(8*(3 + 2*x
^2 + x^4)) - ((352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqr
t[2 - (2*I)*Sqrt[2]]) - ((-352*I + 1339*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2
]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

fricas [B] time = 0.81, size = 519, normalized size = 2.09

$$\frac{242072962564800\,x^{11}-668121376678848\,x^{9}+568064552152064\,x^{7}+13714240239171136\,x^{5}-102773860\cdot 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/338902147590720*(242072962564800*x^11 - 668121376678848*x^9 + 56806455215
2064*x^7 + 13714240239171136*x^5 - 102773860*14158657803^(1/4)*sqrt(68699)*
sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(262771*sqrt(3) + 1854873)*arctan(1/3
145089554732313026311937382*sqrt(50431867201)*14158657803^(3/4)*sqrt(68699)
*sqrt(3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*
sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3))*(329*sqrt(3)*
sqrt(2) - 1339*sqrt(2))*sqrt(262771*sqrt(3) + 1854873) - 1/2078771306904899
4*14158657803^(3/4)*sqrt(68699)*(329*sqrt(3)*sqrt(2)*x - 1339*sqrt(2)*x)*sq
rt(262771*sqrt(3) + 1854873) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 1027738
60*14158657803^(1/4)*sqrt(68699)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(262
771*sqrt(3) + 1854873)*arctan(1/3145089554732313026311937382*sqrt(504318672
01)*14158657803^(3/4)*sqrt(68699)*sqrt(-3*14158657803^(1/4)*sqrt(68699)*(13
39*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 4
53886804809*sqrt(3))*(329*sqrt(3)*sqrt(2) - 1339*sqrt(2))*sqrt(262771*sqrt(
3) + 1854873) - 1/20787713069048994*14158657803^(3/4)*sqrt(68699)*(329*sqrt
(3)*sqrt(2)*x - 1339*sqrt(2)*x)*sqrt(262771*sqrt(3) + 1854873) - 1/2*sqrt(3
)*sqrt(2) + 1/2*sqrt(2)) + 35*14158657803^(1/4)*sqrt(68699)*(1854873*x^4 +
3709746*x^2 - 262771*sqrt(3)*(x^4 + 2*x^2 + 3) + 5564619)*sqrt(262771*sqrt(
3) + 1854873)*log(3*14158657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*
sqrt(262771*sqrt(3) + 1854873) + 453886804809*x^2 + 453886804809*sqrt(3)) -
 35*14158657803^(1/4)*sqrt(68699)*(1854873*x^4 + 3709746*x^2 - 262771*sqrt(
3)*(x^4 + 2*x^2 + 3) + 5564619)*sqrt(262771*sqrt(3) + 1854873)*log(-3*14158
657803^(1/4)*sqrt(68699)*(1339*sqrt(3)*x - 987*x)*sqrt(262771*sqrt(3) + 185
4873) + 453886804809*x^2 + 453886804809*sqrt(3)) + 37491050077223400*x^3 +
41812052459005080*x)/(x^4 + 2*x^2 + 3)
```

giac [B] time = 1.89, size = 585, normalized size = 2.36

$$\frac{5}{7}\,x^{7}-\frac{17}{5}\,x^{5}+\frac{19}{3}\,x^{3}+\frac{1}{20736}\,\sqrt{2}\left(1339\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}}+24102\cdot 3^{\frac{3}{4}}\sqrt{2}\,\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right)-24102\cdot 3^{\frac{3}{4}}\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*(6*sq
rt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3
) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(-6*sq
rt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532*3^(1
/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3
) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/20736*sqrt(2)*(1339*3^(3/4)*sqrt(2)*
(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3
) - 3) - 24102*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 1339*3^(3/4)*(
-6*sqrt(3) + 18)^(3/2) - 35532*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 35532
*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*s
qrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/41472*sqrt(2)*(24102*3^(3/4)*sq
rt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3
) + 18)^(3/2) + 1339*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*
sqrt(3) + 18)*(sqrt(3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) -
 35532*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3
) + 1/2) + sqrt(3)) - 1/41472*sqrt(2)*(24102*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 1339*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 1339
*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 24102*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) - 35532*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 35532*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
+ 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)
```

maple [B] time = 0.12, size = 427, normalized size = 1.72

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x-\frac{505\left(-2+2\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{32\sqrt{2+2\sqrt{3}}}-\frac{11\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{2\sqrt{2+2\sqrt{3}}}-3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/7*x^7-17/5*x^5+19/3*x^3+38*x-(-125/8*x^3-75/8*x)/(x^4+2*x^2+3)-505/64*ln(
x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)-11/4*ln(x^
2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-505/32/(2+2*3^(1/2))
^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2)
)*3^(1/2)-11/2/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))*(-2+2*3^(1/2))-329/8/(2+2*3^(1/2))^(1/2)*arctan((2*x-(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*3^(1/2)+505/64*ln(x^2+3^(1/2)+x*(-2+2*3
^(1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)*3^(1/2)+11/4*ln(x^2+3^(1/2)+x*(-2+2*3^(
1/2))^(1/2))*(-2+2*3^(1/2))^(1/2)-505/32/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*3^(1/2))*3^(1/2)-11/2/(2+2*3
^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-2+2*
3^(1/2))-329/8/(2+2*3^(1/2))^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))*3^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5}{7}\,x^{7}-\frac{17}{5}\,x^{5}+\frac{19}{3}\,x^{3}+38\,x+\frac{25\,\left(5\,x^{3}+3\,x\right)}{8\,\left(x^{4}+2\,x^{2}+3\right)}-\frac{1}{8}\,\int\frac{1339\,x^{2}+987}{x^{4}+2\,x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/7*x^7 - 17/5*x^5 + 19/3*x^3 + 38*x + 25/8*(5*x^3 + 3*x)/(x^4 + 2*x^2 + 3)
 - 1/8*integrate((1339*x^2 + 987)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.11, size = 171, normalized size = 0.69

$$38\,x+\frac{\frac{125\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3}+\frac{19\,x^3}{3}-\frac{17\,x^5}{5}+\frac{5\,x^7}{7}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}\,734099\mathrm{i}}{64\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}+\frac{734099\,\sqrt{2}\,x\,\sqrt{-262771-\sqrt{2}\,734099\mathrm{i}}}{128\left(-\frac{1112159985}{64}+\frac{\sqrt{2}\,724555713\mathrm{i}}{128}\right)}\right)\sqrt{-2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 38*x + (atan((x*(- 2^(1/2)*734099i - 262771)^(1/2)*734099i)/(64*((2^(1/2)*7
24555713i)/128 - 1112159985/64)) + (734099*2^(1/2)*x*(- 2^(1/2)*734099i - 2
62771)^(1/2))/(128*((2^(1/2)*724555713i)/128 - 1112159985/64)))*(- 2^(1/2)*
734099i - 262771)^(1/2)*1i)/16 - (atan((x*(2^(1/2)*734099i - 262771)^(1/2)*
734099i)/(64*((2^(1/2)*724555713i)/128 + 1112159985/64)) - (734099*2^(1/2)*
x*(2^(1/2)*734099i - 262771)^(1/2))/(128*((2^(1/2)*724555713i)/128 + 111215
9985/64)))*(2^(1/2)*734099i - 262771)^(1/2)*1i)/16 + ((75*x)/8 + (125*x^3)/
8)/(2*x^2 + x^4 + 3) + (19*x^3)/3 - (17*x^5)/5 + (5*x^7)/7
```

sympy [A] time = 0.61, size = 71, normalized size = 0.29

$$\frac{5x^7}{7}-\frac{17x^5}{5}+\frac{19x^3}{3}+38x+\frac{125x^3+75x}{8x^4+16x^2+24}+\mathrm{RootSum}\left(1048576t^4+538155008t^2+1146851282043,\left(t\mapsto t\log\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**7/7 - 17*x**5/5 + 19*x**3/3 + 38*x + (125*x**3 + 75*x)/(8*x**4 + 16*x*
*2 + 24) + RootSum(1048576*_t**4 + 538155008*_t**2 + 1146851282043, Lambda(
_t, _t*log(-16547840*_t**3/453886804809 - 11974973632*_t/453886804809 + x))
)
```

3.110 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=237

$$x^5-\frac{17x^3}{3}+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2+\sqrt{2}\right.$$

```
[Out] 19*x-17/3*x^3+x^5+25/8*x*(-x^2+3)/(x^4+2*x^2+3)+3/32*arctan((-2*x+(-2+2*3^(
1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(1/2)-3/32*arctan(
(2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-52014+30066*3^(1/2))^(1/2
)+3/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/2)-3
/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(52014+30066*3^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$x^5-\frac{17x^3}{3}+\frac{25\left(3-x^2\right)x}{8\left(x^4+2x^2+3\right)}+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{3}{32}\sqrt{\frac{3}{2}}\left(8669+5011\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 + (25*x*(3 - x^2))/(8*(3 + 2*x^2 + x^4)) + (3*Sqrt[
(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (3*Sqrt[(3*(-8669 + 5011*Sqrt[3]))/2]*ArcTan[(Sqrt[2*
(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 + (3*Sqrt[(3*(8669 + 5011
*Sqrt[3]))/2]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 - (3*Sqrt[(
3*(8669 + 5011*Sqrt[3]))/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{-450+1050x^2-336x^6+240x^8}{3+2x^2+x^4}\,dx$$

$$= \frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(912-816x^2+240x^4-\frac{54\left(59-31x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{9}{8}\int \frac{59-31x^2}{3+2x^2+x^4}\,dx$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\left(3\sqrt{3\left(1+\sqrt{3}\right)}\right)\int \frac{59\sqrt{2\left(-1+\sqrt{3}\right)}}{\sqrt{3}-\sqrt{2\left(-1\right.}}$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{16}\left(3\sqrt{\frac{3}{2}\left(3182-1829\sqrt{3}\right)}\right)\int \frac{}{\sqrt{3}-\sqrt{}}$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{3}{32}\sqrt{\frac{3}{2}\left(8669+5011\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2}\right.$$

$$= 19x-\frac{17x^3}{3}+x^5+\frac{25x\left(3-x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{3}{16}\sqrt{\frac{3}{2}\left(-8669+5011\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-\right.}}{\sqrt{2}}\right.$$

Mathematica [C] time = 0.16, size = 132, normalized size = 0.56

$$x^5-\frac{17x^3}{3}-\frac{25\left(x^2-3\right)x}{8\left(x^4+2x^2+3\right)}+19x+\frac{9\left(31\sqrt{2}+90i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}+\frac{9\left(31\sqrt{2}-90i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 19*x - (17*x^3)/3 + x^5 - (25*x*(-3 + x^2))/(8*(3 + 2*x^2 + x^4)) + (9*(90*
I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]])
 + (9*(-90*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I
)*Sqrt[2]])
```

fricas [B] time = 0.76, size = 476, normalized size = 2.01

$$\frac{287671488\,x^9-1054795456\,x^7+3068495872\,x^5+3588\cdot 677973267^{\frac{1}{4}}\sqrt{3}\,\sqrt{2}\left(x^4+2\,x^2+3\right)\sqrt{-43440359\,\sqrt{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/287671488*(287671488*x^9 - 1054795456*x^7 + 3068495872*x^5 + 3588*6779732
67^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-43440359*sqrt(3) + 7533036
3)*arctan(1/1822344999502852422*677973267^(3/4)*sqrt(4494867)*sqrt(4494867*
x^2 + 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-43440359*sqrt(3) + 753303
63) + 4494867*sqrt(3))*(59*sqrt(3)*sqrt(2) + 93*sqrt(2))*sqrt(-43440359*sqr
t(3) + 75330363) - 1/405428013666*677973267^(3/4)*(59*sqrt(3)*sqrt(2)*x + 9
3*sqrt(2)*x)*sqrt(-43440359*sqrt(3) + 75330363) - 1/2*sqrt(3)*sqrt(2) + 1/2
*sqrt(2)) + 3588*677973267^(1/4)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-43
440359*sqrt(3) + 75330363)*arctan(1/1822344999502852422*677973267^(3/4)*sqr
t(4494867)*sqrt(4494867*x^2 - 677973267^(1/4)*(31*sqrt(3)*x + 59*x)*sqrt(-4
3440359*sqrt(3) + 75330363) + 4494867*sqrt(3))*(59*sqrt(3)*sqrt(2) + 93*sqr
t(2))*sqrt(-43440359*sqrt(3) + 75330363) - 1/405428013666*677973267^(3/4)*(
59*sqrt(3)*sqrt(2)*x + 93*sqrt(2)*x)*sqrt(-43440359*sqrt(3) + 75330363) + 1
/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 5142127848*x^3 - 3*677973267^(1/4)*(150
33*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^4 + 2*x^2 + 3) + 45099)*sqrt(-43440359
*sqrt(3) + 75330363)*log(4494867*x^2 + 677973267^(1/4)*(31*sqrt(3)*x + 59*x
)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3)) + 3*677973267^(1/4)
*(15033*x^4 + 30066*x^2 + 8669*sqrt(3)*(x^4 + 2*x^2 + 3) + 45099)*sqrt(-434
40359*sqrt(3) + 75330363)*log(4494867*x^2 - 677973267^(1/4)*(31*sqrt(3)*x +
 59*x)*sqrt(-43440359*sqrt(3) + 75330363) + 4494867*sqrt(3)) + 19094195016*
x)/(x^4 + 2*x^2 + 3)
```

giac [B] time = 1.85, size = 576, normalized size = 2.43

$$x^5-\frac{17}{3}\,x^3-\frac{1}{2304}\,\sqrt{2}\left(31\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}}+558\cdot 3^{\frac{3}{4}}\sqrt{2}\,\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right)-558\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] x^5 - 17/3*x^3 - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2)
+ 558*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqr
```

```
t(3) + 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/2304*sqrt(2)*(31*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 558*
3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 558*3^(3/4)*(sqrt(3) +
 3)*sqrt(-6*sqrt(3) + 18) + 31*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 2124*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 2124*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arcta
n(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2
)) - 1/4608*sqrt(2)*(558*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 31*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18
)^(3/2) + 558*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqr
t(2)*sqrt(-6*sqrt(3) + 18) + 2124*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2
*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/4608*sqrt(2)*(558*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 31*3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 31*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 558*3^(3/4)*sqrt(6*sq
rt(3) + 18)*(sqrt(3) - 3) + 2124*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 21
24*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
1/2) + sqrt(3)) + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3)
```

maple [B] time = 0.03, size = 419, normalized size = 1.77

$$x^5-\frac{17x^3}{3}+19x+\frac{57\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{8\sqrt{2+2\sqrt{3}}}+\frac{405\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{32\sqrt{2+2\sqrt{3}}}-\frac{177\sqrt{3}\ \mathrm{arct}}{8\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] x^5-17/3*x^3+19*x+(-25/8*x^3+75/8*x)/(x^4+2*x^2+3)+57/16*(-2+2*3^(1/2))^(1/
2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+405/64*(-2+2*3^(1/2))^(1/
2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+57/8/(2+2*3^(1/2))^(1/2)*(-2+2*3^
(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+405/3
2/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))-177/8/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2
))^(1/2))/(2+2*3^(1/2))^(1/2))-57/16*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-
2+2*3^(1/2))^(1/2)*x+3^(1/2))-405/64*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1
/2))^(1/2)*x+3^(1/2))+57/8/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arcta
n((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+405/32/(2+2*3^(1/2))^(1/2
)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-177
/8/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/
2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x^5-\frac{17}{3}\,x^3+19\,x-\frac{25\left(x^3-3\,x\right)}{8\left(x^4+2\,x^2+3\right)}+\frac{9}{8}\int\frac{31\,x^2-59}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] x^5 - 17/3*x^3 + 19*x - 25/8*(x^3 - 3*x)/(x^4 + 2*x^2 + 3) + 9/8*integrate(
(31*x^2 - 59)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.94, size = 164, normalized size = 0.69

$$19\,x+\frac{\frac{75\,x}{8}-\frac{25\,x^3}{8}}{x^4+2\,x^2+3}-\frac{17\,x^3}{3}+x^5-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,24219\mathrm{i}}{64\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}-\frac{24219\,\sqrt{2}\,x\,\sqrt{26007-\sqrt{2}\,897\mathrm{i}}}{128\left(-\frac{1380483}{16}+\frac{\sqrt{2}\,4286763\mathrm{i}}{128}\right)}\right)\sqrt{26007-\sqrt{2}\,897\mathrm{i}}\,3\mathrm{i}\quad \mathrm{a}}{16}+-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 19*x + ((75*x)/8 - (25*x^3)/8)/(2*x^2 + x^4 + 3) - (atan((x*(26007 - 2^(1/2
)*897i)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 - 1380483/16)) - (24219*2
^(1/2)*x*(26007 - 2^(1/2)*897i)^(1/2))/(128*((2^(1/2)*4286763i)/128 - 13804
83/16)))*(26007 - 2^(1/2)*897i)^(1/2)*3i)/16 + (atan((x*(2^(1/2)*897i + 260
07)^(1/2)*24219i)/(64*((2^(1/2)*4286763i)/128 + 1380483/16)) + (24219*2^(1/
2)*x*(2^(1/2)*897i + 26007)^(1/2))/(128*((2^(1/2)*4286763i)/128 + 1380483/1
6)))*(2^(1/2)*897i + 26007)^(1/2)*3i)/16 - (17*x^3)/3 + x^5
```

sympy [B] time = 1.36, size = 1205, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] x**5 - 17*x**3/3 + 19*x + (-25*x**3 + 75*x)/(8*x**4 + 16*x**2 + 24) - 3*sqr
t(26007/2048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-304*sqrt(2)*sqrt(8669 + 5
011*sqrt(3))/299 - 433349*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/1498289 + 152*s
qrt(3)*sqrt(8669 + 5011*sqrt(3))*sqrt(43440359*sqrt(3) + 75240962)/1498289)
 - 2882918249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521 -
993398584*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 4993637694
9404567/2244869927521 + 17261871038090*sqrt(3)/1343965233) + 3*sqrt(26007/2
048 + 15033*sqrt(3)/2048)*log(x**2 + x*(-152*sqrt(3)*sqrt(8669 + 5011*sqrt(
3))*sqrt(43440359*sqrt(3) + 75240962)/1498289 + 433349*sqrt(6)*sqrt(8669 +
5011*sqrt(3))/1498289 + 304*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/299) - 288291
8249387*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/2244869927521 - 993398584
*sqrt(6)*sqrt(43440359*sqrt(3) + 75240962)/1343965233 + 49936376949404567/2
244869927521 + 17261871038090*sqrt(3)/1343965233) - 2*sqrt(-27*sqrt(2)*sqrt
(43440359*sqrt(3) + 75240962)/1024 + 234063/2048 + 405891*sqrt(3)/2048)*ata
n(2996578*sqrt(3)*x/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) +
75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*s
qrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))) -
 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)*sqrt(-2*sqrt(2)*s
qrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(4344035
9*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8
669 + 15033*sqrt(3))) - 1300047*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/(17641*sq
rt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt
(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*
sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))) + 456*sqrt(8669 + 5011*sqrt(3)
)*sqrt(43440359*sqrt(3) + 75240962)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(434
40359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(
3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 1
5033*sqrt(3)))) - 2*sqrt(-27*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962)/1024
 + 234063/2048 + 405891*sqrt(3)/2048)*atan(2996578*sqrt(3)*x/(17641*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))
+ 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(
3) + 75240962) + 8669 + 15033*sqrt(3))) - 456*sqrt(8669 + 5011*sqrt(3))*sqr
t(43440359*sqrt(3) + 75240962)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359
*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) +
75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*
sqrt(3))) + 1300047*sqrt(2)*sqrt(8669 + 5011*sqrt(3))/(17641*sqrt(2)*sqrt(-
2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3)) + 152*s
qrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75
240962) + 8669 + 15033*sqrt(3))) + 1523344*sqrt(6)*sqrt(8669 + 5011*sqrt(3)
)/(17641*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(43440359*sqrt(3) + 75240962) + 8669 +
 15033*sqrt(3)) + 152*sqrt(43440359*sqrt(3) + 75240962)*sqrt(-2*sqrt(2)*sqr
t(43440359*sqrt(3) + 75240962) + 8669 + 15033*sqrt(3))))
```

3.111 $\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=232

$$\frac{5x^3}{3}-\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2+\sqrt{2\left(\right.}\right.$$

```
[Out] -17*x+5/3*x^3-25/8*x*(x^2+3)/(x^4+2*x^2+3)-1/64*ln(x^2+3^(1/2)-x*(-2+2*3^(1
/2))^(1/2))*(-28790+52998*3^(1/2))^(1/2)+1/64*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2
))^(1/2))*(-28790+52998*3^(1/2))^(1/2)-1/32*arctan((-2*x+(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)+1/32*arctan((2*x+(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(28790+52998*3^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^3}{3}-\frac{25\left(x^2+3\right)x}{8\left(x^4+2x^2+3\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-14395\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{2}\left(26499\sqrt{3}-1439\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) - (Sqrt[(14395 +
 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/16 + (Sqrt[(14395 + 26499*Sqrt[3])/2]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sqrt[(-14395 + 26499*Sqrt[3])/2]*Log[S
qrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-14395 + 26499*Sqrt[3
])/2]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = -\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{450-150x^2-336x^4+240x^6}{3+2x^2+x^4}\,dx$$

$$= -\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(-816+240x^2+\frac{6\left(483+127x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{8}\int \frac{483+127x^2}{3+2x^2+x^4}\,dx$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{\int \frac{483\sqrt{2\left(-1+\sqrt{3}\right)}-\left(483-127\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{\int \frac{483\sqrt{2(}}{\sqrt{}}}{1}$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{32}\left(127+161\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}}$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\log\left(\sqrt{3}-\right.$$

$$= -17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2(-}}{\sqrt{2}}\right.$$

Mathematica [C] time = 0.16, size = 129, normalized size = 0.56

$$\frac{5x^3}{3}-\frac{25(x^2+3)x}{8(x^4+2x^2+3)}-17x+\frac{(127\sqrt{2}-356i)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}+\frac{(127\sqrt{2}+356i)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] -17*x + (5*x^3)/3 - (25*x*(3 + x^2))/(8*(3 + 2*x^2 + x^4)) + ((-356*I + 127
*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) + ((3
56*I + 127*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[
2]])
```

fricas [B] time = 1.09, size = 508, normalized size = 2.19

$$\frac{2159655360\,x^7-17709173952\,x^5-123268\cdot 143883^{\frac{1}{4}}\sqrt{219}\,\sqrt{3}\,\sqrt{2}\left(x^4+2\,x^2+3\right)\sqrt{14395\,\sqrt{3}+79497}\,\arctan\Bigg(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/1295793216*(2159655360*x^7 - 17709173952*x^5 - 123268*143883^(1/4)*sqrt(2
19)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(14395*sqrt(3) + 79497)*arctan(1/
658350237832613766*sqrt(24746051)*143883^(3/4)*sqrt(219)*sqrt(11*143883^(1/
4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 22271445
9*x^2 + 222714459*sqrt(3))*(161*sqrt(3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*s
qrt(3) + 79497) - 1/8868084822*143883^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*
x - 127*sqrt(2)*x)*sqrt(14395*sqrt(3) + 79497) + 1/2*sqrt(3)*sqrt(2) - 1/2*
sqrt(2)) - 123268*143883^(1/4)*sqrt(219)*sqrt(3)*sqrt(2)*(x^4 + 2*x^2 + 3)*
sqrt(14395*sqrt(3) + 79497)*arctan(1/658350237832613766*sqrt(24746051)*1438
83^(3/4)*sqrt(219)*sqrt(-11*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*
sqrt(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3))*(161*sqrt(
3)*sqrt(2) - 127*sqrt(2))*sqrt(14395*sqrt(3) + 79497) - 1/8868084822*143883
^(3/4)*sqrt(219)*(161*sqrt(3)*sqrt(2)*x - 127*sqrt(2)*x)*sqrt(14395*sqrt(3)
 + 79497) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 143883^(1/4)*sqrt(219)*(79
497*x^4 + 158994*x^2 - 14395*sqrt(3)*(x^4 + 2*x^2 + 3) + 238491)*sqrt(14395
*sqrt(3) + 79497)*log(11*143883^(1/4)*sqrt(219)*(127*sqrt(3)*x - 483*x)*sqr
t(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3)) + 143883^(1/4
)*sqrt(219)*(79497*x^4 + 158994*x^2 - 14395*sqrt(3)*(x^4 + 2*x^2 + 3) + 238
491)*sqrt(14395*sqrt(3) + 79497)*log(-11*143883^(1/4)*sqrt(219)*(127*sqrt(3
)*x - 483*x)*sqrt(14395*sqrt(3) + 79497) + 222714459*x^2 + 222714459*sqrt(3
)) - 41627357064*x^3 - 78233515416*x)/(x^4 + 2*x^2 + 3)
```

giac [B] time = 1.85, size = 573, normalized size = 2.47

$$\frac{5}{3}\,x^3-\frac{1}{20736}\,\sqrt{2}\Bigg(127\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\,\sqrt{3}+18\right)^{\frac{3}{2}}+2286\cdot 3^{\frac{3}{4}}\sqrt{2}\,\sqrt{6\,\sqrt{3}+18}\left(\sqrt{3}-3\right)-2286\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\,\sqrt{3}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 228
6*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(3
```

```
) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 17388*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/20736*sqrt(2)*(127*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 22
86*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2286*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 127*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 17388
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 17388*3^(1/4)*sqrt(-6*sqrt(3) + 18)
)*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3
) + 1/2)) - 1/41472*sqrt(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqr
t(3) + 18) - 127*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*s
qrt(3) + 18)^(3/2) + 2286*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1738
8*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18
))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/41472*sqrt
(2)*(2286*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 127*3^(3/4)
*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 127*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 228
6*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 17388*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 17388*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x
*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2
 + 3)
```

maple [B] time = 0.03, size = 416, normalized size = 1.79

$$\frac{5x^3}{3}-17x-\frac{17\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{32\sqrt{2+2\sqrt{3}}}-\frac{89\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{16\sqrt{2+2\sqrt{3}}}+\frac{161\sqrt{3}\ \arctan\left(\right.}{8\sqrt{2+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5/3*x^3-17*x+(-25/8*x^3-75/8*x)/(x^4+2*x^2+3)-17/64*(-2+2*3^(1/2))^(1/2)*3^
(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-89/32*(-2+2*3^(1/2))^(1/2)*ln(
x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-17/32/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2)
)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-89/16/(2+2
*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/
2))^(1/2))+161/8/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/
2))/(2+2*3^(1/2))^(1/2))+17/64*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^
(1/2))^(1/2)*x+3^(1/2))+89/32*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1
/2)*x+3^(1/2))-17/32/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x
+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-89/16/(2+2*3^(1/2))^(1/2)*(-2+2
*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+161/8/(2+2
*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/
2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5}{3}\,x^3-17\,x-\frac{25\left(x^3+3\,x\right)}{8\left(x^4+2\,x^2+3\right)}+\frac{1}{8}\int\frac{127\,x^2+483}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 17*x - 25/8*(x^3 + 3*x)/(x^4 + 2*x^2 + 3) + 1/8*integrate((127*x^
2 + 483)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.09, size = 162, normalized size = 0.70

$$\frac{5\,x^3}{3}-\frac{\frac{25\,x^3}{8}+\frac{75\,x}{8}}{x^4+2\,x^2+3}-17\,x+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,30817\mathrm{i}}{64\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}-\frac{30817\,\sqrt{2}\,x\,\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}}{128\left(-\frac{1571667}{64}+\frac{\sqrt{2}\,14884611\mathrm{i}}{128}\right)}\right)\sqrt{-14395-\sqrt{2}\,30817\mathrm{i}}\,1}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] (atan((x*(- 2^(1/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/
128 - 1571667/64)) - (30817*2^(1/2)*x*(- 2^(1/2)*30817i - 14395)^(1/2))/(12
8*((2^(1/2)*14884611i)/128 - 1571667/64)))*(- 2^(1/2)*30817i - 14395)^(1/2)
*1i)/16 - ((75*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) - 17*x - (atan((x*(2^(1
/2)*30817i - 14395)^(1/2)*30817i)/(64*((2^(1/2)*14884611i)/128 + 1571667/64
)) + (30817*2^(1/2)*x*(2^(1/2)*30817i - 14395)^(1/2))/(128*((2^(1/2)*148846
11i)/128 + 1571667/64)))*(2^(1/2)*30817i - 14395)^(1/2)*1i)/16 + (5*x^3)/3
```

sympy [A] time = 0.61, size = 60, normalized size = 0.26

$$\frac{5x^3}{3}-17x+\frac{-25x^3-75x}{8x^4+16x^2+24}+\mathrm{RootSum}\left(1048576t^4+29480960t^2+2106591003,\left(t\mapsto t\log\left(\frac{557056t^3}{816619683}+\frac{1666}{816}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x**3/3 - 17*x + (-25*x**3 - 75*x)/(8*x**4 + 16*x**2 + 24) + RootSum(10485
76*_t**4 + 29480960*_t**2 + 2106591003, Lambda(_t, _t*log(557056*_t**3/8166
19683 + 166600064*_t/816619683 + x)))
```

3.112 $\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=225

$$-\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}\right.}\right.$$

```
[Out] 5*x+25/8*x*(x^2+1)/(x^4+2*x^2+3)-1/192*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2
))*(-115746+77394*3^(1/2))^(1/2)+1/192*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2
))*(-115746+77394*3^(1/2))^(1/2)+1/96*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)-1/96*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(115746+77394*3^(1/2))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1676, 1169, 634, 618, 204, 628}

$$\frac{25\left(x^2+1\right)x}{8\left(x^4+2x^2+3\right)}-\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{32}\sqrt{\frac{1}{6}\left(12899\sqrt{3}-19291\right)}\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*x*(1 + x^2))/(8*(3 + 2*x^2 + x^4)) + (Sqrt[(19291 + 12899*Sqrt[3]
)/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/16 - (Sq
rt[(19291 + 12899*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/16 - (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/32 + (Sqrt[(-19291 + 12899*Sqrt[3])/6]*Log[Sqrt
[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/32
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{-150-186x^2+240x^4}{3+2x^2+x^4}\,dx$$

$$= \frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(240-\frac{6\left(145+111x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{8}\int \frac{145+111x^2}{3+2x^2+x^4}\,dx$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{\int \frac{145\sqrt{2\left(-1+\sqrt{3}\right)}-\left(145-111\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{16\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int \frac{145\sqrt{2\left(-1+\sqrt{3}\right)}+\left(145-\right.}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}}}{16\sqrt{6\left(-1+\right.}}$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{96}\left(333+145\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx-$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right.}\right.$$

$$= 5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2}{\sqrt{2\left(1+\sqrt{3}\right)}}\right.$$

Mathematica [C] time = 0.16, size = 121, normalized size = 0.54

$$\frac{25\left(x^3+x\right)}{8\left(x^4+2x^2+3\right)}+5x-\frac{\left(111\sqrt{2}-34i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{16\sqrt{2-2i\sqrt{2}}}-\frac{\left(111\sqrt{2}+34i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{16\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] 5*x + (25*(x + x^3))/(8*(3 + 2*x^2 + x^4)) - ((-34*I + 111*Sqrt[2])*ArcTan[
x/Sqrt[1 - I*Sqrt[2]]])/(16*Sqrt[2 - (2*I)*Sqrt[2]]) - ((34*I + 111*Sqrt[2]
)*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(16*Sqrt[2 + (2*I)*Sqrt[2]])
```

fricas [B] time = 0.84, size = 459, normalized size = 2.04

$$98680445760\,x^5+31876\cdot 499152603^{\frac{1}{4}}\sqrt{2}\left(x^4+2\,x^2+3\right)\sqrt{248834609\,\sqrt{3}+499152603}\,\arctan\left(\frac{1}{245328660180}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/19736089152*(98680445760*x^5 + 31876*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2
 + 3)*sqrt(248834609*sqrt(3) + 499152603)*arctan(1/245328660180049420330 2*4
99152603^(3/4)*sqrt(308376393)*sqrt(308376393*x^2 + 499152603^(1/4)*(145*sq
rt(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3))*(
111*sqrt(3)*sqrt(2) - 145*sqrt(2))*sqrt(248834609*sqrt(3) + 499152603) - 1/
7955494186614*499152603^(3/4)*(111*sqrt(3)*sqrt(2)*x - 145*sqrt(2)*x)*sqrt(
248834609*sqrt(3) + 499152603) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 31876
*499152603^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(248834609*sqrt(3) + 4991526
03)*arctan(1/2453286601800494203302*499152603^(3/4)*sqrt(308376393)*sqrt(30
8376393*x^2 - 499152603^(1/4)*(145*sqrt(3)*x - 333*x)*sqrt(248834609*sqrt(3
) + 499152603) + 308376393*sqrt(3))*(111*sqrt(3)*sqrt(2) - 145*sqrt(2))*sqr
t(248834609*sqrt(3) + 499152603) - 1/7955494186614*499152603^(3/4)*(111*sqr
t(3)*sqrt(2)*x - 145*sqrt(2)*x)*sqrt(248834609*sqrt(3) + 499152603) - 1/2*s
qrt(3)*sqrt(2) + 1/2*sqrt(2)) + 259036170120*x^3 + 499152603^(1/4)*(19291*x
^4 + 38582*x^2 - 12899*sqrt(3)*(x^4 + 2*x^2 + 3) + 57873)*sqrt(248834609*sq
rt(3) + 499152603)*log(308376393*x^2 + 499152603^(1/4)*(145*sqrt(3)*x - 333
*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3)) - 499152603^(1
/4)*(19291*x^4 + 38582*x^2 - 12899*sqrt(3)*(x^4 + 2*x^2 + 3) + 57873)*sqrt(
248834609*sqrt(3) + 499152603)*log(308376393*x^2 - 499152603^(1/4)*(145*sqr
t(3)*x - 333*x)*sqrt(248834609*sqrt(3) + 499152603) + 308376393*sqrt(3)) +
357716615880*x)/(x^4 + 2*x^2 + 3)
```

giac [B] time = 1.82, size = 566, normalized size = 2.52

$$\frac{1}{6912}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/6912*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqr
t(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6
*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*
```

```
sqrt(6*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/
4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/6912
*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(2)*s
qrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(
3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 1740*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) + 1740*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x
- 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/13824*sqrt
(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*s
qrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^
(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqr
t(3) + 18) - 1740*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(
-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/13824*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt
(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2)
 + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sq
rt(3) - 3) - 1740*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 1740*3^(1/4)*sqrt
(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
 + 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3)
```

maple [B] time = 0.03, size = 412, normalized size = 1.83

$$5x-\frac{47\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{48\sqrt{2+2\sqrt{3}}}+\frac{17\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{32\sqrt{2+2\sqrt{3}}}-\frac{145\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{24\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] 5*x-(-25/8*x^3-25/8*x)/(x^4+2*x^2+3)-47/96*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(
x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+17/64*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2
*3^(1/2))^(1/2)*x+3^(1/2))-47/48/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)
*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+17/32/(2+2*3^(1/2))
^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2)
)-145/24/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))+47/96*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(
1/2)*x+3^(1/2))-17/64*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^
(1/2))-47/48/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+17/32/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2)
)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-145/24/(2+2*3^(1/2
))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$5\,x+\frac{25\,\left(x^{3}+x\right)}{8\,\left(x^{4}+2\,x^{2}+3\right)}-\frac{1}{8}\int\frac{111\,x^{2}+145}{x^{4}+2\,x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 5*x + 25/8*(x^3 + x)/(x^4 + 2*x^2 + 3) - 1/8*integrate((111*x^2 + 145)/(x^4
 + 2*x^2 + 3), x)
```

mupad [B] time = 0.96, size = 156, normalized size = 0.69

$$5\,x+\frac{\frac{25\,x^{3}}{8}+\frac{25\,x}{8}}{x^{4}+2\,x^{2}+3}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-57873-\sqrt{2}\,23907{}\mathrm{i}}\,7969{}\mathrm{i}}{576\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505{}\mathrm{i}}{384}\right)}+\frac{7969\,\sqrt{2}\,x\,\sqrt{-57873-\sqrt{2}\,23907{}\mathrm{i}}}{1152\,\left(-\frac{374543}{96}+\frac{\sqrt{2}\,1155505{}\mathrm{i}}{384}\right)}\right)\,\sqrt{-57873-\sqrt{2}\,23907{}\mathrm{i}}\,1{}\mathrm{i}}{48}-\ \mathrm{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] 5*x + ((25*x)/8 + (25*x^3)/8)/(2*x^2 + x^4 + 3) + (atan((x*(- 2^(1/2)*23907
i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 - 374543/96)) + (7969*
2^(1/2)*x*(- 2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384 -
 374543/96)))*(- 2^(1/2)*23907i - 57873)^(1/2)*1i)/48 - (atan((x*(2^(1/2)*2
3907i - 57873)^(1/2)*7969i)/(576*((2^(1/2)*1155505i)/384 + 374543/96)) - (7
969*2^(1/2)*x*(2^(1/2)*23907i - 57873)^(1/2))/(1152*((2^(1/2)*1155505i)/384
 + 374543/96)))*(2^(1/2)*23907i - 57873)^(1/2)*1i)/48
```

sympy [A] time = 0.60, size = 51, normalized size = 0.23

$$5x+\frac{25x^3+25x}{8x^4+16x^2+24}+\operatorname{RootSum}\left(3145728t^4+39507968t^2+166384201,\left(t\mapsto t\log\left(-\frac{9240576t^3}{102792131}-\frac{9500348}{1027921}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] 5*x + (25*x**3 + 25*x)/(8*x**4 + 16*x**2 + 24) + RootSum(3145728*_t**4 + 39
507968*_t**2 + 166384201, Lambda(_t, _t*log(-9240576*_t**3/102792131 - 9500
3488*_t/102792131 + x)))
```

3.113 $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=224

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right.}\right.$$

```
[Out] 25/24*x*(-x^2+1)/(x^4+2*x^2+3)-1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/288*arctan((2*x+(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-69402+77382*3^(1/2))^(1/2)+1/576*ln(x^2+
3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)-1/576*ln(x^2+3^
(1/2)+x*(-2+2*3^(1/2))^(1/2))*(69402+77382*3^(1/2))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1678, 1169, 634, 618, 204, 628}

$$\frac{25x\left(1-x^2\right)}{24\left(x^4+2x^2+3\right)}+\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\log$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (25*x*(1 - x^2))/(24*(3 + 2*x^2 + x^4)) - (Sqrt[(-11567 + 12897*Sqrt[3])/6]
*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 + (Sqrt[(
-11567 + 12897*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1
+ Sqrt[3])]])/48 + (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-
1 + Sqrt[3])]*x + x^2])/96 - (Sqrt[(11567 + 12897*Sqrt[3])/6]*Log[Sqrt[3] +
 Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{14+190x^2}{3+2x^2+x^4}\,dx$$

$$= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{\int \frac{14\sqrt{2\left(-1+\sqrt{3}\right)}-\left(14-190\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{\int \frac{14\sqrt{2\left(-1+\sqrt{3}\right)}+\left(14-190\sqrt{3}\right)x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,d}{96\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{\left(7-95\sqrt{3}\right)\int \frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{96\sqrt{6\left(-1+\sqrt{3}\right)}}+\frac{1}{288}\left(285+7\sqrt{3}\right)\int$$

$$= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{1}{96}\sqrt{\frac{11567}{6}+\frac{4299\sqrt{3}}{2}}\,\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2\right)-$$

$$= \frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)+$$

Mathematica [C] time = 0.26, size = 115, normalized size = 0.51

$$\frac{1}{48}\left(-\frac{50x\left(x^2-1\right)}{x^4+2x^2+3}+\frac{\left(95+44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(95-44i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] ((-50*x*(-1 + x^2))/(3 + 2*x^2 + x^4) + ((95 + (44*I)*Sqrt[2])*ArcTan[x/Sqr
t[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((95 - (44*I)*Sqrt[2])*ArcTan[x/Sq
rt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/48
```

fricas [B] time = 0.60, size = 454, normalized size = 2.03

$$-\frac{54052 \cdot 6160467^{\frac{1}{4}} \sqrt{2} \left(x^4 + 2\,x^2 + 3\right) \sqrt{-149179599\,\sqrt{3} + 498997827}\, \arctan\left(\frac{1}{29015889224422097862}\, \sqrt{19364129}\, 61\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/33461214912*(54052*6160467^(1/4)*sqrt(2)*(x^4 + 2*x^2 + 3)*sqrt(-1491795
99*sqrt(3) + 498997827)*arctan(1/29015889224422097862*sqrt(19364129)*616046
7^(3/4)*sqrt(174277161*x^2 + 6160467^(1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-1491
79599*sqrt(3) + 498997827) + 174277161*sqrt(3))*(95*sqrt(3)*sqrt(2) - 7*sqr
t(2))*sqrt(-149179599*sqrt(3) + 498997827) - 1/499478343426*6160467^(3/4)*(
95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(-149179599*sqrt(3) + 498997827) +
1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 54052*6160467^(1/4)*sqrt(2)*(x^4 + 2*x
^2 + 3)*sqrt(-149179599*sqrt(3) + 498997827)*arctan(1/29015889224422097862*
sqrt(19364129)*6160467^(3/4)*sqrt(174277161*x^2 - 6160467^(1/4)*(7*sqrt(3)*
x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 174277161*sqrt(3))*(95*sq
rt(3)*sqrt(2) - 7*sqrt(2))*sqrt(-149179599*sqrt(3) + 498997827) - 1/4994783
43426*6160467^(3/4)*(95*sqrt(3)*sqrt(2)*x - 7*sqrt(2)*x)*sqrt(-149179599*sq
rt(3) + 498997827) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 34855432200*x^3 -
 6160467^(1/4)*(11567*x^4 + 23134*x^2 + 12897*sqrt(3)*(x^4 + 2*x^2 + 3) + 3
4701)*sqrt(-149179599*sqrt(3) + 498997827)*log(174277161*x^2 + 6160467^(1/4
)*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 174277161*sq
rt(3)) + 6160467^(1/4)*(11567*x^4 + 23134*x^2 + 12897*sqrt(3)*(x^4 + 2*x^2
+ 3) + 34701)*sqrt(-149179599*sqrt(3) + 498997827)*log(174277161*x^2 - 6160
467^(1/4)*(7*sqrt(3)*x - 285*x)*sqrt(-149179599*sqrt(3) + 498997827) + 1742
77161*sqrt(3)) - 34855432200*x)/(x^4 + 2*x^2 + 3)
```

giac [B] time = 1.82, size = 565, normalized size = 2.52

$$-\frac{1}{62208}\,\sqrt{2}\left(95 \cdot 3^{\frac{3}{4}} \sqrt{2}\left(6\,\sqrt{3} + 18\right)^{\frac{3}{2}} + 1710 \cdot 3^{\frac{3}{4}} \sqrt{2}\,\sqrt{6\,\sqrt{3} + 18}\left(\sqrt{3} - 3\right) - 1710 \cdot 3^{\frac{3}{4}}\left(\sqrt{3} + 3\right)\sqrt{-6\,\sqrt{3} + 18} + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1/62208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/62
208*sqrt(2)*(95*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(
2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1710*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*
sqrt(3) + 18) + 95*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 252*3^(1/4)*sqrt(2)*sq
rt(6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/124416*
sqrt(2)*(1710*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3
/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1
710*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/124416*sqrt(2)*(1710*3^(3/4)*sqrt(2)
*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 95*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)
^(3/2) + 95*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1710*3^(3/4)*sqrt(6*sqrt(3) +
18)*(sqrt(3) - 3) - 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 252*3^(1/4)
```

```
*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqr
t(3)) - 25/24*(x^3 - x)/(x^4 + 2*x^2 + 3)
```

maple [B] time = 0.03, size = 408, normalized size = 1.82

$$\frac{139\left(-2+2\sqrt{3}\right)\sqrt{3}\,\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{288\sqrt{2+2\sqrt{3}}}+\frac{11\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{24\sqrt{2+2\sqrt{3}}}+\frac{7\sqrt{3}\,\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{72\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x)
```

```
[Out] (-25/24*x^3+25/24*x)/(x^4+2*x^2+3)+139/576*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(
x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+11/48*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2
*3^(1/2))^(1/2)*x+3^(1/2))+139/288/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/
2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+11/24/(2+2*3^(1/2
))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/
2))+7/72/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))-139/576*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))
^(1/2)*x+3^(1/2))-11/48*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+
3^(1/2))+139/288/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2
+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+11/24/(2+2*3^(1/2))^(1/2)*(-2+2*3^(
1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+7/72/(2+2*3^(1
/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{25\left(x^3-x\right)}{24\left(x^4+2\,x^2+3\right)}+\frac{1}{24}\int\frac{95\,x^2+7}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -25/24*(x^3 - x)/(x^4 + 2*x^2 + 3) + 1/24*integrate((95*x^2 + 7)/(x^4 + 2*x
^2 + 3), x)
```

mupad [B] time = 0.13, size = 153, normalized size = 0.68

$$\frac{\frac{25\,x}{24}-\frac{25\,x^3}{24}}{x^4+2\,x^2+3}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,13513\mathrm{i}}{15552\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}+\frac{13513\,\sqrt{2}\,x\,\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}}{31104\left(-\frac{1878307}{5184}+\frac{\sqrt{2}\,94591\mathrm{i}}{10368}\right)}\right)\sqrt{34701-\sqrt{2}\,40539\mathrm{i}}\,1\mathrm{i}}{144}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{\dots}}{15\dots}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] ((25*x)/24 - (25*x^3)/24)/(2*x^2 + x^4 + 3) - (atan((x*(34701 - 2^(1/2)*405
39i)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 - 1878307/5184)) + (13513
*2^(1/2)*x*(34701 - 2^(1/2)*40539i)^(1/2))/(31104*((2^(1/2)*94591i)/10368 -
 1878307/5184)))*(34701 - 2^(1/2)*40539i)^(1/2)*1i)/144 + (atan((x*(2^(1/2)
*40539i + 34701)^(1/2)*13513i)/(15552*((2^(1/2)*94591i)/10368 + 1878307/518
4)) - (13513*2^(1/2)*x*(2^(1/2)*40539i + 34701)^(1/2))/(31104*((2^(1/2)*945
91i)/10368 + 1878307/5184)))*(2^(1/2)*40539i + 34701)^(1/2)*1i)/144
```

sympy [B] time = 1.29, size = 1185, normalized size = 5.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-25*x**3 + 25*x)/(24*x**4 + 48*x**2 + 72) + sqrt(11567/55296 + 1433*sqrt(3
)/6144)*log(x**2 + x*(-556*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/13513 - 1040
345*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 278*sqrt(3)*sqrt(11567
+ 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161) - 476102762
00401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)/30372528846219921 - 43908
31246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658)/7065021829779 + 128104648
1635939181/30372528846219921 + 200684595453464*sqrt(3)/7065021829779) - sqr
t(11567/55296 + 1433*sqrt(3)/6144)*log(x**2 + x*(-278*sqrt(3)*sqrt(11567 +
12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/174277161 + 1040345*sqrt
(6)*sqrt(11567 + 12897*sqrt(3))/174277161 + 556*sqrt(2)*sqrt(11567 + 12897*
sqrt(3))/13513) - 47610276200401*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658
)/30372528846219921 - 4390831246*sqrt(6)*sqrt(149179599*sqrt(3) + 316396658
)/7065021829779 + 1281046481635939181/30372528846219921 + 200684595453464*s
qrt(3)/7065021829779) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqrt(3)*x/(94591*s
qrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*
sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149
179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 7170732*sqrt(6)*sqr
t(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt
(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 31
6396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 3869
1*sqrt(3))) - 3121035*sqrt(2)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sq
rt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))
+ 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sq
rt(3) + 316396658) + 11567 + 38691*sqrt(3))) + 834*sqrt(11567 + 12897*sqrt(
3))*sqrt(149179599*sqrt(3) + 316396658)/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt
(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(1491795
99*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658)
 + 11567 + 38691*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(149179599*sqrt(3) + 3163
96658)/27648 + 11567/55296 + 1433*sqrt(3)/2048)*atan(348554322*sqrt(3)*x/(9
4591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 +
38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sq
rt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3))) - 834*sqrt(1156
7 + 12897*sqrt(3))*sqrt(149179599*sqrt(3) + 316396658)/(94591*sqrt(2)*sqrt(
-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 2
78*sqrt(149179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(
3) + 316396658) + 11567 + 38691*sqrt(3))) + 3121035*sqrt(2)*sqrt(11567 + 12
897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396
658) + 11567 + 38691*sqrt(3)) + 278*sqrt(149179599*sqrt(3) + 316396658)*sqr
t(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)))
+ 7170732*sqrt(6)*sqrt(11567 + 12897*sqrt(3))/(94591*sqrt(2)*sqrt(-2*sqrt(2
)*sqrt(149179599*sqrt(3) + 316396658) + 11567 + 38691*sqrt(3)) + 278*sqrt(1
49179599*sqrt(3) + 316396658)*sqrt(-2*sqrt(2)*sqrt(149179599*sqrt(3) + 3163
96658) + 11567 + 38691*sqrt(3))))
```

3.114 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2} dx$

Optimal. Leaf size=229

$$-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x\right.$$

```
[Out] -4/9/x-25/72*x*(x^2+5)/(x^4+2*x^2+3)+1/288*arctan((-2*x+(-2+2*3^(1/2))^(1/2
))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/288*arctan((2*x+(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5790+4194*3^(1/2))^(1/2)-1/576*ln(x^
2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)+1/576*ln(x^2+3^
(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5790+4194*3^(1/2))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$-\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(x^2\right.$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) + (Sqrt[(-965 + 699*Sqrt
[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/48 -
(Sqrt[(-965 + 699*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*
(1 + Sqrt[3])]])/48 - (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1
 + Sqrt[3])]*x + x^2])/96 + (Sqrt[(965 + 699*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/96
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^2}\,dx = -\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{64+\frac{170x^2}{3}-\frac{50x^4}{3}}{x^2\left(3+2x^2+x^4\right)}\,dx$$

$$= -\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(\frac{64}{3x^2}-\frac{2\left(-7+19x^2\right)}{3+2x^2+x^4}\right)dx$$

$$= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{1}{24}\int \frac{-7+19x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{\int \frac{-7\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-7-19\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,dx}{48\sqrt{6\left(-1+\sqrt{3}\right)}}-\frac{\int \frac{-7\sqrt{2\left(-1+\sqrt{3}\right)}+\left(-7-19\sqrt{3}\right.}{\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}}{48\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(566-133\sqrt{3}\right)}\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,d$$

$$= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\sqrt{3}\right)}\log\left(\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right.$$

$$= -\frac{4}{9x}-\frac{25x\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}+\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right)-$$

Mathematica [C] time = 0.18, size = 126, normalized size = 0.55

$$-\frac{25x\left(x^2+5\right)}{72\left(x^4+2x^2+3\right)}-\frac{4}{9x}-\frac{\left(19\sqrt{2}+26i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{48\sqrt{2-2i\sqrt{2}}}-\frac{\left(19\sqrt{2}-26i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{48\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(9*x) - (25*x*(5 + x^2))/(72*(3 + 2*x^2 + x^4)) - ((26*I + 19*Sqrt[2])*A
rcTan[x/Sqrt[1 - I*Sqrt[2]]])/(48*Sqrt[2 - (2*I)*Sqrt[2]]) - ((-26*I + 19*S
qrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(48*Sqrt[2 + (2*I)*Sqrt[2]])
```

fricas [B] time = 0.83, size = 471, normalized size = 2.06

$$-\frac{164790648\,x^4-2068\cdot 1465803^{\frac{1}{4}}\sqrt{2}\left(x^5+2\,x^3+3\,x\right)\sqrt{-674535\,\sqrt{3}+1465803}\,\arctan\left(\frac{1}{547726639257666}\cdot 146\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/208156608*(164790648*x^4 - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x
)*sqrt(-674535*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sq
rt(120461)*sqrt(1084149*x^2 + 1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqrt(-6745
35*sqrt(3) + 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqrt(2))*s
qrt(-674535*sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt(3)*sqr
t(2)*x + 7*sqrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) - 1/2*sqrt(3)*sqrt(2)
 + 1/2*sqrt(2)) - 2068*1465803^(1/4)*sqrt(2)*(x^5 + 2*x^3 + 3*x)*sqrt(-6745
35*sqrt(3) + 1465803)*arctan(1/547726639257666*1465803^(3/4)*sqrt(120461)*s
qrt(1084149*x^2 - 1465803^(1/4)*(7*sqrt(3)*x + 57*x)*sqrt(-674535*sqrt(3) +
 1465803) + 1084149*sqrt(3))*(19*sqrt(3)*sqrt(2) + 7*sqrt(2))*sqrt(-674535*
sqrt(3) + 1465803) - 1/1515640302*1465803^(3/4)*(19*sqrt(3)*sqrt(2)*x + 7*s
qrt(2)*x)*sqrt(-674535*sqrt(3) + 1465803) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(
2)) - 1465803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 + 2*x^3 + 3*x) +
 2895*x)*sqrt(-674535*sqrt(3) + 1465803)*log(1084149*x^2 + 1465803^(1/4)*(7
*sqrt(3)*x + 57*x)*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3)) + 146
5803^(1/4)*(965*x^5 + 1930*x^3 + 699*sqrt(3)*(x^5 + 2*x^3 + 3*x) + 2895*x)*
sqrt(-674535*sqrt(3) + 1465803)*log(1084149*x^2 - 1465803^(1/4)*(7*sqrt(3)*
x + 57*x)*sqrt(-674535*sqrt(3) + 1465803) + 1084149*sqrt(3)) + 546411096*x^
2 + 277542144)/(x^5 + 2*x^3 + 3*x)
```

giac [B] time = 1.94, size = 572, normalized size = 2.50

$$\frac{1}{62208}\sqrt{2}\left(19\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+342\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-342\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/62208*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-
6*sqrt(3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*
sqrt(6*sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4
)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/62208
```

```
*sqrt(2)*(19*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(2)*s
qrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 342*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(
3) + 18) + 19*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 252*3^(1/4)*sqrt(2)*sqrt(6*
sqrt(3) + 18) - 252*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x -
3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/124416*sqrt(
2)*(342*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sq
rt(2)*(-6*sqrt(3) + 18)^(3/2) + 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(
3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(
3) + 18) + 252*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/
6*sqrt(3) + 1/2) + sqrt(3)) - 1/124416*sqrt(2)*(342*3^(3/4)*sqrt(2)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) - 19*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) +
 19*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 342*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt
(3) - 3) + 252*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 252*3^(1/4)*sqrt(6*s
qrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1
/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x)
```

maple [B] time = 0.03, size = 414, normalized size = 1.81

$$-\frac{\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{9\sqrt{2+2\sqrt{3}}}-\frac{13\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{96\sqrt{2+2\sqrt{3}}}+\frac{7\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{72\sqrt{2+2\sqrt{3}}}-\frac{(-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x)
```

```
[Out] -4/9/x-1/9*(25/8*x^3+125/8*x)/(x^4+2*x^2+3)-1/18*(-2+2*3^(1/2))^(1/2)*3^(1/
2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-13/192*(-2+2*3^(1/2))^(1/2)*ln(x^
2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-1/9/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^
(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-13/96/(2+2*3^(
1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2))+7/72/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(
2+2*3^(1/2))^(1/2))+1/18*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))
^(1/2)*x+3^(1/2))+13/192*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x
+3^(1/2))-1/9/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*
3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-13/96/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2
))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+7/72/(2+2*3^(1/2)
)^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{19\,x^4+63\,x^2+32}{24\left(x^5+2\,x^3+3\,x\right)}-\frac{1}{24}\int\frac{19\,x^2-7}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/24*(19*x^4 + 63*x^2 + 32)/(x^5 + 2*x^3 + 3*x) - 1/24*integrate((19*x^2 -
 7)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.14, size = 159, normalized size = 0.69

$$-\frac{\frac{19\,x^4}{24}+\frac{21\,x^2}{8}+\frac{4}{3}}{x^5+2\,x^3+3\,x}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,517\mathrm{i}}{15552\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}+\frac{517\,\sqrt{2}\,x\,\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}}{31104\left(\frac{517}{162}+\frac{\sqrt{2}\,3619\mathrm{i}}{10368}\right)}\right)\sqrt{2895-\sqrt{2}\,1551\mathrm{i}}\,1\mathrm{i}}{144}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{2895+\sqrt{2}\,\cdots}}{15552\left(-\frac{517}{162}+\cdots\right)}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (atan((x*(2^(1/2)*1551i + 2895)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 -
 517/162)) - (517*2^(1/2)*x*(2^(1/2)*1551i + 2895)^(1/2))/(31104*((2^(1/2)*
3619i)/10368 - 517/162)))*(2^(1/2)*1551i + 2895)^(1/2)*1i)/144 - (atan((x*(
2895 - 2^(1/2)*1551i)^(1/2)*517i)/(15552*((2^(1/2)*3619i)/10368 + 517/162))
 + (517*2^(1/2)*x*(2895 - 2^(1/2)*1551i)^(1/2))/(31104*((2^(1/2)*3619i)/103
68 + 517/162)))*(2895 - 2^(1/2)*1551i)^(1/2)*1i)/144 - ((21*x^2)/8 + (19*x^
4)/24 + 4/3)/(3*x + 2*x^3 + x^5)
```

sympy [B] time = 1.32, size = 1192, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**2,x)
```

```
[Out] (-19*x**4 - 63*x**2 - 32)/(24*x**5 + 48*x**3 + 72*x) - sqrt(965/55296 + 233
*sqrt(3)/18432)*log(x**2 + x*(-128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517 - 21
793*sqrt(6)*sqrt(965 + 699*sqrt(3))/361383 + 64*sqrt(3)*sqrt(965 + 699*sqrt
(3))*sqrt(674535*sqrt(3) + 1198514)/361383) - 8882635459*sqrt(2)*sqrt(67453
5*sqrt(3) + 1198514)/130597672689 - 20458048*sqrt(6)*sqrt(674535*sqrt(3) +
1198514)/560505033 + 18567565928783/130597672689 + 46950427730*sqrt(3)/5605
05033) + sqrt(965/55296 + 233*sqrt(3)/18432)*log(x**2 + x*(-64*sqrt(3)*sqrt
(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/361383 + 21793*sqrt(6)*s
qrt(965 + 699*sqrt(3))/361383 + 128*sqrt(2)*sqrt(965 + 699*sqrt(3))/517) -
8882635459*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/130597672689 - 20458048*s
qrt(6)*sqrt(674535*sqrt(3) + 1198514)/560505033 + 18567565928783/1305976726
89 + 46950427730*sqrt(3)/560505033) + 2*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) +
 1198514)/27648 + 965/55296 + 233*sqrt(3)/6144)*atan(722766*sqrt(3)*x/(-64*
sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 119851
4) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3)
 + 1198514) + 965 + 2097*sqrt(3))) + 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/
(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1
198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sq
rt(3) + 1198514) + 965 + 2097*sqrt(3))) + 65379*sqrt(2)*sqrt(965 + 699*sqrt
(3))/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3
) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(6745
35*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) - 192*sqrt(965 + 699*sqrt(3))*
sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)))) + 2
*sqrt(-sqrt(2)*sqrt(674535*sqrt(3) + 1198514)/27648 + 965/55296 + 233*sqrt(
3)/6144)*atan(722766*sqrt(3)*x/(-64*sqrt(674535*sqrt(3) + 1198514)*sqrt(-2*
sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3)) + 3619*sqrt(2)
*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 2097*sqrt(3))) + 19
2*sqrt(965 + 699*sqrt(3))*sqrt(674535*sqrt(3) + 1198514)/(-64*sqrt(674535*s
qrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965 + 20
97*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) +
 965 + 2097*sqrt(3))) - 65379*sqrt(2)*sqrt(965 + 699*sqrt(3))/(-64*sqrt(674
535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514) + 965
 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 11985
14) + 965 + 2097*sqrt(3))) - 89472*sqrt(6)*sqrt(965 + 699*sqrt(3))/(-64*sqr
t(674535*sqrt(3) + 1198514)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) + 1198514)
+ 965 + 2097*sqrt(3)) + 3619*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(674535*sqrt(3) +
1198514) + 965 + 2097*sqrt(3))))
```

3.115 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=238

$$-\frac{4}{27x^3}+\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2+\sqrt{2\ldots}\right.$$

```
[Out] -4/27/x^3+13/27/x+25/216*x*(5*x^2+7)/(x^4+2*x^2+3)+1/5184*ln(x^2+3^(1/2)-x*
(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/5184*ln(x^2+3^(1/2)+x
*(-2+2*3^(1/2))^(1/2))*(-36438+340038*3^(1/2))^(1/2)-1/2592*arctan((-2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038*3^(1/2))^(1/2)+1/259
2*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(36438+340038*3^(1
/2))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x\left(5x^2+7\right)}{216\left(x^4+2x^2+3\right)}-\frac{4}{27x^3}+\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-6073\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{864}\sqrt{\frac{1}{6}\left(56673\sqrt{3}-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(27*x^3) + 13/(27*x) + (25*x*(7 + 5*x^2))/(216*(3 + 2*x^2 + x^4)) - (Sqr
t[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1
 + Sqrt[3])]])/432 + (Sqrt[(6073 + 56673*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + S
qrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/432 + (Sqrt[(-6073 + 56673*Sqrt[3])
/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864 - (Sqrt[(-6073 + 566
73*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/864
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \frac{64-\frac{80x^2}{3}+\frac{50x^4}{9}+\frac{250x^6}{9}}{x^4\left(3+2x^2+x^4\right)}\,dx$$

$$= \frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)} + \frac{1}{48}\int \left(\frac{64}{3x^4}-\frac{208}{9x^2}+\frac{2\left(137+229x^2\right)}{9\left(3+2x^2+x^4\right)}\right)dx$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{216}\int \frac{137+229x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{\int \frac{137\sqrt{2\left(-1+\sqrt{3}\right)}-\left(137-229\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,dx}{432\sqrt{6\left(-1+\sqrt{3}\right)}} + \cdots$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{432}\sqrt{\frac{1}{6}\left(88046+31373\sqrt{3}\right)}\int \frac{\cdots}{\sqrt{3}-\sqrt{2}\cdots}$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{1}{864}\sqrt{\frac{1}{6}\left(-6073+56673\sqrt{3}\right)}\log\left(\sqrt{3}-\cdots\right.$$

$$= -\frac{4}{27x^3}+\frac{13}{27x}+\frac{25x\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}-\frac{1}{432}\sqrt{\frac{1}{6}\left(6073+56673\sqrt{3}\right)}\tan^{-1}\left(\frac{\sqrt{2\left(-\cdots\right.}}{\sqrt{2}\cdots}\right.$$

Mathematica [C] time = 0.29, size = 131, normalized size = 0.55

$$\frac{1}{864}\left(\frac{4\left(229x^6+351x^4+248x^2-96\right)}{x^3\left(x^4+2x^2+3\right)}+\frac{2\left(229+46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{2\left(229-46i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((4*(-96 + 248*x^2 + 351*x^4 + 229*x^6))/(x^3*(3 + 2*x^2 + x^4)) + (2*(229
+ (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (2*(
229 - (46*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/8
64
```

fricas [B] time = 0.85, size = 528, normalized size = 2.22

$$\frac{2397560030424\,x^6+3674862754056\,x^4-277108\cdot 118956627^{\frac{1}{4}}\sqrt{6297}\,\sqrt{2}\left(x^7+2\,x^5+3\,x^3\right)\sqrt{6073\,\sqrt{3}\,+17001}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/2261454002496*(2397560030424*x^6 + 3674862754056*x^4 - 277108*118956627^(
1/4)*sqrt(6297)*sqrt(2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*a
rctan(1/295480530439458889122*118956627^(3/4)*sqrt(81861)*sqrt(6297)*sqrt(3
*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170
019) + 3926135421*x^2 + 3926135421*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137*sqrt
(2))*sqrt(6073*sqrt(3) + 170019) - 1/16481916497358*118956627^(3/4)*sqrt(62
97)*(229*sqrt(3)*sqrt(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019) + 1
/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 277108*118956627^(1/4)*sqrt(6297)*sqrt(
2)*(x^7 + 2*x^5 + 3*x^3)*sqrt(6073*sqrt(3) + 170019)*arctan(1/2954805304394
58889122*118956627^(3/4)*sqrt(81861)*sqrt(6297)*sqrt(-3*118956627^(1/4)*sqr
t(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^
2 + 3926135421*sqrt(3))*(229*sqrt(3)*sqrt(2) - 137*sqrt(2))*sqrt(6073*sqrt(
3) + 170019) - 1/16481916497358*118956627^(3/4)*sqrt(6297)*(229*sqrt(3)*sqr
t(2)*x - 137*sqrt(2)*x)*sqrt(6073*sqrt(3) + 170019) - 1/2*sqrt(3)*sqrt(2) +
 1/2*sqrt(2)) - 118956627^(1/4)*sqrt(6297)*(6073*x^7 + 12146*x^5 + 18219*x^
3 - 56673*sqrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(6073*sqrt(3) + 170019)*log(3*
118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 687*x)*sqrt(6073*sqrt(3) + 1700
19) + 3926135421*x^2 + 3926135421*sqrt(3)) + 118956627^(1/4)*sqrt(6297)*(60
73*x^7 + 12146*x^5 + 18219*x^3 - 56673*sqrt(3)*(x^7 + 2*x^5 + 3*x^3))*sqrt(
6073*sqrt(3) + 170019)*log(-3*118956627^(1/4)*sqrt(6297)*(137*sqrt(3)*x - 6
87*x)*sqrt(6073*sqrt(3) + 170019) + 3926135421*x^2 + 3926135421*sqrt(3)) +
2596484225088*x^2 - 1005090667776)/(x^7 + 2*x^5 + 3*x^3)
```

giac [B] time = 1.85, size = 579, normalized size = 2.43

$$-\frac{1}{559872}\sqrt{2}\left(229\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+4122\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-4122\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] -1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/
3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/559872*sqrt(2)*(229*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4122*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 229*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 4932*3^(1/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18) + 4932*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/
3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) -
 1/1119744*sqrt(2)*(4122*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18
) - 229*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) +
18)^(3/2) + 4122*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*
sqrt(2)*sqrt(-6*sqrt(3) + 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2
+ 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/1119744*sqrt(2)*(4122
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 229*3^(3/4)*sqrt(2)*
(-6*sqrt(3) + 18)^(3/2) + 229*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 4122*3^(3/4)
*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 4932*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3)
+ 18) - 4932*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*
sqrt(3) + 1/2) + sqrt(3)) + 25/216*(5*x^3 + 7*x)/(x^4 + 2*x^2 + 3) + 1/27*(
13*x^2 - 4)/x^3
```

maple [B] time = 0.04, size = 419, normalized size = 1.76

$$\frac{275\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{2592\sqrt{2+2\sqrt{3}}}+\frac{23\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{432\sqrt{2+2\sqrt{3}}}+\frac{137\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{648\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x)
```

```
[Out] -4/27/x^3+13/27/x+1/27*(125/8*x^3+175/8*x)/(x^4+2*x^2+3)+275/5184*(-2+2*3^(
1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+23/864*(-2+2*3^(
1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+275/2592/(2+2*3^(1/2))^(
1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))
^(1/2))+23/432/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2)
)^(1/2))/(2+2*3^(1/2))^(1/2))+137/648/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2
*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-275/5184*(-2+2*3^(1/2))^(1/2)
*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-23/864*(-2+2*3^(1/2))^(1/2)
*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+275/2592/(2+2*3^(1/2))^(1/2)*(-2+2*
3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+23/
432/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))+137/648/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^
(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{229\,x^6+351\,x^4+248\,x^2-96}{216\left(x^7+2\,x^5+3\,x^3\right)}+\frac{1}{216}\int\frac{229\,x^2+137}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/216*(229*x^6 + 351*x^4 + 248*x^2 - 96)/(x^7 + 2*x^5 + 3*x^3) + 1/216*inte
grate((229*x^2 + 137)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.14, size = 165, normalized size = 0.69

$$\frac{\frac{229\,x^6}{216}+\frac{13\,x^4}{8}+\frac{31\,x^2}{27}-\frac{4}{9}}{x^7+2\,x^5+3\,x^3}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}\,69277\mathrm{i}}{11337408\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}+\frac{69277\,\sqrt{2}\,x\,\sqrt{-18219-\sqrt{2}\,207831\mathrm{i}}}{22674816\left(-\frac{19051175}{3779136}+\frac{\sqrt{2}\,9490949\mathrm{i}}{7558272}\right)}\right)\sqrt{-18219-\sqrt{2}\,2078}}{1296}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] ((31*x^2)/27 + (13*x^4)/8 + (229*x^6)/216 - 4/9)/(3*x^3 + 2*x^5 + x^7) - (a
tan((x*(- 2^(1/2)*207831i - 18219)^(1/2)*69277i)/(11337408*((2^(1/2)*949094
9i)/7558272 - 19051175/3779136)) + (69277*2^(1/2)*x*(- 2^(1/2)*207831i - 18
219)^(1/2))/(22674816*((2^(1/2)*9490949i)/7558272 - 19051175/3779136)))*(-
2^(1/2)*207831i - 18219)^(1/2)*1i)/1296 + (atan((x*(2^(1/2)*207831i - 18219
)^(1/2)*69277i)/(11337408*((2^(1/2)*9490949i)/7558272 + 19051175/3779136))
- (69277*2^(1/2)*x*(2^(1/2)*207831i - 18219)^(1/2))/(22674816*((2^(1/2)*949
0949i)/7558272 + 19051175/3779136)))*(2^(1/2)*207831i - 18219)^(1/2)*1i)/12
96
```

sympy [A] time = 0.65, size = 60, normalized size = 0.25

$$\mathrm{RootSum}\left(2293235712t^4+12437504t^2+4405801,\left(t\mapsto t\log\left(\frac{19707494400t^3}{145412423}+\frac{357152768t}{145412423}+x\right)\right)\right)+\frac{229x^6+}{216x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**2,x)
```

```
[Out] RootSum(2293235712*_t**4 + 12437504*_t**2 + 4405801, Lambda(_t, _t*log(1970
7494400*_t**3/145412423 + 357152768*_t/145412423 + x))) + (229*x**6 + 351*x
**4 + 248*x**2 - 96)/(216*x**7 + 432*x**5 + 648*x**3)
```

3.116 $\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx$

Optimal. Leaf size=245

$$-\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log}{259}$$

```
[Out] -4/45/x^5+13/81/x^3-13/27/x+25/648*x*(-7*x^2+1)/(x^4+2*x^2+3)+1/7776*arctan
((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-6836286+4130514*3^(1/2)
)^(1/2)-1/7776*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-683
6286+4130514*3^(1/2))^(1/2)-1/15552*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*
(6836286+4130514*3^(1/2))^(1/2)+1/15552*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/
2))*(6836286+4130514*3^(1/2))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x\left(1-7x^2\right)}{648\left(x^4+2x^2+3\right)}+\frac{13}{81x^3}-\frac{4}{45x^5}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{2592}+\sqrt{\frac{1}{6}\left(113938\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] -4/(45*x^5) + 13/(81*x^3) - 13/(27*x) + (25*x*(1 - 7*x^2))/(648*(3 + 2*x^2
+ x^4)) + (Sqrt[(-1139381 + 688419*Sqrt[3])/6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3]
)] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sqrt[(-1139381 + 688419*Sqrt[3])/
6]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/1296 - (Sq
rt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x
^2])/2592 + (Sqrt[(1139381 + 688419*Sqrt[3])/6]*Log[Sqrt[3] + Sqrt[2*(-1 +
Sqrt[3])]*x + x^2])/2592
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^6\left(3+2x^2+x^4\right)^2}\,dx = \frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \frac{64-\frac{80x^2}{3}+\frac{400x^4}{9}+\frac{1550x^6}{27}-\frac{350x^8}{27}}{x^6\left(3+2x^2+x^4\right)}\,dx$$

$$= \frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{1}{48}\int \left(\frac{64}{3x^6}-\frac{208}{9x^4}+\frac{208}{9x^2}-\frac{2\left(-463+487x^2\right)}{27\left(3+2x^2+x^4\right)}\right)dx$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{1}{648}\int \frac{-463+487x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{\int \frac{-463\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-463-487\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,dx}{1296\sqrt{6\left(-1+\sqrt{3}\right)}}-$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{\left(1461-463\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+}}{7776}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{6}\left(1139381+688419\sqrt{3}\right)}\log\left(\sqrt{\right.}}{2592}$$

$$= -\frac{4}{45x^5}+\frac{13}{81x^3}-\frac{13}{27x}+\frac{25x\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{6}\left(-1139381+688419\sqrt{3}\right)}\tan^{-}}{1296}$$

Mathematica [C] time = 0.29, size = 140, normalized size = 0.57

$$\frac{-\frac{4\left(2435x^8+2475x^6+3928x^4-984x^2+864\right)}{x^5\left(x^4+2x^2+3\right)}-\frac{10i\left(475\sqrt{2}-487i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{10i\left(475\sqrt{2}+487i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{12960}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^6*(3 + 2*x^2 + x^4)^2),x]
```

```
[Out] ((-4*(864 - 984*x^2 + 3928*x^4 + 2475*x^6 + 2435*x^8))/(x^5*(3 + 2*x^2 + x^
4)) - ((10*I)*(-487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1
- I*Sqrt[2]] + ((10*I)*(487*I + 475*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])
/Sqrt[1 + I*Sqrt[2]])/12960
```

fricas [B] time = 0.79, size = 496, normalized size = 2.02

$$-\frac{1111136748188760\,x^8+1129389507912600\,x^6+1792421004881088\,x^4-4971380\cdot 216699003^{\frac{1}{4}}\sqrt{2}\left(x^9+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] -1/1478473537631040*(1111136748188760*x^8 + 1129389507912600*x^6 + 17924210
04881088*x^4 - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x^7 + 3*x^5)*sqrt(-
784371528639*sqrt(3) + 1421762158683)*arctan(1/614486622356872175645371 8*sq
```

```
rt(704195977)*216699003^(3/4)*sqrt(57039874137*x^2 + 216699003^(1/4)*(463*s
qrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 5703987413
7*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sqrt(-784371528639*sqrt(3) +
 1421762158683) - 1/969563780580726*216699003^(3/4)*(487*sqrt(3)*sqrt(2)*x
+ 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) - 1/2*sqrt(3)*
sqrt(2) + 1/2*sqrt(2)) - 4971380*216699003^(1/4)*sqrt(2)*(x^9 + 2*x^7 + 3*x
^5)*sqrt(-784371528639*sqrt(3) + 1421762158683)*arctan(1/614486622356872175
6453718*sqrt(704195977)*216699003^(3/4)*sqrt(57039874137*x^2 - 216699003^(1
/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) +
57039874137*sqrt(3))*(487*sqrt(3)*sqrt(2) + 463*sqrt(2))*sqrt(-784371528639
*sqrt(3) + 1421762158683) - 1/969563780580726*216699003^(3/4)*(487*sqrt(3)*
sqrt(2)*x + 463*sqrt(2)*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 1/
2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 5*216699003^(1/4)*(1139381*x^9 + 2278762
*x^7 + 3418143*x^5 + 688419*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*sqrt(-7843715286
39*sqrt(3) + 1421762158683)*log(57039874137*x^2 + 216699003^(1/4)*(463*sqrt
(3)*x + 1461*x)*sqrt(-784371528639*sqrt(3) + 1421762158683) + 57039874137*s
qrt(3)) + 5*216699003^(1/4)*(1139381*x^9 + 2278762*x^7 + 3418143*x^5 + 6884
19*sqrt(3)*(x^9 + 2*x^7 + 3*x^5))*sqrt(-784371528639*sqrt(3) + 142176215868
3)*log(57039874137*x^2 - 216699003^(1/4)*(463*sqrt(3)*x + 1461*x)*sqrt(-784
371528639*sqrt(3) + 1421762158683) + 57039874137*sqrt(3)) - 449017889206464
*x^2 + 394259610034944)/(x^9 + 2*x^7 + 3*x^5)
```

giac [B] time = 1.78, size = 584, normalized size = 2.38

$$\frac{1}{1679616}\sqrt{2}\left(487\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+8766\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-8766\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(3/4
)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(
1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/1679616*sqrt(2)*(487*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 8766*3^(
3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 8766*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 487*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 16668*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 16668*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/3359232*sqrt(2)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
 + 18) - 487*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 8766*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^
(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*l
og(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/3359232*sqrt(2
)*(8766*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 487*3^(3/4)*s
qrt(2)*(-6*sqrt(3) + 18)^(3/2) + 487*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 8766*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 16668*3^(1/4)*sqrt(2)*sqrt(-6*
sqrt(3) + 18) + 16668*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*s
qrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 25/648*(7*x^3 - x)/(x^4 + 2*x^2 + 3) -
 1/405*(195*x^4 - 65*x^2 + 36)/x^5
```

maple [B] time = 0.03, size = 424, normalized size = 1.73

$$-\frac{481\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{3888\sqrt{2+2\sqrt{3}}}-\frac{475\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{2592\sqrt{2+2\sqrt{3}}}+\frac{463\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{1944\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x)
```

```
[Out] -4/45/x^5+13/81/x^3-13/27/x-1/27*(175/24*x^3-25/24*x)/(x^4+2*x^2+3)-481/777
6*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-475/5
184*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-481/3888/(2
+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/
(2+2*3^(1/2))^(1/2))-475/2592/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*
x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+463/1944/(2+2*3^(1/2))^(1/2)*3
^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+481/7776*(-2+
2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+475/5184*(-
2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-481/3888/(2+2*3^(
1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))-475/2592/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+463/1944/(2+2*3^(1/2))^(1/2)*3^(1/2)
*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2435\,x^{8}+2475\,x^{6}+3928\,x^{4}-984\,x^{2}+864}{3240\left(x^{9}+2\,x^{7}+3\,x^{5}\right)}-\frac{1}{648}\int\frac{487\,x^{2}-463}{x^{4}+2\,x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^6/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] -1/3240*(2435*x^8 + 2475*x^6 + 3928*x^4 - 984*x^2 + 864)/(x^9 + 2*x^7 + 3*x
^5) - 1/648*integrate((487*x^2 - 463)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.14, size = 171, normalized size = 0.70

$$-\frac{\frac{487\,x^{8}}{648}+\frac{55\,x^{6}}{72}+\frac{491\,x^{4}}{405}-\frac{41\,x^{2}}{135}+\frac{4}{15}}{x^{9}+2\,x^{7}+3\,x^{5}}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}\,248569\mathrm{i}}{306110016\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}+\frac{248569\,\sqrt{2}\,x\,\sqrt{3418143-\sqrt{2}\,745707\mathrm{i}}}{612220032\left(\frac{119561689}{51018336}+\frac{\sqrt{2}\,115087447\mathrm{i}}{204073344}\right)}\right)\sqrt{34}}{3888}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^6*(2*x^2 + x^4 + 3)^2),x)
```

```
[Out] (atan((x*(2^(1/2)*745707i + 3418143)^(1/2)*248569i)/(306110016*((2^(1/2)*11
5087447i)/204073344 - 119561689/51018336)) - (248569*2^(1/2)*x*(2^(1/2)*745
707i + 3418143)^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 - 1195616
89/51018336)))*(2^(1/2)*745707i + 3418143)^(1/2)*1i)/3888 - (atan((x*(34181
43 - 2^(1/2)*745707i)^(1/2)*248569i)/(306110016*((2^(1/2)*115087447i)/20407
3344 + 119561689/51018336)) + (248569*2^(1/2)*x*(3418143 - 2^(1/2)*745707i)
^(1/2))/(612220032*((2^(1/2)*115087447i)/204073344 + 119561689/51018336)))*
(3418143 - 2^(1/2)*745707i)^(1/2)*1i)/3888 - ((491*x^4)/405 - (41*x^2)/135
+ (55*x^6)/72 + (487*x^8)/648 + 4/15)/(3*x^5 + 2*x^7 + x^9)
```

sympy [B] time = 1.33, size = 1202, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**6/(x**4+2*x**2+3)**2,x)
```

```
[Out] -sqrt(1139381/40310784 + 2833*sqrt(3)/165888)*log(x**2 + x*(-3848*sqrt(2)*s
qrt(1139381 + 688419*sqrt(3))/248569 - 769085497*sqrt(6)*sqrt(1139381 + 688
419*sqrt(3))/171119622411 + 1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sqr
```

```
t(784371528639*sqrt(3) + 1359975610922)/171119622411) - 8677510907569510603
*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/29281925174083213452921
 - 21752950947364*sqrt(6)*sqrt(784371528639*sqrt(3) + 1359975610922)/127605
100269239577 + 20196165220927340076543947/29281925174083213452921 + 5094503
6826336313070*sqrt(3)/127605100269239577) + sqrt(1139381/40310784 + 2833*sq
rt(3)/165888)*log(x**2 + x*(-1924*sqrt(3)*sqrt(1139381 + 688419*sqrt(3))*sq
rt(784371528639*sqrt(3) + 1359975610922)/171119622411 + 769085497*sqrt(6)*s
qrt(1139381 + 688419*sqrt(3))/171119622411 + 3848*sqrt(2)*sqrt(1139381 + 68
8419*sqrt(3))/248569) - 8677510907569510603*sqrt(2)*sqrt(784371528639*sqrt(
3) + 1359975610922)/29281925174083213452921 - 21752950947364*sqrt(6)*sqrt(7
84371528639*sqrt(3) + 1359975610922)/127605100269239577 + 20196165220927340
076543947/29281925174083213452921 + 50945036826336313070*sqrt(3)/1276051002
69239577) + 2*sqrt(-sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922)/2015
5392 + 1139381/40310784 + 2833*sqrt(3)/55296)*atan(342239244822*sqrt(3)*x/(
-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(78437
1528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*s
qrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381
 + 2065257*sqrt(3))) + 2649036312*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/(-
1924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371
528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sq
rt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381
+ 2065257*sqrt(3))) + 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1
924*sqrt(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(7843715
28639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqr
t(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 +
 2065257*sqrt(3))) - 5772*sqrt(1139381 + 688419*sqrt(3))*sqrt(784371528639*
sqrt(3) + 1359975610922)/(-1924*sqrt(784371528639*sqrt(3) + 1359975610922)*
sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065
257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3)
+ 1359975610922) + 1139381 + 2065257*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(7843
71528639*sqrt(3) + 1359975610922)/20155392 + 1139381/40310784 + 2833*sqrt(3
)/55296)*atan(342239244822*sqrt(3)*x/(-1924*sqrt(784371528639*sqrt(3) + 135
9975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 11
39381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt(-2*sqrt(2)*sqrt(784371528
639*sqrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3))) + 5772*sqrt(1139
381 + 688419*sqrt(3))*sqrt(784371528639*sqrt(3) + 1359975610922)/(-1924*sqr
t(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*s
qrt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sq
rt(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 206525
7*sqrt(3))) - 2307256491*sqrt(2)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt
(784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sq
rt(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqr
t(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257
*sqrt(3))) - 2649036312*sqrt(6)*sqrt(1139381 + 688419*sqrt(3))/(-1924*sqrt(
784371528639*sqrt(3) + 1359975610922)*sqrt(-2*sqrt(2)*sqrt(784371528639*sqr
t(3) + 1359975610922) + 1139381 + 2065257*sqrt(3)) + 115087447*sqrt(2)*sqrt
(-2*sqrt(2)*sqrt(784371528639*sqrt(3) + 1359975610922) + 1139381 + 2065257*
sqrt(3)))) + (-2435*x**8 - 2475*x**6 - 3928*x**4 + 984*x**2 - 864)/(3240*x*
*9 + 6480*x**7 + 9720*x**5)
```

3.117 $\int \frac{x^{10}(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=243

$$x^5-9x^3+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)-\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\,\log\left(x\right.$$

```
[Out] 58*x-9*x^3+x^5-25/16*x*(7*x^2+15)/(x^4+2*x^2+3)^2+1/64*x*(252*x^2+3305)/(x^
4+2*x^2+3)+3/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-
8595619+7678611*3^(1/2))^(1/2)-3/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2
*3^(1/2))^(1/2))*(-8595619+7678611*3^(1/2))^(1/2)+3/512*ln(x^2+3^(1/2)-x*(-
2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)-3/512*ln(x^2+3^(1/2)+x*
(-2+2*3^(1/2))^(1/2))*(8595619+7678611*3^(1/2))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$x^5-9x^3+\frac{\left(252x^2+3305\right)x}{64\left(x^4+2x^2+3\right)}-\frac{25\left(7x^2+15\right)x}{16\left(x^4+2x^2+3\right)^2}+\frac{3}{512}\sqrt{8595619+7678611\sqrt{3}}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*Sqrt[-8595619 + 7678611*Sqrt[3]]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (3*Sqrt
[-8595619 + 7678611*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(
1 + Sqrt[3])]])/256 + (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[Sqrt[3] - Sqrt
[2*(-1 + Sqrt[3])]*x + x^2])/512 - (3*Sqrt[8595619 + 7678611*Sqrt[3]]*Log[S
qrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^{10}\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx = -\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{2250-2850x^2-4800x^4+2400x^6-672x^{10}+}{\left(3+2x^2+x^4\right)^2}$$

$$= -\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{\int \frac{-201960+193248x^2+87552x^4-78336x^6}{3+2x^2+x^4}}{4608}$$

$$= -\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{\int \left(267264-124416x^2+23040\right.}{4608}$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{3}{64}\int \frac{4647-14}{3+2x^2+}$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{256}\sqrt{3\left(1+\sqrt{3}\right.}$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{256}\left(3\sqrt{722010}\right.$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{3}{512}\sqrt{8595619-}$$

$$= 58x-9x^3+x^5-\frac{25x\left(15+7x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{x\left(3305+252x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{3}{256}\sqrt{-8595619}$$

Mathematica [C] time = 0.22, size = 156, normalized size = 0.64

$$x^5-9x^3+\frac{\left(252x^2+3305\right)x}{64\left(x^4+2x^2+3\right)}-\frac{25\left(7x^2+15\right)x}{16\left(x^4+2x^2+3\right)^2}+58x+\frac{3\left(148\sqrt{2}+4795i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}}+\frac{3\left(148\sqrt{2}-4795\right.}{128\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^10*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 58*x - 9*x^3 + x^5 - (25*x*(15 + 7*x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(330
5 + 252*x^2))/(64*(3 + 2*x^2 + x^4)) + (3*(4795*I + 148*Sqrt[2])*ArcTan[x/S
qrt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (3*(-4795*I + 148*Sqrt
[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

fricas [B] time = 0.76, size = 561, normalized size = 2.31

$$\frac{18808834881088512\,x^{13}-94044174405442560\,x^{11}+601882716194832384\,x^9+2970620359031916864\,x^7+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/18808834881088512*(18808834881088512*x^13 - 94044174405442560*x^11 + 6018
82716194832384*x^9 + 2970620359031916864*x^7 + 10166469141273357744*x^5 + 5
```

```
7410392*2183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqr
t(-66002414605209*sqrt(3) + 176883200667963)*arctan(1/863545621466021963404
537403089353*sqrt(6122667604521)*2183743218123^(3/4)*sqrt(55104008440689*x^
2 + 2183743218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-6600
2414605209*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(1549*sqrt(
3) + 148)*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/4701358281741
8600331*2183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-66002414605209*s
qrt(3) + 176883200667963) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 57410392*2
183743218123^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-660024
14605209*sqrt(3) + 176883200667963)*arctan(1/863545621466021963404537403089
353*sqrt(6122667604521)*2183743218123^(3/4)*sqrt(55104008440689*x^2 - 21837
43218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-6600241460520
9*sqrt(3) + 176883200667963) + 55104008440689*sqrt(3))*(1549*sqrt(3) + 148)
*sqrt(-66002414605209*sqrt(3) + 176883200667963) - 1/47013582817418600331*2
183743218123^(3/4)*(1549*sqrt(3)*x + 148*x)*sqrt(-66002414605209*sqrt(3) +
176883200667963) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 1352649115995281020
8*x^3 - 2183743218123^(1/4)*(8595619*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4
+ 12*x^2 + 9) + 23035833*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(
-66002414605209*sqrt(3) + 176883200667963)*log(55104008440689*x^2 + 2183743
218123^(1/4)*(148*sqrt(3)*sqrt(2)*x + 4647*sqrt(2)*x)*sqrt(-66002414605209*
sqrt(3) + 176883200667963) + 55104008440689*sqrt(3)) + 2183743218123^(1/4)*
(8595619*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 23035833*sqr
t(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-66002414605209*sqrt(3) + 17
6883200667963)*log(55104008440689*x^2 - 2183743218123^(1/4)*(148*sqrt(3)*sq
rt(2)*x + 4647*sqrt(2)*x)*sqrt(-66002414605209*sqrt(3) + 176883200667963) +
 55104008440689*sqrt(3)) + 12291279706746325584*x)/(x^8 + 4*x^6 + 10*x^4 +
12*x^2 + 9)
```

giac [B] time = 2.69, size = 588, normalized size = 2.42

$$x^5 - 9x^3 - \frac{1}{13824}\sqrt{2}\left(37\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 666\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 666\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] x^5 - 9*x^3 - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
666*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 1/13824*sqrt(2)*(37*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 666
*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 666*3^(3/4)*(sqrt(3)
+ 3)*sqrt(-6*sqrt(3) + 18) + 37*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 41823*3^(
1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) - 41823*3^(1/4)*sqrt(-6*sqrt(3) + 18))*ar
ctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
1/2)) - 1/27648*sqrt(2)*(666*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) - 37*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3)
+ 18)^(3/2) + 666*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4
)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x
^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/27648*sqrt(2)*(666
*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 37*3^(3/4)*sqrt(2)*(
-6*sqrt(3) + 18)^(3/2) + 37*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 666*3^(3/4)*sq
rt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 41823*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) +
18) + 41823*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*s
qrt(3) + 1/2) + sqrt(3)) + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 841
5*x)/(x^4 + 2*x^2 + 3)^2
```

maple [B] time = 0.04, size = 429, normalized size = 1.77

$$x^5-9x^3+58x+\frac{5091\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}}+\frac{14385\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] x^5-9*x^3+58*x+(63/16*x^7+3809/64*x^5+3333/32*x^3+8415/64*x)/(x^4+2*x^2+3)^
2+5091/1024*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1
/2))+14385/1024*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))
+5091/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))+14385/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2
))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-4647/64/(2+2*3^(1
/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-5
091/1024*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2)
)-14385/1024*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+50
91/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2)
)^(1/2))/(2+2*3^(1/2))^(1/2))+14385/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*
arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-4647/64/(2+2*3^(1/2)
)^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x^5-9\,x^3+58\,x+\frac{252\,x^7+3809\,x^5+6666\,x^3+8415\,x}{64\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}+\frac{3}{64}\int\frac{148\,x^2-4647}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^10*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] x^5 - 9*x^3 + 58*x + 1/64*(252*x^7 + 3809*x^5 + 6666*x^3 + 8415*x)/(x^8 + 4
*x^6 + 10*x^4 + 12*x^2 + 9) + 3/64*integrate((148*x^2 - 4647)/(x^4 + 2*x^2
+ 3), x)
```

mupad [B] time = 0.11, size = 184, normalized size = 0.76

$$58\,x+\frac{\frac{63\,x^7}{16}+\frac{3809\,x^5}{64}+\frac{3333\,x^3}{32}+\frac{8415\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}-9\,x^3+x^5-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{17191238-\sqrt{2}\,14352598i}\,193760073i}{131072\left(-\frac{986432531643}{131072}+\frac{\sqrt{2}\,900403059231i}{131072}\right)}-\frac{193760073\,\sqrt{2}\,x\,\sqrt{1719123\cdots}}{262144\left(-\frac{986432531643}{131072}+\cdots\right)}\cdots\right)\cdots}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^10*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] 58*x - (atan((x*(17191238 - 2^(1/2)*14352598i)^(1/2)*193760073i)/(131072*((
2^(1/2)*900403059231i)/131072 - 986432531643/131072)) - (193760073*2^(1/2)*
x*(17191238 - 2^(1/2)*14352598i)^(1/2))/(262144*((2^(1/2)*900403059231i)/13
1072 - 986432531643/131072)))*(17191238 - 2^(1/2)*14352598i)^(1/2)*3i)/256
+ (atan((x*(2^(1/2)*14352598i + 17191238)^(1/2)*193760073i)/(131072*((2^(1/
2)*900403059231i)/131072 + 986432531643/131072)) + (193760073*2^(1/2)*x*(2^
(1/2)*14352598i + 17191238)^(1/2))/(262144*((2^(1/2)*900403059231i)/131072
+ 986432531643/131072)))*(2^(1/2)*14352598i + 17191238)^(1/2)*3i)/256 + ((8
415*x)/64 + (3333*x^3)/32 + (3809*x^5)/64 + (63*x^7)/16)/(12*x^2 + 10*x^4 +
 4*x^6 + x^8 + 9) - 9*x^3 + x^5
```

sympy [B] time = 1.35, size = 1204, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**10*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] x**5 - 9*x**3 + 58*x + (252*x**7 + 3809*x**5 + 6666*x**3 + 8415*x)/(64*x**8
 + 256*x**6 + 640*x**4 + 768*x**2 + 576) - 3*sqrt(8595619/262144 + 7678611*
sqrt(3)/262144)*log(x**2 + x*(-6788*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))
/7176299 - 2313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 1697
*sqrt(2)*sqrt(8595619 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 1253
83933330562)/18368002813563) - 1218095240252468879279*sqrt(2)*sqrt(66002414
605209*sqrt(3) + 125383933330562)/1012150582077174852410264907 - 1343534101
96228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/39544284066890
8030011 + 18391902996311867463806959889/1012150582077174852410264907 + 5204
579286823805792980*sqrt(3)/395442840668908030011) + 3*sqrt(8595619/262144 +
 7678611*sqrt(3)/262144)*log(x**2 + x*(-1697*sqrt(2)*sqrt(8595619 + 7678611
*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/18368002813563 + 2
313785528*sqrt(8595619 + 7678611*sqrt(3))/18368002813563 + 6788*sqrt(3)*sqr
t(8595619 + 7678611*sqrt(3))/7176299) - 1218095240252468879279*sqrt(2)*sqrt
(66002414605209*sqrt(3) + 125383933330562)/1012150582077174852410264907 - 1
34353410196228*sqrt(6)*sqrt(66002414605209*sqrt(3) + 125383933330562)/39544
2840668908030011 + 18391902996311867463806959889/10121505820771748524102649
07 + 5204579286823805792980*sqrt(3)/395442840668908030011) - 2*sqrt(-9*sqrt
(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)/131072 + 77360571/262144
 + 207322497*sqrt(3)/262144)*atan(110208016881378*x/(22232174302*sqrt(-2*sq
rt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 23035833*s
qrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)*sqrt(
-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 23035
833*sqrt(3))) - 52122411468*sqrt(3)*sqrt(8595619 + 7678611*sqrt(3))/(222321
74302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595
619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) +
 8595619 + 23035833*sqrt(3))) - 6941356584*sqrt(8595619 + 7678611*sqrt(3))/
(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)
 + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 12538393333
0562) + 8595619 + 23035833*sqrt(3))) + 5091*sqrt(2)*sqrt(8595619 + 7678611*
sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/(22232174302*sqrt(-
2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 230358
33*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)*s
qrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595619 + 2
3035833*sqrt(3)))) - 2*sqrt(-9*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)/131072 + 77360571/262144 + 207322497*sqrt(3)/262144)*atan(110208
016881378*x/(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 1253
83933330562) + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(660024146052
09*sqrt(3) + 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562) + 8595619 + 23035833*sqrt(3))) - 5091*sqrt(2)*sqrt(859561
9 + 7678611*sqrt(3))*sqrt(66002414605209*sqrt(3) + 125383933330562)/(222321
74302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) + 8595
619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383
933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562) +
 8595619 + 23035833*sqrt(3))) + 6941356584*sqrt(8595619 + 7678611*sqrt(3))/
(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 125383933330562)
 + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414605209*sqrt(3) +
 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) + 12538393333
0562) + 8595619 + 23035833*sqrt(3))) + 52122411468*sqrt(3)*sqrt(8595619 + 7
678611*sqrt(3))/(22232174302*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(3) +
```

```
125383933330562) + 8595619 + 23035833*sqrt(3)) + 1697*sqrt(2)*sqrt(66002414
605209*sqrt(3) + 125383933330562)*sqrt(-2*sqrt(2)*sqrt(66002414605209*sqrt(
3) + 125383933330562) + 8595619 + 23035833*sqrt(3))))
```

3.118 $\int \frac{x^8(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3} dx$

Optimal. Leaf size=242

$$\frac{5x^3}{3}-\frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}\right.$$

```
[Out] -27*x+5/3*x^3+25/16*x*(5*x^2+3)/(x^4+2*x^2+3)^2-1/64*x*(835*x^2+1468)/(x^4+
2*x^2+3)-21/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(1/2
))^(1/2)+21/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-34271+22721*3^(1/2
))^(1/2)-21/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(34
271+22721*3^(1/2))^(1/2)+21/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1
/2))^(1/2))*(34271+22721*3^(1/2))^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$\frac{5x^3}{3}-\frac{\left(835x^2+1468\right)x}{64\left(x^4+2x^2+3\right)}+\frac{25\left(5x^2+3\right)x}{16\left(x^4+2x^2+3\right)^2}-\frac{21}{512}\sqrt{22721\sqrt{3}-34271}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)+\frac{21}{512}\sqrt{22\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) - (21*Sqrt[34271 + 22721*Sqrt[3]]*ArcTan
[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (21*Sqrt[3427
1 + 22721*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3
])]])/256 - (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqr
t[3])]*x + x^2])/512 + (21*Sqrt[-34271 + 22721*Sqrt[3]]*Log[Sqrt[3] + Sqrt[
2*(-1 + Sqrt[3])]*x + x^2])/512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{-450-1050x^2+2400x^4-672x^8+480x^{10}}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \frac{98496+27432x^2-78336x^4+23040x^6}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \left(-124416+23040x^2+\frac{1512(312+137}{3+2x^2+x^4}\right.}{4608}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{21}{64}\int \frac{312+137x^2}{3+2x^2+x^4}\,dx$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{1}{256}\left(7\sqrt{3\left(1+\sqrt{3}\right)}\right)$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{1}{512}\left(21\sqrt{-34271+22}\right.$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{21}{512}\sqrt{-34271+22721}$$

$$= -27x + \frac{5x^3}{3} + \frac{25x\left(3+5x^2\right)}{16\left(3+2x^2+x^4\right)^2} - \frac{x\left(1468+835x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{21}{256}\sqrt{34271+22721\sqrt{}}$$

Mathematica [C] time = 0.20, size = 155, normalized size = 0.64

$$\frac{5x^3}{3} - \frac{\left(835x^2+1468\right)x}{64\left(x^4+2x^2+3\right)} + \frac{25\left(5x^2+3\right)x}{16\left(x^4+2x^2+3\right)^2} - 27x + \frac{21\left(137\sqrt{2}-175i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{128\sqrt{2-2i\sqrt{2}}} + \frac{21\left(137\sqrt{2}+175i\right)\tan^{-1}}{128\sqrt{2+2i\sqrt{2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] -27*x + (5*x^3)/3 + (25*x*(3 + 5*x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(1468
+ 835*x^2))/(64*(3 + 2*x^2 + x^4)) + (21*(-175*I + 137*Sqrt[2])*ArcTan[x/Sq
rt[1 - I*Sqrt[2]]])/(128*Sqrt[2 - (2*I)*Sqrt[2]]) + (21*(175*I + 137*Sqrt[2
])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/(128*Sqrt[2 + (2*I)*Sqrt[2]])
```

fricas [B] time = 0.84, size = 557, normalized size = 2.30

$$1591298862080\,x^{11} - 19413846117376\,x^9 - 99660064046704\,x^7 - 285508852710816\,x^5 - 2298072\cdot 154873152$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/954779317248*(1591298862080*x^11 - 19413846117376*x^9 - 99660064046704*x^
7 - 285508852710816*x^5 - 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 1
```

```
0*x^4 + 12*x^2 + 9)*sqrt(778671391*sqrt(3) + 1548731523)*arctan(1/197530213
71716480527209*1548731523^(3/4)*sqrt(932401677)*sqrt(932401677*x^2 + 154873
1523^(1/4)*(137*sqrt(3)*sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) +
 1548731523) + 932401677*sqrt(3))*sqrt(778671391*sqrt(3) + 1548731523)*(104
*sqrt(3) - 137) - 1/21185098503117*1548731523^(3/4)*(104*sqrt(3)*x - 137*x)
*sqrt(778671391*sqrt(3) + 1548731523) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2))
- 2298072*1548731523^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt
(778671391*sqrt(3) + 1548731523)*arctan(1/19753021371716480527209*154873152
3^(3/4)*sqrt(932401677)*sqrt(932401677*x^2 - 1548731523^(1/4)*(137*sqrt(3)*
sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677
*sqrt(3))*sqrt(778671391*sqrt(3) + 1548731523)*(104*sqrt(3) - 137) - 1/2118
5098503117*1548731523^(3/4)*(104*sqrt(3)*x - 137*x)*sqrt(778671391*sqrt(3)
+ 1548731523) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) - 368738756006544*x^3 +
21*1548731523^(1/4)*(34271*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) - 68163*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(778671391*sqr
t(3) + 1548731523)*log(932401677*x^2 + 1548731523^(1/4)*(137*sqrt(3)*sqrt(2
)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677*sqrt(
3)) - 21*1548731523^(1/4)*(34271*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12
*x^2 + 9) - 68163*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(7786713
91*sqrt(3) + 1548731523)*log(932401677*x^2 - 1548731523^(1/4)*(137*sqrt(3)*
sqrt(2)*x - 312*sqrt(2)*x)*sqrt(778671391*sqrt(3) + 1548731523) + 932401677
*sqrt(3)) - 293236597809792*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [B] time = 2.59, size = 585, normalized size = 2.42

$$\frac{5}{3}x^3 - \frac{7}{55296}\sqrt{2}\left(137\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}} + 2466\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right) - 2466\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 5/3*x^3 - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 246
6*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(3
) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232*
3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18))
*arctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3)
 + 1/2)) - 7/55296*sqrt(2)*(137*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 24
66*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 2466*3^(3/4)*(sqrt(
3) + 3)*sqrt(-6*sqrt(3) + 18) + 137*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 11232
*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 11232*3^(1/4)*sqrt(-6*sqrt(3) + 18)
)*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3
) + 1/2)) - 7/110592*sqrt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sq
rt(3) + 18) - 137*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*
sqrt(3) + 18)^(3/2) + 2466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 112
32*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 1
8))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 7/110592*sq
rt(2)*(2466*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 137*3^(3/
4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 137*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 2
466*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11232*3^(1/4)*sqrt(2)*sqrt
(-6*sqrt(3) + 18) - 11232*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)
*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 27*x - 1/64*(835*x^7 + 3138*x^5 +
4941*x^3 + 4104*x)/(x^4 + 2*x^2 + 3)^2
```

maple [B] time = 0.03, size = 426, normalized size = 1.76

$$\frac{5x^3}{3} - 27x + \frac{693\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}} - \frac{3675\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}} + \frac{273\sqrt{3}\ \mathrm{arc}}{8\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] 5/3*x^3-27*x+(-835/64*x^7-1569/32*x^5-4941/64*x^3-513/8*x)/(x^4+2*x^2+3)^2+
693/1024*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2)
)-3675/1024*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+693
/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^
(1/2))/(2+2*3^(1/2))^(1/2))-3675/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arc
tan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+273/8/(2+2*3^(1/2))^(1/
2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-693/1024*
(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+3675/10
24*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+693/512/(2+2
*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))-3675/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+273/8/(2+2*3^(1/2))^(1/2)*3^(1/2
)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{5}{3}x^3 - 27x - \frac{835x^7 + 3138x^5 + 4941x^3 + 4104x}{64\left(x^8 + 4x^6 + 10x^4 + 12x^2 + 9\right)} + \frac{21}{64}\int \frac{137x^2 + 312}{x^4 + 2x^2 + 3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5/3*x^3 - 27*x - 1/64*(835*x^7 + 3138*x^5 + 4941*x^3 + 4104*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9) + 21/64*integrate((137*x^2 + 312)/(x^4 + 2*x^2 + 3)
, x)
```

mupad [B] time = 0.94, size = 182, normalized size = 0.75

$$\frac{5x^3}{3} - \frac{\frac{835x^7}{64} + \frac{1569x^5}{32} + \frac{4941x^3}{64} + \frac{513x}{8}}{x^8 + 4x^6 + 10x^4 + 12x^2 + 9} - 27x + \frac{\mathrm{atan}\left(\frac{x\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}\;126681219\mathrm{i}}{131072\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)} - \frac{126681219\sqrt{2}\,x\sqrt{-68542-\sqrt{2}\,27358\mathrm{i}}}{262144\left(\frac{12541440681}{131072}+\frac{\sqrt{2}\,4940567541\mathrm{i}}{16384}\right)}\right)}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] (atan((x*(- 2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*494
0567541i)/16384 + 12541440681/131072)) - (126681219*2^(1/2)*x*(- 2^(1/2)*27
358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 + 12541440681/131
072)))*(- 2^(1/2)*27358i - 68542)^(1/2)*21i)/256 - ((513*x)/8 + (4941*x^3)/
64 + (1569*x^5)/32 + (835*x^7)/64)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - 27
*x - (atan((x*(2^(1/2)*27358i - 68542)^(1/2)*126681219i)/(131072*((2^(1/2)*
4940567541i)/16384 - 12541440681/131072)) + (126681219*2^(1/2)*x*(2^(1/2)*2
7358i - 68542)^(1/2))/(262144*((2^(1/2)*4940567541i)/16384 - 12541440681/13
1072)))*(2^(1/2)*27358i - 68542)^(1/2)*21i)/256 + (5*x^3)/3
```

sympy [A] time = 0.68, size = 82, normalized size = 0.34

$$\frac{5x^3}{3} - 27x + \frac{-835x^7 - 3138x^5 - 4941x^3 - 4104x}{64x^8 + 256x^6 + 640x^4 + 768x^2 + 576} + 21\,\mathrm{RootSum}\left(17179869184t^4 + 8983937024t^2 + 1548731523,\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x**3/3 - 27*x + (-835*x**7 - 3138*x**5 - 4941*x**3 - 4104*x)/(64*x**8 + 2
56*x**6 + 640*x**4 + 768*x**2 + 576) + 21*RootSum(17179869184*_t**4 + 89839
```

```
37024*_t**2 + 1548731523, Lambda(_t, _t*log(-1107296256*_t**3/310800559 + 4
38857984*_t/310800559 + x)))
```

3.119 $\int \frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$

Optimal. Leaf size=235

$$-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\,\log\left(x^2+\sqrt{2\left(\sqrt{3}-\right.}\right.$$

```
[Out] 5*x+25/16*x*(-x^2+3)/(x^4+2*x^2+3)^2+7/64*x*(58*x^2+11)/(x^4+2*x^2+3)-1/512
*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2)+1/5
12*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-827621+1176531*3^(1/2))^(1/2)+1
/256*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(827621+117653
1*3^(1/2))^(1/2)-1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2
))*(827621+1176531*3^(1/2))^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {1668, 1678, 1676, 1169, 634, 618, 204, 628}

$$\frac{7\left(58x^2+11\right)x}{64\left(x^4+2x^2+3\right)}+\frac{25\left(3-x^2\right)x}{16\left(x^4+2x^2+3\right)^2}-\frac{1}{512}\sqrt{1176531\sqrt{3}-827621}\,\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)+\frac{1}{512}\sqrt{117\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] 5*x + (25*x*(3 - x^2))/(16*(3 + 2*x^2 + x^4)^2) + (7*x*(11 + 58*x^2))/(64*(
3 + 2*x^2 + x^4)) + (Sqrt[827621 + 1176531*Sqrt[3]]*ArcTan[(Sqrt[2*(-1 + Sq
rt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt[827621 + 1176531*Sqrt[3]
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 - (Sqrt
[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512 + (Sqrt[-827621 + 1176531*Sqrt[3]]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]
*x + x^2])/512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^6\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{-450+1650x^2-672x^6+480x^8}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{\int \frac{-12744-49104x^2+23040x^4}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{\int \left(23040-\frac{72\left(1137+1322x^2\right)}{3+2x^2+x^4}\right)dx}{4608}$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{64}\int \frac{1137+1322x^2}{3+2x^2+x^4}\,dx$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{\int \frac{1137\sqrt{2\left(-1+\sqrt{3}\right)}-\left(1137-1322\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,d}{128\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{256}\left(1322+379\sqrt{3}\right)\int \frac{}{\sqrt{3}-\sqrt{}}$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}-\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\ \text{lo}$$

$$= 5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}+\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\ \text{tan}$$

Mathematica [C] time = 0.32, size = 138, normalized size = 0.59

$$\frac{1}{256}\left(\frac{4x\left(320x^8+1686x^6+4089x^4+5112x^2+3411\right)}{\left(x^4+2x^2+3\right)^2}-\frac{i\left(185\sqrt{2}-2644i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{i\left(185\sqrt{2}+2644i\right.}{\sqrt{1+}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(3411 + 5112*x^2 + 4089*x^4 + 1686*x^6 + 320*x^8))/(3 + 2*x^2 + x^4)^
2 - (I*(-2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sq
rt[2]] + (I*(2644*I + 185*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 +
I*Sqrt[2]])/256
```

fricas [B] time = 0.95, size = 551, normalized size = 2.34

$$\frac{23795867690357760\,x^9+125374477893572448\,x^7+304066571830852752\,x^5-10534088\cdot 41526755818883^{\frac{1}{4}}\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/4759173538071552*(23795867690357760*x^9 + 125374477893572448*x^7 + 304066
571830852752*x^5 - 10534088*4152675581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x
^4 + 12*x^2 + 9)*sqrt(973721762751*sqrt(3) + 4152675581883)*arctan(1/847120
69003752172273243024956 33*4152675581883^(3/4)*sqrt(516403378697)*sqrt(46476
30408273*x^2 + 4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x
)*sqrt(973721762751*sqrt(3) + 4152675581883) + 4647630408273*sqrt(3))*sqrt(
973721762751*sqrt(3) + 4152675581883)*(379*sqrt(3) - 1322) - 1/546808125187
5840963*4152675581883^(3/4)*(379*sqrt(3)*x - 1322*x)*sqrt(973721762751*sqrt
(3) + 4152675581883) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 10534088*415267
5581883^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(973721762751
*sqrt(3) + 4152675581883)*arctan(1/847120690037521722732430249563 3*41526755
81883^(3/4)*sqrt(516403378697)*sqrt(4647630408273*x^2 - 4152675581883^(1/4)
*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152
675581883) + 4647630408273*sqrt(3))*sqrt(973721762751*sqrt(3) + 41526755818
83)*(379*sqrt(3) - 1322) - 1/5468081251875840963*4152675581883^(3/4)*(379*s
qrt(3)*x - 1322*x)*sqrt(973721762751*sqrt(3) + 4152675581883) - 1/2*sqrt(3)
*sqrt(2) + 1/2*sqrt(2)) + 380138986353465216*x^3 - 4152675581883^(1/4)*(827
621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) - 3529593*sqrt(2)*(
x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(973721762751*sqrt(3) + 41526755818
83)*log(4647630408273*x^2 + 4152675581883^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1
137*sqrt(2)*x)*sqrt(973721762751*sqrt(3) + 4152675581883) + 4647630408273*s
qrt(3)) + 4152675581883^(1/4)*(827621*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4
 + 12*x^2 + 9) - 3529593*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(
973721762751*sqrt(3) + 4152675581883)*log(4647630408273*x^2 - 4152675581883
^(1/4)*(1322*sqrt(3)*sqrt(2)*x - 1137*sqrt(2)*x)*sqrt(973721762751*sqrt(3)
+ 4152675581883) + 4647630408273*sqrt(3)) + 253649077161907248*x)/(x^8 + 4*
x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [B]   time = 2.61, size = 580, normalized size = 2.47

$$\frac{1}{82944}\sqrt{2}\left(661\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+11898\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-11898\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3/4)
*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3)*s
qrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(
1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2))
 + 1/82944*sqrt(2)*(661*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 11898*3^(3
/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 11898*3^(3/4)*(sqrt(3) + 3
)*sqrt(-6*sqrt(3) + 18) + 661*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 20466*3^(1/
4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 20466*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arct
an(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/
2)) + 1/165888*sqrt(2)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
 + 18) - 661*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(
3) + 18)^(3/2) + 11898*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3
^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*
log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/165888*sqrt(2
)*(11898*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 661*3^(3/4)*
sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 661*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1189
8*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 20466*3^(1/4)*sqrt(2)*sqrt(-
6*sqrt(3) + 18) - 20466*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x
*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 5*x + 1/64*(406*x^7 + 889*x^5 + 1272
*x^3 + 531*x)/(x^4 + 2*x^2 + 3)^2
```

maple [B] time = 0.04, size = 422, normalized size = 1.80

$$5x-\frac{943\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{185\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{379\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2}}{\sqrt{2+2}}\right.}{64\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] 5*x-(-203/32*x^7-889/64*x^5-159/8*x^3-531/64*x)/(x^4+2*x^2+3)^2-943/1024*(-
2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-185/1024*
(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-943/512/(2+2*3^
(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*
3^(1/2))^(1/2))-185/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+
2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-379/64/(2+2*3^(1/2))^(1/2)*3^(1/2)*a
rctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+943/1024*(-2+2*3^(1/2
))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+185/1024*(-2+2*3^(1
/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-943/512/(2+2*3^(1/2))^(1/
2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(
1/2))-185/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))
^(1/2))/(2+2*3^(1/2))^(1/2))-379/64/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x
+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$5\,x+\frac{406\,x^7+889\,x^5+1272\,x^3+531\,x}{64\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}-\frac{1}{64}\int\frac{1322\,x^2+1137}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 5*x + 1/64*(406*x^7 + 889*x^5 + 1272*x^3 + 531*x)/(x^8 + 4*x^6 + 10*x^4 + 1
2*x^2 + 9) - 1/64*integrate((1322*x^2 + 1137)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.99, size = 176, normalized size = 0.75

$$5\,x+\frac{\frac{203\,x^7}{32}+\frac{889\,x^5}{64}+\frac{159\,x^3}{8}+\frac{531\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}\,1316761\mathrm{i}}{131072\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}+\frac{1316761\,\sqrt{2}\,x\,\sqrt{-1655242-\sqrt{2}\,2633522\mathrm{i}}}{262144\left(-\frac{3725116869}{131072}+\frac{\sqrt{2}\,1497157257\mathrm{i}}{131072}\right)}\right)\sqrt{-1}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] 5*x + (atan((x*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(
1/2)*1497157257i)/131072 - 3725116869/131072)) + (1316761*2^(1/2)*x*(- 2^(1
/2)*2633522i - 1655242)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 - 3725
116869/131072)))*(- 2^(1/2)*2633522i - 1655242)^(1/2)*1i)/256 - (atan((x*(2
^(1/2)*2633522i - 1655242)^(1/2)*1316761i)/(131072*((2^(1/2)*1497157257i)/1
31072 + 3725116869/131072)) - (1316761*2^(1/2)*x*(2^(1/2)*2633522i - 165524
2)^(1/2))/(262144*((2^(1/2)*1497157257i)/131072 + 3725116869/131072)))*(2^(
1/2)*2633522i - 1655242)^(1/2)*1i)/256 + ((531*x)/64 + (159*x^3)/8 + (889*x
^5)/64 + (203*x^7)/32)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9)
```

sympy [A] time = 0.66, size = 71, normalized size = 0.30

$$5x+\frac{406x^7+889x^5+1272x^3+531x}{64x^8+256x^6+640x^4+768x^2+576}+\mathrm{RootSum}\left(17179869184t^4+216955879424t^2+4152675581883,\left(t\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] 5*x + (406*x**7 + 889*x**5 + 1272*x**3 + 531*x)/(64*x**8 + 256*x**6 + 640*x
**4 + 768*x**2 + 576) + RootSum(17179869184*_t**4 + 216955879424*_t**2 + 41
52675581883, Lambda(_t, _t*log(-31641829376*_t**3/1549210136091 - 455309168
896*_t/1549210136091 + x)))
```

3.120 $\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=238

$$\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)-\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-\right.}\right.$$

```
[Out] -25/16*x*(x^2+3)/(x^4+2*x^2+3)^2+1/64*x*(-59*x^2+238)/(x^4+2*x^2+3)-1/256*a
rctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-146505+98481*3^(1/
2))^(1/2)+1/256*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-14
6505+98481*3^(1/2))^(1/2)+1/512*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(146
505+98481*3^(1/2))^(1/2)-1/512*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(1465
05+98481*3^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1678, 1169, 634, 618, 204, 628}

$$\frac{x\left(238-59x^2\right)}{64\left(x^4+2x^2+3\right)}-\frac{25x\left(x^2+3\right)}{16\left(x^4+2x^2+3\right)^2}+\frac{1}{512}\sqrt{3\left(48835+32827\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}\,x+\sqrt{3}\right)-\frac{1}{512}\sqrt{3\left(4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (-25*x*(3 + x^2))/(16*(3 + 2*x^2 + x^4)^2) + (x*(238 - 59*x^2))/(64*(3 + 2*
x^2 + x^4)) - (Sqrt[3*(-48835 + 32827*Sqrt[3])]*ArcTan[(Sqrt[2*(-1 + Sqrt[3
])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3*(-48835 + 32827*Sqrt[3])]*
ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[3
*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/51
2 - (Sqrt[3*(48835 + 32827*Sqrt[3])]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x
 + x^2])/512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx = -\frac{25x\left(3+x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{1}{96}\int \frac{450-750x^2-672x^4+480x^6}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= -\frac{25x\left(3+x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(238-59x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \frac{-9936+18792x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= -\frac{25x\left(3+x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(238-59x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{\int \frac{-9936\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-9936-18792\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}x+x^2}\,d}{9216\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{25x\left(3+x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(238-59x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{1}{256}\left(261-46\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1\right.}}$$

$$= -\frac{25x\left(3+x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(238-59x^2\right)}{64\left(3+2x^2+x^4\right)} + \frac{1}{512}\sqrt{146505+98481\sqrt{3}}\,\log\left(\sqrt{3}\right.$$

$$= -\frac{25x\left(3+x^2\right)}{16\left(3+2x^2+x^4\right)^2} + \frac{x\left(238-59x^2\right)}{64\left(3+2x^2+x^4\right)} - \frac{1}{256}\sqrt{3\left(-48835+32827\sqrt{3}\right)}\,\tan^{-1}$$

Mathematica [C] time = 0.30, size = 129, normalized size = 0.54

$$\frac{1}{256}\left(\frac{4x\left(-59x^6+120x^4+199x^2+414\right)}{\left(x^4+2x^2+3\right)^2} + \frac{3\left(174+133i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{3\left(174-133i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{}}}\right.}{\sqrt{1+i\sqrt{2}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(414 + 199*x^2 + 120*x^4 - 59*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(174 + (
133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(17
4 - (133*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/25
6
```

fricas [B] time = 0.88, size = 546, normalized size = 2.29

$$-\frac{1914264223824\,x^7-3893418760320\,x^5+164728\cdot 29095522083^{\frac{1}{4}}\sqrt{3}\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)\sqrt{-160310}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/2076490005504*(1914264223824*x^7 - 3893418760320*x^5 + 164728*2909552208
3^(1/4)*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3
) + 3232835787)*arctan(1/121488027699636551876l363*29095522083^(3/4)*sqrt(2
027822271)*sqrt(2027822271*x^2 + 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x +
46*sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3))*
(46*sqrt(3) + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/5991058952110
53*29095522083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqrt(3) + 3232
```

```
835787) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 164728*29095522083^(1/4)*sqr
t(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1603106545*sqrt(3) + 3232835
787)*arctan(1/1214880276996365518761363*29095522083^(3/4)*sqrt(2027822271)*
sqrt(2027822271*x^2 - 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*
x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3))*(46*sqrt(3)
 + 261)*sqrt(-1603106545*sqrt(3) + 3232835787) - 1/599105895211053*29095522
083^(3/4)*(46*sqrt(3)*x + 261*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 1
/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - 6456586110864*x^3 + 29095522083^(1/4)*(
48835*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 98481*sqrt(2)*(
x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-1603106545*sqrt(3) + 3232835787)*
log(2027822271*x^2 + 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*sqrt(2)*x
)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3)) - 2909552208
3^(1/4)*(48835*sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 98481*
sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9))*sqrt(-1603106545*sqrt(3) + 323
2835787)*log(2027822271*x^2 - 29095522083^(1/4)*(87*sqrt(3)*sqrt(2)*x + 46*
sqrt(2)*x)*sqrt(-1603106545*sqrt(3) + 3232835787) + 2027822271*sqrt(3)) - 1
3432294723104*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [B] time = 2.60, size = 577, normalized size = 2.42

$$-\frac{1}{18432}\sqrt{2}\left(29\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+522\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-522\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/18432*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*s
qrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)
*sqrt(6*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/
4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/1843
2*sqrt(2)*(29*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(2)*
sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 522*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt
(3) + 18) + 29*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 552*3^(1/4)*sqrt(2)*sqrt(6
*sqrt(3) + 18) - 552*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x -
 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/36864*sqrt(
2)*(522*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sq
rt(2)*(-6*sqrt(3) + 18)^(3/2) + 29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(
3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(
3) + 18) + 552*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/
6*sqrt(3) + 1/2) + sqrt(3)) + 1/36864*sqrt(2)*(522*3^(3/4)*sqrt(2)*(sqrt(3)
 + 3)*sqrt(-6*sqrt(3) + 18) - 29*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) +
29*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 522*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(
3) - 3) + 552*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 552*3^(1/4)*sqrt(6*sq
rt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/
64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^4 + 2*x^2 + 3)^2
```

maple [B] time = 0.03, size = 418, normalized size = 1.76

$$\frac{307\left(-2+2\sqrt{3}\right)\sqrt{3}\;\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}}+\frac{399\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{23\sqrt{3}\;\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{32\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (-59/64*x^7+15/8*x^5+199/64*x^3+207/32*x)/(x^4+2*x^2+3)^2+307/1024*(-2+2*3^
(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+399/1024*(-2+2*
3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+307/512/(2+2*3^(1/2))
^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2
))^(1/2))+399/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1
/2))^(1/2))/(2+2*3^(1/2))^(1/2))-23/32/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((
2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-307/1024*(-2+2*3^(1/2))^(1/2
)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-399/1024*(-2+2*3^(1/2))^(1
/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+307/512/(2+2*3^(1/2))^(1/2)*(-2+
2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+3
99/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))
/(2+2*3^(1/2))^(1/2))-23/32/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3
^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{59\,x^7-120\,x^5-199\,x^3-414\,x}{64\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}+\frac{3}{64}\int\frac{87\,x^2-46}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(59*x^7 - 120*x^5 - 199*x^3 - 414*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 3/64*integrate((87*x^2 - 46)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.15, size = 173, normalized size = 0.73

$$\frac{-\frac{59\,x^7}{64}+\frac{15\,x^5}{8}+\frac{199\,x^3}{64}+\frac{207\,x}{32}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{293010-\sqrt{2}\,123546{}\mathrm{i}}\,61773{}\mathrm{i}}{131072\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337{}\mathrm{i}}{65536}\right)}+\frac{61773\,\sqrt{2}\,x\,\sqrt{293010-\sqrt{2}\,123546{}\mathrm{i}}}{262144\left(\frac{56892933}{131072}+\frac{\sqrt{2}\,4262337{}\mathrm{i}}{65536}\right)}\right)\sqrt{293010-\sqrt{2}\,123}}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] ((207*x)/32 + (199*x^3)/64 + (15*x^5)/8 - (59*x^7)/64)/(12*x^2 + 10*x^4 + 4
*x^6 + x^8 + 9) + (atan((x*(293010 - 2^(1/2)*123546i)^(1/2)*61773i)/(131072
*((2^(1/2)*4262337i)/65536 + 56892933/131072)) + (61773*2^(1/2)*x*(293010 -
 2^(1/2)*123546i)^(1/2))/(262144*((2^(1/2)*4262337i)/65536 + 56892933/13107
2)))*(293010 - 2^(1/2)*123546i)^(1/2)*1i)/256 - (atan((x*(2^(1/2)*123546i +
 293010)^(1/2)*61773i)/(131072*((2^(1/2)*4262337i)/65536 - 56892933/131072)
) - (61773*2^(1/2)*x*(2^(1/2)*123546i + 293010)^(1/2))/(262144*((2^(1/2)*42
62337i)/65536 - 56892933/131072)))*(2^(1/2)*123546i + 293010)^(1/2)*1i)/256
```

sympy [B] time = 1.31, size = 1198, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-59*x**7 + 120*x**5 + 199*x**3 + 414*x)/(64*x**8 + 256*x**6 + 640*x**4 + 7
68*x**2 + 576) - sqrt(146505/262144 + 98481*sqrt(3)/262144)*log(x**2 + x*(-
307*sqrt(6)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884650
6)/675940757 + 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 122
8*sqrt(48835 + 32827*sqrt(3))/20591) - 941929306825573*sqrt(2)*sqrt(1603106
545*sqrt(3) + 2808846506)/456895906973733049 - 47771215762*sqrt(6)*sqrt(160
3106545*sqrt(3) + 2808846506)/41754888382161 + 97477949666790882353/4568959
```

```
06973733049 + 5200450130596150*sqrt(3)/41754888382161) + sqrt(146505/262144
 + 98481*sqrt(3)/262144)*log(x**2 + x*(-1228*sqrt(48835 + 32827*sqrt(3))/20
591 - 10626354*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/675940757 + 307*sqrt(6)*
sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/675940757
) - 941929306825573*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/456895906
973733049 - 47771215762*sqrt(6)*sqrt(1603106545*sqrt(3) + 2808846506)/41754
888382161 + 97477949666790882353/456895906973733049 + 5200450130596150*sqrt
(3)/41754888382161) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 280884650
6)/131072 + 146505/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*
x/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2
*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) -
40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt
(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sq
rt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3)
 + 2808846506) + 48835 + 98481*sqrt(3))) - 31879062*sqrt(48835 + 32827*sqrt
(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 4883
5 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)))
 + 921*sqrt(2)*sqrt(48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 280884
6506)/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 488
35 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqr
t(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))
)) + 2*sqrt(-3*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)/131072 + 14650
5/262144 + 295443*sqrt(3)/262144)*atan(1351881514*sqrt(3)*x/(-1894372*sqrt(
-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3)) +
 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(160
3106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) - 921*sqrt(2)*sqrt(
48835 + 32827*sqrt(3))*sqrt(1603106545*sqrt(3) + 2808846506)/(-1894372*sqrt
(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))
+ 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt(16
03106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))) + 31879062*sqrt(48
835 + 32827*sqrt(3))/(-1894372*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 28
08846506) + 48835 + 98481*sqrt(3)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) +
2808846506)*sqrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 +
 98481*sqrt(3))) + 40311556*sqrt(3)*sqrt(48835 + 32827*sqrt(3))/(-1894372*s
qrt(-2*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3
)) + 307*sqrt(2)*sqrt(1603106545*sqrt(3) + 2808846506)*sqrt(-2*sqrt(2)*sqrt
(1603106545*sqrt(3) + 2808846506) + 48835 + 98481*sqrt(3))))
```

3.121 $\int \frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=246

$$-\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\right.}{1536}$$

```
[Out] 25/16*x*(x^2+1)/(x^4+2*x^2+3)^2-1/192*x*(88*x^2+353)/(x^4+2*x^2+3)-11/2304*
arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-5475+3267*3^(1/2)
)^(1/2)+11/2304*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-54
75+3267*3^(1/2))^(1/2)-11/4608*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(5475
+3267*3^(1/2))^(1/2)+11/4608*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(5475+3
267*3^(1/2))^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1668, 1678, 1169, 634, 618, 204, 628}

$$\frac{25x\left(x^2+1\right)}{16\left(x^4+2x^2+3\right)^2}-\frac{x\left(88x^2+353\right)}{192\left(x^4+2x^2+3\right)}-\frac{11\sqrt{\frac{1}{3}\left(1825+1089\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{1536}+\frac{11\sqrt{\frac{1}{3}}\left(1825\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 + x^2))/(16*(3 + 2*x^2 + x^4)^2) - (x*(353 + 88*x^2))/(192*(3 + 2*
x^2 + x^4)) - (11*Sqrt[(-1825 + 1089*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[
3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 + (11*Sqrt[(-1825 + 1089*Sqrt[3])/3
]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/768 - (11*S
qrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])
/1536 + (11*Sqrt[(1825 + 1089*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/1536
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^2(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}dx = \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} + \frac{1}{96}\int \frac{-150+78x^2+480x^4}{(3+2x^2+x^4)^2}dx$$

$$= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} + \frac{\int \frac{6072-2112x^2}{3+2x^2+x^4}dx}{4608}$$

$$= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} + \frac{\int \frac{6072\sqrt{2(-1+\sqrt{3})}-(6072+2112\sqrt{3})x}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+x^2}dx}{9216\sqrt{6(-1+\sqrt{3})}} +$$

$$= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} - \frac{(11(24-23\sqrt{3}))\int \frac{1}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}x+}}{2304}$$

$$= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} - \frac{11}{768}\sqrt{\frac{1825}{12}+\frac{363\sqrt{3}}{4}}\log\left(\sqrt{3}-\sqrt{}\right.$$

$$= \frac{25x(1+x^2)}{16(3+2x^2+x^4)^2} - \frac{x(353+88x^2)}{192(3+2x^2+x^4)} - \frac{11}{768}\sqrt{\frac{1}{3}(-1825+1089\sqrt{3})}\tan^{-1}\left(\right.$$

Mathematica [C] time = 0.30, size = 133, normalized size = 0.54

$$\frac{1}{768}\left(-\frac{4x(88x^6+529x^4+670x^2+759)}{(x^4+2x^2+3)^2} - \frac{11i(31\sqrt{2}-16i)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}} + \frac{11i(31\sqrt{2}+16i)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right.}{\sqrt{1+i\sqrt{2}}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(4 + x^2 + 3*x^4 + 5*x^6))/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((-4*x*(759 + 670*x^2 + 529*x^4 + 88*x^6))/(3 + 2*x^2 + x^4)^2 - ((11*I)*(-
16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + ((1
1*I)*(16*I + 31*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]]
)/768
```

fricas [B] time = 0.87, size = 570, normalized size = 2.32

$$-\frac{12811392x^7+77013936x^5+1348\sqrt{6}\,3^{\frac{3}{4}}\sqrt{2}\left(x^8+4x^6+10x^4+12x^2+9\right)\sqrt{-1987425\sqrt{3}+3557763}\,\arctan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/27952128*(12811392*x^7 + 77013936*x^5 + 1348*sqrt(6)*3^(3/4)*sqrt(2)*(x^
8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763)*arctan(1/
2226179538*sqrt(3707)*sqrt(6)*3^(3/4)*sqrt(sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 2
3*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3))*(23*sqrt
(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987425*sqrt(3) + 3557763) - 1/200178*sqrt(
```

```
6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 24*sqrt(2)*x)*sqrt(-1987425*sqrt(3) + 35
57763) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) + 1348*sqrt(6)*3^(3/4)*sqrt(2)*
(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1987425*sqrt(3) + 3557763)*arctan
(1/2226179538*sqrt(3707)*sqrt(6)*3^(3/4)*sqrt(-sqrt(6)*3^(1/4)*(8*sqrt(3)*x
 + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*sqrt(3))*(23*
sqrt(3)*sqrt(2) + 24*sqrt(2))*sqrt(-1987425*sqrt(3) + 3557763) - 1/200178*s
qrt(6)*3^(3/4)*(23*sqrt(3)*sqrt(2)*x + 24*sqrt(2)*x)*sqrt(-1987425*sqrt(3)
+ 3557763) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) - sqrt(6)*3^(1/4)*(3267*x^8
 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1825*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 +
 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt(3) + 3557763)*log(sqrt(6)*3^(1/4)*
(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt(3) + 3557763) + 33363*x^2 + 33363*s
qrt(3)) + sqrt(6)*3^(1/4)*(3267*x^8 + 13068*x^6 + 32670*x^4 + 39204*x^2 + 1
825*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 29403)*sqrt(-1987425*sqrt
(3) + 3557763)*log(-sqrt(6)*3^(1/4)*(8*sqrt(3)*x + 23*x)*sqrt(-1987425*sqrt
(3) + 3557763) + 33363*x^2 + 33363*sqrt(3)) + 97541280*x^3 + 110498256*x)/(
x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)
```

giac [B] time = 2.69, size = 577, normalized size = 2.35

$$\frac{11}{124416}\sqrt{2}\left(2\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+36\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-36\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}+2\cdot$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 11/124416*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6
*sqrt(3) + 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sq
rt(6*sqrt(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*
(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/124416
*sqrt(2)*(2*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(2)*sqr
t(6*sqrt(3) + 18)*(sqrt(3) - 3) - 36*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sqrt(3)
+ 18) + 2*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 207*3^(1/4)*sqrt(2)*sqrt(6*sqrt
(3) + 18) - 207*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x - 3^(1
/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 11/248832*sqrt(2)*
(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)
*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sq
rt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18
) + 207*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(
3) + 1/2) + sqrt(3)) - 11/248832*sqrt(2)*(36*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*
sqrt(-6*sqrt(3) + 18) - 2*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2*3^(3/
4)*(6*sqrt(3) + 18)^(3/2) + 36*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) +
 207*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 207*3^(1/4)*sqrt(6*sqrt(3) + 1
8))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/192*(88*x
^7 + 529*x^5 + 670*x^3 + 759*x)/(x^4 + 2*x^2 + 3)^2
```

maple [B] time = 0.03, size = 418, normalized size = 1.70

$$-\frac{517\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{4608\sqrt{2+2\sqrt{3}}}-\frac{341\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{1536\sqrt{2+2\sqrt{3}}}+\frac{253\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2}}{\sqrt{2+}}\right.}{576\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (-11/24*x^7-529/192*x^5-335/96*x^3-253/64*x)/(x^4+2*x^2+3)^2-517/9216*(-2+2
*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-341/3072*(-2
```

```
+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-517/4608/(2+2*3^(1
/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^
(1/2))^(1/2))-341/1536/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2
*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+253/576/(2+2*3^(1/2))^(1/2)*3^(1/2)*a
rctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+517/9216*(-2+2*3^(1/2
))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+341/3072*(-2+2*3^(1
/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-517/4608/(2+2*3^(1/2))^(1
/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^
(1/2))-341/1536/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2
))^(1/2))/(2+2*3^(1/2))^(1/2))+253/576/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((
2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{88\,x^7+529\,x^5+670\,x^3+759\,x}{192\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}-\frac{11}{192}\int\frac{8\,x^2-23}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/192*(88*x^7 + 529*x^5 + 670*x^3 + 759*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2
+ 9) - 11/192*integrate((8*x^2 - 23)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 1.01, size = 174, normalized size = 0.71

$$-\frac{\frac{11\,x^7}{24}+\frac{529\,x^5}{192}+\frac{335\,x^3}{96}+\frac{253\,x}{64}}{x^8+4\,x^6+10\,x^4+12\,x^2+9}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}\,448547\mathrm{i}}{31850496\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}-\frac{448547\,\sqrt{2}\,x\,\sqrt{10950-\sqrt{2}\,2022\mathrm{i}}}{63700992\left(-\frac{21081709}{10616832}+\frac{\sqrt{2}\,10316581\mathrm{i}}{10616832}\right)}\right)\sqrt{10950-\sqrt{2}}}{2304}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(x^2 + 3*x^4 + 5*x^6 + 4))/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] (atan((x*(10950 - 2^(1/2)*2022i)^(1/2)*448547i)/(31850496*((2^(1/2)*1031658
1i)/10616832 - 21081709/10616832)) - (448547*2^(1/2)*x*(10950 - 2^(1/2)*202
2i)^(1/2))/(63700992*((2^(1/2)*10316581i)/10616832 - 21081709/10616832)))*(
10950 - 2^(1/2)*2022i)^(1/2)*11i)/2304 - ((253*x)/64 + (335*x^3)/96 + (529*
x^5)/192 + (11*x^7)/24)/(12*x^2 + 10*x^4 + 4*x^6 + x^8 + 9) - (atan((x*(2^(
1/2)*2022i + 10950)^(1/2)*448547i)/(31850496*((2^(1/2)*10316581i)/10616832
+ 21081709/10616832)) + (448547*2^(1/2)*x*(2^(1/2)*2022i + 10950)^(1/2))/(6
3700992*((2^(1/2)*10316581i)/10616832 + 21081709/10616832)))*(2^(1/2)*2022i
 + 10950)^(1/2)*11i)/2304
```

sympy [B] time = 1.33, size = 1200, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-88*x**7 - 529*x**5 - 670*x**3 - 759*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
 2304*x**2 + 1728) - sqrt(220825/7077888 + 14641*sqrt(3)/786432)*log(x**2 +
 x*(-47*sqrt(6)*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/3
66993 + 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 188*sqrt(1825 + 10
89*sqrt(3))/337) - 24765218375*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)/1346
83862049 - 38128468*sqrt(6)*sqrt(1987425*sqrt(3) + 3444194)/371029923 + 904
13874433403/134683862049 + 144251139148*sqrt(3)/371029923) + sqrt(220825/70
77888 + 14641*sqrt(3)/786432)*log(x**2 + x*(-188*sqrt(1825 + 1089*sqrt(3))/
```

```
337 - 52016*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/366993 + 47*sqrt(6)*sqrt(1825
 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/366993) - 24765218375*sqrt
(2)*sqrt(1987425*sqrt(3) + 3444194)/134683862049 - 38128468*sqrt(6)*sqrt(19
87425*sqrt(3) + 3444194)/371029923 + 90413874433403/134683862049 + 14425113
9148*sqrt(3)/371029923) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqrt(3) + 344419
4)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(733986*sqrt(3)*x/(
15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)
) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425
*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 204732*sqrt(3)*sqrt(1825 + 10
89*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 +
 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)
*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 156048*sqrt(1825
 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1
825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sq
rt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) + 141*sqrt(2)
*sqrt(1825 + 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/(15502*sqrt(-2*s
qrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*
sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444
194) + 1825 + 3267*sqrt(3)))) + 2*sqrt(-121*sqrt(2)*sqrt(1987425*sqrt(3) +
3444194)/3538944 + 220825/7077888 + 14641*sqrt(3)/262144)*atan(733986*sqrt(
3)*x/(15502*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*s
qrt(3)) + 47*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1
987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3))) - 141*sqrt(2)*sqrt(1825 +
 1089*sqrt(3))*sqrt(1987425*sqrt(3) + 3444194)/(15502*sqrt(-2*sqrt(2)*sqrt(
1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(1987425
*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825
+ 3267*sqrt(3))) + 156048*sqrt(1825 + 1089*sqrt(3))/(15502*sqrt(-2*sqrt(2)*
sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sqrt(2)*sqrt(19
87425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) +
1825 + 3267*sqrt(3))) + 204732*sqrt(3)*sqrt(1825 + 1089*sqrt(3))/(15502*sqr
t(-2*sqrt(2)*sqrt(1987425*sqrt(3) + 3444194) + 1825 + 3267*sqrt(3)) + 47*sq
rt(2)*sqrt(1987425*sqrt(3) + 3444194)*sqrt(-2*sqrt(2)*sqrt(1987425*sqrt(3)
+ 3444194) + 1825 + 3267*sqrt(3))))
```

3.122 $\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3} \, dx$

Optimal. Leaf size=248

$$\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x\right.$$

```
[Out] 25/48*x*(-x^2+1)/(x^4+2*x^2+3)^2+1/192*x*(51*x^2+64)/(x^4+2*x^2+3)-1/768*ar
ctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-3873+3057*3^(1/2))^
(1/2)+1/768*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(-3873+3
057*3^(1/2))^(1/2)+1/1536*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(3873+3057
*3^(1/2))^(1/2)-1/1536*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(3873+3057*3^
(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1678, 1178, 1169, 634, 618, 204, 628}

$$\frac{25x\left(1-x^2\right)}{48\left(x^4+2x^2+3\right)^2}+\frac{x\left(51x^2+64\right)}{192\left(x^4+2x^2+3\right)}+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)-\frac{1}{512}\sqrt{\frac{1}{3}\left(1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] (25*x*(1 - x^2))/(48*(3 + 2*x^2 + x^4)^2) + (x*(64 + 51*x^2))/(192*(3 + 2*x
^2 + x^4)) - (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
 - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(-1291 + 1019*Sqrt[3])/3]*ArcTa
n[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])/256 + (Sqrt[(1291
+ 1019*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/512 - (Sq
rt[(1291 + 1019*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/
512
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
```

```
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{78+230x^2}{\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{\int \frac{-288+1224x^2}{3+2x^2+x^4}\,dx}{4608}$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{\int \frac{-288\sqrt{2\left(-1+\sqrt{3}\right)}-\left(-288-1224\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,dx}{9216\sqrt{6\left(-1+\sqrt{3}\right)}}+$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{1}{768}\left(51-4\sqrt{3}\right)\int \frac{1}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{}\right.}}$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}+\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\sqrt{3}\right)}\,\log\left(\sqrt{3}-\right.$$

$$= \frac{25x\left(1-x^2\right)}{48\left(3+2x^2+x^4\right)^2}+\frac{x\left(64+51x^2\right)}{192\left(3+2x^2+x^4\right)}-\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\sqrt{3}\right)}\,\tan^{-1}\left(\right.$$

Mathematica [C] time = 0.29, size = 129, normalized size = 0.52

$$\frac{1}{768}\left(\frac{4x\left(51x^6+166x^4+181x^2+292\right)}{\left(x^4+2x^2+3\right)^2}+\frac{3\left(34+21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{3\left(34-21i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(3 + 2*x^2 + x^4)^3,x]
```

```
[Out] ((4*x*(292 + 181*x^2 + 166*x^4 + 51*x^6))/(3 + 2*x^2 + x^4)^2 + (3*(34 + (2
1*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*Sqrt[2]] + (3*(34 -
 (21*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[1 + I*Sqrt[2]])/768
```

fricas [B] time = 0.86, size = 576, normalized size = 2.32

$$\frac{2122829712\,x^7+6909602592\,x^5-3404\cdot 3115083^{\frac{1}{4}}\sqrt{6}\,\sqrt{3}\,\sqrt{2}\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)\sqrt{-1315529\,\sqrt{3}+3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/7991829504*(2122829712*x^7 + 6909602592*x^5 - 3404*3115083^(1/4)*sqrt(6)*
sqrt(3)*sqrt(2)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) +
 3115083)*arctan(1/41378565634793586*3115083^(3/4)*sqrt(2601507)*sqrt(6)*sq
rt(3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 31150
83) + 2601507*x^2 + 2601507*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2))*sqrt(
-1315529*sqrt(3) + 3115083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4*sqrt(3
)*sqrt(2)*x + 51*sqrt(2)*x)*sqrt(-1315529*sqrt(3) + 3115083) - 1/2*sqrt(3)*
sqrt(2) + 1/2*sqrt(2)) - 3404*3115083^(1/4)*sqrt(6)*sqrt(3)*sqrt(2)*(x^8 +
4*x^6 + 10*x^4 + 12*x^2 + 9)*sqrt(-1315529*sqrt(3) + 3115083)*arctan(1/4137
8565634793586*3115083^(3/4)*sqrt(2601507)*sqrt(6)*sqrt(-3115083^(1/4)*sqrt(
6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 2601507*x^2 + 26
01507*sqrt(3))*(4*sqrt(3)*sqrt(2) + 51*sqrt(2))*sqrt(-1315529*sqrt(3) + 311
5083) - 1/15905613798*3115083^(3/4)*sqrt(6)*(4*sqrt(3)*sqrt(2)*x + 51*sqrt(
2)*x)*sqrt(-1315529*sqrt(3) + 3115083) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2))
 - 3115083^(1/4)*sqrt(6)*(3057*x^8 + 12228*x^6 + 30570*x^4 + 36684*x^2 + 12
91*sqrt(3)*(x^8 + 4*x^6 + 10*x^4 + 12*x^2 + 9) + 27513)*sqrt(-1315529*sqrt(
3) + 3115083)*log(3115083^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*
sqrt(3) + 3115083) + 2601507*x^2 + 2601507*sqrt(3)) + 3115083^(1/4)*sqrt(6)
*(3057*x^8 + 12228*x^6 + 30570*x^4 + 36684*x^2 + 1291*sqrt(3)*(x^8 + 4*x^6
+ 10*x^4 + 12*x^2 + 9) + 27513)*sqrt(-1315529*sqrt(3) + 3115083)*log(-31150
83^(1/4)*sqrt(6)*(17*sqrt(3)*x + 4*x)*sqrt(-1315529*sqrt(3) + 3115083) + 26
01507*x^2 + 2601507*sqrt(3)) + 7533964272*x^3 + 12154240704*x)/(x^8 + 4*x^6
 + 10*x^4 + 12*x^2 + 9)
```

giac [B] time = 2.62, size = 577, normalized size = 2.33

$$-\frac{1}{165888}\sqrt{2}\left(17\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+306\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-306\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/165888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqrt
(-6*sqrt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3
/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/165
888*sqrt(2)*(17*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 306*3^(3/4)*(sqrt(3) + 3)*sqrt(-6*sq
rt(3) + 18) + 17*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) + 144*3^(1/4)*sqrt(2)*sqrt
(6*sqrt(3) + 18) - 144*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(3/4)*(x
 - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) - 1/331776*sq
rt(2)*(306*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)
*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*
3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sq
rt(3) + 18) + 144*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(
-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/331776*sqrt(2)*(306*3^(3/4)*sqrt(2)*(sqr
t(3) + 3)*sqrt(-6*sqrt(3) + 18) - 17*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2
) + 17*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 306*3^(3/4)*sqrt(6*sqrt(3) + 18)*(s
qrt(3) - 3) + 144*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) + 144*3^(1/4)*sqrt(
6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3))
+ 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^4 + 2*x^2 + 3)^2
```

maple [B] time = 0.03, size = 418, normalized size = 1.69

$$\frac{55\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{1536\sqrt{2+2\sqrt{3}}}+\frac{21\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{512\sqrt{2+2\sqrt{3}}}-\frac{\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{48\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x)
```

```
[Out] (17/64*x^7+83/96*x^5+181/192*x^3+73/48*x)/(x^4+2*x^2+3)^2+55/3072*(-2+2*3^(
1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+21/1024*(-2+2*3^
(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+55/1536/(2+2*3^(1/2))^(
1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))
^(1/2))+21/512/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2)
)^(1/2))/(2+2*3^(1/2))^(1/2))-1/48/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-55/3072*(-2+2*3^(1/2))^(1/2)*3^(
1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-21/1024*(-2+2*3^(1/2))^(1/2)*ln
(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))+55/1536/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1
/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+21/512/
(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))-1/48/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^
(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{51\,x^7+166\,x^5+181\,x^3+292\,x}{192\left(x^8+4\,x^6+10\,x^4+12\,x^2+9\right)}+\frac{1}{64}\int\frac{17\,x^2-4}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(51*x^7 + 166*x^5 + 181*x^3 + 292*x)/(x^8 + 4*x^6 + 10*x^4 + 12*x^2 +
 9) + 1/64*integrate((17*x^2 - 4)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 1.01, size = 173, normalized size = 0.70

$$\frac{\frac{17x^7}{64}+\frac{83x^5}{96}+\frac{181x^3}{192}+\frac{73x}{48}}{x^8+4x^6+10x^4+12x^2+9}+\frac{\mathrm{atan}\left(\frac{x\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,851\mathrm{i}}{1179648\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}+\frac{851\sqrt{2}\,x\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}}{2359296\left(\frac{46805}{393216}+\frac{\sqrt{2}\,851\mathrm{i}}{98304}\right)}\right)\sqrt{7746-\sqrt{2}\,5106\mathrm{i}}\,1\mathrm{i}}{768}-\ \mathrm{atan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(2*x^2 + x^4 + 3)^3,x)
```

```
[Out] ((73*x)/48 + (181*x^3)/192 + (83*x^5)/96 + (17*x^7)/64)/(12*x^2 + 10*x^4 +
4*x^6 + x^8 + 9) + (atan((x*(7746 - 2^(1/2)*5106i)^(1/2)*851i)/(1179648*((2
^(1/2)*851i)/98304 + 46805/393216)) + (851*2^(1/2)*x*(7746 - 2^(1/2)*5106i)
^(1/2))/(2359296*((2^(1/2)*851i)/98304 + 46805/393216)))*(7746 - 2^(1/2)*51
06i)^(1/2)*1i)/768 - (atan((x*(2^(1/2)*5106i + 7746)^(1/2)*851i)/(1179648*(
(2^(1/2)*851i)/98304 - 46805/393216)) - (851*2^(1/2)*x*(2^(1/2)*5106i + 774
6)^(1/2))/(2359296*((2^(1/2)*851i)/98304 - 46805/393216)))*(2^(1/2)*5106i +
 7746)^(1/2)*1i)/768
```

sympy [B] time = 1.34, size = 1195, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/(x**4+2*x**2+3)**3,x)
```

```
[Out] (51*x**7 + 166*x**5 + 181*x**3 + 292*x)/(192*x**8 + 768*x**6 + 1920*x**4 +
2304*x**2 + 1728) - sqrt(1291/786432 + 1019*sqrt(3)/786432)*log(x**2 + x*(-
55*sqrt(6)*sqrt(1291 + 1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169
 + 49606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 220*sqrt(1291 + 1019*sq
rt(3))/851) - 26628761029*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/751982074
561 - 40176070*sqrt(6)*sqrt(1315529*sqrt(3) + 2390882)/2213882457 + 7609499
4709709/751982074561 + 133967471914*sqrt(3)/2213882457) + sqrt(1291/786432
+ 1019*sqrt(3)/786432)*log(x**2 + x*(-220*sqrt(1291 + 1019*sqrt(3))/851 - 4
9606*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/867169 + 55*sqrt(6)*sqrt(1291 + 1019
*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/867169) - 26628761029*sqrt(2)*sqr
t(1315529*sqrt(3) + 2390882)/751982074561 - 40176070*sqrt(6)*sqrt(1315529*s
qrt(3) + 2390882)/2213882457 + 76094994709709/751982074561 + 133967471914*s
qrt(3)/2213882457) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/393216
 + 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/(-6808*sqrt(-2
*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2
)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 23
90882) + 1291 + 3057*sqrt(3))) - 224180*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/(
-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)
) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529
*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) - 148818*sqrt(1291 + 1019*sqrt(
3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sq
rt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(13
15529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) + 165*sqrt(2)*sqrt(1291 +
1019*sqrt(3))*sqrt(1315529*sqrt(3) + 2390882)/(-6808*sqrt(-2*sqrt(2)*sqrt(1
315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*
sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 +
 3057*sqrt(3)))) + 2*sqrt(-sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)/393216 +
 1291/786432 + 1019*sqrt(3)/262144)*atan(1734338*sqrt(3)*x/(-6808*sqrt(-2*s
qrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*
sqrt(1315529*sqrt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390
882) + 1291 + 3057*sqrt(3))) - 165*sqrt(2)*sqrt(1291 + 1019*sqrt(3))*sqrt(1
315529*sqrt(3) + 2390882)/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 239
0882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882)*s
```

```
qrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3))) + 14
8818*sqrt(1291 + 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3)
+ 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sqrt(3) + 23908
82)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)))
 + 224180*sqrt(3)*sqrt(1291 + 1019*sqrt(3))/(-6808*sqrt(-2*sqrt(2)*sqrt(131
5529*sqrt(3) + 2390882) + 1291 + 3057*sqrt(3)) + 55*sqrt(2)*sqrt(1315529*sq
rt(3) + 2390882)*sqrt(-2*sqrt(2)*sqrt(1315529*sqrt(3) + 2390882) + 1291 + 3
057*sqrt(3))))
```

3.123 $\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3} dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2+\sqrt{2\left(\sqrt{3}-1\right)}x+\right.}{4608}$$

```
[Out] -4/27/x-25/144*x*(x^2+5)/(x^4+2*x^2+3)^2-1/1728*x*(242*x^2+325)/(x^4+2*x^2+
3)-1/13824*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(1/2))^
(1/2)+1/13824*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-179133+165483*3^(1/2
))^(1/2)+1/6912*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(17
9133+165483*3^(1/2))^(1/2)-1/6912*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^
(1/2))^(1/2))*(179133+165483*3^(1/2))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$-\frac{25x\left(x^2+5\right)}{144\left(x^4+2x^2+3\right)^2}-\frac{x\left(242x^2+325\right)}{1728\left(x^4+2x^2+3\right)}-\frac{\sqrt{\frac{1}{3}\left(55161\sqrt{3}-59711\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{4608}+\sqrt{\frac{1}{3}}\left(551\right.$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] -4/(27*x) - (25*x*(5 + x^2))/(144*(3 + 2*x^2 + x^4)^2) - (x*(325 + 242*x^2)
)/(1728*(3 + 2*x^2 + x^4)) + (Sqrt[(59711 + 55161*Sqrt[3])/3]*ArcTan[(Sqrt[
2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/2304 - (Sqrt[(59711 + 5516
1*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] + 2*x)/Sqrt[2*(1 + Sqrt[3])]])
/2304 - (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3
])]*x + x^2])/4608 + (Sqrt[(-59711 + 55161*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2
*(-1 + Sqrt[3])]*x + x^2])/4608
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
```

```
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^2\left(3+2x^2+x^4\right)^3}\,dx = -\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{128+30x^2-\frac{250x^4}{3}}{x^2\left(3+2x^2+x^4\right)^2}\,dx$$

$$= -\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}+\frac{\int \frac{2048-\frac{56x^2}{3}-\frac{1936x^4}{3}}{x^2(3+2x^2+x^4)}\,dx}{4608}$$

$$= -\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}+\frac{\int \left(\frac{2048}{3x^2}-\frac{8(173+166x^2)}{3+2x^2+x^4}\right)dx}{4608}$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{1}{576}\int \frac{173+166x^2}{3+2x^2+x^4}\,dx$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{\int \frac{173\sqrt{2\left(-1+\sqrt{3}\right)}-\left(173-166\sqrt{3}\right)x}{\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2}\,d}{1152\sqrt{6\left(-1+\sqrt{3}\right)}}$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\int -}{4608}$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\log}{46}$$

$$= -\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\tan^{-1}}{2304}$$

Mathematica [C] time = 0.37, size = 140, normalized size = 0.55

$$\frac{-\frac{12\left(166x^8+611x^6+1412x^4+1849x^2+768\right)}{x\left(x^4+2x^2+3\right)^2}+\frac{3i\left(7\sqrt{2}+332i\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}-\frac{3i\left(7\sqrt{2}-332i\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{6912}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^2*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((-12*(768 + 1849*x^2 + 1412*x^4 + 611*x^6 + 166*x^8))/(x*(3 + 2*x^2 + x^4)
^2) + ((3*I)*(332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[2]]])/Sqrt[1 - I*
Sqrt[2]] - ((3*I)*(-332*I + 7*Sqrt[2])*ArcTan[x/Sqrt[1 + I*Sqrt[2]]])/Sqrt[
1 + I*Sqrt[2]])/6912
```

fricas [B] time = 0.91, size = 630, normalized size = 2.49

$$-\frac{858518351136\,x^8+3159968147856\,x^6+210956\cdot 1391283^{\frac{1}{4}}\sqrt{681}\,\sqrt{6}\,\sqrt{3}\,\sqrt{2}\left(x^9+4\,x^7+10\,x^5+12\,x^3+9\,x\right)\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] -1/2978955242496*(858518351136*x^8 + 3159968147856*x^6 + 210956*1391283^(1/
4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)*
sqrt(59711*sqrt(3) + 165483)*arctan(1/15811665652336538898*sqrt(11971753)*1
391283^(3/4)*sqrt(681)*sqrt(6)*sqrt(1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sq
rt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 107745777*x^2 + 107745777*s
qrt(3))*(173*sqrt(3)*sqrt(2) - 498*sqrt(2))*sqrt(59711*sqrt(3) + 165483) -
1/440249244822*1391283^(3/4)*sqrt(681)*sqrt(6)*(173*sqrt(3)*sqrt(2)*x - 498
*sqrt(2)*x)*sqrt(59711*sqrt(3) + 165483) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2
)) + 210956*1391283^(1/4)*sqrt(681)*sqrt(6)*sqrt(3)*sqrt(2)*(x^9 + 4*x^7 +
10*x^5 + 12*x^3 + 9*x)*sqrt(59711*sqrt(3) + 165483)*arctan(1/47434996957009
616694*sqrt(11971753)*1391283^(3/4)*sqrt(681)*sqrt(6)*sqrt(-9*1391283^(1/4)
*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(59711*sqrt(3) + 165483) + 9
69711993*x^2 + 969711993*sqrt(3))*(173*sqrt(3)*sqrt(2) - 498*sqrt(2))*sqrt(
59711*sqrt(3) + 165483) - 1/440249244822*1391283^(3/4)*sqrt(681)*sqrt(6)*(1
73*sqrt(3)*sqrt(2)*x - 498*sqrt(2)*x)*sqrt(59711*sqrt(3) + 165483) - 1/2*sq
rt(3)*sqrt(2) + 1/2*sqrt(2)) + 7302577781952*x^4 - 1391283^(1/4)*sqrt(681)*
sqrt(6)*(165483*x^9 + 661932*x^7 + 1654830*x^5 + 1985796*x^3 - 59711*sqrt(3
)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) + 1489347*x)*sqrt(59711*sqrt(3) + 1
65483)*log(9*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 173*x)*sqrt(5
9711*sqrt(3) + 165483) + 969711993*x^2 + 969711993*sqrt(3)) + 1391283^(1/4)
*sqrt(681)*sqrt(6)*(165483*x^9 + 661932*x^7 + 1654830*x^5 + 1985796*x^3 - 5
9711*sqrt(3)*(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x) + 1489347*x)*sqrt(59711*
sqrt(3) + 165483)*log(-9*1391283^(1/4)*sqrt(681)*sqrt(6)*(166*sqrt(3)*x - 1
73*x)*sqrt(59711*sqrt(3) + 165483) + 969711993*x^2 + 969711993*sqrt(3)) + 9
562653200304*x^2 + 3971940323328)/(x^9 + 4*x^7 + 10*x^5 + 12*x^3 + 9*x)
```

giac [B] time = 3.27, size = 582, normalized size = 2.30

$$\frac{1}{746496}\sqrt{2}\left(83\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+1494\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-1494\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6\sqrt{3}+18}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] 1/746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*
sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqr
t(-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt
(2)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3
^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/
746496*sqrt(2)*(83*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sq
rt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 1494*3^(3/4)*(sqrt(3) + 3)*sqrt(
-6*sqrt(3) + 18) + 83*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 3114*3^(1/4)*sqrt(2
)*sqrt(6*sqrt(3) + 18) + 3114*3^(1/4)*sqrt(-6*sqrt(3) + 18))*arctan(1/3*3^(
3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) + 1/2)) + 1/14
92992*sqrt(2)*(1494*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 8
3*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/
2) + 1494*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)
*sqrt(-6*sqrt(3) + 18) - 3114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 + 2*3^(
1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) - 1/1492992*sqrt(2)*(1494*3^(3/4
)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 18) - 83*3^(3/4)*sqrt(2)*(-6*sqrt
(3) + 18)^(3/2) + 83*3^(3/4)*(6*sqrt(3) + 18)^(3/2) + 1494*3^(3/4)*sqrt(6*s
qrt(3) + 18)*(sqrt(3) - 3) - 3114*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 3
114*3^(1/4)*sqrt(6*sqrt(3) + 18))*log(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) +
 1/2) + sqrt(3)) - 1/1728*(242*x^7 + 809*x^5 + 1676*x^3 + 2475*x)/(x^4 + 2*
x^2 + 3)^2 - 4/27/x
```

maple [B] time = 0.03, size = 424, normalized size = 1.68

$$-\frac{325\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{13824\sqrt{2+2\sqrt{3}}}+\frac{7\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{4608\sqrt{2+2\sqrt{3}}}-\frac{173\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{1728\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x)
```

```
[Out] -4/27/x-1/27*(121/32*x^7+809/64*x^5+419/16*x^3+2475/64*x)/(x^4+2*x^2+3)^2-3
25/27648*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2)
)+7/9216*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-325/13
824/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(
1/2))/(2+2*3^(1/2))^(1/2))+7/4608/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan
((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-173/1728/(2+2*3^(1/2))^(1/
2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+325/27648
*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-7/9216
*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1/2)*x+3^(1/2))-325/13824/(2+2
*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2
+2*3^(1/2))^(1/2))+7/4608/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*arctan((2*x+(-
2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))-173/1728/(2+2*3^(1/2))^(1/2)*3^(1/
2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{166\,x^8+611\,x^6+1412\,x^4+1849\,x^2+768}{576\left(x^9+4\,x^7+10\,x^5+12\,x^3+9\,x\right)}-\frac{1}{576}\int\frac{166\,x^2+173}{x^4+2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^2/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] -1/576*(166*x^8 + 611*x^6 + 1412*x^4 + 1849*x^2 + 768)/(x^9 + 4*x^7 + 10*x^
5 + 12*x^3 + 9*x) - 1/576*integrate((166*x^2 + 173)/(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 0.99, size = 179, normalized size = 0.71

$$-\frac{\frac{83\,x^8}{288}+\frac{611\,x^6}{576}+\frac{353\,x^4}{144}+\frac{1849\,x^2}{576}+\frac{4}{3}}{x^9+4\,x^7+10\,x^5+12\,x^3+9\,x}+\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}\,52739\mathrm{i}}{859963392\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}+\frac{52739\,\sqrt{2}\,x\,\sqrt{-358266-\sqrt{2}\,316434\mathrm{i}}}{1719926784\left(-\frac{17140175}{286654464}+\frac{\sqrt{2}\,9123847\mathrm{i}}{286654464}\right)}\right)\sqrt{-35}}{6912}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^2*(2*x^2 + x^4 + 3)^3),x)
```

```
[Out] (atan((x*(- 2^(1/2)*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*91
23847i)/286654464 - 17140175/286654464)) + (52739*2^(1/2)*x*(- 2^(1/2)*3164
34i - 358266)^(1/2))/(1719926784*((2^(1/2)*9123847i)/286654464 - 17140175/2
86654464)))*(- 2^(1/2)*316434i - 358266)^(1/2)*1i)/6912 - (atan((x*(2^(1/2)
*316434i - 358266)^(1/2)*52739i)/(859963392*((2^(1/2)*9123847i)/286654464 +
 17140175/286654464)) - (52739*2^(1/2)*x*(2^(1/2)*316434i - 358266)^(1/2))/
(1719926784*((2^(1/2)*9123847i)/286654464 + 17140175/286654464)))*(2^(1/2)*
316434i - 358266)^(1/2)*1i)/6912 - ((1849*x^2)/576 + (353*x^4)/144 + (611*x
^6)/576 + (83*x^8)/288 + 4/3)/(9*x + 12*x^3 + 10*x^5 + 4*x^7 + x^9)
```

sympy [A] time = 0.67, size = 75, normalized size = 0.30

$$\frac{-166x^8-611x^6-1412x^4-1849x^2-768}{576x^9+2304x^7+5760x^5+6912x^3+5184x}+\mathrm{RootSum}\left(4174708211712t^4+15652880384t^2+37564641,\left(t\mapsto\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**2/(x**4+2*x**2+3)**3,x)
```

```
[Out] (-166*x**8 - 611*x**6 - 1412*x**4 - 1849*x**2 - 768)/(576*x**9 + 2304*x**7
+ 5760*x**5 + 6912*x**3 + 5184*x) + RootSum(4174708211712*_t**4 + 156528803
84*_t**2 + 37564641, Lambda(_t, _t*log(-98146713600*_t**3/11971753 - 963936
4864*_t/323237331 + x)))
```

3.124 $\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$

Optimal. Leaf size=262

$$-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right)}{41472}-\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x\right.}{41472}$$

```
[Out] -4/81/x^3+7/27/x+25/432*x*(5*x^2+7)/(x^4+2*x^2+3)^2+1/5184*x*(1025*x^2+1474
)/(x^4+2*x^2+3)+1/124416*ln(x^2+3^(1/2)-x*(-2+2*3^(1/2))^(1/2))*(-30014223+
33721353*3^(1/2))^(1/2)-1/124416*ln(x^2+3^(1/2)+x*(-2+2*3^(1/2))^(1/2))*(-3
0014223+33721353*3^(1/2))^(1/2)-1/62208*arctan((-2*x+(-2+2*3^(1/2))^(1/2))/
(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2))^(1/2)+1/62208*arctan((2*x+
(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))*(30014223+33721353*3^(1/2))^(1/2
)
```

Rubi [A] time = 0.37, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {1669, 1664, 1169, 634, 618, 204, 628}

$$\frac{25x\left(5x^2+7\right)}{432\left(x^4+2x^2+3\right)^2}+\frac{x\left(1025x^2+1474\right)}{5184\left(x^4+2x^2+3\right)}-\frac{4}{81x^3}+\frac{\sqrt{\frac{1}{3}\left(11240451\sqrt{3}-10004741\right)}\log\left(x^2-\sqrt{2\left(\sqrt{3}-1\right)}x+\sqrt{3}\right.}{41472}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] -4/(81*x^3) + 7/(27*x) + (25*x*(7 + 5*x^2))/(432*(3 + 2*x^2 + x^4)^2) + (x*
(1474 + 1025*x^2))/(5184*(3 + 2*x^2 + x^4)) - (Sqrt[(10004741 + 11240451*Sq
rt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])] - 2*x)/Sqrt[2*(1 + Sqrt[3])]])/207
36 + (Sqrt[(10004741 + 11240451*Sqrt[3])/3]*ArcTan[(Sqrt[2*(-1 + Sqrt[3])]
+ 2*x)/Sqrt[2*(1 + Sqrt[3])]])/20736 + (Sqrt[(-10004741 + 11240451*Sqrt[3])
/3]*Log[Sqrt[3] - Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/41472 - (Sqrt[(-10004741
 + 11240451*Sqrt[3])/3]*Log[Sqrt[3] + Sqrt[2*(-1 + Sqrt[3])]*x + x^2])/4147
2
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx = \frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{1}{96}\int \frac{128-\frac{160x^2}{3}+50x^4+\frac{1250x^6}{9}}{x^4\left(3+2x^2+x^4\right)^2}\,dx$$

$$= \frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \frac{2048-\frac{6656x^2}{3}+\frac{2576x^4}{9}+\frac{8200x^6}{9}}{x^4(3+2x^2+x^4)}\,dx}{4608}$$

$$= \frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \left(\frac{2048}{3x^4}-\frac{3584}{3x^2}+\frac{8(2242+2369x^2)}{9(3+2x^2+x^4)}\right)dx}{4608}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \frac{2242+2369x^2}{3+2x^2+x^4}\,dx}{5184}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\int \frac{2242\sqrt{2(-1+\sqrt{3})}-(2242\cdots}{\sqrt{3}-\sqrt{2(-1+\sqrt{3})}\cdots}}{10368\sqrt{6\left(-1+\cdots\right.}}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\left(2242-2369\sqrt{3}\right)\int \cdots}{20736\sqrt{6\left(\cdots\right.}}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}+\frac{\sqrt{-\frac{10004741}{12}+\frac{3746817\cdots}{4}\cdots}}{\cdots}$$

$$= -\frac{4}{81x^3}+\frac{7}{27x}+\frac{25x\left(7+5x^2\right)}{432\left(3+2x^2+x^4\right)^2}+\frac{x\left(1474+1025x^2\right)}{5184\left(3+2x^2+x^4\right)}-\frac{\sqrt{\frac{1}{3}\left(10004741+1124\cdots\right.}}{\cdots}$$

Mathematica [C] time = 0.32, size = 139, normalized size = 0.53

$$\frac{\frac{4\left(2369x^{10}+8644x^8+19939x^6+20090x^4+9024x^2-2304\right)}{x^3\left(x^4+2x^2+3\right)^2}+\frac{\left(4738+127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1-i\sqrt{2}}}\right)}{\sqrt{1-i\sqrt{2}}}+\frac{\left(4738-127i\sqrt{2}\right)\tan^{-1}\left(\frac{x}{\sqrt{1+i\sqrt{2}}}\right)}{\sqrt{1+i\sqrt{2}}}}{20736}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + x^2 + 3*x^4 + 5*x^6)/(x^4*(3 + 2*x^2 + x^4)^3),x]
```

```
[Out] ((4*(-2304 + 9024*x^2 + 20090*x^4 + 19939*x^6 + 8644*x^8 + 2369*x^10))/(x^3
*(3 + 2*x^2 + x^4)^2) + ((4738 + (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 - I*Sqrt[
2]]])/Sqrt[1 - I*Sqrt[2]] + ((4738 - (127*I)*Sqrt[2])*ArcTan[x/Sqrt[1 + I*S
qrt[2]]])/Sqrt[1 + I*Sqrt[2]])/20736
```

fricas [B] time = 0.87, size = 652, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="fricas")
```

```
[Out] 1/135934787413472256*(62119890312985296*x^10 + 226662866975704896*x^8 + 522
840224968600176*x^6 + 47239676*713236683^(1/4)*sqrt(15419)*sqrt(6)*sqrt(3)*
sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)*sqrt(10004741*sqrt(3) + 33
721353)*arctan(1/276093525919725583675206533346*sqrt(182097141061)*713236683
^(3/4)*sqrt(15419)*sqrt(6)*sqrt(3)*sqrt(713236683^(1/4)*sqrt(15419)*sqrt(6)
*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 33721353) + 546291423183
*x^2 + 546291423183*sqrt(3))*(2242*sqrt(3)*sqrt(2) - 7107*sqrt(2))*sqrt(100
04741*sqrt(3) + 33721353) - 1/50539604724352062*713236683^(3/4)*sqrt(15419)
*sqrt(6)*(2242*sqrt(3)*sqrt(2)*x - 7107*sqrt(2)*x)*sqrt(10004741*sqrt(3) +
33721353) + 1/2*sqrt(3)*sqrt(2) - 1/2*sqrt(2)) + 47239676*713236683^(1/4)*s
qrt(15419)*sqrt(6)*sqrt(3)*sqrt(2)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3)
*sqrt(10004741*sqrt(3) + 33721353)*arctan(1/828280577759176751025619600038*s
qrt(182097141061)*713236683^(3/4)*sqrt(15419)*sqrt(6)*sqrt(-27*713236683^(1
/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10004741*sqrt(3) + 3
3721353) + 14749868425941*x^2 + 14749868425941*sqrt(3))*(2242*sqrt(3)*sqrt(
2) - 7107*sqrt(2))*sqrt(10004741*sqrt(3) + 33721353) - 1/50539604724352062*
713236683^(3/4)*sqrt(15419)*sqrt(6)*(2242*sqrt(3)*sqrt(2)*x - 7107*sqrt(2)*
x)*sqrt(10004741*sqrt(3) + 33721353) - 1/2*sqrt(3)*sqrt(2) + 1/2*sqrt(2)) +
 526799745203830560*x^4 - 713236683^(1/4)*sqrt(15419)*sqrt(6)*(33721353*x^1
1 + 134885412*x^9 + 337213530*x^7 + 404656236*x^5 + 303492177*x^3 - 1000474
1*sqrt(3)*(x^11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3))*sqrt(10004741*sqrt(3) +
 33721353)*log(27*713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 224
2*x)*sqrt(10004741*sqrt(3) + 33721353) + 14749868425941*x^2 + 1474986842594
1*sqrt(3)) + 713236683^(1/4)*sqrt(15419)*sqrt(6)*(33721353*x^11 + 134885412
*x^9 + 337213530*x^7 + 404656236*x^5 + 303492177*x^3 - 10004741*sqrt(3)*(x^
11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3))*sqrt(10004741*sqrt(3) + 33721353)*lo
g(-27*713236683^(1/4)*sqrt(15419)*sqrt(6)*(2369*sqrt(3)*x - 2242*x)*sqrt(10
004741*sqrt(3) + 33721353) + 14749868425941*x^2 + 14749868425941*sqrt(3)) +
 236627222534562816*x^2 - 60415461072654336)/(x^11 + 4*x^9 + 10*x^7 + 12*x^
5 + 9*x^3)
```

giac [B] time = 2.99, size = 589, normalized size = 2.25

$$-\frac{1}{13436928}\sqrt{2}\left(2369\cdot 3^{\frac{3}{4}}\sqrt{2}\left(6\sqrt{3}+18\right)^{\frac{3}{2}}+42642\cdot 3^{\frac{3}{4}}\sqrt{2}\sqrt{6\sqrt{3}+18}\left(\sqrt{3}-3\right)-42642\cdot 3^{\frac{3}{4}}\left(\sqrt{3}+3\right)\sqrt{-6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="giac")
```

```
[Out] -1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) + 42642*3^
(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(sqrt(3) +
 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) - 80712*3^
(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3) + 18))*a
rctan(1/3*3^(3/4)*(x + 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*sqrt(3) +
 1/2)) - 1/13436928*sqrt(2)*(2369*3^(3/4)*sqrt(2)*(6*sqrt(3) + 18)^(3/2) +
42642*3^(3/4)*sqrt(2)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 42642*3^(3/4)*(s
qrt(3) + 3)*sqrt(-6*sqrt(3) + 18) + 2369*3^(3/4)*(-6*sqrt(3) + 18)^(3/2) -
80712*3^(1/4)*sqrt(2)*sqrt(6*sqrt(3) + 18) + 80712*3^(1/4)*sqrt(-6*sqrt(3)
+ 18))*arctan(1/3*3^(3/4)*(x - 3^(1/4)*sqrt(-1/6*sqrt(3) + 1/2))/sqrt(1/6*s
qrt(3) + 1/2)) - 1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sq
rt(-6*sqrt(3) + 18) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3
^(3/4)*(6*sqrt(3) + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3)
 - 3) - 80712*3^(1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*
sqrt(3) + 18))*log(x^2 + 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) +
1/26873856*sqrt(2)*(42642*3^(3/4)*sqrt(2)*(sqrt(3) + 3)*sqrt(-6*sqrt(3) + 1
8) - 2369*3^(3/4)*sqrt(2)*(-6*sqrt(3) + 18)^(3/2) + 2369*3^(3/4)*(6*sqrt(3)
 + 18)^(3/2) + 42642*3^(3/4)*sqrt(6*sqrt(3) + 18)*(sqrt(3) - 3) - 80712*3^(
1/4)*sqrt(2)*sqrt(-6*sqrt(3) + 18) - 80712*3^(1/4)*sqrt(6*sqrt(3) + 18))*lo
```

```
g(x^2 - 2*3^(1/4)*x*sqrt(-1/6*sqrt(3) + 1/2) + sqrt(3)) + 1/5184*(1025*x^7
+ 3524*x^5 + 7523*x^3 + 6522*x)/(x^4 + 2*x^2 + 3)^2 + 1/81*(21*x^2 - 4)/x^3
```

maple [B] time = 0.04, size = 429, normalized size = 1.64

$$\frac{4865\left(-2+2\sqrt{3}\right)\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{124416\sqrt{2+2\sqrt{3}}}+\frac{127\left(-2+2\sqrt{3}\right)\arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{41472\sqrt{2+2\sqrt{3}}}+\frac{1121\sqrt{3}\ \arctan\left(\frac{2x-\sqrt{-2+2\sqrt{3}}}{\sqrt{2+2\sqrt{3}}}\right)}{7776\sqrt{2+2\sqrt{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x)
```

```
[Out] -4/81/x^3+7/27/x+1/27*(1025/192*x^7+881/48*x^5+7523/192*x^3+1087/32*x)/(x^4
+2*x^2+3)^2+4865/248832*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2-(-2+2*3^(1/2))^
(1/2)*x+3^(1/2))+127/82944*(-2+2*3^(1/2))^(1/2)*ln(x^2-(-2+2*3^(1/2))^(1/2)
*x+3^(1/2))+4865/124416/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arctan((
2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+127/41472/(2+2*3^(1/2))^(1/2
)*(-2+2*3^(1/2))*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+112
1/7776/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x-(-2+2*3^(1/2))^(1/2))/(2+2*3
^(1/2))^(1/2))-4865/248832*(-2+2*3^(1/2))^(1/2)*3^(1/2)*ln(x^2+(-2+2*3^(1/2
))^(1/2)*x+3^(1/2))-127/82944*(-2+2*3^(1/2))^(1/2)*ln(x^2+(-2+2*3^(1/2))^(1
/2)*x+3^(1/2))+4865/124416/(2+2*3^(1/2))^(1/2)*(-2+2*3^(1/2))*3^(1/2)*arcta
n((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+127/41472/(2+2*3^(1/2))^(
1/2)*(-2+2*3^(1/2))*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+2*3^(1/2))^(1/2))+
1121/7776/(2+2*3^(1/2))^(1/2)*3^(1/2)*arctan((2*x+(-2+2*3^(1/2))^(1/2))/(2+
2*3^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{2369\,x^{10}+8644\,x^{8}+19939\,x^{6}+20090\,x^{4}+9024\,x^{2}-2304}{5184\left(x^{11}+4\,x^{9}+10\,x^{7}+12\,x^{5}+9\,x^{3}\right)}+\frac{1}{5184}\int\frac{2369\,x^{2}+2242}{x^{4}+2\,x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x^6+3*x^4+x^2+4)/x^4/(x^4+2*x^2+3)^3,x, algorithm="maxima")
```

```
[Out] 1/5184*(2369*x^10 + 8644*x^8 + 19939*x^6 + 20090*x^4 + 9024*x^2 - 2304)/(x^
11 + 4*x^9 + 10*x^7 + 12*x^5 + 9*x^3) + 1/5184*integrate((2369*x^2 + 2242)/
(x^4 + 2*x^2 + 3), x)
```

mupad [B] time = 1.02, size = 185, normalized size = 0.71

$$\frac{\frac{2369\,x^{10}}{5184}+\frac{2161\,x^{8}}{1296}+\frac{19939\,x^{6}}{5184}+\frac{10045\,x^{4}}{2592}+\frac{47\,x^{2}}{27}-\frac{4}{9}}{x^{11}+4\,x^{9}+10\,x^{7}+12\,x^{5}+9\,x^{3}}-\frac{\mathrm{atan}\left(\frac{x\,\sqrt{-60028446-\sqrt{2}\,70859514\mathrm{i}}\,11809919\mathrm{i}}{626913312768\left(-\frac{57455255935}{208971104256}+\frac{\sqrt{2}\,13238919199\mathrm{i}}{104485552128}\right)}+\frac{11809919\,\sqrt{2}\,x\,\sqrt{-60\ldots}}{1253826625536\left(-\frac{5745\ldots}{2089\ldots}\right.}\right.}{62208}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 3*x^4 + 5*x^6 + 4)/(x^4*(2*x^2 + x^4 + 3)^3),x)
```

```
[Out] ((47*x^2)/27 + (10045*x^4)/2592 + (19939*x^6)/5184 + (2161*x^8)/1296 + (236
9*x^10)/5184 - 4/9)/(9*x^3 + 12*x^5 + 10*x^7 + 4*x^9 + x^11) - (atan((x*(-
2^(1/2)*70859514i - 60028446)^(1/2)*11809919i)/(626913312768*((2^(1/2)*1323
8919199i)/104485552128 - 57455255935/208971104256)) + (11809919*2^(1/2)*x*(
- 2^(1/2)*70859514i - 60028446)^(1/2))/(1253826625536*((2^(1/2)*13238919199
i)/104485552128 - 57455255935/208971104256)))*(- 2^(1/2)*70859514i - 600284
46)^(1/2)*1i)/62208 + (atan((x*(2^(1/2)*70859514i - 60028446)^(1/2)*1180991
9i)/(626913312768*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971
```

```
104256)) - (11809919*2^(1/2)*x*(2^(1/2)*70859514i - 60028446)^(1/2))/(12538
26625536*((2^(1/2)*13238919199i)/104485552128 + 57455255935/208971104256)))
*(2^(1/2)*70859514i - 60028446)^(1/2)*1i)/62208
```

sympy [A] time = 0.69, size = 80, normalized size = 0.31

$$\text{RootSum}\left(338151365148672t^4 + 2622682824704t^2 + 19257390441, \left(t \mapsto t\log\left(\frac{357010935644160t^3}{182097141061} + \frac{260}{163}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5*x**6+3*x**4+x**2+4)/x**4/(x**4+2*x**2+3)**3,x)
```

```
[Out] RootSum(338151365148672*_t**4 + 2622682824704*_t**2 + 19257390441, Lambda(_
t, _t*log(357010935644160*_t**3/182097141061 + 26016957890816*_t/1638874269
549 + x))) + (2369*x**10 + 8644*x**8 + 19939*x**6 + 20090*x**4 + 9024*x**2
- 2304)/(5184*x**11 + 20736*x**9 + 51840*x**7 + 62208*x**5 + 46656*x**3)
```

3.125 $\int \frac{x\left(d+ex^2+fx^4+gx^6\right)}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=149

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)}{4c^3}-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right.}{2c^3\sqrt{b^2-4ac}}$$

```
[Out] 1/2*(-b*g+c*f)*x^2/c^2+1/4*g*x^4/c+1/4*(c^2*e+b^2*g-c*(a*g+b*f))*ln(c*x^4+b
*x^2+a)/c^3-1/2*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*c*(3*a*g+b*f))*arctanh((2*
c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1663, 1657, 634, 618, 206, 628}

$$-\frac{\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{2c^3\sqrt{b^2-4ac}}+\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2\right.}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*f - b*g)*x^2)/(2*c^2) + (g*x^4)/(4*c) - ((2*c^3*d - c^2*(b*e + 2*a*f) -
 b^3*g + b*c*(b*f + 3*a*g))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^
3*Sqrt[b^2 - 4*a*c]) + ((c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x
^4])/(4*c^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1657

```
Int[(Pq_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq
, x] && IGtQ[p, -2]
```

Rule 1663

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :
> Dist[1/2, Subst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x + c*x^2)^
p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x^2] && IntegerQ[
(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(d+ex^2+fx^4+gx^6\right)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\,\text{Subst}\left(\int \frac{d+ex+fx^2+gx^3}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\,\text{Subst}\left(\int \left(\frac{cf-bg}{c^2}+\frac{gx}{c}+\frac{c^2d-acf+abg+\left(c^2e+b^2g-c(bf+ag)\right)x}{c^2\left(a+bx+cx^2\right)}\right.\right.\\ &= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\text{Subst}\left(\int \frac{c^2d-acf+abg+\left(c^2e+b^2g-c(bf+ag)\right)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}\\ &= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\left(c^2e+b^2g-c(bf+ag)\right)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}+\\ &= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}+\frac{\left(c^2e+b^2g-c(bf+ag)\right)\log\left(a+bx^2+cx^4\right)}{4c^3}-\frac{\left(2c^3d-\right.}{}\\ &= \frac{(cf-bg)x^2}{2c^2}+\frac{gx^4}{4c}-\frac{\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)\tanh^{-1}\left(\frac{b+}{\sqrt{b^2}}\right.}{2c^3\sqrt{b^2-4ac}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 142, normalized size = 0.95

$$\frac{\log\left(a+bx^2+cx^4\right)\left(-c(ag+bf)+b^2g+c^2e\right)+\frac{2\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{4ac-b^2}}\right)\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)}{\sqrt{4ac-b^2}}+2cx^2(cf-bg}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(c*f - b*g)*x^2 + c^2*g*x^4 + (2*(2*c^3*d - c^2*(b*e + 2*a*f) - b^3*g
+ b*c*(b*f + 3*a*g))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 +
4*a*c] + (c^2*e + b^2*g - c*(b*f + a*g))*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

fricas [A] time = 1.07, size = 486, normalized size = 3.26

$$\left[\frac{\left(b^2c^2-4\,ac^3\right)gx^4+2\left(\left(b^2c^2-4\,ac^3\right)f-\left(b^3c-4\,abc^2\right)g\right)x^2+\left(2\,c^3d-bc^2e+\left(b^2c-2\,ac^2\right)f-\left(b^3-3\,abc\right)g\right)\sqrt{}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3)*f - (b^3*c - 4*a*b
*c^2)*g)*x^2 + (2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c
 - 4*a*b*c^2)*f + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/
```

```
(b^2*c^3 - 4*a*c^4), 1/4*((b^2*c^2 - 4*a*c^3)*g*x^4 + 2*((b^2*c^2 - 4*a*c^3
)*f - (b^3*c - 4*a*b*c^2)*g)*x^2 - 2*(2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)
*f - (b^3 - 3*a*b*c)*g)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
+ 4*a*c)/(b^2 - 4*a*c)) + ((b^2*c^2 - 4*a*c^3)*e - (b^3*c - 4*a*b*c^2)*f +
(b^4 - 5*a*b^2*c + 4*a^2*c^2)*g)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4
)]
```

giac [A] time = 1.90, size = 146, normalized size = 0.98

$$\frac{cgx^4 + 2\,cfx^2 - 2\,bgx^2}{4\,c^2} - \frac{\left(bcf - b^2g + acg - c^2e\right)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} + \frac{\left(2\,c^3d + b^2cf - 2\,ac^2f - b^3g + 3\,abcg - bc^2\right.}{2\,\sqrt{-b^2 + 4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(c*g*x^4 + 2*c*f*x^2 - 2*b*g*x^2)/c^2 - 1/4*(b*c*f - b^2*g + a*c*g - c^
2*e)*log(c*x^4 + b*x^2 + a)/c^3 + 1/2*(2*c^3*d + b^2*c*f - 2*a*c^2*f - b^3*
g + 3*a*b*c*g - b*c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^
2 + 4*a*c)*c^3)
```

maple [B] time = 0.01, size = 357, normalized size = 2.40

$$\frac{g\,x^4}{4c} + \frac{3abg\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2} - \frac{af\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}\;c} - \frac{b^3g\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^3} + \frac{b^2f\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{2\sqrt{4ac-b^2}\;c^2} - \frac{be\arctan\left(\frac{2c\,x}{\sqrt{4a}}\right.}{2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] 1/4*g*x^4/c-1/2/c^2*x^2*b*g+1/2/c*f*x^2-1/4/c^2*ln(c*x^4+b*x^2+a)*a*g+1/4/c
^3*ln(c*x^4+b*x^2+a)*b^2*g-1/4*b/c^2*f*ln(c*x^4+b*x^2+a)+1/4/c*e*ln(c*x^4+b
*x^2+a)+3/2/c^2/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*a*b
*g-1/(4*a*c-b^2)^(1/2)*a/c*f*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))+1/(4*a*c
-b^2)^(1/2)*d*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))-1/2/c^3/(4*a*c-b^2)^(1/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))*b^3*g+1/2/(4*a*c-b^2)^(1/2)*b^2/c^
2*f*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))-1/2/(4*a*c-b^2)^(1/2)*b/c*e*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re details)Is 4*a*c-b^2 positive or negative?
```

mupad [B] time = 1.68, size = 1834, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(f/(2*c) - (b*g)/(2*c^2)) + (g*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 1
```

```
0*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*
(((((4*c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a*b*c
^4*g)/c^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b
^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*d - b^
3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^2)^(1/2
)) - (b*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)*(2*b^
4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*
a*b^2*c*g))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((((4*
c^6*d + 6*b^2*c^4*f - 6*b^3*c^3*g - 4*a*c^5*f - 6*b*c^5*e + 10*a*b*c^4*g)/c
^4 - (4*b*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f
+ 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*g + 2*b^2*c^2
*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(2*
(16*a*c^4 - 4*b^2*c^3)) - (b^5*g^2 + b*c^4*e^2 + b^3*c^2*f^2 - c^5*d*e + 2*
a^2*b*c^2*g^2 + a*c^4*d*g + a*c^4*e*f + b*c^4*d*f - 2*b^4*c*f*g - a*b*c^3*f
^2 - 3*a*b^3*c*g^2 - b^2*c^3*d*g - 2*b^2*c^3*e*f - a^2*c^3*f*g + 2*b^3*c^2*
e*g + 4*a*b^2*c^2*f*g - 3*a*b*c^3*e*g)/c^4 + (b*(2*c^3*d - b^3*g - 2*a*c^2*
f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2)/(2*c^4*(4*a*c - b^2))))/(2*a*(4*a*c -
 b^2)^(1/2))) + ((((8*a^2*c^4*g - 8*a*c^5*e + 8*a*b*c^4*f - 8*a*b^2*c^3*g)/
c^4 - (8*a*c^2*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f
 + 8*a*b*c^2*f - 10*a*b^2*c*g))/(16*a*c^4 - 4*b^2*c^3))*(2*c^3*d - b^3*g -
2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(8*c^3*(4*a*c - b^2)^(1/2)) - (
a*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)*(2*b^4*g +
2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*
c*g))/(c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a + (b*((((8*a^2*c^4*
g - 8*a*c^5*e + 8*a*b*c^4*f - 8*a*b^2*c^3*g)/c^4 - (8*a*c^2*(2*b^4*g + 2*b^
2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e - 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g)
)/(16*a*c^4 - 4*b^2*c^3))*(2*b^4*g + 2*b^2*c^2*e + 8*a^2*c^2*g - 8*a*c^3*e
- 2*b^3*c*f + 8*a*b*c^2*f - 10*a*b^2*c*g))/(2*(16*a*c^4 - 4*b^2*c^3)) - (a*
c^4*e^2 + a*b^4*g^2 + a^3*c^2*g^2 + a*b^2*c^2*f^2 - 2*a^2*b^2*c*g^2 - 2*a^2
*c^3*e*g + 2*a*b^2*c^2*e*g + 2*a^2*b*c^2*f*g - 2*a*b*c^3*e*f - 2*a*b^3*c*f*
g)/c^4 + (a*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g)^2
)/(c^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(4*c^6*d^2 + b^6*g^2 +
4*a^2*c^4*f^2 + b^2*c^4*e^2 + b^4*c^2*f^2 - 4*a*b^2*c^3*f^2 - 8*a*c^5*d*f -
 4*b*c^5*d*e - 2*b^5*c*f*g + 9*a^2*b^2*c^2*g^2 - 6*a*b^4*c*g^2 + 4*b^2*c^4*
d*f - 4*b^3*c^3*d*g - 2*b^3*c^3*e*f + 2*b^4*c^2*e*g - 6*a*b^2*c^3*e*g + 10*
a*b^3*c^2*f*g - 12*a^2*b*c^3*f*g + 12*a*b*c^4*d*g + 4*a*b*c^4*e*f))*(2*c^3*
d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*c^3*(4*a*c - b^2
)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

3.126 $\int \frac{x^4\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=594

$$\frac{x\left(a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)+x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^4(-\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] (-2*b*g+c*f)*x/c^3+1/3*g*x^3/c^2+1/2*x*(a*(2*c^3*d-c^2*(2*a*f+b*e)-b^3*g+b*
c*(3*a*g+b*f))+(b^3*c*f+b*c^2*(-3*a*f+c*d)-b^4*g-b^2*c*(-4*a*g+c*e)+2*a*c^2
*(-a*g+c*e))*x^2)/c^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*f-b*c^2*(13*a*f+c*d)-5*b^4*g-b^
2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)+(-3*b^4*c*f+4*a*c^3*(-5*a*f+c*d)+b
^2*c^2*(19*a*f+c*d)+5*b^5*g+b^3*c*(-34*a*g+c*e)-4*a*b*c^2*(-13*a*g+2*c*e))/
(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*c*f-b*
c^2*(13*a*f+c*d)-5*b^4*g-b^2*c*(-24*a*g+c*e)+2*a*c^2*(-7*a*g+3*c*e)+(3*b^4*
c*f-4*a*c^3*(-5*a*f+c*d)-b^2*c^2*(19*a*f+c*d)-5*b^5*g-b^3*c*(-34*a*g+c*e)+4
*a*b*c^2*(-13*a*g+2*c*e))/(-4*a*c+b^2)^(1/2))/c^(7/2)/(-4*a*c+b^2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 14.11, antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1668, 1676, 1166, 205}

$$\frac{x\left(x^2\left(-b^2c(ce-4ag)+bc^2(cd-3af)+2ac^2(ce-ag)+b^3cf+b^4(-g)\right)+a\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((c*f - 2*b*g)*x)/c^3 + (g*x^3)/(3*c^2) + (x*(a*(2*c^3*d - c^2*(b*e + 2*a*f
) - b^3*g + b*c*(b*f + 3*a*g)) + (b^3*c*f + b*c^2*(c*d - 3*a*f) - b^4*g - b
^2*c*(c*e - 4*a*g) + 2*a*c^2*(c*e - a*g))*x^2))/(2*c^3*(b^2 - 4*a*c)*(a + b
*x^2 + c*x^4)) - ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e
- 24*a*g) + 2*a*c^2*(3*c*e - 7*a*g) - (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) -
b^2*c^2*(c*d + 19*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e
- 13*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/(2*Sqrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) -
 ((3*b^3*c*f - b*c^2*(c*d + 13*a*f) - 5*b^4*g - b^2*c*(c*e - 24*a*g) + 2*a*
c^2*(3*c*e - 7*a*g) + (3*b^4*c*f - 4*a*c^3*(c*d - 5*a*f) - b^2*c^2*(c*d + 1
9*a*f) - 5*b^5*g - b^3*c*(c*e - 34*a*g) + 4*a*b*c^2*(2*c*e - 13*a*g))/Sqrt[
b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*S
qrt[2]*c^(7/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^4\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}dx = \frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3cf+bc^2(cd-3af)-b\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3cf+bc^2(cd-3af)-b\right.\right.}{2c^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3c\right.\right.}{2c^3\left(b^2-4a\right.}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3c\right.\right.}{2c^3\left(b^2-4a\right.}$$

$$= \frac{(cf-2bg)x}{c^3}+\frac{gx^3}{3c^2}+\frac{x\left(a\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+3ag)\right)+\left(b^3c\right.\right.}{2c^3\left(b^2-4a\right.}$$

Mathematica [A] time = 2.64, size = 721, normalized size = 1.21

$$\frac{\frac{6\sqrt{c}\,x\left(a^2c\left(3bg-2c\left(f+gx^2\right)\right)+a\left(b^3(-g)+b^2c\left(f+4gx^2\right)-bc^2\left(e+3fx^2\right)+2c^3\left(d+ex^2\right)\right)+bx^2\left(b^3(-g)+b^2cf-bc^2e+c^3d\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-b\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (12*Sqrt[c]*(c*f - 2*b*g)*x + 4*c^(3/2)*g*x^3 + (6*Sqrt[c]*x*(b*(c^3*d - b*
c^2*e + b^2*c*f - b^3*g)*x^2 + a^2*c*(3*b*g - 2*c*(f + g*x^2)) + a*(-(b^3*g
) + 2*c^3*(d + e*x^2) - b*c^2*(e + 3*f*x^2) + b^2*c*(f + 4*g*x^2))))/((b^2
- 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*(-5*b^5*g - b^3*c*(c*e + 3*Sqrt[
b^2 - 4*a*c]*f - 34*a*g) + b^4*(3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + 2*a*c^2*(-
2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e + 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*c]*g) - b
^2*c*(c^2*d - c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f + 24*a*Sqrt[b^2 - 4*a*c]*g)
+ b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d + 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f - 4*a*
g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c
)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*(5*b^5*g + b^3*c*(c*e - 3
*Sqrt[b^2 - 4*a*c]*f - 34*a*g) + b^4*(-3*c*f + 5*Sqrt[b^2 - 4*a*c]*g) + b^2
*c*(c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 19*a*c*f - 24*a*Sqrt[b^2 - 4*a*c]*g) +
2*a*c^2*(2*c^2*d - 3*c*Sqrt[b^2 - 4*a*c]*e - 10*a*c*f + 7*a*Sqrt[b^2 - 4*a*
c]*g) + b*c^2*(c*(Sqrt[b^2 - 4*a*c]*d - 8*a*e) + 13*a*(Sqrt[b^2 - 4*a*c]*f
+ 4*a*g)))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 -
 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*c^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 9.95, size = 10761, normalized size = 18.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^3*d*x^3 + b^3*c*f*x^3 - 3*a*b*c^2*f*x^3 - b^4*g*x^3 + 4*a*b^2*c*g*
x^3 - 2*a^2*c^2*g*x^3 - b^2*c^2*x^3*e + 2*a*c^3*x^3*e + 2*a*c^3*d*x + a*b^2
*c*f*x - 2*a^2*c^2*f*x - a*b^3*g*x + 3*a^2*b*c*g*x - a*b*c^2*x*e)/((b^2*c^3
 - 4*a*c^4)*(c*x^4 + b*x^2 + a)) + 1/16*((2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*(b^2*c^3 - 4
*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2 - 4*a*c)*a
*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*f + (10*b^6*c^2 - 88*a*b^4*c^3 + 220*a^2*b^2*
c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c -
 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2
- 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 -
5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 56*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 28*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 24*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14*sqrt(2)*sq
```

```
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^2 - 4*a*c)*
b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)*(b^2*c^3 -
 4*a*c^4)^2*g + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*a*c^5)*(b^2
*c^3 - 4*a*c^4)^2*e - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7
- 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 - 2*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 - 2*a*b^4*c^8 + 16*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^3*c^9 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^9 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^9 + 16*a^2*b^2*c^9
- 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^10 - 32*a^3*c^10 + 2*(b^2
 - 4*a*c)*a*b^2*c^8 - 8*(b^2 - 4*a*c)*a^2*c^9)*d*abs(b^2*c^3 - 4*a*c^4) - 2
*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a*b^5*c^6 - 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 + 3*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 + 68*a^2*b^4*c^7 - 160*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
2*c^8 - 256*a^3*b^2*c^8 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^
9 + 320*a^4*c^9 + 6*(b^2 - 4*a*c)*a*b^4*c^6 - 44*(b^2 - 4*a*c)*a^2*b^2*c^7
+ 80*(b^2 - 4*a*c)*a^3*c^8)*f*abs(b^2*c^3 - 4*a*c^4) + 2*(5*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 1
0*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^6 + 78*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 + 118*a^2*b^5*c^6 - 304*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^7 - 464
*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 608*a
^4*b*c^8 + 10*(b^2 - 4*a*c)*a*b^5*c^5 - 78*(b^2 - 4*a*c)*a^2*b^3*c^6 + 152*
(b^2 - 4*a*c)*a^3*b*c^7)*g*abs(b^2*c^3 - 4*a*c^4) + 2*(sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 2*a*b^5*
c^7 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b^3*c^8 + 16*a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^9 - 32*a^3*b*c^9 + 2*(b^2 - 4*a*c)*a*b^3*c^7 - 8*(b^2 - 4*a*c)*a
^2*b*c^8)*abs(b^2*c^3 - 4*a*c^4)*e - (2*b^7*c^11 - 8*a*b^5*c^12 - 32*a^2*b^
3*c^13 + 128*a^3*b*c^14 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^7*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^5*c^10 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^6*c^10 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^2*b^3*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c^11 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^12 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^12 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^13 - 2*(b^2 - 4*a*c)*b^5*c^11 + 32*(b^2 - 4*a*c)*a^2*b*c^13)*d + (6*b^9*c
^9 - 86*a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a^4*b*c^13 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^7 + 43*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 + 6*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 - 220*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 - 62*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3*sqrt(2)*sq
```

rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 + 464*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 96*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 80*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6*(b^2 - 4*a
*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^2*b^3*c^11
+ 160*(b^2 - 4*a*c)*a^3*b*c^12)*f - (10*b^10*c^8 - 148*a*b^8*c^9 + 808*a^2*
b^6*c^10 - 1920*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*c)*b^8*c^8 + 108*
(b^2 - 4*a*c)*a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 + 416*(b^2 - 4*a*c
)*a^3*b^2*c^11)*g - (2*b^8*c^10 - 32*a*b^6*c^11 + 160*a^2*b^4*c^12 - 256*a^
3*b^2*c^13 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*
c^8 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^
9 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^9 - 8
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 -
24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^10 + 128*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 + 64*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 + 12*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^11 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^12 - 2*(b
^2 - 4*a*c)*b^6*c^10 + 24*(b^2 - 4*a*c)*a*b^4*c^11 - 64*(b^2 - 4*a*c)*a^2*b
^2*c^12)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*a*b*c^4 + sqrt((b^3*c^3
- 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 - 4*a*c^5)))/(b^2*c^4 -
 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*c^8 + 48*a^3*b^2*c^9 + 1
6*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*c^10 - 8*a^2*b^2*c^10 +
16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/16*((2*b^3*c^5 - 8*a*b*c^6
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*(
b^2*c^3 - 4*a*c^4)^2*d - (6*b^5*c^3 - 50*a*b^3*c^4 + 104*a^2*b*c^5 - 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 25*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 52*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 26*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 6*(b^2 - 4*a*c)*b^3*c^3 + 26*(b^2
 - 4*a*c)*a*b*c^4)*(b^2*c^3 - 4*a*c^4)^2*f + (10*b^6*c^2 - 88*a*b^4*c^3 + 2
20*a^2*b^2*c^4 - 112*a^3*c^5 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*b^6 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^5*c - 110*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a

```
^2*b^2*c^2 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^3*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*
c^2 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3
 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +
24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 14
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 10*(b^
2 - 4*a*c)*b^4*c^2 + 48*(b^2 - 4*a*c)*a*b^2*c^3 - 28*(b^2 - 4*a*c)*a^2*c^4)
*(b^2*c^3 - 4*a*c^4)^2*g + (2*b^4*c^4 - 20*a*b^2*c^5 + 48*a^2*c^6 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^5 - 2*(b^2 - 4*a*c)*b^2*c^4 + 12*(b^2 - 4*a*c)*
a*c^5)*(b^2*c^3 - 4*a*c^4)^2*e - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a*b^4*c^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 - 2*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^8 + 2*a*b^4*c^8 + 16*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^9 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^9 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^9 - 16*a
^2*b^2*c^9 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^10 + 32*a^3*c^
10 - 2*(b^2 - 4*a*c)*a*b^2*c^8 + 8*(b^2 - 4*a*c)*a^2*c^9)*d*abs(b^2*c^3 - 4
*a*c^4) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^5 - 34*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 - 6*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^5*c^6 + 6*a*b^6*c^6 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b^2*c^7 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3
*c^7 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7 - 68*a^2*b^4*c^7
 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^8 - 80*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^8 + 256*a^3*b^2*c^8 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*c^9 - 320*a^4*c^9 - 6*(b^2 - 4*a*c)*a*b^4*c^6 + 44*(b^2 - 4*a*c)*a
^2*b^2*c^7 - 80*(b^2 - 4*a*c)*a^3*c^8)*f*abs(b^2*c^3 - 4*a*c^4) + 2*(5*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^4 - 59*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^5*c^5 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^6*c^5 + 10*a*b^7*c^5 + 232*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^
3*c^6 + 78*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^6 + 5*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 118*a^2*b^5*c^6 - 304*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^7 - 152*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c^7 - 39*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^
3*c^7 + 464*a^3*b^3*c^7 + 76*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^8 - 608*a^4*b*c^8 - 10*(b^2 - 4*a*c)*a*b^5*c^5 + 78*(b^2 - 4*a*c)*a^2*b^3
*c^6 - 152*(b^2 - 4*a*c)*a^3*b*c^7)*g*abs(b^2*c^3 - 4*a*c^4) + 2*(sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^6 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^7 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^7
 + 2*a*b^5*c^7 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^8 + 8*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^8 + sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^8 - 16*a^2*b^3*c^8 - 4*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^9 + 32*a^3*b*c^9 - 2*(b^2 - 4*a*c)*a*b^3*c^7 + 8*(b^2
 - 4*a*c)*a^2*b*c^8)*abs(b^2*c^3 - 4*a*c^4)*e - (2*b^7*c^11 - 8*a*b^5*c^12
- 32*a^2*b^3*c^13 + 128*a^3*b*c^14 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^7*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^5*c^10 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^6*c^10 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^5*c^11 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*b*c^12 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^12 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^13 - 2*(b^2 - 4*a*c)*b^5*c^11 + 32*(b^2 - 4*a*c)*a^2*b*c^13)*d
 + (6*b^9*c^9 - 86*a*b^7*c^10 + 440*a^2*b^5*c^11 - 928*a^3*b^3*c^12 + 640*a
```

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^4*b*c^13 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9
*c^7 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c
^8 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 -
220*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 -
 62*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 3
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^9 + 464*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 + 192*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 31
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^10 - 320
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^11 - 160
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 9
6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^11 +
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^12 - 6
*(b^2 - 4*a*c)*b^7*c^9 + 62*(b^2 - 4*a*c)*a*b^5*c^10 - 192*(b^2 - 4*a*c)*a^
2*b^3*c^11 + 160*(b^2 - 4*a*c)*a^3*b*c^12)*f - (10*b^10*c^8 - 148*a*b^8*c^9
 + 808*a^2*b^6*c^10 - 1920*a^3*b^4*c^11 + 1664*a^4*b^2*c^12 - 5*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^10*c^6 + 74*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^8*c^7 + 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^7 - 404*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^8 - 108*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^8 - 5*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^8 + 960*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^9 + 376*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^9 + 54*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^9 - 832*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^10 - 416*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^10 - 188*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^10 + 208*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^11 - 10*(b^2 - 4*a*c)*b^8
*c^8 + 108*(b^2 - 4*a*c)*a*b^6*c^9 - 376*(b^2 - 4*a*c)*a^2*b^4*c^10 + 416*(
b^2 - 4*a*c)*a^3*b^2*c^11)*g - (2*b^8*c^10 - 32*a*b^6*c^11 + 160*a^2*b^4*c^
12 - 256*a^3*b^2*c^13 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*b^8*c^8 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a*b^6*c^9 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^7*c^9 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^4*c^10 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^1
0 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
^11 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*
c^11 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^11 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*
c^12 - 2*(b^2 - 4*a*c)*b^6*c^10 + 24*(b^2 - 4*a*c)*a*b^4*c^11 - 64*(b^2 - 4
*a*c)*a^2*b^2*c^12)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^3 - 4*a*b*c^4 - sqr
t((b^3*c^3 - 4*a*b*c^4)^2 - 4*(a*b^2*c^3 - 4*a^2*c^4)*(b^2*c^4 - 4*a*c^5)))
/(b^2*c^4 - 4*a*c^5)))/((a*b^6*c^7 - 12*a^2*b^4*c^8 - 2*a*b^5*c^8 + 48*a^3*
b^2*c^9 + 16*a^2*b^3*c^9 + a*b^4*c^9 - 64*a^4*c^10 - 32*a^3*b*c^10 - 8*a^2*
b^2*c^10 + 16*a^3*c^11)*abs(b^2*c^3 - 4*a*c^4)*abs(c)) + 1/3*(c^4*g*x^3 + 3
*c^4*f*x - 6*b*c^3*g*x)/c^6
```

maple [B] time = 0.06, size = 3028, normalized size = 5.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*b^3/c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+13/4/c/(4*a*
c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*f-13/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2
```

```
)^(1/2))*c)^(1/2)*a*b/c*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)+1/3*g*x^3/c^2-5/4/c^3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^5*g+6/c^2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*g-5/4/c^3/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^5*g-6/c^2/(4*a*c-b^2)*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*a*b^2*g-2/c^3*x*b*g+1/c^2*f*x-2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^
3*a*b^2*g-13/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*b*g
+17/2/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^3*g-13/c/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*b*g+17/2/c^2/(4*a*c-b^2
)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^3*g+1/2/(c*x^4+b*x^2+a)/(4*a
*c-b^2)*a*b/c*e*x+3/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*a*b/c*f*x^3-1/2/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*a*b^2/c^2*f*x-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
b^3*f+1/4/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e+3/4/(4*a*c-b^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b^3/c^2*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*b^2/c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/4/(4*a*c
-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d-1/4/(4*a*c-b^2)/(-4*a*
c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b^2*d*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*a^2*f+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b^4*f-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
b^3*e+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*a*b*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/(c*x^4+b*
x^2+a)/(4*a*c-b^2)*a*d*x-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*a*e*x^3-1/2/(c*x^4+b
*x^2+a)/(4*a*c-b^2)*b*d*x^3-3/2/c^2/(c*x^4+b*x^2+a)*a^2/(4*a*c-b^2)*x*b*g+1
/2/c^3/(c*x^4+b*x^2+a)*a/(4*a*c-b^2)*x*b^3*g-5/4/c^3/(4*a*c-b^2)*2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)*b^4*g+7/2/c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*g+5/4/c^3/(4*a*c
-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*g-7/2/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*
g-1/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*a*c*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+3/4/(4*a*c-b^2)
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b^4/c^2*f*arct
an(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+2/(4*a*c-b^2)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*e-c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)*a*d-19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*a*b^2*f-19/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*a*b^2/c*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b^3/c^2*f*x^3+1/2/(c*x^4+b*x^2+a)/(4*a*c-
b^2)*b^2/c*e*x^3+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*a^2/c*f*x-3/2/(4*a*c-b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
```

```
1/2))*c)^(1/2)*c*x)*a*e+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+3/2/(4*a*
c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a*e*arctan(2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*b*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+5/(4*a
*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a^2*f*a
rctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/c/(c*x^4+b*x^2+a)/(4*
a*c-b^2)*x^3*a^2*g+1/2/c^3/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^4*g
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bc^3d-\left(b^2c^2-2\,ac^3\right)e+\left(b^3c-3\,abc^2\right)f-\left(b^4-4\,ab^2c+2\,a^2c^2\right)g\right)x^3+\left(2\,ac^3d-abc^2e+\left(ab^2c-2\,a^2c^2\right)f-\left(ab^3\right.\right.}{2\left(ab^2c^3-4\,a^2c^4+\left(b^2c^4-4\,ac^5\right)x^4+\left(b^3c^3-4\,abc^4\right)x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*((b*c^3*d - (b^2*c^2 - 2*a*c^3)*e + (b^3*c - 3*a*b*c^2)*f - (b^4 - 4*a*
b^2*c + 2*a^2*c^2)*g)*x^3 + (2*a*c^3*d - a*b*c^2*e + (a*b^2*c - 2*a^2*c^2)*
f - (a*b^3 - 3*a^2*b*c)*g)*x)/(a*b^2*c^3 - 4*a^2*c^4 + (b^2*c^4 - 4*a*c^5)*
x^4 + (b^3*c^3 - 4*a*b*c^4)*x^2) + 1/2*integrate(-(2*a*c^3*d - a*b*c^2*e -
(b*c^3*d + (b^2*c^2 - 6*a*c^3)*e - (3*b^3*c - 13*a*b*c^2)*f + (5*b^4 - 24*a
*b^2*c + 14*a^2*c^2)*g)*x^2 + (3*a*b^2*c - 10*a^2*c^2)*f - (5*a*b^3 - 19*a^
2*b*c)*g)/(c*x^4 + b*x^2 + a), x)/(b^2*c^3 - 4*a*c^4) + 1/3*(c*g*x^3 + 3*(c
*f - 2*b*g)*x)/c^3
```

mupad [B] time = 4.73, size = 47339, normalized size = 79.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(b^4*g + b^2*c^2*e + 2*a^2*c^2*g - 2*a*c^3*e - b*c^3*d - b^3*c*f + 3*
a*b*c^2*f - 4*a*b^2*c*g))/(2*(4*a*c - b^2)) + (x*(2*a^2*c^2*f - 2*a*c^3*d +
 a*b^3*g + a*b*c^2*e - a*b^2*c*f - 3*a^2*b*c*g))/(2*(4*a*c - b^2)))/(a*c^3
+ c^4*x^4 + b*c^3*x^2) + x*(f/c^2 - (2*b*g)/c^3) + atan(((((2048*a^4*c^10*d
 - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c
^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a
^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*
g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 8
0*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6
- 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b
^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9
)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*
c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2
 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c
^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2
+ 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44
800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 +
 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 -
 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)
^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c
^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*
e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5
*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5
```

```
*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 2
58*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g
*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) +
192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^
4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d
*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 26
88*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^
4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*
f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g +
 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119
616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*
b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^
(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c -
 b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*
f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/
(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*
b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192
*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 -
 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(
1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640
*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 +
 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a
^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*
b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*
a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*
c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) -
 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g -
 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35
840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1
536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(
4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^1
1*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*
c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*
b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*
(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512
*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b
^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^
7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a
*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a
^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^
(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c -
b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*
b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*
e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) +
12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)
^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4
*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*g^2 + 8*a^2
*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 3
92*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a
*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 71
8*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*
b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*
```

d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 1
4*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 8
6*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g
 + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*
c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g
 - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 -
8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1
/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) -
 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(
-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*
a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 +
288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^
2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b
^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a
^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c
^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) -
615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g -
15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 358
40*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 15
36*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4
*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11
*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c
 - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b
^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(
-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*
a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^
6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7
*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*
c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^
4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(
1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b
^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b
^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e
g(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 1
2*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^
(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*
a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096
*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i - (((2048*a^4*c^10*d -
10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*
e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*
b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g -
 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a
*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 4
8*a^2*b^2*c^7)) + (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)
^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(
1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5
e^2(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 -
80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*
d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2
077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800
*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 11
6928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49
*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*
d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f
 + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*

```
f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*
e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*
a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-
(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192
*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c
*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f
- 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*
a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b
^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(
-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44
352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2
)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a
*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616
*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4
*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/
2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^
2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g
*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6
*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*
b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*
a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2
) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 7
68*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(
4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^
7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 28
8*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*
b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3
*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c -
 b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4
*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3
*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 61
5*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15
360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840
*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536
*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a
*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c
^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c -
 b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6
*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(
4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^
4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*
c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e
*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*
b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/
2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2
)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6
*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g
*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*
a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1
/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*
c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 +
3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*g^2 + 8*a^2*c^
8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*
a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^
6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a
^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2
```

*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f +

35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f -
1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(
-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b
^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*
a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^
2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^
2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 5
12*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3
*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*
c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4
*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352
*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9
)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c
- b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^
4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^
2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^
9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-
(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^
10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5
*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b
^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) +
b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*
a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a
*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b
*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a
^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9
*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^
7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^
7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a
*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360
*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^
7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^
5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c
- b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*
e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^
2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^
7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a
*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b
^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6
*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f
+ 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b
^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5
*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2)
+ 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9
)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^
5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-
(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b
*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2)
- 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c -
b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*
c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 384
0*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*g^2 + 8*a^2*c^8*d
^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5
*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c
^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*

b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2*c^
4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f +
336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^
3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^
5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394
*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*
g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804
*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^
2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) +
b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a
^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*
c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*
c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^
2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*
c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7
*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2
)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7
*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2
*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*
b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*
a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7
*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5
*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e
*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2
)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7
*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*
c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^
3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*
d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f +
6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^
2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*
c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) +
10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)
^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5
*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(
4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*
c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2)
- 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c -
b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c
^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840
*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((2048*a^4*c^10*d - 10240*a^5
*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a
^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f
- 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6
*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*
g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2
c^7)) + (x(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 76
8*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4
*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7
*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288
*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b
^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*
c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c -
b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*
b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*
g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615
*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 153

```
60*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*
a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*
a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^
3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c -
b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*
c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4
*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c
^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*
f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c -
 b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b
^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2
) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)
^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*
c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*
(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a
*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/
2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c
 - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^
6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3
840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 -
 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6
)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*
c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*
c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g
^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7
*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f
^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2
+ 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*
g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4
*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*
c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c
^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*
f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^
9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)
^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g +
43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^
(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e
- 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b
^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8
*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g +
 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144
*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e
*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b
^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2
) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g
+ 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c
 - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d
*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*
a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^
9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 +
 b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*
b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 7
2*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g
^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2
```

- 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 + 512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g - 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e + 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^10*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (2744*a^7*c^3*g^3 - 225*a^4*b^6*g^3 - 216*a^4*c^6*e^3 + 3*a*b^3*c^6*d^3 + 4*a^2*b*c^7*d^3 + 1300*a^5*b*c^4*f^3 - 24*a^3*c^7*d^2*e + 2060*a^5*b^4*c*g^3 - 125*a^2*b^8*e*g^2 + 56*a^4*c^6*d^2*g - 600*a^5*c^5*e*f^2 + 175*a^3*b^7*f*g^2 + 1512*a^5*c^5*e^2*g - 3528*a^6*c^4*e*g^2 + 1400*a^6*c^4*f^2*g - 5*a^2*b^4*c^4*e^3 + 66*a^3*b^2*c^5*e^3 + 63*a^3*b^5*c^2*f^3 - 573*a^4*b^3*c^3*f^3 - 5334*a^6*b^2*c^2*g^3 + 75*a*b^9*d*g^2 + 240*a^4*c^6*d*e*f - 560*a^5*c^5*d*f*g + 6*a*b^4*c^5*d^2*e + 3*a*b^5*c^4*d*e^2 + 204*a^3*b*c^6*d*e^2 - 18*a*b^5*c^4*d^2*f + 27*a*b^7*c^2*d*f^2 + 12*a^3*b*c^6*d^2*f - 420*a^4*b*c^5*d*f^2 + 30*a*b^6*c^3*d^2*g - 845*a^2*b^7*c*d*g^2 + 924*a^4*b*c^5*e^2*f + 2044*a^5*b*c^4*d*g^2 + 1350*a^3*b^6*c*e*g^2 - 210*a^3*b^6*c*f^2*g - 1485*a^4*b^5*c*f*g^2 + 364*a^6*b*c^3*f*g^2 - 42*a^2*b^2*c^6*d^2*e - 51*a^2*b^3*c^5*d*e^2 + 81*a^2*b^3*c^5*d^2*f - 279*a^2*b^5*c^3*d*f^2 + 801*a^3*b^3*c^4*d*f^2 - 149*a^2*b^4*c^4*d^2*g + 30*a^2*b^5*c^3*e^2*f - 45*a^2*b^6*c^2*e*f^2 + 78*a^3*b^2*c^5*d^2*g - 339*a^3*b^3*c^4*e^2*f + 402*a^3*b^4*c^3*e*f^2 + 3198*a^3*b^5*c^2*d*g^2 - 762*a^4*b^2*c^4*e*f^2 - 4571*a^4*b^3*c^3*d*g^2 - 50*a^2*b^6*c^2*e^2*g + 600*a^3*b^4*c^3*e^2*g - 2002*a^4*b^2*c^4*e^2*g - 4835*a^4*b^4*c^2*e*g^2 + 6598*a^5*b^2*c^3*e*g^2

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+ 1927*a^4*b^4*c^2*f^2*g - 4722*a^5*b^2*c^3*f^2*g + 3061*a^5*b^3*c^2*f*g^2
- 90*a*b^8*c*d*f*g - 18*a*b^6*c^3*d*e*f + 30*a*b^7*c^2*d*e*g - 1352*a^4*b*c
^5*d*e*g + 150*a^2*b^7*c*e*f*g - 2312*a^5*b*c^4*e*f*g + 246*a^2*b^4*c^4*d*e
*f - 804*a^3*b^2*c^5*d*e*f - 424*a^2*b^5*c^3*d*e*g + 1578*a^3*b^3*c^4*d*e*g
 + 972*a^2*b^6*c^2*d*f*g - 3244*a^3*b^4*c^3*d*f*g + 3276*a^4*b^2*c^4*d*f*g
- 1480*a^3*b^5*c^2*e*f*g + 4122*a^4*b^3*c^3*e*f*g)/(4*(64*a^3*c^8 - b^6*c^5
 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))))*(-(25*b^15*g^2 + b^9*c^6*d^2 + c^6*d^2
*(-(4*a*c - b^2)^9)^(1/2) + b^11*c^4*e^2 + 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 768*a^4*b*c^10*d^2 - 27*a*b^9*c^5*e^2 - 3840*a^5*b*c
^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) - 213*a*b^11*c^3*f^2 + 26880*
a^6*b*c^8*f^2 - 80640*a^7*b*c^7*g^2 - 30*b^14*c*f*g - 96*a^2*b^5*c^8*d^2 +
512*a^3*b^3*c^9*d^2 + 288*a^2*b^7*c^6*e^2 - 1504*a^3*b^5*c^7*e^2 + 3840*a^4
*b^3*c^8*e^2 + 2077*a^2*b^9*c^4*f^2 - 10656*a^3*b^7*c^5*f^2 + 30240*a^4*b^5
*c^6*f^2 - 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6366*a^2*b^11*c^2*g^2 - 35767*a^3*
b^9*c^3*g^2 + 116928*a^4*b^7*c^4*g^2 - 219744*a^5*b^5*c^5*g^2 + 215040*a^6*
b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 615*a*b^13*c*g^2 + 3072*a^5*c^10*d*e + 2*b^10*c^5*d*e
 - 7168*a^6*c^9*d*g - 15360*a^6*c^9*e*f - 6*b^11*c^4*d*f + 10*b^12*c^3*d*g
- 6*b^12*c^3*e*f + 35840*a^7*c^8*f*g + 10*b^13*c^2*e*g - 36*a*b^8*c^6*d*e +
 98*a*b^9*c^5*d*f - 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 168*a*b^10*c^4*d*g + 152*a*b^1
0*c^4*e*f - 258*a*b^11*c^3*e*g + 43520*a^6*b*c^8*e*g + 724*a*b^12*c^2*f*g -
 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) + 192*a^2*b^6*c^7*d*e - 128*a^3*b^4*c^8*d*e - 1536*a^4*b^2*c^9*d
*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 576*a^2*b^7*c^6*d*f + 1344*
a^3*b^5*c^7*d*f - 512*a^4*b^3*c^8*d*f + 1044*a^2*b^8*c^5*d*g - 1548*a^2*b^8
*c^5*e*f - 2688*a^3*b^6*c^6*d*g + 8064*a^3*b^6*c^6*e*f + 1152*a^4*b^4*c^7*d
*g - 22400*a^4*b^4*c^7*e*f + 6144*a^5*b^2*c^8*d*g + 30720*a^5*b^2*c^8*e*f -
 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^2*b^9*c^4*e*g - 14784*a^3*
b^7*c^5*e*g + 44352*a^4*b^5*c^6*e*g - 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*
g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^
3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^2*b^10*c^3*f*g + 39132*a^3*b^8*
c^4*f*g - 119616*a^4*b^6*c^5*f*g + 201600*a^5*b^4*c^6*f*g - 161280*a^6*b^2*
c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a
*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e
*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^
9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*2i +
 atan(((((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*
b^2*c^9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 422
4*a^3*b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c
^6*g - 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c
^9*e - 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8
- b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*((c^6*d^2*(-(4*a*c - b^2)^
9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b
^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3
840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f
^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5
*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2
 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30
240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2
)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 +
 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 -
215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^
2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b
^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^
12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^
```

8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*

```
b^4*c^3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b
^5*c^5*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e
*f + 10*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f
 - 58*a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*
e*f - 148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^
2*c^6*d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g
- 1132*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) -
b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4
*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c
^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*
a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 -
512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4
*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5
*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*
b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*
b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*
a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e
 + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g
+ 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e -
 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1
/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^1
0*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g -
 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d
*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*
a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8
*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d
*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f -
 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*
b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*
g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^
3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*
c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*
c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a
*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e
*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^
2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^
9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i -
 (((2048*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^
9*d - 192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*
b^4*c^7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g -
 19712*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e -
 48*a*b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*
c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7)) + (x*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/
2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2
*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^
5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 2
6880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d
^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 384
0*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^
4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(
1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767
*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040
*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^
5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3
```

*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*
d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^
9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*
a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*
f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c
- b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*
c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f -
1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^
2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*
c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*
e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784
*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c
^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) -
6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3
*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6
*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(
-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*
c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2
) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b
^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)
*(16*b^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a
^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*
c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c
- b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e
^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*
b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*
a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3
*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6
*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^
2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*
c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*
c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7
168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*
b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*
a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2)
+ 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^
4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*
b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)
^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e -
165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*
b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5
*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g +
22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b
^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*
c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-
(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^
3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*
f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*
f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(
-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a
*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9
)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 -
1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25
*b^10*g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2
+ b^6*c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^
4*c^5*e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2

```
*b^4*c^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^
3*g^2 + 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5
*d*e - 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 1
0*b^8*c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*
a*b^5*c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f -
148*a*b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*
d*f + 26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132
*a^3*b^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^
6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c -
 b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2
 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*
c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^
3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c
^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f
^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*
c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^
3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^
6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 716
8*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^
12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*
b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) +
2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*
e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^
5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(
1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 1
65*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^
5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e
*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 2
2400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2
*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^
5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4
*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*
g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*
g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b
^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b
^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 12
80*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*1i)/((((20
48*a^4*c^10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d -
192*a^2*b^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^
7*f + 10752*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712
*a^4*b^3*c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*
b^8*c^5*f + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 +
12*a*b^4*c^6 - 48*a^2*b^2*c^7)) - (x*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b
^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*
a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^
9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a
^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 5
12*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*
b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*
c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) +
 b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b
^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b
^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a
*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e
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+ 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g +
 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e -
98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/
2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10
*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g -
30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)
^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*
e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a
^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*
c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*
g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f -
6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b
^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g
*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3
*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c
^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c
^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*
c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*
f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 18
4*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9
 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b
^7*c^7 - 192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7
 + b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^
2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2
)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9
*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*
f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^
3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e
^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 +
 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^
2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^
2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^
6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c
^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*
c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*
c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f
+ 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*
f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2)
 - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a
*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^
7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f +
 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400
*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4
*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*
g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c
 - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*
(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g +
119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g +
10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*
c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c
^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a
^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (x*(25*b^10*
g^2 + 8*a^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*
c^4*e^2 - 392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*
```

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e^2 - 114*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c
^4*f^2 - 718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2
+ 2794*a^4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e -
 6*b^6*c^4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*
c^2*e*g - 14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*
c^4*d*g + 86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*
b^6*c^3*e*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f +
26*a^2*b^3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b
^2*c^5*e*g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 +
 b^4*c^5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2
- 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^
9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a
*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^
2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*
c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2
 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 4
4800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^
2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2
- 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2
- 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9
)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*
c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3
*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^
5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^
5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f +
258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*
g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) -
 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b
^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*
d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2
688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a
^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d
*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g
- 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c -
 b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-
(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 11
9616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10
*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)
^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c
- b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2
*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3
*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (((2048*a^4*c^
10*d - 10240*a^5*c^9*f + 384*a^2*b^4*c^8*d - 1536*a^3*b^2*c^9*d - 192*a^2*b
^5*c^7*e + 768*a^3*b^3*c^8*e + 736*a^2*b^6*c^6*f - 4224*a^3*b^4*c^7*f + 107
52*a^4*b^2*c^8*f - 1264*a^2*b^7*c^5*g + 7488*a^3*b^5*c^6*g - 19712*a^4*b^3*
c^7*g - 32*a*b^6*c^7*d + 16*a*b^7*c^6*e - 1024*a^4*b*c^9*e - 48*a*b^8*c^5*f
 + 80*a*b^9*c^4*g + 19456*a^5*b*c^8*g)/(8*(64*a^3*c^8 - b^6*c^5 + 12*a*b^4*
c^6 - 48*a^2*b^2*c^7)) + (x*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^
2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2
)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9
*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*
f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^
3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e
^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 +
 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*
e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^
2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^
2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)
```

^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^
6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c
^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*
c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*
c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f
+ 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*
f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a
*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^
7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f +
2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400
*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4
*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*
g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c
- b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*
(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g +
119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g +
10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*
c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c
^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a
^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2)*(16*b^7*c^7 -
192*a*b^5*c^8 - 1024*a^3*b*c^10 + 768*a^2*b^3*c^9))/(2*(16*a^2*c^7 + b^4*c^
5 - 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^
15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 806
40*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2
- 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077
*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^
5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 11692
8*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^
3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g
+ 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f -
35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f +
1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(
-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b
^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*
a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^
2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 5
12*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3
*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*
c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4
*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352
*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9
)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c
- b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^
4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^
2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2)
+ 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^
9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-
(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4
096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^
10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) + (x*(25*b^10*g^2 + 8*a
^2*c^8*d^2 - 72*a^3*c^7*e^2 + b^4*c^6*d^2 + 200*a^4*c^6*f^2 + b^6*c^4*e^2 -

392*a^5*c^5*g^2 + 9*b^8*c^2*f^2 + 2*a*b^2*c^7*d^2 - 16*a*b^4*c^5*e^2 - 114
*a*b^6*c^3*f^2 - 30*b^9*c*f*g + 74*a^2*b^2*c^6*e^2 + 481*a^2*b^4*c^4*f^2 -
718*a^3*b^2*c^5*f^2 + 1676*a^2*b^6*c^2*g^2 - 3536*a^3*b^4*c^3*g^2 + 2794*a^
4*b^2*c^4*g^2 - 340*a*b^8*c*g^2 - 80*a^3*c^7*d*f + 2*b^5*c^5*d*e - 6*b^6*c^
4*d*f + 336*a^4*c^6*e*g + 10*b^7*c^3*d*g - 6*b^7*c^3*e*f + 10*b^8*c^2*e*g -
14*a*b^3*c^6*d*e - 8*a^2*b*c^7*d*e + 32*a*b^4*c^5*d*f - 58*a*b^5*c^4*d*g +
86*a*b^5*c^4*e*f + 152*a^3*b*c^6*d*g + 472*a^3*b*c^6*e*f - 148*a*b^6*c^3*e
*g + 394*a*b^7*c^2*f*g - 1768*a^4*b*c^5*f*g + 4*a^2*b^2*c^6*d*f + 26*a^2*b^
3*c^5*d*g - 374*a^2*b^3*c^5*e*f + 698*a^2*b^4*c^4*e*g - 1132*a^3*b^2*c^5*e*
g - 1804*a^2*b^5*c^3*f*g + 3266*a^3*b^3*c^4*f*g))/(2*(16*a^2*c^7 + b^4*c^5
- 8*a*b^2*c^6)))*((c^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^6*d^2 - 25*b^15
*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*b^6*g^2*(-(4*a*c - b^2)^9)^(1/2)
+ 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 + 3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*f^2 - 26880*a^6*b*c^8*f^2 + 80640
*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 -
288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^2 - 3840*a^4*b^3*c^8*e^2 - 2077*a
^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 30240*a^4*b^5*c^6*f^2 + 44800*a^5*
b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*
c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2 + 35767*a^3*b^9*c^3*g^2 - 116928*
a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 - 215040*a^6*b^3*c^6*g^2 - 49*a^3*
c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) +
615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*b^10*c^5*d*e + 7168*a^6*c^9*d*g +
15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b^12*c^3*d*g + 6*b^12*c^3*e*f - 35
840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1
536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(
4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g - 152*a*b^10*c^4*e*f + 258*a*b^1
1*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*
c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 192*a^2*
b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*
(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6*d*f - 1344*a^3*b^5*c^7*d*f + 512
*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g + 1548*a^2*b^8*c^5*e*f + 2688*a^3*b
^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a^4*b^4*c^7*d*g + 22400*a^4*b^4*c^
7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a
*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g + 14784*a^3*b^7*c^5*e*g - 44352*a
^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 42*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^
(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*b^3*c^3*e*f*(-(4*a*c -
b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39132*a^3*b^8*c^4*f*g + 119616*a^4*
b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161280*a^6*b^2*c^7*f*g + 10*b^4*c^2*
e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) +
12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)
^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4
*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 240*a^2*b^8*c^9 - 1280*a^3*b^6*c^10
+ 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12)))^(1/2) - (2744*a^7*c^3*g^3 - 225
*a^4*b^6*g^3 - 216*a^4*c^6*e^3 + 3*a*b^3*c^6*d^3 + 4*a^2*b*c^7*d^3 + 1300*a
^5*b*c^4*f^3 - 24*a^3*c^7*d^2*e + 2060*a^5*b^4*c*g^3 - 125*a^2*b^8*e*g^2 +
56*a^4*c^6*d^2*g - 600*a^5*c^5*e*f^2 + 175*a^3*b^7*f*g^2 + 1512*a^5*c^5*e^2
*g - 3528*a^6*c^4*e*g^2 + 1400*a^6*c^4*f^2*g - 5*a^2*b^4*c^4*e^3 + 66*a^3*b
^2*c^5*e^3 + 63*a^3*b^5*c^2*f^3 - 573*a^4*b^3*c^3*f^3 - 5334*a^6*b^2*c^2*g^
3 + 75*a*b^9*d*g^2 + 240*a^4*c^6*d*e*f - 560*a^5*c^5*d*f*g + 6*a*b^4*c^5*d^
2*e + 3*a*b^5*c^4*d*e^2 + 204*a^3*b*c^6*d*e^2 - 18*a*b^5*c^4*d^2*f + 27*a*b
^7*c^2*d*f^2 + 12*a^3*b*c^6*d^2*f - 420*a^4*b*c^5*d*f^2 + 30*a*b^6*c^3*d^2*
g - 845*a^2*b^7*c*d*g^2 + 924*a^4*b*c^5*e^2*f + 2044*a^5*b*c^4*d*g^2 + 1350
*a^3*b^6*c*e*g^2 - 210*a^3*b^6*c*f^2*g - 1485*a^4*b^5*c*f*g^2 + 364*a^6*b*c
^3*f*g^2 - 42*a^2*b^2*c^6*d^2*e - 51*a^2*b^3*c^5*d*e^2 + 81*a^2*b^3*c^5*d^2
*f - 279*a^2*b^5*c^3*d*f^2 + 801*a^3*b^3*c^4*d*f^2 - 149*a^2*b^4*c^4*d^2*g
+ 30*a^2*b^5*c^3*e^2*f - 45*a^2*b^6*c^2*e*f^2 + 78*a^3*b^2*c^5*d^2*g - 339*
a^3*b^3*c^4*e^2*f + 402*a^3*b^4*c^3*e*f^2 + 3198*a^3*b^5*c^2*d*g^2 - 762*a^
4*b^2*c^4*e*f^2 - 4571*a^4*b^3*c^3*d*g^2 - 50*a^2*b^6*c^2*e^2*g + 600*a^3*b

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^4*c^3*e^2*g - 2002*a^4*b^2*c^4*e^2*g - 4835*a^4*b^4*c^2*e*g^2 + 6598*a^5*b
^2*c^3*e*g^2 + 1927*a^4*b^4*c^2*f^2*g - 4722*a^5*b^2*c^3*f^2*g + 3061*a^5*b
^3*c^2*f*g^2 - 90*a*b^8*c*d*f*g - 18*a*b^6*c^3*d*e*f + 30*a*b^7*c^2*d*e*g -
 1352*a^4*b*c^5*d*e*g + 150*a^2*b^7*c*e*f*g - 2312*a^5*b*c^4*e*f*g + 246*a^
2*b^4*c^4*d*e*f - 804*a^3*b^2*c^5*d*e*f - 424*a^2*b^5*c^3*d*e*g + 1578*a^3*
b^3*c^4*d*e*g + 972*a^2*b^6*c^2*d*f*g - 3244*a^3*b^4*c^3*d*f*g + 3276*a^4*b
^2*c^4*d*f*g - 1480*a^3*b^5*c^2*e*f*g + 4122*a^4*b^3*c^3*e*f*g)/(4*(64*a^3*
c^8 - b^6*c^5 + 12*a*b^4*c^6 - 48*a^2*b^2*c^7))))*((c^6*d^2*(-(4*a*c - b^2)
^9)^(1/2) - b^9*c^6*d^2 - 25*b^15*g^2 - b^11*c^4*e^2 - 9*b^13*c^2*f^2 + 25*
b^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^4*b*c^10*d^2 + 27*a*b^9*c^5*e^2 +
3840*a^5*b*c^9*e^2 - 9*a*c^5*e^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a*b^11*c^3*
f^2 - 26880*a^6*b*c^8*f^2 + 80640*a^7*b*c^7*g^2 + 30*b^14*c*f*g + 96*a^2*b^
5*c^8*d^2 - 512*a^3*b^3*c^9*d^2 - 288*a^2*b^7*c^6*e^2 + 1504*a^3*b^5*c^7*e^
2 - 3840*a^4*b^3*c^8*e^2 - 2077*a^2*b^9*c^4*f^2 + 10656*a^3*b^7*c^5*f^2 - 3
0240*a^4*b^5*c^6*f^2 + 44800*a^5*b^3*c^7*f^2 + 25*a^2*c^4*f^2*(-(4*a*c - b^
2)^9)^(1/2) + b^2*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6366*a^2*b^11*c^2*g^2
+ 35767*a^3*b^9*c^3*g^2 - 116928*a^4*b^7*c^4*g^2 + 219744*a^5*b^5*c^5*g^2 -
 215040*a^6*b^3*c^6*g^2 - 49*a^3*c^3*g^2*(-(4*a*c - b^2)^9)^(1/2) + 9*b^4*c
^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 615*a*b^13*c*g^2 - 3072*a^5*c^10*d*e - 2*
b^10*c^5*d*e + 7168*a^6*c^9*d*g + 15360*a^6*c^9*e*f + 6*b^11*c^4*d*f - 10*b
^12*c^3*d*g + 6*b^12*c^3*e*f - 35840*a^7*c^8*f*g - 10*b^13*c^2*e*g + 36*a*b
^8*c^6*d*e - 98*a*b^9*c^5*d*f + 1536*a^5*b*c^9*d*f - 10*a*c^5*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 2*b*c^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 168*a*b^10*c^4*d*g
 - 152*a*b^10*c^4*e*f + 258*a*b^11*c^3*e*g - 43520*a^6*b*c^8*e*g - 724*a*b^
12*c^2*f*g - 30*b^5*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 246*a^2*b^2*c^2*g^2*(-
(4*a*c - b^2)^9)^(1/2) - 192*a^2*b^6*c^7*d*e + 128*a^3*b^4*c^8*d*e + 1536*a
^4*b^2*c^9*d*e - 165*a*b^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 576*a^2*b^7*c^6
*d*f - 1344*a^3*b^5*c^7*d*f + 512*a^4*b^3*c^8*d*f - 1044*a^2*b^8*c^5*d*g +
1548*a^2*b^8*c^5*e*f + 2688*a^3*b^6*c^6*d*g - 8064*a^3*b^6*c^6*e*f - 1152*a
^4*b^4*c^7*d*g + 22400*a^4*b^4*c^7*e*f - 6144*a^5*b^2*c^8*d*g - 30720*a^5*b
^2*c^8*e*f - 6*b^2*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^2*b^9*c^4*e*g
+ 14784*a^3*b^7*c^5*e*g - 44352*a^4*b^5*c^6*e*g + 69120*a^5*b^3*c^7*e*g + 4
2*a^2*c^4*e*g*(-(4*a*c - b^2)^9)^(1/2) + 10*b^3*c^3*d*g*(-(4*a*c - b^2)^9)^
(1/2) - 6*b^3*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^2*b^10*c^3*f*g - 39
132*a^3*b^8*c^4*f*g + 119616*a^4*b^6*c^5*f*g - 201600*a^5*b^4*c^6*f*g + 161
280*a^6*b^2*c^7*f*g + 10*b^4*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 51*a*b^2*c^
3*f^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a*b*c^4*d*g*(-(4*a*c - b^2)^9)^(1/2) +
44*a*b*c^4*e*f*(-(4*a*c - b^2)^9)^(1/2) - 78*a*b^2*c^3*e*g*(-(4*a*c - b^2)^
9)^(1/2) + 184*a*b^3*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2) - 186*a^2*b*c^3*f*g*(
-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^13 + b^12*c^7 - 24*a*b^10*c^8 + 24
0*a^2*b^8*c^9 - 1280*a^3*b^6*c^10 + 3840*a^4*b^4*c^11 - 6144*a^5*b^2*c^12))
)^(1/2)*2i + (g*x^3)/(3*c^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.127 $\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=471

$$-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\ldots}{\sqrt{b-\sqrt{b^2-\ldots}}}\right)\ldots}{\ldots}$$

```
[Out] g*x/c^2-1/2*x*(b*c*(a*f+c*d)-a*b^2*g-2*a*c*(-a*g+c*e)+(2*c^3*d-c^2*(2*a*f+b
*e)-b^3*g+b*c*(3*a*g+b*f))*x^2)/c^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*e)+
3*b^3*g-b*c*(13*a*g+b*f)+(b^3*c*f-4*b*c^2*(2*a*f+c*d)-3*b^4*g+4*a*c^2*(-5*a
*g+c*e)+b^2*c*(19*a*g+c*e))/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*(2*c^3*d-c^2*(-6*a*f+b*e)+3*b^3*g-b*c*(13*a*g+b*f)+(-b^3*c*f
+4*b*c^2*(2*a*f+c*d)+3*b^4*g-4*a*c^2*(-5*a*g+c*e)-b^2*c*(19*a*g+c*e))/(-4*a
*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 6.66, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1668, 1676, 1166, 205}

$$-\frac{x\left(x^2\left(-c^2(2af+be)+bc(3ag+bf)+b^3(-g)+2c^3d\right)-ab^2g+bc(af+cd)-2ac(ce-ag)\right)}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\ldots}{\sqrt{b-\sqrt{b^2-\ldots}}}\right)\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (g*x)/c^2 - (x*(b*c*(c*d + a*f) - a*b^2*g - 2*a*c*(c*e - a*g) + (2*c^3*d -
c^2*(b*e + 2*a*f) - b^3*g + b*c*(b*f + 3*a*g))*x^2))/(2*c^2*(b^2 - 4*a*c)*(
a + b*x^2 + c*x^4)) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f +
13*a*g) + (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g
) + b^2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b
^2 - 4*a*c]]) - ((2*c^3*d - c^2*(b*e - 6*a*f) + 3*b^3*g - b*c*(b*f + 13*a*g
) - (b^3*c*f - 4*b*c^2*(c*d + 2*a*f) - 3*b^4*g + 4*a*c^2*(c*e - 5*a*g) + b^
2*c*(c*e + 19*a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4
*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1668

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*Polyno
mialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p
 + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b,
c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] && NeQ[b^2 - 4*a*c, 0] &
& LtQ[p, -1] && IGtQ[m/2, 0]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rubi steps

$$\int \frac{x^2\left(d+ex^2+fx^4+gx^6\right)}{\left(a+bx^2+cx^4\right)^2}dx = -\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(bf+\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(b\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(b\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{gx}{c^2}-\frac{x\left(bc(cd+af)-ab^2g-2ac(ce-ag)+\left(2c^3d-c^2(be+2af)-b^3g+bc(b\right.\right.}{2c^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Mathematica [A] time = 1.95, size = 575, normalized size = 1.22

$$-\frac{2\sqrt{c}x\left(2c\left(a^2g-ac\left(e+fx^2\right)+c^2dx^2\right)+b^2\left(cfx^2-ag\right)+bc\left(a\left(f+3gx^2\right)+c\left(d-ex^2\right)\right)+b^3(-g)x^2\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2c^2\left(-10a^2g+cd\sqrt{b^2-4ac}+3af\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*Sqrt[c]*g*x - (2*Sqrt[c]*x*(-(b^3*g*x^2) + b^2*(-(a*g) + c*f*x^2) + 2*c*
(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2)) + b*c*(c*(d - e*x^2) + a*(f + 3*g*x^2
))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) - (Sqrt[2]*(-3*b^4*g + b^2*c*(c*e
- Sqrt[b^2 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d + 2*a*c*e +
3*a*Sqrt[b^2 - 4*a*c]*f - 10*a^2*g) + b^3*(c*f + 3*Sqrt[b^2 - 4*a*c]*g) - b
*c*(4*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f + 13*a*Sqrt[b^2 - 4*a*c]*g))*
```

```
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/
2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3*b^4*g - b^2*c*(c*e + Sqrt[b^2
 - 4*a*c]*f + 19*a*g) + 2*c^2*(c*Sqrt[b^2 - 4*a*c]*d - 2*a*c*e + 3*a*Sqrt[b
^2 - 4*a*c]*f + 10*a^2*g) + b^3*(-(c*f) + 3*Sqrt[b^2 - 4*a*c]*g) + b*c*(4*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 8*a*c*f - 13*a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt
[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [**B**] time = 8.92, size = 9170, normalized size = 19.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] g*x/c^2 - 1/2*(2*c^3*d*x^3 + b^2*c*f*x^3 - 2*a*c^2*f*x^3 - b^3*g*x^3 + 3*a*
b*c*g*x^3 - b*c^2*x^3*e + b*c^2*d*x + a*b*c*f*x - a*b^2*g*x + 2*a^2*c*g*x -
 2*a*c^2*x*e)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/16*(2*(2*b^2*c^
5 - 8*a*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2
*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^4 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*c^5 - 2*(b^2 - 4*a*c)*c^
5)*(b^2*c^2 - 4*a*c^3)^2*d - (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*
a*c^4)*(b^2*c^2 - 4*a*c^3)^2*f + (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4
- 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b
^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*g - (2*b^3*c^4 - 8*a*b*c^5 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(b^2*c
^2 - 4*a*c^3)^2*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c^6 + 2*b^5*c^6 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 +
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^7 - 16*a*b^3*c^7 - 4*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^8 + 32*a^2*b*c^8 - 2*(b^2 - 4*a*c)*b^
3*c^6 + 8*(b^2 - 4*a*c)*a*b*c^7)*d*abs(-b^2*c^2 + 4*a*c^3) - 2*(sqrt(2)*sqr
```

t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 +
 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 -
 4*a*c)*a^2*b*c^6)*f*abs(-b^2*c^2 + 4*a*c^3) + 2*(3*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^
4 + 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 2
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 4
0*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2 -
4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)
*g*abs(-b^2*c^2 + 4*a*c^3) + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^4*c^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*a*b^4*c^6 + 16*sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^7 - 16*a^2*b
^2*c^7 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^8 + 32*a^3*c^8 - 2
*(b^2 - 4*a*c)*a*b^2*c^6 + 8*(b^2 - 4*a*c)*a^2*c^7)*abs(-b^2*c^2 + 4*a*c^3)
e - 4(2*b^6*c^9 - 16*a*b^4*c^10 + 32*a^2*b^2*c^11 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^2*c^10 - 2*(b^2 - 4*a*c)*b^4*c^9 + 8*(b^2 - 4*a*c)*a*b^2*
c^10)*d + (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a^3*b^2*c^10 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 16*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 80*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 24*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4*a*c)*b^6*c^7
+ 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*f - (6*b^9*c^6
 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6
+ 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4
*a*c)*a^3*b*c^9)*g + (2*b^7*c^8 - 8*a*b^5*c^9 - 32*a^2*b^3*c^10 + 128*a^3*b

```
*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 +
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 16*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 64*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 32*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 16*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b
^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*e)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2
 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^
2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a
*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^
3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c)) + 1/1
6*(2*(2*b^2*c^5 - 8*a*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^5 - 2*(b
^2 - 4*a*c)*c^5)*(b^2*c^2 - 4*a*c^3)^2*d - (2*b^4*c^3 - 20*a*b^2*c^4 + 48*a
^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c +
10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*
(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4*a*c^3)^2*f + (6*b^5*c^2 - 50*a*b^3*c^3 +
104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b
^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3)*(b^2*c^2 - 4*a*c^3)^2*g - (2*b^3*c^4 - 8
*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)
*b*c^4)*(b^2*c^2 - 4*a*c^3)^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^5*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^6 - 2*b^5*c^6 + 16*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^7 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^7 + 16*a*b^3*c^
7 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^8 - 32*a^2*b*c^8 + 2*(b
^2 - 4*a*c)*b^3*c^6 - 8*(b^2 - 4*a*c)*a*b*c^7)*d*abs(-b^2*c^2 + 4*a*c^3) +
2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b^4*c^5 - 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3
*b*c^6 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 16*a^2*b^3*c^6 - 4*sqrt(2)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 32*a^3*b*c^7 + 2*(b^2 - 4*a*c)*a*b^3*
c^5 - 8*(b^2 - 4*a*c)*a^2*b*c^6)*f*abs(-b^2*c^2 + 4*a*c^3) - 2*(3*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^
4 - 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 +
 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + 3*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 68*a^2*b^4*c^5 - 160*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 256*a
```

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^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 320*a^4*c
^7 + 6*(b^2 - 4*a*c)*a*b^4*c^4 - 44*(b^2 - 4*a*c)*a^2*b^2*c^5 + 80*(b^2 - 4
*a*c)*a^3*c^6)*g*abs(-b^2*c^2 + 4*a*c^3) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^4*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^
6 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 2*a*b^4*c^6 + 16*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^7 + 8*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^7 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^7 + 16*a^2*b^2*c^7 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^8 -
32*a^3*c^8 + 2*(b^2 - 4*a*c)*a*b^2*c^6 - 8*(b^2 - 4*a*c)*a^2*c^7)*abs(-b^2*
c^2 + 4*a*c^3)*e - 4*(2*b^6*c^9 - 16*a*b^4*c^10 + 32*a^2*b^2*c^11 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^8 - 16*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 8*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^10 - 2*(b^2 - 4*a*c)*b^4*c^9 + 8*(b^2 -
 4*a*c)*a*b^2*c^10)*d + (2*b^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a
^3*b^2*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8
*c^5 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c
^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 -
80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 -
24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4
*a*c)*b^6*c^7 + 24*(b^2 - 4*a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*
f - (6*b^9*c^6 - 86*a*b^7*c^7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4
*b*c^10 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^9*c
^4 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5
 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 22
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 6
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4
*a*c)*b^7*c^6 + 62*(b^2 - 4*a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8
+ 160*(b^2 - 4*a*c)*a^3*b*c^9)*g + (2*b^7*c^8 - 8*a*b^5*c^9 - 32*a^2*b^3*c^
10 + 128*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*b^7*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6
*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3
*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^8 -
64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 32
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(
b^2 - 4*a*c)*b^5*c^8 + 32*(b^2 - 4*a*c)*a^2*b*c^10)*e)*arctan(2*sqrt(1/2)*x
/sqrt((b^3*c^2 - 4*a*b*c^3 - sqrt((b^3*c^2 - 4*a*b*c^3)^2 - 4*(a*b^2*c^2 -
4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c^4)))/((a*b^6*c^5 - 12*a^2
*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*b^3*c^7 + a*b^4*c^7 - 64*a
^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^9)*abs(-b^2*c^2 + 4*a*c^3)
*abs(c))
```

maple [B] time = 0.05, size = 2300, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -19/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*g-19/4/c
/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b^2*g+g*x/c^2+3/4/c^2/
(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*g+5/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*c*x)*a^2*g-3/4/c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*g+5/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a^2*g+1/4/(4*a*c-b^2)/c*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)*b^2*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2*e-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b^2/c
*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/4/(4*a*c-b^2)/(-4
*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b^2*e*arctan(2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^
3*b*e-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*a*e+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x
*b*d+1/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*b*c*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+13/4/c/(4*a*
c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*g+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*c*x)*b^4*g+3/4/c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*c*x)*b^4*g-13/4/c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*b*g-1/2/c^2/(c*x^4+
b*x^2+a)/(4*a*c-b^2)*x*a*b^2*g+2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*c*x)*a*b*f-1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a
*e-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*f+1/(4*a*
c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d+2/(4*a*c-b^2)/(-4*a*c
+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a*b*f*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a*c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)-1/2/c^2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^3*g+1/2/c/(c*
x^4+b*x^2+a)/(4*a*c-b^2)*x^3*b^2*f+1/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*a^2*g+
3/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*b*g+1/2/c/(c*x^4+b*x^2+a)/(4*a*c-b^
2)*x*a*b*f-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*b^3/c*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+c/
(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*d-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x^3*a*f-3/2
/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*a*f+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*b*e-1/2/(4*a*c-b^2)*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2
^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+3/2/(4*a*c-b^2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b*e*ar
```

```
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/2/(4*a*c-b^2)*2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*((2*c^3*d - b*c^2*e + (b^2*c - 2*a*c^2)*f - (b^3 - 3*a*b*c)*g)*x^3 + (
b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x)/(a*b^2*c^2 - 4*a^2*
c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*c^3)*x^2) + g*x/c^2 + 1/2*
integrate((b*c^2*d - 2*a*c^2*e + a*b*c*f - (2*c^3*d - b*c^2*e - (b^2*c - 6*
a*c^2)*f + (3*b^3 - 13*a*b*c)*g)*x^2 - (3*a*b^2 - 10*a^2*c)*g)/(c*x^4 + b*x
^2 + a), x)/(b^2*c^2 - 4*a*c^3)
```

mupad [B] time = 4.22, size = 36589, normalized size = 77.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2 + f*x^4 + g*x^6))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x^3*(2*c^3*d - b^3*g - 2*a*c^2*f - b*c^2*e + b^2*c*f + 3*a*b*c*g))/(2*(4*
a*c - b^2)) + (x*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*
(4*a*c - b^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) - atan(((((10240*a^5*c^7*g -
16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 153
6*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g
 + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*
c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g
)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((c^5*d^2
*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2
- a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) -
a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2)
+ 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b
^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2
- 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2
*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^
3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e
*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^
8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10
*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 38
4*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^
3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g
 - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752
*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*
c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*
a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^
4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*
f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c -
b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^
2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))
/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280
*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 -
192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3
```

- 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^
13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*
a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^
7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3
840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9
*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^
5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a
^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*
b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1
8*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g
 - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^
3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4
*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g +
128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2
*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6
*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 154
8*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^
6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 5
1*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9
)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c -
 b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*
g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c
^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c
^10)))^(1/2) - (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2
 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c
^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*
c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^
5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6
*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 18
4*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 4
72*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e)
)/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1
/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*
a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*
a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 -
 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^
5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2
+ 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^
2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*
c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) -
 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g
 + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-
(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^
2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 12
8*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^
8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f
 + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 153
6*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5
*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) -
152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*
a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b
*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^
9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 +
a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^
5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*g - 16*b^7*c^5

*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 -

718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*
b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e
*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^
5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c
^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a
^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*
c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*
e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*
f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7
*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2
- 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4
*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 +
44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*
c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^
8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b
^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 15
36*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*
c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g
+ 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^
4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*
c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g +
512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^
10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^
5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b
^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(
-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*
c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) -
44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5
- 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9
- 6144*a^6*b^2*c^10)))^(1/2)*1i)/((((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*
a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 19
2*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*
g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5
*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b
^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((c^5*d^2*(-(4*a*c - b^2)^9)^
(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 76
8*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 384
0*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2
- 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*
b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^
2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^
5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f
*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*
(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*
c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e -
128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*
b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e
*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1
536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b
^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2)
- 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 2240
0*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a
*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2
)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11

+ a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*
a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024
*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((
c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c
^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(
1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)
^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 51
2*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c
^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2
+ 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4
*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g
^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^
6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a
^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2
*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d
*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072
*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*
c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f
- 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a
^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*
g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g -
8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*
b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4
*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a
*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7
- 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (x*(9
*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 +
b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^
3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^
2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*
f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*
b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*
a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4
*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b
^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 -
9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^
4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*
b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2
+ 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*
b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f
^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a
^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*
b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e +
5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e
+ 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/
2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c
^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f +
960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b
^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*
d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f -
98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b
^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g
+ 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30
720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/
2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c -
b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4
*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*

c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*g^2 - 8* a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7 *c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 1 14*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^ 5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4* e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a* b^2*c^4)))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 - a*c^4*e^2*(-(4*a* c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 - 9*a*b^4*g^2*(-(4* a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5 *c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5 *b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^ 2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8* d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4 *d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^1 0*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128* a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c ^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4 *b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^ 3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b ^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c ^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b ^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4* a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 24 0*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^ (1/2) - (8*a*c^7*d^3 + 9*b^8*d*g^2 + 6*b^2*c^6*d^3 - 63*a^3*b^5*g^3 + 216*a ^4*c^4*f^3 - 3*a*b^3*c^4*e^3 - 4*a^2*b*c^5*e^3 + 8*a^2*c^6*d*e^2 + 573*a^4* b^3*c*g^3 - 1300*a^5*b*c^2*g^3 + 72*a^2*c^6*d^2*f + 216*a^3*c^5*d*f^2 - 5*b ^3*c^5*d^2*e + b^4*c^4*d*e^2 + 24*a^3*c^5*e^2*f + 200*a^4*c^4*d*g^2 - 5*b^4 *c^4*d^2*f + b^6*c^2*d*f^2 + 45*a^2*b^6*f*g^2 + 15*b^5*c^3*d^2*g + 600*a^5* c^3*f*g^2 + 5*a^2*b^4*c^2*f^3 - 66*a^3*b^2*c^3*f^3 - 27*a*b^7*e*g^2 - 28*a* b*c^6*d^2*e - 78*a*b^6*c*d*g^2 - 80*a^3*c^5*d*e*g + 2*b^5*c^3*d*e*f - 6*b^6 *c^2*d*e*g - 240*a^4*c^4*e*f*g + 18*a*b^2*c^5*d*e^2 + 26*a*b^2*c^5*d^2*f - 12*a*b^4*c^3*d*f^2 - 53*a*b^3*c^4*d^2*g - 6*a*b^4*c^3*e^2*f - 3*a*b^5*c^2*e *f^2 - 76*a^2*b*c^5*d^2*g - 204*a^3*b*c^4*e*f^2 + 18*a*b^5*c^2*e^2*g + 279* a^2*b^5*c*e*g^2 - 12*a^3*b*c^4*e^2*g + 420*a^4*b*c^3*e*g^2 - 30*a^2*b^5*c*f ^2*g - 402*a^3*b^4*c*f*g^2 - 924*a^4*b*c^3*f^2*g - 6*b^7*c*d*f*g + 2*a^2*b^ 2*c^4*d*f^2 + 42*a^2*b^2*c^4*e^2*f + 51*a^2*b^3*c^3*e*f^2 + 133*a^2*b^4*c^2 *d*g^2 + 114*a^3*b^2*c^3*d*g^2 - 81*a^2*b^3*c^3*e^2*g - 801*a^3*b^3*c^2*e*g ^2 + 339*a^3*b^3*c^2*f^2*g + 762*a^4*b^2*c^2*f*g^2 + 18*a*b^6*c*e*f*g + 6*a *b^3*c^4*d*e*f - 152*a^2*b*c^5*d*e*f - 28*a*b^4*c^3*d*e*g + 62*a*b^5*c^2*d* f*g - 536*a^3*b*c^4*d*f*g + 276*a^2*b^2*c^4*d*e*g - 42*a^2*b^3*c^3*d*f*g - 246*a^2*b^4*c^2*e*f*g + 804*a^3*b^2*c^3*e*f*g)/(4*(64*a^3*c^6 - b^6*c^3 + 1 2*a*b^4*c^4 - 48*a^2*b^2*c^5))))*((c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - b^9*c ^5*d^2 - 9*a*b^13*g^2 + 768*a^4*b*c^9*d^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e ^2 - a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f ^2 - 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7* b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4* b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 + 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 4

4800*a^6*b^3*c^5*g^2 - 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f + 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g + 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g - a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*2i - atan(((((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3

584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*1i - (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) + (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1

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504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^
9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4
*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 10
24*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g +
12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*
a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e
*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a
^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c
^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f +
7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a
^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^
5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152
*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5
*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*
a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^
3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6
*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^
(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b
^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b
^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*
b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a
^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^
2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
- a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2
) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3
*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^
2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a
^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*
c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-
(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8
*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*
c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^
10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e +
384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*
b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d
*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 107
52*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^
7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(
4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*
a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*
c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c
- b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(
-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*
c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 12
80*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*
g^2 - 8*a*c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c
^4*e^2 + 200*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2
 - 6*b^7*c*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3
*g^2 - 114*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 8
0*a^3*c^5*e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c
^5*d*f - 126*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b
*c^5*e*f + 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*
b^2*c^4*e*g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^
3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2
*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g
^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96
*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c
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^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*1i)/((((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 192*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 -

30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b
^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 153
60*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f -
6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f
 + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^
4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*
f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a
^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2
*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g
- 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^
2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^
4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*
b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^
(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c -
b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-
(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6
*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (x*(9*b^8*g^2 - 8*a*
c^7*d^2 + 8*a^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 2
00*a^4*c^4*g^2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c
*f*g + 74*a^2*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114
*a*b^6*c*g^2 - 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*
e*g + 18*b^5*c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 1
26*a*b^3*c^4*d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f +
 32*a*b^4*c^3*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*
g - 374*a^2*b^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^
2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c
- b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*
c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c
^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 2
7*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b
^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2
+ 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 2
5*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*
g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d
*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*
c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^
2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6
*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 22
40*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b
^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*
e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10
*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8
*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6
*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2
*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) +
 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^
(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*
a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1
/2) + (((10240*a^5*c^7*g - 16*b^7*c^5*d - 2048*a^4*c^8*e - 768*a^2*b^3*c^7*
d - 384*a^2*b^4*c^6*e + 1536*a^3*b^2*c^7*e + 192*a^2*b^5*c^5*f - 768*a^3*b^
3*c^6*f - 736*a^2*b^6*c^4*g + 4224*a^3*b^4*c^5*g - 10752*a^4*b^2*c^6*g + 19
2*a*b^5*c^6*d + 1024*a^3*b*c^8*d + 32*a*b^6*c^5*e - 16*a*b^7*c^4*f + 1024*a
^4*b*c^7*f + 48*a*b^8*c^3*g)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 48*a
^2*b^2*c^5)) + (x*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^

```
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*
a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^
7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3
840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9
*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^
5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a
^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*
b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1
8*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g
 - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^
3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4
*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g +
128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2
*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6
*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 154
8*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^
6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 5
1*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9
)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c -
 b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*
g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c
^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c
^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^8 + 768*a^2*b^3*c^7
))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((768*a^4*b*c^9*d^2 - b^9*c^5*
d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768
*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840
*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2
- 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b
^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2
 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b
^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5
*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*
g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(
-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c
^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 1
28*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b
^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*
f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 15
36*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^
5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) -
 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400
*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(
-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*
b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2)
 - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)
^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 +
 a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a
^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) + (x*(9*b^8*g^2 - 8*a*c^7*d^2 + 8*a
^2*c^6*e^2 + 10*b^2*c^6*d^2 - 72*a^3*c^5*f^2 + b^4*c^4*e^2 + 200*a^4*c^4*g^
2 + b^6*c^2*f^2 + 2*a*b^2*c^5*e^2 - 16*a*b^4*c^3*f^2 - 6*b^7*c*f*g + 74*a^2
*b^2*c^4*f^2 + 481*a^2*b^4*c^2*g^2 - 718*a^3*b^2*c^3*g^2 - 114*a*b^6*c*g^2
- 48*a^2*c^6*d*f - 6*b^3*c^5*d*e - 6*b^4*c^4*d*f - 80*a^3*c^5*e*g + 18*b^5*
c^3*d*g + 2*b^5*c^3*e*f - 6*b^6*c^2*e*g + 52*a*b^2*c^5*d*f - 126*a*b^3*c^4*
d*g - 14*a*b^3*c^4*e*f + 184*a^2*b*c^5*d*g - 8*a^2*b*c^5*e*f + 32*a*b^4*c^3
*e*g + 86*a*b^5*c^2*f*g + 472*a^3*b*c^4*f*g + 4*a^2*b^2*c^4*e*g - 374*a^2*b
^3*c^3*f*g - 8*a*b*c^6*d*e))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((76
8*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a*b^13
*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*(-(4*a*c - b^2)^9)^(1/
```

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2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^2*(-(4*a*c - b^2)^9)^(
1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*a^2*b^5*c^7*d^2 - 512*
a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^7*e^2 + 27*a^2*b^9*c^3
*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3840*a^5*b^3*c^6*f^2 -
9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9*c^2*g^2 + 10656*a^4*b
^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^5*g^2 + 25*a^3*c^2*g^2
*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a^6*c^8*d*g - 3072*a^6*
c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*b^9*c^4*d*f + 3584*a^5
*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 18*a*b^10*c^3*d*g - 2*a
*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g - 128*a^2*b^6*c^6*d*e
 + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^3*b^5*c^6*d*f - 3072*a
^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4*e*f - 2240*a^3*b^6*c^
5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g + 128*a^4*b^4*c^6*e*f -
10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2*b^9*c^3*e*g + 576*a^3
*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6*e*g - 10*a^2*c^3*e*g*
(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 1548*a^3*b^8*c^3*f*g - 80
64*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^6*b^2*c^6*f*g + 6*a*b^
12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 51*a^2*b^2*c*g^2*(-(4*a
*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a*b*c^3*e*
f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b
^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*g*(-(4*a*c - b^2)^9)^(
1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c^6 + 240*a^3*b^8*c^7 -
 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c^10)))^(1/2) - (8*a*c^
7*d^3 + 9*b^8*d*g^2 + 6*b^2*c^6*d^3 - 63*a^3*b^5*g^3 + 216*a^4*c^4*f^3 - 3*
a*b^3*c^4*e^3 - 4*a^2*b*c^5*e^3 + 8*a^2*c^6*d*e^2 + 573*a^4*b^3*c*g^3 - 130
0*a^5*b*c^2*g^3 + 72*a^2*c^6*d^2*f + 216*a^3*c^5*d*f^2 - 5*b^3*c^5*d^2*e +
b^4*c^4*d*e^2 + 24*a^3*c^5*e^2*f + 200*a^4*c^4*d*g^2 - 5*b^4*c^4*d^2*f + b^
6*c^2*d*f^2 + 45*a^2*b^6*f*g^2 + 15*b^5*c^3*d^2*g + 600*a^5*c^3*f*g^2 + 5*a
^2*b^4*c^2*f^3 - 66*a^3*b^2*c^3*f^3 - 27*a*b^7*e*g^2 - 28*a*b*c^6*d^2*e - 7
8*a*b^6*c*d*g^2 - 80*a^3*c^5*d*e*g + 2*b^5*c^3*d*e*f - 6*b^6*c^2*d*e*g - 24
0*a^4*c^4*e*f*g + 18*a*b^2*c^5*d*e^2 + 26*a*b^2*c^5*d^2*f - 12*a*b^4*c^3*d*
f^2 - 53*a*b^3*c^4*d^2*g - 6*a*b^4*c^3*e^2*f - 3*a*b^5*c^2*e*f^2 - 76*a^2*b
*c^5*d^2*g - 204*a^3*b*c^4*e*f^2 + 18*a*b^5*c^2*e^2*g + 279*a^2*b^5*c*e*g^2
 - 12*a^3*b*c^4*e^2*g + 420*a^4*b*c^3*e*g^2 - 30*a^2*b^5*c*f^2*g - 402*a^3*
b^4*c*f*g^2 - 924*a^4*b*c^3*f^2*g - 6*b^7*c*d*f*g + 2*a^2*b^2*c^4*d*f^2 + 4
2*a^2*b^2*c^4*e^2*f + 51*a^2*b^3*c^3*e*f^2 + 133*a^2*b^4*c^2*d*g^2 + 114*a^
3*b^2*c^3*d*g^2 - 81*a^2*b^3*c^3*e^2*g - 801*a^3*b^3*c^2*e*g^2 + 339*a^3*b^
3*c^2*f^2*g + 762*a^4*b^2*c^2*f*g^2 + 18*a*b^6*c*e*f*g + 6*a*b^3*c^4*d*e*f
- 152*a^2*b*c^5*d*e*f - 28*a*b^4*c^3*d*e*g + 62*a*b^5*c^2*d*f*g - 536*a^3*b
*c^4*d*f*g + 276*a^2*b^2*c^4*d*e*g - 42*a^2*b^3*c^3*d*f*g - 246*a^2*b^4*c^2
*e*f*g + 804*a^3*b^2*c^3*e*f*g)/(4*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 - 4
8*a^2*b^2*c^5))))*((768*a^4*b*c^9*d^2 - b^9*c^5*d^2 - c^5*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 9*a*b^13*g^2 - a*b^9*c^4*e^2 + 768*a^5*b*c^8*e^2 + a*c^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - a*b^11*c^2*f^2 + 3840*a^6*b*c^7*f^2 + 9*a*b^4*g^
2*(-(4*a*c - b^2)^9)^(1/2) + 213*a^2*b^11*c*g^2 - 26880*a^7*b*c^6*g^2 + 96*
a^2*b^5*c^7*d^2 - 512*a^3*b^3*c^8*d^2 + 96*a^3*b^5*c^6*e^2 - 512*a^4*b^3*c^
7*e^2 + 27*a^2*b^9*c^3*f^2 - 288*a^3*b^7*c^4*f^2 + 1504*a^4*b^5*c^5*f^2 - 3
840*a^5*b^3*c^6*f^2 - 9*a^2*c^3*f^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^3*b^9
*c^2*g^2 + 10656*a^4*b^7*c^3*g^2 - 30240*a^5*b^5*c^4*g^2 + 44800*a^6*b^3*c^
5*g^2 + 25*a^3*c^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - 1024*a^5*c^9*d*e + 5120*a
^6*c^8*d*g - 3072*a^6*c^8*e*f + 15360*a^7*c^7*f*g + 12*a*b^8*c^5*d*e + 6*a*
b^9*c^4*d*f + 3584*a^5*b*c^8*d*f - 6*a*c^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1
8*a*b^10*c^3*d*g - 2*a*b^10*c^3*e*f + 6*a*b^11*c^2*e*g + 1536*a^6*b*c^7*e*g
 - 128*a^2*b^6*c^6*d*e + 384*a^3*b^4*c^7*d*e - 128*a^2*b^7*c^5*d*f + 960*a^
3*b^5*c^6*d*f - 3072*a^4*b^3*c^7*d*f + 324*a^2*b^8*c^4*d*g + 36*a^2*b^8*c^4
*e*f - 2240*a^3*b^6*c^5*d*g - 192*a^3*b^6*c^5*e*f + 7296*a^4*b^4*c^6*d*g +
128*a^4*b^4*c^6*e*f - 10752*a^5*b^2*c^7*d*g + 1536*a^5*b^2*c^7*e*f - 98*a^2
*b^9*c^3*e*g + 576*a^3*b^7*c^4*e*g - 1344*a^4*b^5*c^5*e*g + 512*a^5*b^3*c^6
*e*g - 10*a^2*c^3*e*g*(-(4*a*c - b^2)^9)^(1/2) - 152*a^2*b^10*c^2*f*g + 154
```

```
8*a^3*b^8*c^3*f*g - 8064*a^4*b^6*c^4*f*g + 22400*a^5*b^4*c^5*f*g - 30720*a^
6*b^2*c^6*f*g + 6*a*b^12*c*f*g + a*b^2*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 5
1*a^2*b^2*c*g^2*(-(4*a*c - b^2)^9)^(1/2) + 18*a*b*c^3*d*g*(-(4*a*c - b^2)^9
)^(1/2) + 2*a*b*c^3*e*f*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*f*g*(-(4*a*c -
 b^2)^9)^(1/2) - 6*a*b^2*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*f*
g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^7*c^11 + a*b^12*c^5 - 24*a^2*b^10*c
^6 + 240*a^3*b^8*c^7 - 1280*a^4*b^6*c^8 + 3840*a^5*b^4*c^9 - 6144*a^6*b^2*c
^10)))^(1/2)*2i + (g*x)/c^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.128 $\int \frac{d+ex^2+fx^4+gx^6}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=449

$$\frac{x\left(x^2\left(-ab^2g+bc(af+cd)-2ac(ce-ag)\right)+c\left(-\frac{ab(ag+ce)}{c}-2a(cd-af)+b^2d\right)\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{ab^2g}{c}+\cdots\right.}{\cdots}$$

```
[Out] 1/2*x*(c*(b^2*d-2*a*(-a*f+c*d)-a*b*(a*g+c*e)/c)+(b*c*(a*f+c*d)-a*b^2*g-2*a*
c*(-a*g+c*e))*x^2)/a/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d)+a*b^2*g/c-2*a*(3*a*g+c*e)+
(b^2*c*(-a*f+c*d)-4*a*c^2*(a*f+3*c*d)-a*b^3*g+4*a*b*c*(2*a*g+c*e))/c/(-4*a*
c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1
/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*(a*f+c*d)+a*b^
2*g/c-2*a*(3*a*g+c*e)+(-b^2*c*(-a*f+c*d)+4*a*c^2*(a*f+3*c*d)+a*b^3*g-4*a*b*
c*(2*a*g+c*e))/c/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*
a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.87, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1678, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{b^2c(cd-af)-ab^3g+4abc(2ag+ce)-4ac^2(af+3cd)}{c\sqrt{b^2-4ac}}+\frac{ab^2g}{c}+b(af+cd)-2a(3ag+ce)\right)}{2\sqrt{2}\,a\sqrt{c}\left(b^2-4ac\right)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{\cdots}}{\sqrt{\sqrt{b^2-4\cdots}}}\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(c*(b^2*d - 2*a*(c*d - a*f) - (a*b*(c*e + a*g))/c) + (b*c*(c*d + a*f) -
a*b^2*g - 2*a*c*(c*e - a*g))*x^2))/(2*a*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)
) + ((b*(c*d + a*f) + (a*b^2*g)/c - 2*a*(c*e + 3*a*g) + (b^2*c*(c*d - a*f)
- 4*a*c^2*(3*c*d + a*f) - a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*
a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*
a*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*(c*d + a*f) + (a
*b^2*g)/c - 2*a*(c*e + 3*a*g) - (b^2*c*(c*d - a*f) - 4*a*c^2*(3*c*d + a*f)
- a*b^3*g + 4*a*b*c*(c*e + 2*a*g))/(c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*Sqrt[c]*(b^2 - 4*a*c)*
Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \int$$

$$= \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b(\right.}{}$$

$$= \frac{x\left(c\left(b^2d-2a(cd-af)-\frac{ab(ce+ag)}{c}\right)+\left(bc(cd+af)-ab^2g-2ac(ce-ag)\right)x^2\right)}{2ac\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(b(\right.}{}$$

Mathematica [A] time = 1.65, size = 512, normalized size = 1.14

$$\frac{2\sqrt{c}x\left(b\left(a^2(-g)-ace+acfx^2+c^2dx^2\right)+b^2\left(cd-agx^2\right)+2ac\left(a\left(f+gx^2\right)-c\left(d+ex^2\right)\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(bc\left(8a^2g+cd\sqrt{b^2-4ac}+af\sqrt{b^2-4ac}+4ace\right)\right.}{\left(b^2-4a\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*x*(b*(-(a*c*e) - a^2*g + c^2*d*x^2 + a*c*f*x^2) + b^2*(c*d - a*
g*x^2) + 2*a*c*(-(c*(d + e*x^2)) + a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (Sqrt[2]*(-(a*b^3*g) + b*c*(c*Sqrt[b^2 - 4*a*c]*d + 4*a*c*e
+ a*Sqrt[b^2 - 4*a*c]*f + 8*a^2*g) + b^2*(c^2*d - a*c*f + a*Sqrt[b^2 - 4*a*
c]*g) - 2*a*c*(6*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f + 3*a*Sqrt[b^2 - 4
*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 -
4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(a*b^3*g + b*c*(c*Sqrt
[b^2 - 4*a*c]*d - 4*a*c*e + a*Sqrt[b^2 - 4*a*c]*f - 8*a^2*g) + 2*a*c*(6*c^2
*d - c*Sqrt[b^2 - 4*a*c]*e + 2*a*c*f - 3*a*Sqrt[b^2 - 4*a*c]*g) + b^2*(-(c^
2*d) + a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b +
Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*
a*c^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

<u>**giac**</u> **[B]** time = 8.51, size = 8913, normalized size = 19.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*c^2*d*x^3 + a*b*c*f*x^3 - a*b^2*g*x^3 + 2*a^2*c*g*x^3 - 2*a*c^2*x^3*
e + b^2*c*d*x - 2*a*c^2*d*x + 2*a^2*c*f*x - a^2*b*g*x - a*b*c*x*e)/((c*x^4
+ b*x^2 + a)*(a*b^2*c - 4*a^2*c^2)) + 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c
- 4*a^2*c^2)^2*d + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^
2*f + (2*a*b^4*c^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a
*b^2*c - 4*a^2*c^2)^2*g - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)
^2*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^5*c^4 - 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^2*c^5 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 28*a^2*b^4*c^5 - 96*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^6 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^6 - 128*a^3*b^2*c^6 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
c^7 + 192*a^4*c^7 + 2*(b^2 - 4*a*c)*a*b^4*c^4 - 20*(b^2 - 4*a*c)*a^2*b^2*c^
5 + 48*(b^2 - 4*a*c)*a^3*c^6)*d*abs(a*b^2*c - 4*a^2*c^2) - 4*(sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^4*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4
 - 2*a^3*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c^5 + 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*c^6 - 32*a^5*c^6 + 2*(b^2 - 4*a*c)*a^3*b^2*c^4 - 8*(b^2 -
 4*a*c)*a^4*c^5)*f*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*a^3*b^5*
c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^3*c^4 + 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^4*b*c^5 - 32*a^5*b*c^5 + 2*(b^2 - 4*a*c)*a^3*b^3*c^3 - 8*(b^2 - 4*a*
c)*a^4*b*c^4)*g*abs(a*b^2*c - 4*a^2*c^2) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^5*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*
c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*a^2*b^5*c^4
```

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 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 + 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^3*b*c^6 - 32*a^4*b*c^6 + 2*(b^2 - 4*a*c)*a^2*b^3*c^4 - 8*(b^2 - 4*a*c)*
a^3*b*c^5)*abs(a*b^2*c - 4*a^2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 2
24*a^4*b^3*c^8 - 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^4*b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*
b^3*c^7 - 96*(b^2 - 4*a*c)*a^4*b*c^8)*d - (2*a^3*b^7*c^5 - 8*a^4*b^5*c^6 -
32*a^5*b^3*c^7 + 128*a^6*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^4*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^3*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^5*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^6*b*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b^2*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^7 - 2*(b^2 - 4*a*c)*a^3*b^5*c^5 + 32*(b^2 - 4*a*c)*a^5*b*c
^7)*f - (2*a^3*b^8*c^4 - 32*a^4*b^6*c^5 + 160*a^5*b^4*c^6 - 256*a^6*b^2*c^7
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 +
16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 8
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 2
4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 128*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 64*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 + 12*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 32*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 2*(
b^2 - 4*a*c)*a^3*b^6*c^4 + 24*(b^2 - 4*a*c)*a^4*b^4*c^5 - 64*(b^2 - 4*a*c)*
a^5*b^2*c^6)*g + 4*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^2*c^8 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 16*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)
*a^3*b^4*c^6 + 8*(b^2 - 4*a*c)*a^4*b^2*c^7)*e)*arctan(2*sqrt(1/2)*x/sqrt((a
*b^3*c - 4*a^2*b*c^2 + sqrt((a*b^3*c - 4*a^2*b*c^2)^2 - 4*(a^2*b^2*c - 4*a^
3*c^2)*(a*b^2*c^2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^2*c^3)))/((a^3*b^6*c^3 -
12*a^4*b^4*c^4 - 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 + 16*a^4*b^3*c^5 + a^3*b^4*
c^5 - 64*a^6*c^6 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 + 16*a^5*c^7)*abs(a*b^2*c -
 4*a^2*c^2)*abs(c)) - 1/16*((2*b^3*c^4 - 8*a*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b*c^4 - 2*(b^2 - 4*a*c)*b*c^4)*(a*b^2*c - 4*a^2*c^2)^2*d
 + (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
```

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*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a*b^2*c - 4*a^2*c^2)^2*f + (2*a*b^4*c
^2 - 20*a^2*b^2*c^3 + 48*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3
 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 12*(b^2 - 4*a*c)*a^2*c^3)*(a*b^2*c - 4*a^2*c
^2)^2*g - 2*(2*a*b^2*c^4 - 8*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*a*c^4)*(a*b^2*c - 4*a^2*c^2)^2*e - 2*(sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 14*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^4*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5
*c^4 + 2*a*b^6*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5
 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 28*a^2*b^4*c^5 - 96*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b*c^6 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 128*a^
3*b^2*c^6 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^7 - 192*a^4*c^
7 - 2*(b^2 - 4*a*c)*a*b^4*c^4 + 20*(b^2 - 4*a*c)*a^2*b^2*c^5 - 48*(b^2 - 4*
a*c)*a^3*c^6)*d*abs(a*b^2*c - 4*a^2*c^2) + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^3*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*
c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 2*a^3*b^4*c^4
 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 8*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^2*c^5 - 16*a^4*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
4*c^6 + 32*a^5*c^6 - 2*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2 - 4*a*c)*a^4*c^5)
*f*abs(a*b^2*c - 4*a^2*c^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 2*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*a^3*b^5*c^3 + 16*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^4*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c
^4 - 16*a^4*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 +
 32*a^5*b*c^5 - 2*(b^2 - 4*a*c)*a^3*b^3*c^3 + 8*(b^2 - 4*a*c)*a^4*b*c^4)*g*
abs(a*b^2*c - 4*a^2*c^2) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b
^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 2*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 2*a^2*b^5*c^4 + 16*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5
- 16*a^3*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 32
*a^4*b*c^6 - 2*(b^2 - 4*a*c)*a^2*b^3*c^4 + 8*(b^2 - 4*a*c)*a^3*b*c^5)*abs(a
*b^2*c - 4*a^2*c^2)*e + (2*a^2*b^7*c^6 - 40*a^3*b^5*c^7 + 224*a^4*b^3*c^8 -
 384*a^5*b*c^9 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b^7*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^5*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^6*c^5 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^4*b^3*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^3*b^4*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^6 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b*c^7 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b^2*c^7 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^3*c^7 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*
b*c^8 - 2*(b^2 - 4*a*c)*a^2*b^5*c^6 + 32*(b^2 - 4*a*c)*a^3*b^3*c^7 - 96*(b^
2 - 4*a*c)*a^4*b*c^8)*d - (2*a^3*b^7*c^5 - 8*a^4*b^5*c^6 - 32*a^5*b^3*c^7 +
 128*a^6*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
```

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a^3*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^4*b^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
3*b^6*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
5*b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b
^5*c^5 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b
*c^6 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2
*c^6 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c
^7 - 2*(b^2 - 4*a*c)*a^3*b^5*c^5 + 32*(b^2 - 4*a*c)*a^5*b*c^7)*f - (2*a^3*b
^8*c^4 - 32*a^4*b^6*c^5 + 160*a^5*b^4*c^6 - 256*a^6*b^2*c^7 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^8*c^2 + 16*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^3 - 80*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 24*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^4 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 128*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 + 64*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^5 + 12*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 - 32*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 2*(b^2 - 4*a*c)*a^3
*b^6*c^4 + 24*(b^2 - 4*a*c)*a^4*b^4*c^5 - 64*(b^2 - 4*a*c)*a^5*b^2*c^6)*g +
 4*(2*a^3*b^6*c^6 - 16*a^4*b^4*c^7 + 32*a^5*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^4 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^6 - 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^7 - 2*(b^2 - 4*a*c)*a^3*b^4*c^6 + 8
*(b^2 - 4*a*c)*a^4*b^2*c^7)*e)*arctan(2*sqrt(1/2)*x/sqrt((a*b^3*c - 4*a^2*b
*c^2 - sqrt((a*b^3*c - 4*a^2*b*c^2)^2 - 4*(a^2*b^2*c - 4*a^3*c^2)*(a*b^2*c^
2 - 4*a^2*c^3)))/(a*b^2*c^2 - 4*a^2*c^3)))/((a^3*b^6*c^3 - 12*a^4*b^4*c^4 -
 2*a^3*b^5*c^4 + 48*a^5*b^2*c^5 + 16*a^4*b^3*c^5 + a^3*b^4*c^5 - 64*a^6*c^6
 - 32*a^5*b*c^6 - 8*a^4*b^2*c^6 + 16*a^5*c^7)*abs(a*b^2*c - 4*a^2*c^2)*abs(
c))
```

maple [B] time = 0.05, size = 1760, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
c^2*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+(-1/2/a*(2*a^2*c
*g-a*b^2*g+a*b*c*f-2*a*c^2*e+b*c^2*d)/(4*a*c-b^2)/c*x^3+1/2*(a^2*b*g-2*a^2*
c*f+a*b*c*e+2*a*c^2*d-b^2*c*d)/a/c/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+1/4/a/(4*
a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/4/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*b^2*c*d*arctan
(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/4/(4*a*c-b^2)/c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*g+2*a/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*c*x)*b*g-1/4/(4*a*c-b^2)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b^3*g+1/4/(4*a*c-b^2)/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arcta
nh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*g-1/4/(4*a*c-b^2)/c*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)*b^2*g-3/2*a/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*g+3/2*a/(4*
```

```
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*c*x)*g-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*b^2*f-3/(4*a*c-b^2)*c^2/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x
)*d-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*b^2*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+2*a/(4*a*c-
b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*g-1/2/(4*a*c-b^2)*c*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*e-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b*f*
arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/a/(4*a*c-b^2)*c*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)*b*d-a/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*c*x)*f+1/(4*a*c-b^2)*c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-1/4/(4
*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*b*c*d*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*a*c*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*c*x)+1/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*b*c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f+1/2/(4*a*c-b^2)*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(bc^2d-2\,ac^2e+abcf-\left(ab^2-2\,a^2c\right)g\right)x^3-\left(abce-2\,a^2cf+a^2bg-\left(b^2c-2\,ac^2\right)d\right)x}{2\left(a^2b^2c-4\,a^3c^2+\left(ab^2c^2-4\,a^2c^3\right)x^4+\left(ab^3c-4\,a^2bc^2\right)x^2\right)}-\int\frac{abce-2\,a^2cf+a^2bg+\left(bc^2d\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((b*c^2*d - 2*a*c^2*e + a*b*c*f - (a*b^2 - 2*a^2*c)*g)*x^3 - (a*b*c*e -
 2*a^2*c*f + a^2*b*g - (b^2*c - 2*a*c^2)*d)*x)/(a^2*b^2*c - 4*a^3*c^2 + (a*
b^2*c^2 - 4*a^2*c^3)*x^4 + (a*b^3*c - 4*a^2*b*c^2)*x^2) - 1/2*integrate(-(a
*b*c*e - 2*a^2*c*f + a^2*b*g + (b*c^2*d - 2*a*c^2*e + a*b*c*f + (a*b^2 - 6*
a^2*c)*g)*x^2 + (b^2*c - 6*a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a*b^2*c - 4*a
^2*c^2)
```

mupad [B] time = 5.82, size = 32587, normalized size = 72.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(2*a*c^2*d - b^2*c*d + a^2*b*g - 2*a^2*c*f + a*b*c*e))/(2*a*c*(4*a*c -
b^2)) - (x^3*(b*c^2*d - 2*a*c^2*e - a*b^2*g + 2*a^2*c*g + a*b*c*f))/(2*a*c*
(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((6144*a^5*c^7*d + 2048*a^6*c^
6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*
b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*
a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g +
 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/
(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*
b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^
```

2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4
*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 28
8*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c
^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c -
b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96
*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/
2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*
b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g -
1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f
 + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6
*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f -
 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c
^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f +
 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5
*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e
*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*
f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3
*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^
4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a
^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 76
8*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^
4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4
*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c
*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*
b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2
 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9
)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b
^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a
^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^
3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a
^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 358
4*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d
*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a
^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f
*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a
^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c
^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 9
60*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9
)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g +
1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f
g(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10
*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2
c^8)))^(1/2) + (x(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*
g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2
+ 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2
*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e
 - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g +
 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2
*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c
 - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840
*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 +
 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-
(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*
a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^
2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(
1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2
g^2(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*
a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^

```
8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c
^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^
2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^
7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 307
2*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3
*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 268
8*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*
c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g
+ 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6
*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9
*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6
+ 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i - (((6144*a^5*c^7*d + 204
8*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d +
 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*
f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5
*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b
*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x
*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*
a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2
+ 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a
^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(
4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*
f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1
504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^
7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b
*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192
*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c
^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f -
 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c
^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g +
 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b
^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(
4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*
b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
 - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7
- 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*
c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*
a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a
^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) -
288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9
*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c
- b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 +
96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(
1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^
6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g
- 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d
*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b
^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f
 - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2
*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f
 + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a
^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3
*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c
- b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^
```

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4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a
^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*
a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144
*a^8*b^2*c^8)))^(1/2) - (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 +
a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^
4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4
*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b
*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c
^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2
*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a
^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^
2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c
^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*
c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2
 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^
3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b
^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 -
 a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 307
2*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*
a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g
 + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^
5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d
*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3
*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e
*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*
a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*
c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192
*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d
*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(
4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*
b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*1i)/((((6144*a^5*c^7
*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2
*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*
b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192
*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 10
24*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^
3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*
d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*
c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^
2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7
*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3
*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*
b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a
*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2
*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 921
6*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 35
84*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d
*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a
^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c
^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a
^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c
^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 1
28*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2
*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g +
128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^
9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*
b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*
b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*
```

a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)
))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 -
9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^
2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^
(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 -
a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(
-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^
2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b
^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 +
1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*
c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6
*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 1
92*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9
*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f
- 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8
*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g
+ 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4
*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(
-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^
6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/
2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c
^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^
7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^
4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 1
6*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2
+ 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e -
40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8
*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f
- 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4
*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^1
1*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768
*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2
+ 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5
*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 51
2*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4
*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c
^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g
^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*
f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^
10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e +
1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b
^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2)
- 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b
^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*
d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*
a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f
*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a
*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1
280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (((6144*a^
5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c^5*d - 5632*a^
4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32
*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g
- 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e
- 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b
^2*c^3)) + (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b
*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a
^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c

- b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^
3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^
2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) -
a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(
-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^
7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e
- 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e
+ 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*
c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e
+ 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*
b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g +
12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*
b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*
g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^
3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f
*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 +
a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840
*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 +
192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2
c^2)))((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d
^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c
^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2
)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*
d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 + a^2*c^3*
e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b
^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*
c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*
g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216
*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 358
4*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*
e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^
2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^
6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^
3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^
5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 12
8*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*
e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 1
28*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g + 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9
)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b
^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b
^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b
^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^
2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3
*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d
*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*
g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^
3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(1
6*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2
- b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5
*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^
3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2
- 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2
*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*
b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*
c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8
*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a

^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*
d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*
a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*
a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2
*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g -
3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*
c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g
+ 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^
5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (8*a^
3*c^5*e^3 + 5*b^3*c^5*d^3 + 5*a^4*b^4*g^3 + 216*a^6*c^2*g^3 - 4*a^4*b*c^3*f
^3 + 72*a^2*c^6*d^2*e - 66*a^5*b^2*c*g^3 - 3*b^4*c^4*d^2*e + a^2*b^6*e*g^2
+ 216*a^3*c^5*d^2*g + 8*a^4*c^4*e*f^2 + b^5*c^3*d^2*f - 3*a^3*b^5*f*g^2 + 7
2*a^4*c^4*e^2*g + 216*a^5*c^3*e*g^2 + b^6*c^2*d^2*g + 24*a^5*c^3*f^2*g + 6*
a^2*b^2*c^4*e^3 - 3*a^3*b^3*c^2*f^3 - 36*a*b*c^6*d^3 + a*b^7*d*g^2 + 48*a^3
*c^5*d*e*f + 144*a^4*c^4*d*f*g + 18*a*b^2*c^5*d^2*e + 3*a*b^3*c^4*d*e^2 - 6
0*a^2*b*c^5*d*e^2 - a*b^3*c^4*d^2*f + a*b^5*c^2*d*f^2 - 60*a^2*b*c^5*d^2*f
- 28*a^3*b*c^4*d*f^2 - 10*a*b^4*c^3*d^2*g - 21*a^2*b^5*c*d*g^2 - 28*a^3*b*c
^4*e^2*f - 396*a^4*b*c^3*d*g^2 - 12*a^3*b^4*c*e*g^2 - 6*a^3*b^4*c*f^2*g + 5
1*a^4*b^3*c*f*g^2 - 204*a^5*b*c^2*f*g^2 - 9*a^2*b^3*c^3*d*f^2 - 6*a^2*b^2*c
^4*d^2*g - 5*a^2*b^3*c^3*e^2*f + a^2*b^4*c^2*e*f^2 + 18*a^3*b^2*c^3*e*f^2 +
155*a^3*b^3*c^2*d*g^2 - 5*a^2*b^4*c^2*e^2*g + 26*a^3*b^2*c^3*e^2*g + 2*a^4
*b^2*c^2*e*g^2 + 42*a^4*b^2*c^2*f^2*g + 2*a*b^6*c*d*f*g - 4*a*b^4*c^3*d*e*f
- 4*a*b^5*c^2*d*e*g - 312*a^3*b*c^4*d*e*g + 2*a^2*b^5*c*e*f*g - 152*a^4*b*
c^3*e*f*g + 52*a^2*b^2*c^4*d*e*f + 70*a^2*b^3*c^3*d*e*g - 30*a^2*b^4*c^2*d*
f*g + 100*a^3*b^2*c^3*d*f*g + 6*a^3*b^3*c^2*e*f*g)/(4*(64*a^5*c^4 - a^2*b^6
*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3))))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2
- b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 - 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2)
+ 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5
*g^2 + 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^
3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2
- 512*a^5*b^3*c^6*e^2 + a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) + b^2*c^3*d^2
*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*
b^3*c^5*f^2 - a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*c^2*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*
c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8
*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a
^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*
d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*
a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f - 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(
1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*
a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2
*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g -
3072*a^6*b^3*c^5*e*g + 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*
c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g
+ 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)
^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^
5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*2i - at
an(((((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a^3*b^4*c
^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e + 768*a^4*
b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5*f + 16*
a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^3*d - 10
24*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*
c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^
2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c
^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*
c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2
- 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^

3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b
^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 +
a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2
) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 307
2*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*
a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g
+ 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^
5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d
*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3
*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e
*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*
a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*
c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192
*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d
e(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(
4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*
b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*c^6 - 16
*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 + a^2*b^4
*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 38
40*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2
+ 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*
(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 384
0*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*
e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b
^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 28
8*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*
c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10
*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*
a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*
c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3
072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c
^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2
688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^
9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*
g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b
^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(
4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a
^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^
6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a^2*c^6*d^2 - 8*a
^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3*g^2 - 1
4*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c^2*f^2 +
2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c^4*e*g +
2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^4*e*f +
2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*c^3*d*g
- 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a^3*b^3*c
^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 -
a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 +
3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5
*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a
^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2)
- b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*
f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*
f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 -
3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*
e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b
*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 12
8*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*

```
c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a
*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*
c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f
+ 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*
b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2)
 + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^
7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4
*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 +
240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))
^(1/2)*1i - (((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d + 1920*a
^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*c^4*e +
768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5*b^2*c^5
*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*a*b^8*c^
3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6*c + 12*
a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^1
1*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768
*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2
- 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5
*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 51
2*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4
*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c
^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)
^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g
^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*
f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^
10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e +
 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b
^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2)
- 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b
^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*
d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*
a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f
*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a
*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2
))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1
280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(1024*a^5*b*
c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a^4*c^3 +
 a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*
d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b
*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^
4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d
^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*
b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2
 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3
072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g -
2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f
*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*
a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5
*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a
^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4
*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g +
6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^
3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 1
92*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3
*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32
*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^
6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*(72*a^2*c^6*d
```

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^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72*a^5*c^3
*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a^2*b^4*c
^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f - 48*a^4*c
^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*a^3*b*c^
4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*a^2*b^3*
c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*g - 14*a
^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c
^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*
c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2
*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^
2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^
9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*
b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) +
a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c
^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*
a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 35
84*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*
d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*
a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*
f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*
a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*
c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g +
960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^
9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g +
 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*
f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^1
0*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^
2*c^8)))^(1/2)*1i)/((((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c^4*d
+ 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3*b^5*
c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 1536*a^5
*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g + 16*
a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^2*b^6
*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) - (x*((27*a*b^9*c^4*d^2 - a^3*b^11*g
^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*
c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504
*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*
e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*
d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a
^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*
c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b
^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*
a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g -
2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c
^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 9
60*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9
)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 1
28*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*
b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g
 - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b
^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f
*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)
^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8
*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)*(102
4*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*(16*a
^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b
^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7
68*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^
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2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b
^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 -
512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-
(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3
*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^
2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4
*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^
6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*
b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e
 + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4
*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2
) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4
*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^
6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 307
2*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2
*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2
*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1
/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 -
 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) + (x*(72*a
^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2 - 72
*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^2 + a
^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d*f -
48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g - 8*
a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f + 10*
a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c^3*e*
g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27
*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4
*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*
a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) -
 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^
9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 +
 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^
(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a
^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g
 - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*
d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*
b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*
f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^
2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*
f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*
a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^
3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c
 - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c
^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*
a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24
*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 614
4*a^8*b^2*c^8)))^(1/2) + (((6144*a^5*c^7*d + 2048*a^6*c^6*f - 288*a^2*b^6*c
^4*d + 1920*a^3*b^4*c^5*d - 5632*a^4*b^2*c^6*d + 16*a^2*b^7*c^3*e - 192*a^3
*b^5*c^4*e + 768*a^4*b^3*c^5*e - 32*a^3*b^6*c^3*f + 384*a^4*b^4*c^4*f - 153
6*a^5*b^2*c^5*f + 16*a^3*b^7*c^2*g - 192*a^4*b^5*c^3*g + 768*a^5*b^3*c^4*g
+ 16*a*b^8*c^3*d - 1024*a^5*b*c^6*e - 1024*a^6*b*c^5*g)/(8*(64*a^5*c^4 - a^
2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3)) + (x*((27*a*b^9*c^4*d^2 - a^3*b
^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9
)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a
^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 +
 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5
*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2
*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 -
```

512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-
(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*
a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f -
3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e
*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*
b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*
f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b
^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*
g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192
*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^
4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*
a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*
c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c -
b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^
5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2)
*(1024*a^5*b*c^6 - 16*a^2*b^7*c^3 + 192*a^3*b^5*c^4 - 768*a^4*b^3*c^5))/(2*
(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))*((27*a*b^9*c^4*d^2 - a^3*b^11*g^
2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2
) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2 + 27*a^4*b^9*c*g^2 + 3840*a^8*b*c
^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^2*b^7*c^5*d^2 + 1504*
a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e
^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(4*a*c - b^2)^9)^(1/2) - b^2*c^3*d
^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^
6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1504*a^6*b^5*c^3*g^2 - 3840*a^7*b^
3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^7*d*g - 1024*a^7*c^7*e*f - 3072*a
^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b*c^7*d*f + 3584*a^7*b*c^6*e*g - 2
*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^
6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c^3*d*f - 128*a^3*b^7*c^4*d*f + 96
0*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f + 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)
^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c^3*e*f + 384*a^4*b^6*c^4*d*g - 12
8*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g + 384*a^5*b^4*c^5*e*f + 8192*a^6*b
^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b^7*c^3*e*g + 960*a^5*b^5*c^4*e*g
- 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^4*b^
8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*
g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^
9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3 - 24*a^4*b^10*c^4 + 240*a^5*b^8*
c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7 - 6144*a^8*b^2*c^8)))^(1/2) - (x*
(72*a^2*c^6*d^2 - 8*a^3*c^5*e^2 + b^4*c^4*d^2 + a^2*b^6*g^2 + 8*a^4*c^4*f^2
- 72*a^5*c^3*g^2 - 14*a*b^2*c^5*d^2 - 16*a^3*b^4*c*g^2 + 10*a^2*b^2*c^4*e^
2 + a^2*b^4*c^2*f^2 + 2*a^3*b^2*c^3*f^2 + 74*a^4*b^2*c^2*g^2 + 48*a^3*c^5*d
*f - 48*a^4*c^4*e*g + 2*a*b^3*c^4*d*e - 40*a^2*b*c^5*d*e - 72*a^3*b*c^4*d*g
- 8*a^3*b*c^4*e*f + 2*a^2*b^5*c*f*g - 8*a^4*b*c^3*f*g + 4*a^2*b^2*c^4*d*f
+ 10*a^2*b^3*c^3*d*g - 6*a^2*b^3*c^3*e*f - 6*a^2*b^4*c^2*e*g + 52*a^3*b^2*c
^3*e*g - 14*a^3*b^3*c^2*f*g))/(2*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))
*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*
a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2
+ 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a
^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(
4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*
f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1
504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^
7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b
*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192
*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c
^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f +
6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c

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^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g +
 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b
^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(
4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*
b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
 + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7
- 6144*a^8*b^2*c^8)))^(1/2) + (8*a^3*c^5*e^3 + 5*b^3*c^5*d^3 + 5*a^4*b^4*g^
3 + 216*a^6*c^2*g^3 - 4*a^4*b*c^3*f^3 + 72*a^2*c^6*d^2*e - 66*a^5*b^2*c*g^3
 - 3*b^4*c^4*d^2*e + a^2*b^6*e*g^2 + 216*a^3*c^5*d^2*g + 8*a^4*c^4*e*f^2 +
b^5*c^3*d^2*f - 3*a^3*b^5*f*g^2 + 72*a^4*c^4*e^2*g + 216*a^5*c^3*e*g^2 + b^
6*c^2*d^2*g + 24*a^5*c^3*f^2*g + 6*a^2*b^2*c^4*e^3 - 3*a^3*b^3*c^2*f^3 - 36
*a*b*c^6*d^3 + a*b^7*d*g^2 + 48*a^3*c^5*d*e*f + 144*a^4*c^4*d*f*g + 18*a*b^
2*c^5*d^2*e + 3*a*b^3*c^4*d*e^2 - 60*a^2*b*c^5*d*e^2 - a*b^3*c^4*d^2*f + a*
b^5*c^2*d*f^2 - 60*a^2*b*c^5*d^2*f - 28*a^3*b*c^4*d*f^2 - 10*a*b^4*c^3*d^2*
g - 21*a^2*b^5*c*d*g^2 - 28*a^3*b*c^4*e^2*f - 396*a^4*b*c^3*d*g^2 - 12*a^3*
b^4*c*e*g^2 - 6*a^3*b^4*c*f^2*g + 51*a^4*b^3*c*f*g^2 - 204*a^5*b*c^2*f*g^2
- 9*a^2*b^3*c^3*d*f^2 - 6*a^2*b^2*c^4*d^2*g - 5*a^2*b^3*c^3*e^2*f + a^2*b^4
*c^2*e*f^2 + 18*a^3*b^2*c^3*e*f^2 + 155*a^3*b^3*c^2*d*g^2 - 5*a^2*b^4*c^2*e
^2*g + 26*a^3*b^2*c^3*e^2*g + 2*a^4*b^2*c^2*e*g^2 + 42*a^4*b^2*c^2*f^2*g +
2*a*b^6*c*d*f*g - 4*a*b^4*c^3*d*e*f - 4*a*b^5*c^2*d*e*g - 312*a^3*b*c^4*d*e
*g + 2*a^2*b^5*c*e*f*g - 152*a^4*b*c^3*e*f*g + 52*a^2*b^2*c^4*d*e*f + 70*a^
2*b^3*c^3*d*e*g - 30*a^2*b^4*c^2*d*f*g + 100*a^3*b^2*c^3*d*f*g + 6*a^3*b^3*
c^2*e*f*g)/(4*(64*a^5*c^4 - a^2*b^6*c + 12*a^3*b^4*c^2 - 48*a^4*b^2*c^3))))
*((27*a*b^9*c^4*d^2 - a^3*b^11*g^2 - b^11*c^3*d^2 + 3840*a^5*b*c^8*d^2 + 9*
a*c^4*d^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^6*b*c^7*e^2 + 768*a^7*b*c^6*f^2
+ 27*a^4*b^9*c*g^2 + 3840*a^8*b*c^5*g^2 - 9*a^4*c*g^2*(-(4*a*c - b^2)^9)^(1
/2) - 288*a^2*b^7*c^5*d^2 + 1504*a^3*b^5*c^6*d^2 - 3840*a^4*b^3*c^7*d^2 - a
^2*b^9*c^3*e^2 + 96*a^4*b^5*c^5*e^2 - 512*a^5*b^3*c^6*e^2 - a^2*c^3*e^2*(-(
4*a*c - b^2)^9)^(1/2) - b^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^3*b^9*c^2*
f^2 + 96*a^5*b^5*c^4*f^2 - 512*a^6*b^3*c^5*f^2 + a^3*b^2*g^2*(-(4*a*c - b^2
)^9)^(1/2) + a^3*c^2*f^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^5*b^7*c^2*g^2 + 1
504*a^6*b^5*c^3*g^2 - 3840*a^7*b^3*c^4*g^2 - 3072*a^6*c^8*d*e - 9216*a^7*c^
7*d*g - 1024*a^7*c^7*e*f - 3072*a^8*c^6*f*g - 2*a*b^10*c^3*d*e + 3584*a^6*b
*c^7*d*f + 3584*a^7*b*c^6*e*g - 2*a^3*b^10*c*f*g + 36*a^2*b^8*c^4*d*e - 192
*a^3*b^6*c^5*d*e + 128*a^4*b^4*c^6*d*e + 1536*a^5*b^2*c^7*d*e + 6*a^2*b^9*c
^3*d*f - 128*a^3*b^7*c^4*d*f + 960*a^4*b^5*c^5*d*f - 3072*a^5*b^3*c^6*d*f +
 6*a^2*c^3*d*f*(-(4*a*c - b^2)^9)^(1/2) - 20*a^3*b^8*c^3*d*g + 12*a^3*b^8*c
^3*e*f + 384*a^4*b^6*c^4*d*g - 128*a^4*b^6*c^4*e*f - 2688*a^5*b^4*c^5*d*g +
 384*a^5*b^4*c^5*e*f + 8192*a^6*b^2*c^6*d*g + 6*a^3*b^9*c^2*e*g - 128*a^4*b
^7*c^3*e*g + 960*a^5*b^5*c^4*e*g - 3072*a^6*b^3*c^5*e*g - 6*a^3*c^2*e*g*(-(
4*a*c - b^2)^9)^(1/2) + 36*a^4*b^8*c^2*f*g - 192*a^5*b^6*c^3*f*g + 128*a^6*
b^4*c^4*f*g + 1536*a^7*b^2*c^5*f*g - 2*a*b*c^3*d*e*(-(4*a*c - b^2)^9)^(1/2)
 + 2*a^3*b*c*f*g*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^9*c^9 + a^3*b^12*c^3
 - 24*a^4*b^10*c^4 + 240*a^5*b^8*c^5 - 1280*a^6*b^6*c^6 + 3840*a^7*b^4*c^7
- 6144*a^8*b^2*c^8)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.129 $\int \frac{d+ex^2+fx^4+gx^6}{x^2(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=460

$$-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{}$$

```
[Out] -d/a^2/x-1/2*x*(a*(b^3*d/a-b*(b*e+3*c*d)+a*(b*f+2*c*e)-2*a^2*g)+(b^2*c*d-2*
a*c*(-a*f+c*d)-a*b*(a*g+c*e))*x^2)/a^2/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/4*arc
tan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^2*c*d-2*a*c*(-a*f+
5*c*d)-a*b*(a*g+c*e)+(3*b^3*c*d-4*a*b*c*(a*f+4*c*d)-a*b^2*(-a*g+c*e)+4*a^2*
c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(
1/2))*(3*b^2*c*d-2*a*c*(-a*f+5*c*d)-a*b*(a*g+c*e)+(-3*b^3*c*d+4*a*b*c*(a*f+
4*c*d)+a*b^2*(-a*g+c*e)-4*a^2*c*(a*g+3*c*e))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A] time = 2.79, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {1669, 1664, 1166, 205}

$$-\frac{x\left(a\left(-2a^2g+\frac{b^3d}{a}+a(bf+2ce)-b(be+3cd)\right)+x^2\left(-ab(ag+ce)-2ac(cd-af)+b^2cd\right)\right)}{2a^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -(d/(a^2*x)) - (x*(a*((b^3*d)/a - b*(3*c*d + b*e) + a*(2*c*e + b*f) - 2*a^2
*g) + (b^2*c*d - 2*a*c*(c*d - a*f) - a*b*(c*e + a*g))*x^2))/(2*a^2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e +
 a*g) + (3*b^3*c*d - 4*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3
*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^
2 - 4*a*c]]])/(2*Sqrt[2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) - ((3*b^2*c*d - 2*a*c*(5*c*d - a*f) - a*b*(c*e + a*g) - (3*b^3*c*d - 4
*a*b*c*(4*c*d + a*f) - a*b^2*(c*e - a*g) + 4*a^2*c*(3*c*e + a*g))/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[
2]*a^2*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d + ex^2 + fx^4 + gx^6}{x^2\left(a + bx^2 + cx^4\right)^2}\,dx = -\frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2g\right) + \left(b^2cd - 2ac(cd - af) - ab(ce +\right.\right.}{2a^2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)}$$

$$= -\frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2g\right) + \left(b^2cd - 2ac(cd - af) - ab(ce +\right.\right.}{2a^2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2g\right) + \left(b^2cd - 2ac(cd - af) - a\right.\right.}{2a^2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2g\right) + \left(b^2cd - 2ac(cd - af) - a\right.\right.}{2a^2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)}$$

$$= -\frac{d}{a^2x} - \frac{x\left(a\left(\frac{b^3d}{a} - b(3cd + be) + a(2ce + bf) - 2a^2g\right) + \left(b^2cd - 2ac(cd - af) - a\right.\right.}{2a^2\left(b^2 - 4ac\right)\left(a + bx^2 + cx^4\right)}$$

Mathematica [A] time = 2.43, size = 529, normalized size = 1.15

$$-\frac{-\frac{2x\left(2a\left(a^2g-ac\left(e+fx^2\right)+c^2dx^2\right)+b^2\left(ae-cdx^2\right)+ab\left(-af+agx^2+3cd+cex^2\right)+b^3(-d)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{2}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(2ac\left(2a^2g-5cd\sqrt{b^2-4ac}+af\sqrt{b^2\cdots}\right.\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/4*((4*d)/x - (2*x*(-(b^3*d) + b^2*(a*e - c*d*x^2) + a*b*(3*c*d - a*f + c
*e*x^2 + a*g*x^2) + 2*a*(a^2*g + c^2*d*x^2 - a*c*(e + f*x^2))))/((b^2 - 4*a
*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]
*d - a*c*e + a^2*g) + 2*a*c*(-5*c*Sqrt[b^2 - 4*a*c]*d + 6*a*c*e + a*Sqrt[b^
2 - 4*a*c]*f + 2*a^2*g) - a*b*(16*c^2*d + c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f +
```

```
a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a
*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]
*(-3*b^3*c*d + b^2*(3*c*Sqrt[b^2 - 4*a*c]*d + a*c*e - a^2*g) - 2*a*c*(5*c*S
qrt[b^2 - 4*a*c]*d + 6*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 2*a^2*g) + a*b*(16*c
^2*d - c*Sqrt[b^2 - 4*a*c]*e + 4*a*c*f - a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)
*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a^2
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 8.30, size = 9176, normalized size = 19.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*b^2*c*d*x^4 - 10*a*c^2*d*x^4 + 2*a^2*c*f*x^4 - a^2*b*g*x^4 - a*b*c*
x^4*e + 3*b^3*d*x^2 - 11*a*b*c*d*x^2 + a^2*b*f*x^2 - 2*a^3*g*x^2 - a*b^2*x^
2*e + 2*a^2*c*x^2*e + 2*a*b^2*d - 8*a^2*c*d)/((c*x^5 + b*x^3 + a*x)*(a^2*b^
2 - 4*a^3*c)) - 1/16*((6*b^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 22*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*c^4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2 - 4*a*c)*a*
c^4)*(a^2*b^2 - 4*a^3*c)^2*d + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a^2*c^3)*(a^2*b^2
 - 4*a^3*c)^2*f - (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^2*b^2 - 4*a^3*c)
^2*g - (2*a*b^3*c^3 - 8*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^3*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*a*b*c^3)*(a^2*b^2 - 4*a^3*c)^2*e + 2*(3*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 37*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^6*c^2 - 6*a^2*b^7*c^2 + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b^3*c^3 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 3*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 74*a^3*b^5*c^3 - 208*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 104*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^3*c^4 - 304*a^4*b^3*c^4 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b*c^5 + 416*a^5*b*c^5 + 6*(b^2 - 4*a*c)*a^2*b^5*c^2 - 50*(b^2 - 4*a*c)*a^
```

3*b^3*c^3 + 104*(b^2 - 4*a*c)*a^4*b*c^4)*d*abs(a^2*b^2 - 4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 - 2*a^4*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 16*a^5*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 32*a^6*b*c^4 + 2*(b^2 - 4*a*c)*a^4*b^3*c^2 - 8*(b^2 - 4*a*c)*a^5*b*c^3)*f*abs(a^2*b^2 - 4*a^3*c) + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 - 2*a^5*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 16*a^6*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 32*a^7*c^4 + 2*(b^2 - 4*a*c)*a^5*b^2*c^2 - 8*(b^2 - 4*a*c)*a^6*c^3)*g*abs(a^2*b^2 - 4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 - 2*a^3*b^6*c^2 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 28*a^4*b^4*c^3 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 128*a^5*b^2*c^4 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 192*a^6*c^5 + 2*(b^2 - 4*a*c)*a^3*b^4*c^2 - 20*(b^2 - 4*a*c)*a^4*b^2*c^3 + 48*(b^2 - 4*a*c)*a^5*c^4)*abs(a^2*b^2 - 4*a^3*c)*e + (6*a^4*b^8*c^3 - 80*a^5*b^6*c^4 + 352*a^6*b^4*c^5 - 512*a^7*b^2*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 - 6*(b^2 - 4*a*c)*a^4*b^6*c^3 + 56*(b^2 - 4*a*c)*a^5*b^4*c^4 - 128*(b^2 - 4*a*c)*a^6*b^2*c^5)*d - 4*(2*a^6*b^6*c^3 - 16*a^7*b^4*c^4 + 32*a^8*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 2*(b^2 - 4*a*c)*a^6*b^4*c^3 + 8*(b^2 - 4*a*c)*a^7*b^2*c^4)*f + (2*a^6*b^7*c^2 - 8*a^7*b^5*c^3 - 32*a^8*b^3*c^4 + 128*a^9*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^4 - 2*(b^2 - 4*a*c)*a^6*b^5*c^2 + 32*(b^2 - 4*a*c)*a^8*b*c^4)*g - (2*a^5*b^7*c^3 - 40*a^6*b^5*c^4 + 224*a^7*b^3*c^5 - 384*a^8*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c

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)*a^7*b^3*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^6*b^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^8*b*c^4 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
7*b^2*c^4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
6*b^3*c^4 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
7*b*c^5 - 2*(b^2 - 4*a*c)*a^5*b^5*c^3 + 32*(b^2 - 4*a*c)*a^6*b^3*c^4 - 96*(
b^2 - 4*a*c)*a^7*b*c^5)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c +
 sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^
2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6*c - 12*a^6*b^4*c^2 - 2*a^5*b^5*c^2
 + 48*a^7*b^2*c^3 + 16*a^6*b^3*c^3 + a^5*b^4*c^3 - 64*a^8*c^4 - 32*a^7*b*c^
4 - 8*a^6*b^2*c^4 + 16*a^7*c^5)*abs(a^2*b^2 - 4*a^3*c)*abs(c)) + 1/16*((6*b
^4*c^3 - 44*a*b^2*c^4 + 80*a^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^4*c + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^2*c^3 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^
4 - 6*(b^2 - 4*a*c)*b^2*c^3 + 20*(b^2 - 4*a*c)*a*c^4)*(a^2*b^2 - 4*a^3*c)^2
*d + 2*(2*a^2*b^2*c^3 - 8*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a^2*c^3)*(a^2*b^2 - 4*a^3*c)^2*f - (2*a^2*b
^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2
 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^2*b^2 - 4*a^3*c)^2*g - (2*a*b^3*c^3 - 8*a^
2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 +
2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4
*a*c)*a*b*c^3)*(a^2*b^2 - 4*a^3*c)^2*e - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^7*c - 37*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c
^2 - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 6*a^2*b^7*c^2
+ 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 + 50*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c^3 - 74*a^3*b^5*c^3 - 208*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^4 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^
4 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 304*a^4*b^3*c^
4 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 416*a^5*b*c^5 -
6*(b^2 - 4*a*c)*a^2*b^5*c^2 + 50*(b^2 - 4*a*c)*a^3*b^3*c^3 - 104*(b^2 - 4*a
*c)*a^4*b*c^4)*d*abs(a^2*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^4*b^5*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2
 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 + 2*a^4*b^5*c^2 +
16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^3 + 8*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^3*c^3 - 16*a^5*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b*c^4 + 32*a^6*b*c^4 - 2*(b^2 - 4*a*c)*a^4*b^3*c^2 + 8*(b^2 - 4*a*c)*a^5
*b*c^3)*f*abs(a^2*b^2 - 4*a^3*c) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^5*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^2 - 2*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^2 + 2*a^5*b^4*c^2 + 16*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^6*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^
3 - 16*a^6*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 + 32
*a^7*c^4 - 2*(b^2 - 4*a*c)*a^5*b^2*c^2 + 8*(b^2 - 4*a*c)*a^6*c^3)*g*abs(a^2
*b^2 - 4*a^3*c) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 14
```

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*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^2 - 2*sqrt(2)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + 2*a^3*b^6*c^2 + 64*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^4*b^3*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 28*a^
4*b^4*c^3 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 48*sqrt(2)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 10*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^4*b^2*c^4 + 128*a^5*b^2*c^4 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^5*c^5 - 192*a^6*c^5 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 20*(b^2
- 4*a*c)*a^4*b^2*c^3 - 48*(b^2 - 4*a*c)*a^5*c^4)*abs(a^2*b^2 - 4*a^3*c)*e +
 (6*a^4*b^8*c^3 - 80*a^5*b^6*c^4 + 352*a^6*b^4*c^5 - 512*a^7*b^2*c^6 - 3*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^8*c + 40*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 + 6*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c^2 - 176*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 - 56*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 - 3*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^3 + 256*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 128*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 + 28*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^4 - 64*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^5 - 6*(b^2
 - 4*a*c)*a^4*b^6*c^3 + 56*(b^2 - 4*a*c)*a^5*b^4*c^4 - 128*(b^2 - 4*a*c)*a^
6*b^2*c^5)*d - 4*(2*a^6*b^6*c^3 - 16*a^7*b^4*c^4 + 32*a^8*b^2*c^5 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^6*c + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^4*c^2 + 2*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 - 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^2*c^3 - 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^3 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 + 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 - 2*(b^2 - 4*a*c)*a^6
*b^4*c^3 + 8*(b^2 - 4*a*c)*a^7*b^2*c^4)*f + (2*a^6*b^7*c^2 - 8*a^7*b^5*c^3
- 32*a^8*b^3*c^4 + 128*a^9*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^6*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^7*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^6*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^8*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^6*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^9*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^8*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^8*b*c^4 - 2*(b^2 - 4*a*c)*a^6*b^5*c^2 + 32*(b^2 - 4*a*c)*a^8*b*c^4)*g
- (2*a^5*b^7*c^3 - 40*a^6*b^5*c^4 + 224*a^7*b^3*c^5 - 384*a^8*b*c^6 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^7*c + 20*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c^2 + 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^2 - 112*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^3 - 32*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^3 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^3 + 192*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b*c^4 + 96*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^4 + 16*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^4 - 48*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^5 - 2*(b^2 - 4*a*c)*
a^5*b^5*c^3 + 32*(b^2 - 4*a*c)*a^6*b^3*c^4 - 96*(b^2 - 4*a*c)*a^7*b*c^5)*e)
*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c - sqrt((a^2*b^3 - 4*a^3*b*c
)^2 - 4*(a^3*b^2 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^
2)))/((a^5*b^6*c - 12*a^6*b^4*c^2 - 2*a^5*b^5*c^2 + 48*a^7*b^2*c^3 + 16*a^6
*b^3*c^3 + a^5*b^4*c^3 - 64*a^8*c^4 - 32*a^7*b*c^4 - 8*a^6*b^2*c^4 + 16*a^7
*c^5)*abs(a^2*b^2 - 4*a^3*c)*abs(c))
```

maple [B] time = 0.06, size = 2045, normalized size = 4.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] 4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a
*b*c^2*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+4/a*c^2/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*d-3/4/a^2*c/(4*a*c-b^2)/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d-3/4/(4*a*c-b^2)/(-4*a*c+b^2)^
(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^2*b^3*c*d*arctan(2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)
*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*c*x)*b^2*e+1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*b^2*c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)-1/a^2*d/x-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2
)*c*x)*b^2*g+1/4/a*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*e-a*c/(4*a*c-b^2)/
(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/
((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*g-a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)*g-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*c^2*d*x^3-1/2/(c*x^
4+b*x^2+a)/(4*a*c-b^2)/a*b^2*e*x+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^3*d*
x-1/2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*f+1/2/(4*a*c-b^2)*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/
2)*c*x)-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*b*c*e*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+c/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*f+1/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*b*c*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+3/4/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^2*b^2*c*d*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
b*g+1/4/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/
2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*g+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)
*c*e*x+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*b*f*x-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2
)*x^3*b*g-a/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*g+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c
*f*x^3-1/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*g+1/2/(
c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^2*c*d*x^3+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)*c*x)*d-5/2/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*c^
2*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c^2*e*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2
)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*c*x)*e-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*b*c*e*x^3-3/2/(
c*x^4+b*x^2+a)/(4*a*c-b^2)/a*b*c*d*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/2*((a*b*c*e - 2*a^2*c*f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^4 - (a^2*b*
f - 2*a^3*g + (3*b^3 - 11*a*b*c)*d - (a*b^2 - 2*a^2*c)*e)*x^2 - 2*(a*b^2 -
4*a^2*c)*d)/((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3
*b^2 - 4*a^4*c)*x) + 1/2*integrate((a^2*b*f - 2*a^3*g + (a*b*c*e - 2*a^2*c*
f + a^2*b*g - (3*b^2*c - 10*a*c^2)*d)*x^2 - (3*b^3 - 13*a*b*c)*d + (a*b^2 -
 6*a^2*c)*e)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

mupad [B] time = 7.76, size = 40860, normalized size = 88.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(x^2*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan(((((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2
) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^
2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^
2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c
^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*
d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^
4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a
^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g
 - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*
d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c
^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*
b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*
c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f
- 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d
*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 107
52*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b
^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f
*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c
*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18
*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1
/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^
7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2
)*(x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2)
- 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2
+ 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2
+ a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3
*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^
2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*
b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7
*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g -
 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*
f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5
*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^
10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^
5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f -
1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g
 - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752
*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5
*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g
 + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a
```

^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c -
b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*
b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*
(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b
^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)
- 131072*a^16*c^7*g - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^1
1*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^
6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 1
7920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 55705
6*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^
7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g
 - 2560*a^12*b^8*c^3*g + 20480*a^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 1638
40*a^15*b^2*c^6*g + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a
^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 +
16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a
^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*
a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^
6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*
c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^
6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2
 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568
*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*
b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*
d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d
*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f +
 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a
^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^1
1*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b
^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c
^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((213*a*b^11*c^2*d
^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*
a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c
 - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^
4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4
*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*
b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*
b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 102
4*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g -
 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4
*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b
^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c
^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f +
512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8
*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f
+ 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536
*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3
*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*
a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(
1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b
^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-
(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*
c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*1i + (((213*a*b^11*c^2*
d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880
*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c
 - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c -

b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c
^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(
4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5
*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2
) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8
*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 10
24*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g
- 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(
4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*
b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*
c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f +
512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^
8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f
+ 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 153
6*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^
3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51
*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^
(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c -
b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(
-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(40
96*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6
*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(393216*a^15*c^8*e + 1
31072*a^16*c^7*g + x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c
- b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 384
0*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 76
8*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 -
2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 4480
0*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^
2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 -
9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*
c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 51
20*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1
536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g
+ 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*
f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e +
22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^
4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f
*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240
*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4
*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e
*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384
*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/
2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)
^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*
c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c
*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^1
0*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b
^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^
3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 157286
4*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9
*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c
^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 10
2400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^
10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5
*c^5*f + 81920*a^14*b^3*c^6*f - 128*a^11*b^10*c^2*g + 2560*a^12*b^8*c^3*g -
20480*a^13*b^6*c^4*g + 81920*a^14*b^4*c^5*g - 163840*a^15*b^2*c^6*g - 8519
68*a^14*b*c^8*d - 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13
*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144
*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*

a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*1i)/((((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c

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- b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-
(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*
c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2)
- 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c
 - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5
- 6144*a^10*b^2*c^6)))^(1/2)*(393216*a^15*c^8*e + 131072*a^16*c^7*g + x*((2
13*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13
*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*
c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*
f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 1
0656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a
^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*
e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c
- b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2
*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^
8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a
^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6
*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d
*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e -
30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5
*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2
) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a
^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2
*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g
 - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b
^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*
a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d
*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a
^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^
(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3
- 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576
*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 -
 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^
8*b^13*c^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7
*c^5*d - 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e
 - 1664*a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 32
7680*a^13*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^1
1*b^9*c^3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*
c^6*f - 128*a^11*b^10*c^2*g + 2560*a^12*b^8*c^3*g - 20480*a^13*b^6*c^4*g +
81920*a^14*b^4*c^5*g - 163840*a^15*b^2*c^6*g - 851968*a^14*b*c^8*d - 65536*
a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7
*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264
*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568
*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9
*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*
b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*
c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*
g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f -
49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*
a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c
^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*
c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*
f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g +
32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*
a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a
^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*
b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c
^4*f*g))*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1
/2) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*
```

e^2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*
f^2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9
*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^
6*d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*
a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e
^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96
*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d
*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^
6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b
*c^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^
2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^
4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*
f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c -
b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3
*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 1
0752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6
*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3
*f*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2
*c*e^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) +
18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^
(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*
c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*
a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1
/2) - (((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2
) - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^
2 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^
2 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c
^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*
d^2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^
4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2
*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a
^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g
 - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*
d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c
^5*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*
b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*
c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f
 - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^
2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d
*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 107
52*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b
^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f
*g + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c
e^2(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18
*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1
/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c
 - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^
7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2
)*(x*((213*a*b^11*c^2*d^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2)
 - 9*b^13*c*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2
 + 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2
 + a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3
*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^
2 + 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*
b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7
*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g -
 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*
f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5

*e*g - 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^
10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^
5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f -
1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)
^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g
 - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752
*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5
*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g
 + 6*a*b^12*c*d*e - 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a
^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c -
b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*
b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*
(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b
^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7)
 - 131072*a^16*c^7*g - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^1
1*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^
6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 1
7920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 55705
6*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^
7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g
 - 2560*a^12*b^8*c^3*g + 20480*a^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 1638
40*a^15*b^2*c^6*g + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a
^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 +
16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a
^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*
a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^
6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*
c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^
6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2
 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568
*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*
b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*
d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d
*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f +
 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a
^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^1
1*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b
^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c
^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((213*a*b^11*c^2*d
^2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*
a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c
- b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c -
b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^
4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4
*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*
b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
 + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*
b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 102
4*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g -
 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4
*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b
^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c
^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f +
512*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8
*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f
+ 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536
*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3

```
*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*
a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(
1/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b
^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-
(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(409
6*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*
c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2) - 128000*a^10*c^9*d^3 +
 1024*a^13*c^6*f^3 - 4608*a^11*b*c^7*e^3 - 24*a^11*b^7*c*g^3 - 46080*a^11*c
^8*d*e^2 - 512*a^14*b*c^4*g^3 + 76800*a^11*c^8*d^2*f - 15360*a^12*c^7*d*f^2
 + 9216*a^12*c^7*e^2*f - 5120*a^13*c^6*d*g^2 + 1024*a^14*c^5*f*g^2 - 504*a^
6*b^8*c^5*d^3 + 8112*a^7*b^6*c^6*d^3 - 48704*a^8*b^4*c^7*d^3 + 129280*a^9*b
^2*c^8*d^3 + 40*a^8*b^7*c^4*e^3 - 608*a^9*b^5*c^5*e^3 + 2944*a^10*b^3*c^6*e
^3 + 48*a^10*b^6*c^3*f^3 - 320*a^11*b^4*c^4*f^3 + 256*a^12*b^2*c^5*f^3 + 16
0*a^12*b^5*c^2*g^3 - 128*a^13*b^3*c^3*g^3 - 30720*a^12*c^7*d*e*g + 6144*a^1
3*c^6*e*f*g + 84480*a^10*b*c^8*d^2*e - 24*a^8*b^10*c*d*g^2 + 2560*a^11*b*c^
7*d^2*g - 7680*a^12*b*c^6*e*f^2 + 8*a^9*b^9*c*e*g^2 - 7680*a^12*b*c^6*e^2*g
 - 3584*a^13*b*c^5*e*g^2 + 8*a^10*b^8*c*f*g^2 - 3584*a^13*b*c^5*f^2*g + 360
*a^6*b^9*c^4*d^2*e - 5736*a^7*b^7*c^5*d^2*e - 240*a^7*b^8*c^4*d*e^2 + 33888
*a^8*b^5*c^6*d^2*e + 3792*a^8*b^6*c^5*d*e^2 - 87936*a^9*b^3*c^7*d^2*e - 216
96*a^9*b^4*c^6*d*e^2 + 52992*a^10*b^2*c^7*d*e^2 - 216*a^6*b^10*c^3*d^2*f +
3744*a^7*b^8*c^4*d^2*f - 25200*a^8*b^6*c^5*d^2*f - 72*a^8*b^8*c^3*d*f^2 + 8
1984*a^9*b^4*c^6*d^2*f + 1296*a^9*b^6*c^4*d*f^2 - 128256*a^10*b^2*c^7*d^2*f
 - 7872*a^10*b^4*c^5*d*f^2 + 19200*a^11*b^2*c^6*d*f^2 + 72*a^6*b^11*c^2*d^2
*g - 1128*a^7*b^9*c^3*d^2*g + 6488*a^8*b^7*c^4*d^2*g - 24*a^8*b^8*c^3*e^2*f
 - 16032*a^9*b^5*c^5*d^2*g + 336*a^9*b^6*c^4*e^2*f + 24*a^9*b^7*c^3*e*f^2 +
 368*a^9*b^8*c^2*d*g^2 + 13440*a^10*b^3*c^6*d^2*g - 960*a^10*b^4*c^5*e^2*f
- 672*a^10*b^5*c^4*e*f^2 - 1840*a^10*b^6*c^3*d*g^2 - 2304*a^11*b^2*c^6*e^2*
f + 4224*a^11*b^3*c^5*e*f^2 + 2880*a^11*b^4*c^4*d*g^2 + 1792*a^12*b^2*c^5*d
*g^2 + 8*a^8*b^9*c^2*e^2*g - 72*a^9*b^7*c^3*e^2*g - 288*a^10*b^5*c^4*e^2*g
- 136*a^10*b^7*c^2*e*g^2 + 3712*a^11*b^3*c^5*e^2*g + 480*a^11*b^5*c^3*e*g^2
 + 640*a^12*b^3*c^4*e*g^2 - 40*a^10*b^7*c^2*f^2*g + 96*a^11*b^5*c^3*f^2*g +
 80*a^11*b^6*c^2*f*g^2 + 1152*a^12*b^3*c^4*f^2*g - 960*a^12*b^4*c^3*f*g^2 +
 1792*a^13*b^2*c^4*f*g^2 + 21504*a^11*b*c^7*d*e*f + 17408*a^12*b*c^6*d*f*g
+ 144*a^7*b^9*c^3*d*e*f - 2256*a^8*b^7*c^4*d*e*f + 12480*a^9*b^5*c^5*d*e*f
- 28416*a^10*b^3*c^6*d*e*f - 48*a^7*b^10*c^2*d*e*g + 592*a^8*b^8*c^3*d*e*g
- 1632*a^9*b^6*c^4*d*e*g - 4992*a^10*b^4*c^5*d*e*g + 28160*a^11*b^2*c^6*d*e
*g + 96*a^8*b^9*c^2*d*f*g - 1616*a^9*b^7*c^3*d*f*g + 9408*a^10*b^5*c^4*d*f*
g - 22272*a^11*b^3*c^5*d*f*g - 32*a^9*b^8*c^2*e*f*g + 672*a^10*b^6*c^3*e*f*
g - 3456*a^11*b^4*c^4*e*f*g + 3584*a^12*b^2*c^5*e*f*g))*((213*a*b^11*c^2*d^
2 - a^5*b^9*g^2 - a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - 9*b^13*c*d^2 - 26880*a
^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 + 9*b^4*c*d^2*(-(4*a*c -
 b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 + a^4*c*f^2*(-(4*a*c - b
^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4
*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 + 25*a^2*c^3*d^2*(-(4*
a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b
^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 - 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b
^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024
*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g -
2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g - 6*a^4*c*e*g*(-(4*
a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^
8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^
6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 5
12*a^6*b^3*c^5*d*f - 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*
c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f +
 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*
a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*
c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e - 51*a
*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1
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/2) - 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) + 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*2i - (d/a - (x^2*(3*b^3*d - 2*a^3*g - a*b^2*e + a^2*b*f + 2*a^2*c*e - 11*a*b*c*d))/(2*a^2*(4*a*c - b^2)) + (x^4*(10*a*c^2*d - 3*b^2*c*d + a^2*b*g - 2*a^2*c*f + a*b*c*e))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5) + atan(((((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(x*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 9

83040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 131072*a^16*c^7*g - 393216*a^1
5*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d -
 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 6
4*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^1
2*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*
c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f -
 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g - 2560*a^12*b^8*c^3*g + 20480*a
^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 163840*a^15*b^2*c^6*g + 851968*a^14*
b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2
 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^1
2*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*
c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c
^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e
^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2
+ 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 5
12*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920
*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*
c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e
+ 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e -
253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168
*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^
9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*
c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c
^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e
*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g -
 3072*a^12*b^5*c^4*f*g))*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 -
 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 +
 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2
+ 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^
2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 +
44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^
9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e
^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*
b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e
+ 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f
 + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*
e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^
8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*
e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 57
6*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2
*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f -
2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6
*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c
^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g +
 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b
^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6
*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^
10*b^2*c^6)))^(1/2)*1i + (((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2
- 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2
+ 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2
 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g
^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 +
 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b
^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*
e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7
*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e

```
 + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*
f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c
*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b
^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d
*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 5
76*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^
2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f -
 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^
6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*
c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g
+ 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9
)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c -
 b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-
(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*
b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^
6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a
^10*b^2*c^6)))^(1/2)*(393216*a^15*c^8*e + 131072*a^16*c^7*g + x*((a^5*g^2*(
-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 -
 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-
(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4
*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3
*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d
^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 15
04*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 5
12*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*
f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*
c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e
*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 154
8*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^
6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4
*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*
a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c
^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g
 + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*
a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*
e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^
2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*
a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2
*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(
32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a
^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*
c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*
a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c
^2*d + 4672*a^9*b^11*c^3*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d -
 778240*a^12*b^5*c^6*d + 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*
a^10*b^10*c^3*e + 17920*a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^1
3*b^4*c^6*e - 557056*a^14*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^
3*f + 10240*a^12*b^7*c^4*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f -
128*a^11*b^10*c^2*g + 2560*a^12*b^8*c^3*g - 20480*a^13*b^6*c^4*g + 81920*a^
14*b^4*c^5*g - 163840*a^15*b^2*c^6*g - 851968*a^14*b*c^8*d - 65536*a^15*b*c
^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8
192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^1
0*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^
4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4
*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*
e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2
- 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 10
24*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^
```

14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c
^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e -
22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e
- 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144
*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b
^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5
*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*
c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*
e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))
*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^
11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*
b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^
4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2
+ 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 -
25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*
c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5
*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 307
2*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f -
18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g
+ 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c
^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*
e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344
*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^
(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 1
92*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7
*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3
*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6
*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(
-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b
*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) -
44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)
^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*
c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*1i)/
((((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b
^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9
*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a
^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^
2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 -
25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7
*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4
*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^
5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 30
72*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f -
18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*
g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*
c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d
*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 134
4*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)
^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g -
192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^
7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^
3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g +
6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*
(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*
b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) -
44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2
)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8
*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(39

```
3216*a^15*c^8*e + 131072*a^16*c^7*g + x*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2)
- a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a
^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) -
 a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 7
68*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4
*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1
/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 384
0*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3
*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 153
60*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6
*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f
 + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1
/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064
*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b
^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d
*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4
*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^
4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f
- 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^
6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(
4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*
d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*
a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^
(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*
b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^
4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2
- 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^
14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 192*a^8*b^13*c^2*d + 4672*a^9*b^11*c^3
*d - 47360*a^10*b^9*c^4*d + 256000*a^11*b^7*c^5*d - 778240*a^12*b^5*c^6*d +
 1261568*a^13*b^3*c^7*d + 64*a^9*b^12*c^2*e - 1664*a^10*b^10*c^3*e + 17920*
a^11*b^8*c^4*e - 102400*a^12*b^6*c^5*e + 327680*a^13*b^4*c^6*e - 557056*a^1
4*b^2*c^7*e + 64*a^10*b^11*c^2*f - 1280*a^11*b^9*c^3*f + 10240*a^12*b^7*c^4
*f - 40960*a^13*b^5*c^5*f + 81920*a^14*b^3*c^6*f - 128*a^11*b^10*c^2*g + 25
60*a^12*b^8*c^3*g - 20480*a^13*b^6*c^4*g + 81920*a^14*b^4*c^5*g - 163840*a^
15*b^2*c^6*g - 851968*a^14*b*c^8*d - 65536*a^15*b*c^7*f) + x*(204800*a^12*c
^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^
10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^
8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*
b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5
*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f
^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2
 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 40
96*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12
*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6
*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e +
 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f +
1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 4096
0*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b
^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5
*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^
3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*
g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((a^5*g^2*(-(4*a*c - b^
2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b
*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2
)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2
 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c
- b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c
^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96
```

*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c
^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9
*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^
3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c
- b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^
3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*
e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a
^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*
d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 729
6*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*
b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*
e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2
*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9
)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*
c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^1
1*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 +
 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2) - (((a^5*g^2*(-(4*a*c - b^2)
^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c
^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^
9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)
^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 -
 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c -
b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4
*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a
^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3
*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c
^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*
b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*
d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e
- 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6
*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*
g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*
a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^
2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*
g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c
^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) +
6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^
(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c
- b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*
c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3
840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(x*((a^5*g^2*(-(4*a*c - b^2)^9
)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7
*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)
^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(
1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 3
0240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^
2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e
^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6
*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g
^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5
*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^
10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^
2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*
e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e -
98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b
^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g
+ 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^

6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 131072*a^16*c^7*g - 393216*a^15*c^8*e + 192*a^8*b^13*c^2*d - 4672*a^9*b^11*c^3*d + 47360*a^10*b^9*c^4*d - 256000*a^11*b^7*c^5*d + 778240*a^12*b^5*c^6*d - 1261568*a^13*b^3*c^7*d - 64*a^9*b^12*c^2*e + 1664*a^10*b^10*c^3*e - 17920*a^11*b^8*c^4*e + 102400*a^12*b^6*c^5*e - 327680*a^13*b^4*c^6*e + 557056*a^14*b^2*c^7*e - 64*a^10*b^11*c^2*f + 1280*a^11*b^9*c^3*f - 10240*a^12*b^7*c^4*f + 40960*a^13*b^5*c^5*f - 81920*a^14*b^3*c^6*f + 128*a^11*b^10*c^2*g - 2560*a^12*b^8*c^3*g + 20480*a^13*b^6*c^4*g - 81920*a^14*b^4*c^5*g + 163840*a^15*b^2*c^6*g + 851968*a^14*b*c^8*d + 65536*a^15*b*c^7*f) + x*(204800*a^12*c^9*d^2 - 73728*a^13*c^8*e^2 + 8192*a^14*c^7*f^2 - 8192*a^15*c^6*g^2 + 16*a^10*b^10*c*g^2 + 144*a^6*b^12*c^3*d^2 - 3264*a^7*b^10*c^4*d^2 + 30112*a^8*b^8*c^5*d^2 - 143360*a^9*b^6*c^6*d^2 + 365568*a^10*b^4*c^7*d^2 - 458752*a^11*b^2*c^8*d^2 + 16*a^8*b^10*c^3*e^2 - 416*a^9*b^8*c^4*e^2 + 4608*a^10*b^6*c^5*e^2 - 25600*a^11*b^4*c^6*e^2 + 69632*a^12*b^2*c^7*e^2 + 160*a^10*b^8*c^3*f^2 - 2048*a^11*b^6*c^4*f^2 + 9216*a^12*b^4*c^5*f^2 - 16384*a^13*b^2*c^6*f^2 - 160*a^11*b^8*c^2*g^2 + 512*a^12*b^6*c^3*g^2 - 1024*a^13*b^4*c^4*g^2 + 4096*a^14*b^2*c^5*g^2 - 81920*a^13*c^8*d*f - 49152*a^14*c^7*e*g + 237568*a^12*b*c^8*d*e + 106496*a^13*b*c^7*d*g + 40960*a^13*b*c^7*e*f + 8192*a^14*b*c^6*f*g - 96*a^7*b^11*c^3*d*e + 2336*a^8*b^9*c^4*d*e - 22528*a^9*b^7*c^5*d*e + 107520*a^10*b^5*c^6*d*e - 253952*a^11*b^3*c^7*d*e - 96*a^8*b^10*c^3*d*f + 1472*a^9*b^8*c^4*d*f - 7168*a^10*b^6*c^5*d*f + 6144*a^11*b^4*c^6*d*f + 40960*a^12*b^2*c^7*d*f + 288*a^9*b^9*c^3*d*g + 32*a^9*b^9*c^3*e*f - 5120*a^10*b^7*c^4*d*g - 1024*a^10*b^7*c^4*e*f + 33792*a^11*b^5*c^5*d*g + 9216*a^11*b^5*c^5*e*f - 98304*a^12*b^3*c^6*d*g - 32768*a^12*b^3*c^6*e*f + 64*a^10*b^8*c^3*e*g - 6144*a^12*b^4*c^5*e*g + 32768*a^13*b^2*c^6*e*g - 96*a^10*b^9*c^2*f*g + 1024*a^11*b^7*c^3*f*g - 3072*a^12*b^5*c^4*f*g))*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6 *b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2) - 128000*a^10*c^9*d^3 + 1024*a^13*c^6*f^3 - 4608*a^11*b*c^7*e^3 - 24*a^11*b^7*c*g^3 - 46080*a^11*c^8*d*e^2 - 512*a^14*b*c^4*g^3 + 76800*a^11*c^8*d^2*f - 15360*a^12*c^7*d*f^2 + 9216*a^12*c^7*e^2*f - 5120*a^13*c^6*d*g^2 + 1024*a^14*c^5*f*g^2 - 504*a^6*b^8*c^5*d^3 + 8112*a^7*b^6*c^6*d^3 - 48704*a^8*b^4*c^7*d^3 + 129280*a^9*b^2*c^8*d^3 + 40*a^8*b^7*c^4*e^3 - 608*a^9*b^5*c^5*e^3 + 2944*a^10*b^3*c^6*e^3 + 48*a^10*b^6*c^3*f^3 - 320*a^11*b^4*c^4*f^3 + 256*a^12*b^2*c^5*f^3 + 160*a^12*b^5*c^2*g^3 - 128*a^13*b^3*c^3*g^3 - 30720*a^12*c^7*d*e*g + 6144*a^13*c^6*e*f*g + 84480*a^10*b*c^8*d^2*e - 24*a^8*b^10*c*d*g^2 + 2560*a^11*b*c^7*d^2*g - 7680*a^12*b*c^6*e*f^2 + 8*a^9*b^9*c*e*g^2 - 7680*a^12*b*c^6*e^2*g - 3584*a^13*b*c^5*e*g^2 + 8*a^10*b^8*c*f*g^2 - 3584*a^13*b*c^5*f^2*g + 360*a^6*b^9*c^4*d^2*e - 5736*a^7*b^7*c^5*d^2*e - 240*a^7*b^8*c^4*d*e^2 + 33888*a^8*b^5*c^6*d^2*e + 3792*a^8*b^6*c^5*d*e^2 - 87936*a^9*b^3*c^7*d^2*e - 21696*a^9*b^4*c^6*d*e^2 + 52992*a^10*b^2*c^7*d*e^2 - 216*a^6*b^10*c^3*d^2*f + 3744*a^7*b^8*c^4*d^2*f - 25200*a^8*b^6*c^5*d^2*f - 72*a^8*b^8*c^3*d*f^2 + 81984*a^9*b^4*c^6*d^2*f + 1296*a^9*b^6*c^4*d*f^2 - 128256*a^10*b^2*c^7*d^2*f - 7872*a^10*b^4*c^5*d*f^2 + 19200*a^11*b^2*c^6*d*f^2 + 72*a^6*b^11*c^2*d^2*g - 1128*a^7*b^9*c^3*d^2*g + 6488*a^8*b^7*c^4*d^2*g - 24*a^8*b^8*c^3*e^2*f - 16032*a^9*b^5*c^5*d^2*g + 336*a^9*b^6*c^4*e^2*f + 24*a^9*b^7*c^3*e*f^2 + 368*a^9*b^8*c^2*d*g^2 + 13440*a^10*b^3*c^6*d^2*g - 960*a^10*b^4*c^5*e^2*f - 672*a^10*b^5*c^4*e*f^2 - 1840*a^10*b^6*c^3*d*g^2 - 2304*a^11*b^2*c^6*e^2*f + 4224*a^11*b^3*c^5*e*f^2 + 2880*a^11*b^4*c^4*d*g^2 + 1792*a^12*b^2*c^5*d*g^2 + 8*a^8*b^9*c^2*e^2*g - 72*a^9*b^7*c^3*e^2*g - 288*a^10*b^5*c^4*e^2*g - 136*a^10*b^7*c^2*e*g^2 + 3712*a^11*b^3*c^5*e^2*g + 480*a^11*b^5*c^3*e*g^2 + 640*a^12*b^3*c^4*e*g^2 - 40*a^10*b^7*c^2*f^2*g + 96*a^11*b^5*c^3*f^2*g + 80*a^11*b^6*c^2*f*g^2 + 1152*a^12*b^3*c^4*f^2*g - 960*a^12*b^4*c^3*f*g^2 + 1792*a^13*b^2*c^4*f*g^2 + 21504*a^11*b*c^7*d*e*f + 17408*a^12*b*c^6*d*f*g + 144*a^7*b^9*c^3*d*e*f - 2256*a^8*b^7*c^4*d*e*f + 12480*a^9*b^5*c^5*d*e*f - 28416*a^10*b^3*c^6*d*e*f - 48*a^7*b^10*c^2*d*e*g + 592*a^8*b^8*c^3*d*e*g - 1632*a^9*b^6*c^4*d*e*g - 4992*a^10*b^4*c^5*d*e*g + 28160*a^11*b^2*c^6*d*e*g + 96*a^8*b^9*c^2*d*f*g - 1616*a^9*b^7*c^3*d*f*g + 9408*a^10*b^5*c^4*d*f*g - 22272*a^11*b^3*c^5*d*f*g - 32*a^9*b^8*c^2*e*f*g + 672*a^10*b^6*c^3*e*f*g - 3456*a^11*b^4*c^4*e*f*g + 3584*a^12*b^2*c^5*e*f*g))*((a^5*g^2*(-(4*a*c - b^2)^9)^(1/2) - a^5*b^9*g^2 - 9*b^13*c*d^2 + 213*a*b^11*c^2*d^2 - 26880*a^6*b*c^7*d^2 - a^2*b^11*c*e^2 + 3840*a^7*b*c^6*e^2 - 9*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^4*b^9*c*f^2 + 768*a^8*b*c^5*f^2 - a^4*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^9*b*c^4*g^2 - 2077*a^2*b^9*c^3*d^2 + 10656*a^3*b^7*c^4*d^2 - 30240*a^4*b^5*c^5*d^2 + 44800*a^5*b^3*c^6*d^2 - 25*a^2*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 27*a^3*b^9*c^2*e^2 - 288*a^4*b^7*c^3*e^2 + 1504*a^5*b^5*c^4*e^2 - 3840*a^6*b^3*c^5*e^2 + 9*a^3*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 96*a^6*b^5*c^3*f^2 - 512*a^7*b^3*c^4*f^2 + 96*a^7*b^5*c^2*g^2 - 512*a^8*b^3*c^3*g^2 + 15360*a^7*c^7*d*e + 5120*a^8*c^6*d*g - 3072*a^8*c^6*e*f - 1024*a^9*c^5*f*g + 6*a^2*b^11*c*d*f + 1536*a^7*b*c^6*d*f - 18*a^3*b^10*c*d*g - 2*a^3*b^10*c*e*f + 6*a^4*b^9*c*e*g + 3584*a^8*b*c^5*e*g + 6*a^4*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 12*a^5*b^8*c*f*g - 152*a^2*b^10*c^2*d*e + 1548*a^3*b^8*c^3*d*e - 8064*a^4*b^6*c^4*d*e + 22400*a^5*b^4*c^5*d*e - 30720*a^6*b^2*c^6*d*e - 98*a^3*b^9*c^2*d*f + 576*a^4*b^7*c^3*d*f - 1344*a^5*b^5*c^4*d*f + 512*a^6*b^3*c^5*d*f + 10*a^3*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 324*a^4*b^8*c^2*d*g + 36*a^4*b^8*c^2*e*f - 2240*a^5*b^6*c^3*d*g - 192*a^5*b^6*c^3*e*f + 7296*a^6*b^4*c^4*d*g + 128*a^6*b^4*c^4*e*f - 10752*a^7*b^2*c^5*d*g + 1536*a^7*b^2*c^5*e*f - 128*a^5*b^7*c^2*e*g + 960*a^6*b^5*c^3*e*g - 3072*a^7*b^3*c^4*e*g - 128*a^6*b^6*c^2*f*g + 384*a^7*b^4*c^3*f*g + 6*a*b^12*c*d*e + 51*a*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - a^2*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 6*a*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 18*a^3*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) - 2*a^3*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) - 44*a^2*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) + 6*a^2*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^11*c^7 + a^5*b^12*c - 24*a^6*b^10*c^2 + 240*a^7*b^8*c^3 - 1280*a^8*b^6*c^4 + 3840*a^9*b^4*c^5 - 6144*a^10*b^2*c^6)))^(1/2)*2i

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.130 $\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=542

$$\frac{2bd-ae}{a^3x}-\frac{d}{3a^2x^3}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

```
[Out] -1/3*d/a^2/x^3+(-a*e+2*b*d)/a^3/x+1/2*x*(a^2*(b^4*d/a^2+2*c^2*d+3*b*c*e-b^2
*(b*e+4*c*d)/a+b^2*f-a*(b*g+2*c*f))+c*(b^3*d-a*b^2*e-a*b*(-a*f+3*c*d)+2*a^2
*(-a*g+c*e))*x^2)/a^3/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+19*
c*d)+2*a^2*(-a*g+5*c*e)+(5*b^4*d-3*a*b^3*e+4*a^2*c*(-3*a*f+7*c*d)-a*b^2*(-a
*f+29*c*d)+4*a^2*b*(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*2^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2))*c^(1/2)*(5*b^3*d-3*a*b^2*e-a*b*(-a*f+19*c*d)+2*a^2*(-a*g+5*c
*e)+(-5*b^4*d+3*a*b^3*e-4*a^2*c*(-3*a*f+7*c*d)+a*b^2*(-a*f+29*c*d)-4*a^2*b*
(a*g+4*c*e))/(-4*a*c+b^2)^(1/2))/a^3/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2)
```

Rubi [A] time = 7.26, antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1669, 1664, 1166, 205}

$$\frac{x\left(a^2\left(\frac{b^4d}{a^2}-\frac{b^2(be+4cd)}{a}-a(bg+2cf)+b^2f+3bce+2c^2d\right)+cx^2\left(2a^2(ce-ag)-ab^2e-ab(3cd-af)+b^3d\right)\right)}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -d/(3*a^2*x^3) + (2*b*d - a*e)/(a^3*x) + (x*(a^2*((b^4*d)/a^2 + 2*c^2*d + 3
*b*c*e - (b^2*(4*c*d + b*e))/a + b^2*f - a*(2*c*f + b*g)) + c*(b^3*d - a*b^
2*e - a*b*(3*c*d - a*f) + 2*a^2*(c*e - a*g))*x^2))/(2*a^3*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f) + 2*
a^2*(5*c*e - a*g) + (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) - a*b^2*
(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*
Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(5*b^3*d - 3*a*b^2*e - a*b*(19*c*d - a*f
) + 2*a^2*(5*c*e - a*g) - (5*b^4*d - 3*a*b^3*e + 4*a^2*c*(7*c*d - 3*a*f) -
a*b^2*(29*c*d - a*f) + 4*a^2*b*(4*c*e + a*g))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)
*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rubi steps

$$\int \frac{d+ex^2+fx^4+gx^6}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c\left(b^3d-ab^2e-ab(3c\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= \frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)+c\left(b^3d-ab^2e-ab(3c\right.\right.}{2a^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)\right.}{2a^3\left(b^2-4ac\right)\left(a+\right.}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)\right.}{2a^3\left(b^2-4ac\right)\left(a+\right.}$$

$$= -\frac{d}{3a^2x^3}+\frac{2bd-ae}{a^3x}+\frac{x\left(a^2\left(\frac{b^4d}{a^2}+2c^2d+3bce-\frac{b^2(4cd+be)}{a}+b^2f-a(2cf+bg)\right)\right.}{2a^3\left(b^2-4ac\right)\left(a+\right.}$$

Mathematica [A] time = 2.15, size = 612, normalized size = 1.13

$$\frac{\frac{6x\left(ab\left(a^2(-g)+ac\left(3e+fx^2\right)-3c^2dx^2\right)+2a^2c\left(c\left(d+ex^2\right)-a\left(f+gx^2\right)\right)+b^3\left(cdx^2-ae\right)+ab^2\left(af-c\left(4d+ex^2\right)\right)+b^4d\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{3\sqrt{2}\sqrt{c}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-2a^2\right(}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*d)/x^3 + (24*b*d - 12*a*e)/x + (6*x*(b^4*d + b^3*(-(a*e) + c*d*x^2)
+ a*b^2*(a*f - c*(4*d + e*x^2)) + a*b*(-(a^2*g) - 3*c^2*d*x^2 + a*c*(3*e +
```

```
f*x^2)) + 2*a^2*c*(c*(d + e*x^2) - a*(f + g*x^2))))/((b^2 - 4*a*c)*(a + b*x
^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(5*b^4*d + b^3*(5*Sqrt[b^2 - 4*a*c]*d - 3
*a*e) + a*b^2*(-29*c*d - 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(-19*c*Sqrt[b^2
 - 4*a*c]*d + 16*a*c*e + a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) - 2*a^2*(-14*c^2*
d - 5*c*Sqrt[b^2 - 4*a*c]*e + 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTan[(Sqr
t[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b -
 Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(5*b^4*d - b^3*(5*Sqrt[b^2 - 4*a*
c]*d + 3*a*e) + a*b^2*(-29*c*d + 3*Sqrt[b^2 - 4*a*c]*e + a*f) + a*b*(19*c*S
qrt[b^2 - 4*a*c]*d + 16*a*c*e - a*Sqrt[b^2 - 4*a*c]*f + 4*a^2*g) + 2*a^2*(1
4*c^2*d - 5*c*Sqrt[b^2 - 4*a*c]*e - 6*a*c*f + a*Sqrt[b^2 - 4*a*c]*g))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sq
rt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

giac [B] time = 9.06, size = 10422, normalized size = 19.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*c*d*x^3 - 3*a*b*c^2*d*x^3 + a^2*b*c*f*x^3 - 2*a^3*c*g*x^3 - a*b^2*
c*x^3*e + 2*a^2*c^2*x^3*e + b^4*d*x - 4*a*b^2*c*d*x + 2*a^2*c^2*d*x + a^2*b
^2*f*x - 2*a^3*c*f*x - a^3*b*g*x - a*b^3*x*e + 3*a^2*b*c*x*e)/((a^3*b^2 - 4
*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b
*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 39
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2
+ 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^
3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3
 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4
*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(2*a^3*b^2*c^2 - 8*a^4*c^3 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 2*(b^2 - 4*a*c)*a^3*c^2)*(a
^3*b^2 - 4*a^4*c)^2*g - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b
^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e + 2*(5*sqrt(2)*sqrt(b*c + sqrt
```

```
(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c
+ 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3
*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4
*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4
*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c
^4)*d*abs(a^3*b^2 - 4*a^4*c) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c - 2*a^5*b^6*c + 64*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 +
28*a^6*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*c^3 - 48*sq
rt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^7*c^4 + 192*a^8*c^4 + 2*(b^2 - 4*a*c)*a^5*b^4*c - 20*(b
^2 - 4*a*c)*a^6*b^2*c^2 + 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4*a^4*c
) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5 - 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^4*c - 2*a^6*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8
*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 16*a^7*b^3*c^2 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 32*a^8*b*c^3 + 2*(b^2 - 4*a*c)*a^6*
b^3*c - 8*(b^2 - 4*a*c)*a^7*b*c^2)*g*abs(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(2)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c -
 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 + 50
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 3
04*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 416
*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 + 104
*(b^2 - 4*a*c)*a^6*b*c^3)*abs(a^3*b^2 - 4*a^4*c)*e + (10*a^6*b^9*c^2 - 138*
a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 - 4*a
*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b^3*
c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d + (2*a^8*b^7*c^2 - 40*a^9*b^5*c^3 + 22
4*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
```

*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(b^2 - 4*a*c)*a^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f + 4*(2*a^9*b^6*c^2 - 16*a^10*b^4*c^3 + 32*a^11*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^11*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 - 2*(b^2 - 4*a*c)*a^9*b^4*c^2 + 8*(b^2 - 4*a*c)*a^10*b^2*c^3)*g - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^2*c^4)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c + sqrt((a^3*b^3 - 4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c - 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) - 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*d + (2*a^2*b^3*c^2 - 8*a^3*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^2)*(a^3*b^2 - 4*a^4*c)^2*f - 2*(2*a^3*b^2*c^2 - 8*a^4*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 2*(b^2 - 4*a*c)*a^3*c^2)*(a^3*b^2 - 4*a^4*c)^2*g - (6*a*b^4*c^2 - 44*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^2*e - 2*(5*sqrt(2)*sqrt(b*c

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- sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c + 10*a^3*
b^8*c + 286*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 88*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 5*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^6*c^2 - 128*a^4*b^6*c^2 - 496*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^3 + 572*a
^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 + 112*sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 + 110*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^5*b^2*c^4 - 992*a^6*b^2*c^4 - 56*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^6*c^5 + 448*a^7*c^5 - 10*(b^2 - 4*a*c)*a^3*b^6*c + 88*(b
^2 - 4*a*c)*a^4*b^4*c^2 - 220*(b^2 - 4*a*c)*a^5*b^2*c^3 + 112*(b^2 - 4*a*c)
*a^6*c^4)*d*abs(a^3*b^2 - 4*a^4*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^5*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c - 2*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c + 2*a^5*b^6*c + 64*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^6*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*
c^2 - 28*a^6*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*c^3 -
 48*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 10*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 + 128*a^7*b^2*c^3 + 24*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^7*c^4 - 192*a^8*c^4 - 2*(b^2 - 4*a*c)*a^5*b^4*c +
 20*(b^2 - 4*a*c)*a^6*b^2*c^2 - 48*(b^2 - 4*a*c)*a^7*c^3)*f*abs(a^3*b^2 - 4
*a^4*c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^5 - 8*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^6*b^4*c + 2*a^6*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*
c)*a^8*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^2 + sqrt
(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 - 16*a^7*b^3*c^2 - 4*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 + 32*a^8*b*c^3 - 2*(b^2 - 4*a*c
)*a^6*b^3*c + 8*(b^2 - 4*a*c)*a^7*b*c^2)*g*abs(a^3*b^2 - 4*a^4*c) + 2*(3*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b
^6*c + 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^
2 + 50*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 - 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c
^3 + 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4
 - 416*a^7*b*c^4 - 6*(b^2 - 4*a*c)*a^4*b^5*c + 50*(b^2 - 4*a*c)*a^5*b^3*c^2
 - 104*(b^2 - 4*a*c)*a^6*b*c^3)*abs(a^3*b^2 - 4*a^4*c)*e + (10*a^6*b^9*c^2
- 138*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2
 - 4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^
8*b^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*d + (2*a^8*b^7*c^2 - 40*a^9*b^5*c^
3 + 224*a^10*b^3*c^4 - 384*a^11*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^8*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^9*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^8*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
```

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^2 - 4*a*c)*c)*a^10*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^9*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^8*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^11*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^10*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^9*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^10*b*c^4 - 2*(b^2 - 4*a*c)*a^8*b^5*c^2 + 32*(b^2 - 4*a*c)*a
^9*b^3*c^3 - 96*(b^2 - 4*a*c)*a^10*b*c^4)*f + 4*(2*a^9*b^6*c^2 - 16*a^10*b^
4*c^3 + 32*a^11*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^9*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^10*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^9*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^11*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^10*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
9*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^1
0*b^2*c^3 - 2*(b^2 - 4*a*c)*a^9*b^4*c^2 + 8*(b^2 - 4*a*c)*a^10*b^2*c^3)*g -
 (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 + 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 - 64*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 6*(b^2 - 4
*a*c)*a^7*b^6*c^2 + 56*(b^2 - 4*a*c)*a^8*b^4*c^3 - 128*(b^2 - 4*a*c)*a^9*b^
2*c^4)*e)*arctan(2*sqrt(1/2)*x/sqrt((a^3*b^3 - 4*a^4*b*c - sqrt((a^3*b^3 -
4*a^4*b*c)^2 - 4*(a^4*b^2 - 4*a^5*c)*(a^3*b^2*c - 4*a^4*c^2)))/(a^3*b^2*c -
 4*a^4*c^2)))/((a^7*b^6 - 12*a^8*b^4*c - 2*a^7*b^5*c + 48*a^9*b^2*c^2 + 16*
a^8*b^3*c^2 + a^7*b^4*c^2 - 64*a^10*c^3 - 32*a^9*b*c^3 - 8*a^8*b^2*c^3 + 16
*a^9*c^4)*abs(a^3*b^2 - 4*a^4*c)*abs(c)) + 1/3*(6*b*d*x^2 - 3*a*x^2*e - a*d
)/(a^3*x^3)
```

maple [B] time = 0.07, size = 2503, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x)
```

```
[Out] -1/3/a^2*d/x^3+c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*g+c
/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*ar
ctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b*g+2/a^3*b*d/x-1/2*c/(4
*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*g+1/2*c/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2)*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*g-5/
4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^3*b^3*c*d*arctan(2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+7/a*c^3/(4*a*c-b^2)/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*d+1/4/a*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b
^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*
b*f-3/4/a^2*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*e-1/4/(4*a*c-b^2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*b*c*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)
^(1/2))*c)^(1/2)*c*x)+3/4/(4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)/a^2*b^2*c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+19/4/(
4*a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^2*b*c^2*d*arctan(2^(1
```

```
/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+7/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*c^3*d*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)-19/4/a^2*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b
*d+5/4/a^3*c/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*d+1/(c*x^4+b*x^2+a)*c/(4
*a*c-b^2)*x^3*g+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)*x*b*g-29/4/a^2*c^2/(4*a*c-b
^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*d+1/4/(4*a*c-b^2)/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*b^2*c*f*arctan(2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^
(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*b*c^2*e*arctan(2^(1/2)/((b+(-4*a*c
+b^2)^(1/2))*c)^(1/2)*c*x)-3/4/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)/a^2*b^3*c*e*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)*c*x)+4/a*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c
*x)*b*e-3/4/a^2*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^
(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^3*
e+1/4/a*c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c
)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^2*f+5/4/(4
*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^3*b
^4*c*d*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)+5/4/a^3*c/(4*a*
c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*b^4*d-29/4/(4*a*c-b^2)/(-4*
a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a^2*b^2*c^2*d*arcta
n(2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-1/2/(c*x^4+b*x^2+a)/(4*a*c-
b^2)/a*b*c*f*x^3+1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^2*c*e*x^3+3/2/(c*x^4
+b*x^2+a)/(4*a*c-b^2)/a^2*b*c^2*d*x^3-3/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*b*c
*e*x+2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^2*b^2*c*d*x-1/2/(c*x^4+b*x^2+a)/(4*a*c
-b^2)/a^3*b^3*c*d*x^3+5/2/a*c^2/(4*a*c-b^2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)*e-5/2/(4*
a*c-b^2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)/a*c^2*e*arctan(2^(1/2)/((
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*x)-3/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c^2*f*arctan(2^(1/2)/((b+(-4*a*c+b^2)^(1
/2))*c)^(1/2)*c*x)-3*c^2/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*
c+b^2)^(1/2))*c)^(1/2)*arctanh(2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*c*
x)*f+1/(c*x^4+b*x^2+a)/(4*a*c-b^2)*c*f*x-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*c^
2*e*x^3-1/2/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a^3*b^4*d*x-1/2/(c*x^4+b*x^2+a)/(4*
a*c-b^2)/a*b^2*f*x-1/(c*x^4+b*x^2+a)/(4*a*c-b^2)/a*c^2*d*x+1/2/(c*x^4+b*x^2
+a)/(4*a*c-b^2)/a^2*b^3*e*x-1/a^2*e/x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{3\left(a^2bcf-2\,a^3cg+\left(5\,b^3c-19\,abc^2\right)d-\left(3\,ab^2c-10\,a^2c^2\right)e\right)x^6-\left(3\,a^3bg-\left(15\,b^4-62\,ab^2c+14\,a^2c^2\right)d+3\left(3\right.\right.}{6\left(\left(a^3b^2c-4\,a^4c^2\right)x^7+\left(a^3b^3-4\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x^6+f*x^4+e*x^2+d)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima"
)
```

```
[Out] 1/6*(3*(a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*d - (3*a*b^2*c - 10*
a^2*c^2)*e)*x^6 - (3*a^3*b*g - (15*b^4 - 62*a*b^2*c + 14*a^2*c^2)*d + 3*(3*
a*b^3 - 11*a^2*b*c)*e - 3*(a^2*b^2 - 2*a^3*c)*f)*x^4 + 2*(5*(a*b^3 - 4*a^2*
b*c)*d - 3*(a^2*b^2 - 4*a^3*c)*e)*x^2 - 2*(a^2*b^2 - 4*a^3*c)*d)/((a^3*b^2*
c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3) -
 1/2*integrate(-(a^3*b*g + (a^2*b*c*f - 2*a^3*c*g + (5*b^3*c - 19*a*b*c^2)*
d - (3*a*b^2*c - 10*a^2*c^2)*e)*x^2 + (5*b^4 - 24*a*b^2*c + 14*a^2*c^2)*d -
 (3*a*b^3 - 13*a^2*b*c)*e + (a^2*b^2 - 6*a^3*c)*f)/(c*x^4 + b*x^2 + a), x)/
(a^3*b^2 - 4*a^4*c)
```

mupad [B] time = 8.47, size = 51386, normalized size = 94.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2 + f*x^4 + g*x^6)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] atan((((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2
) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a
^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2
- 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^
4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116
928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a
^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/
2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 -
 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*
e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 +
 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^
13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^
12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10
*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*
b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g
+ 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*
(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b
^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 1
19616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 1
0*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b
^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g
*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4
*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2
*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g -
 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a
^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^
3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1
536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*
c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) +
184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b
^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4
096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a
^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c
^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1
/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640
*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^
2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*
c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 1
16928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9
*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(
1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2
 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^
2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2
 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*
b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*
b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^
10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^
2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*
g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*
g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6
*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2
```

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*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e -
 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e +
 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5
*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d
*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a
^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c
^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g
 - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6
*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*
c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g -
 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*
b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
+ 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c -
 b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 +
 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840
*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b
^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 9
83040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^1
3*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16
*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^1
3*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5
*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f -
1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 32768
0*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b
^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6
*g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 4
01408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14
*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b
^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*
a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a
^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45875
2*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^1
5*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*
b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^
2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d
*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g -
480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e +
 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*
d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d
*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^
8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*
g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f
 - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g
 - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g -
 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 3
2*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*
a^17*b^3*c^6*f*g))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c -
b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/
2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^
5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 7
68*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c
^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c
^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c -
 b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b
^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) +
 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b
^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^
2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*
g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f
```

+ 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2
) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4
*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g +
2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2
) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a
*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8
*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2
*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f -
14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10
*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1
/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548
*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b
^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c
^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 134
4*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4
*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2)
+ 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)
^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*
(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(
a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*
c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i + ((-(25*b^15*d^2 +
9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*
b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*
b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 -
9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e +
6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 2
19744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b
^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e
^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2
+ a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^
(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 -
96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*
d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g
- 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*
g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^
11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f +
98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2
) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*
c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2)
- 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e +
201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c
- b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*
b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/
2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2
)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3
*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f
+ 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b
^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4
*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 5
1*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9
)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*
a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b
^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b
^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12
*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^20*c^8*f + x*(-(25*b^15*d^2
+ 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^
6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^
3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2
- 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e

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 + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 -
 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c -
 b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2
*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^
2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9
)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2
 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^1
3*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e
*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*
f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*
b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f
+ 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1
/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^
2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/
2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e
 + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*
c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^
6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(
1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b
^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c
^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*
f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c -
 b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c
^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g -
 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)
^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(
4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3
*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8
*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^
12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^1
1*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 15
72864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^
14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^
17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^1
1*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^
6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f -
17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 5570
56*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^16*b^9*c^3*g - 10240*a^17*b
^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*c^6*g + 851968*a^19*b*c^8*
e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 7
3728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^
12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^
13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*
a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 14336
0*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16
*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a
^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16
*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17
*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d
*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e +
 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*
e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3
*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^
6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c
^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e
*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d
*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*
e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g
+ 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 10
```

24*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*(-(2
5*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^1
1*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^
2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*
b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*
a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7
*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*
(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a
^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*
b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b
^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 +
10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*
a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072
*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e
- 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b
^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c -
b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g +
246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b
^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b
^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d
f(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f
+ 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c -
b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-
(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688
*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*
b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*
(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512
*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2
*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*
a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3
*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2
) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^
6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2)*1i)/(((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^
6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*
a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*
b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c -
b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2
- 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 +
215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c
^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*
e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c
- b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2
*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 5
12*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e
+ 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 1
5360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a
*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*
c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536
*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a
*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)
^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*
e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e
- 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*
a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7
*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4
*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b
^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3

*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g
+ 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*
b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*
f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*
c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*
f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 1
86*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2
)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2
 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(3932
16*a^20*c^8*f - 917504*a^19*c^9*d + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*
b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(
4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^
8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c
 - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^
2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2
 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3
*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^
3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*
c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c
^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 +
 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d
*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g -
 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4
*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*
b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 15
36*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4
*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*
d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d
*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 270
6*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a
^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-
(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5
*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c
^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*
g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^
6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^
2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) -
 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c
^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(10
48576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*
c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 3
20*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*
a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*
a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^
9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*
c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f -
102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*
a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b
^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g)
 + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 819
2*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11
*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 240128
0*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 326
4*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 36
5568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 41

6*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632
*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^1
7*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c
^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*
e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*
e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^
7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c
^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4
*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c
^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5
*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^
6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*
e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g
 + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9
216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*(-(25*b^15*d^2 + 9*a^2*b^13
*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a
^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2
 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2
*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b
^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b
^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2
) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*
a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f
^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288
*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5
*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840
*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9
*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^
5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f +
43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*
c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*
c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(
4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3
*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6
*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(
1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f
 - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*
b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2)
 + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064
*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*
b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2
) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*
a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c
e^2(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 4
4*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^
9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240
*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))
^(1/2) - ((-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(
1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 8064
0*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f
^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b
*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 +
116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 +
9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^
(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^
2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c
^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^
2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a

*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3
*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b
^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a
^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d
*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f
g(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^
6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^
2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e
- 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e
+ 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^
5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*
d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*
a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*
c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*
g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f -
6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5
*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g
- 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4
*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2)
 + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c
- b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
+ 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 384
0*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^2
0*c^8*f + x*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)
^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80
640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c
*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10
*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2
+ 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2
+ 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9
)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*
e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4
*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*
f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615
*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a
^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5
*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724
*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c
*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b
*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*
a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*
d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*
e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*
e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*
a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^
3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 4
2*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^
8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*
d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f
- 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b
^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*
g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a
^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^1
2 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3
840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^1
5*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5
+ 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*

```
a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a
^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*
b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*
c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f
 + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 32
7680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^1
6*b^9*c^3*g - 10240*a^17*b^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*
c^6*g + 851968*a^19*b*c^8*e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2
- 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b
^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^1
2*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 18718
72*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 3011
2*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45
8752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*
a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^
15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18
*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^
9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g
 - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*
e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c
^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^
5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2
*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4
*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*
e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*
d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*
g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g
+ 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 327
68*a^17*b^3*c^6*f*g))*(-(25*b^15*d^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c
 - b^2)^9)^(1/2) + a^4*b^11*f^2 + a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^
(1/2) - 80640*a^7*b*c^7*d^2 - 213*a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27
*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2)
- 768*a^10*b*c^4*g^2 - 30*a*b^14*d*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^
9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^
3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*
c - b^2)^9)^(1/2) + 2077*a^4*b^9*c^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^
6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2
) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^
7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3
*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12
*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e
*f + 2*a^5*b^10*f*g + 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(
1/2) + 724*a^2*b^12*c*d*e - 258*a^3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*
a^4*b^10*c*d*g + 152*a^4*b^10*c*e*f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g
 + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(
1/2) - 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 16
5*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*
b^8*c^3*d*e - 119616*a^5*b^6*c^4*d*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*
b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*
f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f +
 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)
^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1
548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^
7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^
2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g +
1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*
b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1
/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b
^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d
```

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*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(3
2*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b
^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2) - 128000*a^15*c^9*e
^3 + 1024*a^18*c^6*g^3 + 476672*a^13*b*c^10*d^3 - 4608*a^16*b*c^7*f^3 - 250
880*a^14*c^10*d^2*e + 50176*a^15*c^9*d^2*g - 46080*a^16*c^8*e*f^2 + 76800*a
^16*c^8*e^2*g - 15360*a^17*c^7*e*g^2 + 9216*a^17*c^7*f^2*g + 1800*a^9*b^9*c
^6*d^3 - 29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^3 - 473216*a^12*b^3
*c^9*d^3 - 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3 - 48704*a^13*b^4*c^
7*e^3 + 129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 - 608*a^14*b^5*c^5*f^
3 + 2944*a^15*b^3*c^6*f^3 + 48*a^15*b^6*c^3*g^3 - 320*a^16*b^4*c^4*g^3 + 25
6*a^17*b^2*c^5*g^3 + 215040*a^15*c^9*d*e*f - 43008*a^16*c^8*d*f*g + 442880*
a^14*b*c^9*d*e^2 - 433664*a^14*b*c^9*d^2*f + 109056*a^15*b*c^8*d*f^2 + 8448
0*a^15*b*c^8*e^2*f + 43520*a^16*b*c^7*d*g^2 - 7680*a^17*b*c^6*f*g^2 - 1400*
a^9*b^10*c^5*d^2*e + 21680*a^10*b^8*c^6*d^2*e + 1680*a^10*b^9*c^5*d*e^2 - 1
21648*a^11*b^6*c^7*d^2*e - 27176*a^11*b^7*c^6*d*e^2 + 275264*a^12*b^4*c^8*d
^2*e + 164448*a^12*b^5*c^7*d*e^2 - 121088*a^13*b^2*c^9*d^2*e - 441216*a^13*
b^3*c^8*d*e^2 + 1000*a^9*b^11*c^4*d^2*f - 17800*a^10*b^9*c^5*d^2*f + 124280
*a^11*b^7*c^6*d^2*f + 400*a^11*b^9*c^4*d*f^2 - 422944*a^12*b^5*c^7*d^2*f -
6600*a^12*b^7*c^5*d*f^2 + 694912*a^13*b^3*c^8*d^2*f + 40416*a^13*b^5*c^6*d*
f^2 - 108928*a^14*b^3*c^7*d*f^2 - 600*a^9*b^12*c^3*d^2*g + 10960*a^10*b^10*
c^4*d^2*g - 78904*a^11*b^8*c^5*d^2*g + 360*a^11*b^9*c^4*e^2*f + 278096*a^12
*b^6*c^6*d^2*g - 5736*a^12*b^7*c^5*e^2*f - 240*a^12*b^8*c^4*e*f^2 + 120*a^1
2*b^9*c^3*d*g^2 - 472000*a^13*b^4*c^7*d^2*g + 33888*a^13*b^5*c^6*e^2*f + 37
92*a^13*b^6*c^5*e*f^2 - 2216*a^13*b^7*c^4*d*g^2 + 284416*a^14*b^2*c^8*d^2*g
 - 87936*a^14*b^3*c^7*e^2*f - 21696*a^14*b^4*c^6*e*f^2 + 14688*a^14*b^5*c^5
*d*g^2 + 52992*a^15*b^2*c^7*e*f^2 - 41856*a^15*b^3*c^6*d*g^2 - 216*a^11*b^1
0*c^3*e^2*g + 3744*a^12*b^8*c^4*e^2*g - 25200*a^13*b^6*c^5*e^2*g - 72*a^13*
b^8*c^3*e*g^2 + 81984*a^14*b^4*c^6*e^2*g + 1296*a^14*b^6*c^4*e*g^2 - 128256
*a^15*b^2*c^7*e^2*g - 7872*a^15*b^4*c^5*e*g^2 + 19200*a^16*b^2*c^6*e*g^2 -
24*a^13*b^8*c^3*f^2*g + 336*a^14*b^6*c^4*f^2*g + 24*a^14*b^7*c^3*f*g^2 - 96
0*a^15*b^4*c^5*f^2*g - 672*a^15*b^5*c^4*f*g^2 - 2304*a^16*b^2*c^6*f^2*g + 4
224*a^16*b^3*c^5*f*g^2 - 306176*a^15*b*c^8*d*e*g + 21504*a^16*b*c^7*e*f*g -
 1200*a^10*b^10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f - 130656*a^12*b^6*c^6*
d*e*f + 394368*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8*d*e*f + 720*a^10*b^
11*c^3*d*e*g - 12816*a^11*b^9*c^4*d*e*g + 89264*a^12*b^7*c^5*d*e*g - 302400
*a^13*b^5*c^6*d*e*g + 493824*a^14*b^3*c^7*d*e*g - 240*a^11*b^10*c^3*d*f*g +
 3872*a^12*b^8*c^4*d*f*g - 22368*a^13*b^6*c^5*d*f*g + 51840*a^14*b^4*c^6*d*
f*g - 25088*a^15*b^2*c^7*d*f*g + 144*a^12*b^9*c^3*e*f*g - 2256*a^13*b^7*c^4
*e*f*g + 12480*a^14*b^5*c^5*e*f*g - 28416*a^15*b^3*c^6*e*f*g))*(-(25*b^15*d
^2 + 9*a^2*b^13*e^2 + 25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) + a^4*b^11*f^2 +
a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) - 80640*a^7*b*c^7*d^2 - 213*
a^3*b^11*c*e^2 + 26880*a^8*b*c^6*e^2 - 27*a^5*b^9*c*f^2 - 3840*a^9*b*c^5*f^
2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) - 768*a^10*b*c^4*g^2 - 30*a*b^14*d
*e + 6366*a^2*b^11*c^2*d^2 - 35767*a^3*b^9*c^3*d^2 + 116928*a^4*b^7*c^4*d^2
 - 219744*a^5*b^5*c^5*d^2 + 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) + 2077*a^4*b^9*c
^2*e^2 - 10656*a^5*b^7*c^3*e^2 + 30240*a^6*b^5*c^4*e^2 - 44800*a^7*b^3*c^5*
e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)
^9)^(1/2) + 288*a^6*b^7*c^2*f^2 - 1504*a^7*b^5*c^3*f^2 + 3840*a^8*b^3*c^4*f
^2 - 96*a^8*b^5*c^2*g^2 + 512*a^9*b^3*c^3*g^2 - 615*a*b^13*c*d^2 + 10*a^2*b
^13*d*f + 35840*a^8*c^7*d*e + 10*a^3*b^12*d*g - 6*a^3*b^12*e*f - 6*a^4*b^11
*e*g - 7168*a^9*c^6*d*g - 15360*a^9*c^6*e*f + 2*a^5*b^10*f*g + 3072*a^10*c^
5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) + 724*a^2*b^12*c*d*e - 258*a^
3*b^11*c*d*f + 43520*a^8*b*c^6*d*f - 168*a^4*b^10*c*d*g + 152*a^4*b^10*c*e*
f + 98*a^5*b^9*c*e*g - 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^
(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) - 36*a^6*b^8*c*f*g + 246*a^2*
b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(
1/2) - 7278*a^3*b^10*c^2*d*e + 39132*a^4*b^8*c^3*d*e - 119616*a^5*b^6*c^4*d
*e + 201600*a^6*b^4*c^5*d*e - 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*
```

a*c - b^2)^9)^(1/2) + 2706*a^4*b^9*c^2*d*f - 14784*a^5*b^7*c^3*d*f + 44352*a^6*b^5*c^4*d*f - 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) + 1044*a^5*b^8*c^2*d*g - 1548*a^5*b^8*c^2*e*f - 2688*a^6*b^6*c^3*d*g + 8064*a^6*b^6*c^3*e*f + 1152*a^7*b^4*c^4*d*g - 22400*a^7*b^4*c^4*e*f + 6144*a^8*b^2*c^5*d*g + 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) - 576*a^6*b^7*c^2*e*g + 1344*a^7*b^5*c^3*e*g - 512*a^8*b^3*c^4*e*g + 192*a^7*b^6*c^2*f*g - 128*a^8*b^4*c^3*f*g - 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i - (d/(3*a) + (x^2*(3*a*e - 5*b*d))/(3*a^2) + (x^4*(15*b^4*d + 14*a^2*c^2*d + 3*a^2*b^2*f - 9*a*b^3*e - 3*a^3*b*g - 6*a^3*c*f - 62*a*b^2*c*d + 33*a^2*b*c*e))/(6*a^3*(4*a*c - b^2)) + (c*x^6*(5*b^3*d - 2*a^3*g - 3*a*b^2*e + a^2*b*f + 10*a^2*c*e - 19*a*b*c*d))/(2*a^3*(4*a*c - b^2)))/(a*x^3 + b*x^5 + c*x^7) + atan(((((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^

3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*
c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c
^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 -
 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d
*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g +
 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4
*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*
b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 15
36*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4
*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^
2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*
d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d
*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 270
6*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a
^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-
(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5
*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c
^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*
g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^
6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^
2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*
a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*
e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) -
 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b
^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c
^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(10
48576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*
c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 3
20*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*
a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*
a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^
9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^6*e + 1261568*a^18*b^3*
c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f + 17920*a^16*b^8*c^4*f -
102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 557056*a^19*b^2*c^7*f + 64*
a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b^7*c^4*g - 40960*a^18*b
^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8*e - 65536*a^20*b*c^7*g)
 + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 819
2*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11
*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 240128
0*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 326
4*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 36
5568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 41
6*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632
*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^1
7*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c
^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*
e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*
e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^
7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c
^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4
*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c
^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5
*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^
6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*
e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g
 + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9
216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2
)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^
6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 -

26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*
(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^
11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^
5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2)
 - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a
^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^
2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*
a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*
c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*
a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*
c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5
*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 4
3520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c
*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c
*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4
*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*
b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*
b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1
/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f
 + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b
^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) -
 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*
a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b
^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2)
 + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a
^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*
e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44
*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9
)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(
4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*
a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^
(1/2)*1i + (((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^1
5*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 806
40*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*
f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*
b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 -
 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 +
 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)
^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e
^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*
c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f
^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*
a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^
3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*
b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*
a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*
d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*
f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a
^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d
^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e
 + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e
 + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a
^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3
*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42
*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8
*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d
*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f -
 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^
5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g

```
 + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^
4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2
) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c
 - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12
 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 38
40*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^19*c^9*d - 393216*a^
20*c^8*f + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^
15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80
640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c
*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10
*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2
- 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2
+ 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9
)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*
e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4
*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*
f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615
*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a
^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5
*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724
*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c
*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b
*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*
a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*
d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*
e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*
e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*
a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^
3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 4
2*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^
8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*
d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f
- 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b
^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*
g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a
^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/
2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*
c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^1
2 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3
840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^1
5*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5
+ 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*
a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a
^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*
b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*
c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f
 + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 32
7680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^1
6*b^9*c^3*g - 10240*a^17*b^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*
c^6*g + 851968*a^19*b*c^8*e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2
- 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b
^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^1
2*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 18718
72*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 3011
2*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 45
8752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*
a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^
15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18
*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^
```

9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g
 - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*
e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c
^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^
5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2
*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4
*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*
e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*
d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*
g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g
 + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 327
68*a^17*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^
2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(
1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*
a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) +
 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9
*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3
*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c
 - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6
*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2)
 + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7
*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*
g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*
d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*
f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1
/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a
^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g
 + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1
/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165
*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b
^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b
^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f
 + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f +
10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^
(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 15
48*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7
*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2
*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1
344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b
^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/
2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^
2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*
e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32
*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^
6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*1i)/((((25*b^6*d^2*(
-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6
*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3
*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 -
 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e
- 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 +
219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c -
b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*
e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2
 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)
^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2
+ 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13
*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*
g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f
*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b

^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(393216*a^20*c^8*f - 917504*a^19*c^9*d + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) + 320*a^12*b^14*c^2*d - 7936*a^13*b^12*c^3*d + 82816*a^14*b^10*c^4*d - 468480*a^15*b^8*c^5*d + 1536000*a^16*b^6*c^6*d - 2867200*a^17*b^4*c^7*d + 2719744*a^18*b^2*c^8*d - 192*a^13*b^13*c^2*e + 4672*a^14*b^11*c^3*e - 47360*a^15*b^9*c^4*e + 256000*a^16*b^7*c^5*e - 778240*a^17*b^5*c^

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6*e + 1261568*a^18*b^3*c^7*e + 64*a^14*b^12*c^2*f - 1664*a^15*b^10*c^3*f +
17920*a^16*b^8*c^4*f - 102400*a^17*b^6*c^5*f + 327680*a^18*b^4*c^6*f - 5570
56*a^19*b^2*c^7*f + 64*a^15*b^11*c^2*g - 1280*a^16*b^9*c^3*g + 10240*a^17*b
^7*c^4*g - 40960*a^18*b^5*c^5*g + 81920*a^19*b^3*c^6*g - 851968*a^19*b*c^8*
e - 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 7
3728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^
12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^
13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*
a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 14336
0*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16
*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a
^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16
*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17
*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d
*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e +
 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*
e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3
*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^
6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c
^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e
*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d
*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*
e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g
 + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 10
24*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25
*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11
*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2
 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b
*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a
*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*
c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(
-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^
4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b
^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c
 - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^
3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 1
0*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a
^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*
a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e +
 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^
10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b
^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 2
46*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^
2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^
6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*
f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f -
 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c -
b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(
4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*
a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b
^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(
-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*
a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*
c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a
*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*
c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2)
 - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4
- 6144*a^12*b^2*c^5)))^(1/2) - (((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a
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^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c -
 b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6
*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^
9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 357
67*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 2150
40*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^
2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 -
 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2
)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2
+ 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^
9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10
*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*
a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c -
b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d
*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*
b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c -
b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(
1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 3
9132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 16
1280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b
^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*
c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c
- b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^
2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f
- 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 307
20*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c
^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g +
 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b
^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(
4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^
3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^
(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 12
80*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*a^
19*c^9*d - 393216*a^20*c^8*f + x*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*
a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c
- b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^
6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)
^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35
767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215
040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d
^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2
- 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^
2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2
 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a
^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 1
0*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360
*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c -
 b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*
d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9
*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c -
 b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^
(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e -
39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 1
61280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*
b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3
*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c
 - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c
^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f
 - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30
```

720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^3*c^7) - 320*a^12*b^14*c^2*d + 7936*a^13*b^12*c^3*d - 82816*a^14*b^10*c^4*d + 468480*a^15*b^8*c^5*d - 1536000*a^16*b^6*c^6*d + 2867200*a^17*b^4*c^7*d - 2719744*a^18*b^2*c^8*d + 192*a^13*b^13*c^2*e - 4672*a^14*b^11*c^3*e + 47360*a^15*b^9*c^4*e - 256000*a^16*b^7*c^5*e + 778240*a^17*b^5*c^6*e - 1261568*a^18*b^3*c^7*e - 64*a^14*b^12*c^2*f + 1664*a^15*b^10*c^3*f - 17920*a^16*b^8*c^4*f + 102400*a^17*b^6*c^5*f - 327680*a^18*b^4*c^6*f + 557056*a^19*b^2*c^7*f - 64*a^15*b^11*c^2*g + 1280*a^16*b^9*c^3*g - 10240*a^17*b^7*c^4*g + 40960*a^18*b^5*c^5*g - 81920*a^19*b^3*c^6*g + 851968*a^19*b*c^8*e + 65536*a^20*b*c^7*g) + x*(204800*a^17*c^9*e^2 - 401408*a^16*c^10*d^2 - 73728*a^18*c^8*f^2 + 8192*a^19*c^7*g^2 + 400*a^9*b^14*c^3*d^2 - 9440*a^10*b^12*c^4*d^2 + 92816*a^11*b^10*c^5*d^2 - 488096*a^12*b^8*c^6*d^2 + 1458688*a^13*b^6*c^7*d^2 - 2401280*a^14*b^4*c^8*d^2 + 1871872*a^15*b^2*c^9*d^2 + 144*a^11*b^12*c^3*e^2 - 3264*a^12*b^10*c^4*e^2 + 30112*a^13*b^8*c^5*e^2 - 143360*a^14*b^6*c^6*e^2 + 365568*a^15*b^4*c^7*e^2 - 458752*a^16*b^2*c^8*e^2 + 16*a^13*b^10*c^3*f^2 - 416*a^14*b^8*c^4*f^2 + 4608*a^15*b^6*c^5*f^2 - 25600*a^16*b^4*c^6*f^2 + 69632*a^17*b^2*c^7*f^2 + 160*a^15*b^8*c^3*g^2 - 2048*a^16*b^6*c^4*g^2 + 9216*a^17*b^4*c^5*g^2 - 16384*a^18*b^2*c^6*g^2 + 344064*a^17*c^9*d*f - 81920*a^18*c^8*e*g - 1236992*a^16*b*c^9*d*e + 40960*a^17*b*c^8*d*g + 237568*a^17*b*c^8*e*f + 40960*a^18*b*c^7*f*g - 480*a^10*b^13*c^3*d*e + 11104*a^11*b^11*c^4*d*e - 105824*a^12*b^9*c^5*d*e + 530432*a^13*b^7*c^6*d*e - 1469440*a^14*b^5*c^7*d*e + 2121728*a^15*b^3*c^8*d*e + 160*a^11*b^12*c^3*d*f - 3968*a^12*b^10*c^4*d*f + 39488*a^13*b^8*c^5*d*f - 200704*a^14*b^6*c^6*d*f + 542720*a^15*b^4*c^7*d*f - 720896*a^16*b^2*c^8*d*f + 160*a^12*b^11*c^3*d*g - 96*a^12*b^11*c^3*e*f - 2528*a^13*b^9*c^4*d*g + 2336*a^13*b^9*c^4*e*f + 14336*a^14*b^7*c^5*d*g - 22528*a^14*b^7*c^5*e*f - 31744*a^15*b^5*c^6*d*g + 107520*a^15*b^5*c^6*e*f + 8192*a^16*b^3*c^7*d*g - 253952*a^16*b^3*c^7*e*f - 96*a^13*b^10*c^3*e*g + 1472*a^14*b^8*c^4*e*g - 7168*a^15*b^6*c^5*e*g + 6144*a^16*b^4*c^6*e*g + 40960*a^17*b^2*c^7*e*g + 32*a^14*b^9*c^3*f*g - 1024*a^15*b^7*c^4*f*g + 9216*a^16*b^5*c^5*f*g - 32768*a^17*b^3*c^6*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2 - a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 + 3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 116928*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 + 44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 - 3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^13*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^12*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g - 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 119616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) -

2706*a^4*b^9*c^2*d*f + 14784*a^5*b^7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 6912
0*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f
*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*
a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^
6*c^3*e*f - 1152*a^7*b^4*c^4*d*g + 22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5
*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576
*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6
*c^2*f*g + 128*a^8*b^4*c^3*f*g + 1536*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-
(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b
*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 184*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2
) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c
- b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^
8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)
- 128000*a^15*c^9*e^3 + 1024*a^18*c^6*g^3 + 476672*a^13*b*c^10*d^3 - 4608*a
^16*b*c^7*f^3 - 250880*a^14*c^10*d^2*e + 50176*a^15*c^9*d^2*g - 46080*a^16*
c^8*e*f^2 + 76800*a^16*c^8*e^2*g - 15360*a^17*c^7*e*g^2 + 9216*a^17*c^7*f^2
*g + 1800*a^9*b^9*c^6*d^3 - 29080*a^10*b^7*c^7*d^3 + 176032*a^11*b^5*c^8*d^
3 - 473216*a^12*b^3*c^9*d^3 - 504*a^11*b^8*c^5*e^3 + 8112*a^12*b^6*c^6*e^3
- 48704*a^13*b^4*c^7*e^3 + 129280*a^14*b^2*c^8*e^3 + 40*a^13*b^7*c^4*f^3 -
608*a^14*b^5*c^5*f^3 + 2944*a^15*b^3*c^6*f^3 + 48*a^15*b^6*c^3*g^3 - 320*a^
16*b^4*c^4*g^3 + 256*a^17*b^2*c^5*g^3 + 215040*a^15*c^9*d*e*f - 43008*a^16*
c^8*d*f*g + 442880*a^14*b*c^9*d*e^2 - 433664*a^14*b*c^9*d^2*f + 109056*a^15
*b*c^8*d*f^2 + 84480*a^15*b*c^8*e^2*f + 43520*a^16*b*c^7*d*g^2 - 7680*a^17*
b*c^6*f*g^2 - 1400*a^9*b^10*c^5*d^2*e + 21680*a^10*b^8*c^6*d^2*e + 1680*a^1
0*b^9*c^5*d*e^2 - 121648*a^11*b^6*c^7*d^2*e - 27176*a^11*b^7*c^6*d*e^2 + 27
5264*a^12*b^4*c^8*d^2*e + 164448*a^12*b^5*c^7*d*e^2 - 121088*a^13*b^2*c^9*d
^2*e - 441216*a^13*b^3*c^8*d*e^2 + 1000*a^9*b^11*c^4*d^2*f - 17800*a^10*b^9
*c^5*d^2*f + 124280*a^11*b^7*c^6*d^2*f + 400*a^11*b^9*c^4*d*f^2 - 422944*a^
12*b^5*c^7*d^2*f - 6600*a^12*b^7*c^5*d*f^2 + 694912*a^13*b^3*c^8*d^2*f + 40
416*a^13*b^5*c^6*d*f^2 - 108928*a^14*b^3*c^7*d*f^2 - 600*a^9*b^12*c^3*d^2*g
+ 10960*a^10*b^10*c^4*d^2*g - 78904*a^11*b^8*c^5*d^2*g + 360*a^11*b^9*c^4*
e^2*f + 278096*a^12*b^6*c^6*d^2*g - 5736*a^12*b^7*c^5*e^2*f - 240*a^12*b^8*
c^4*e*f^2 + 120*a^12*b^9*c^3*d*g^2 - 472000*a^13*b^4*c^7*d^2*g + 33888*a^13
*b^5*c^6*e^2*f + 3792*a^13*b^6*c^5*e*f^2 - 2216*a^13*b^7*c^4*d*g^2 + 284416
*a^14*b^2*c^8*d^2*g - 87936*a^14*b^3*c^7*e^2*f - 21696*a^14*b^4*c^6*e*f^2 +
14688*a^14*b^5*c^5*d*g^2 + 52992*a^15*b^2*c^7*e*f^2 - 41856*a^15*b^3*c^6*d
*g^2 - 216*a^11*b^10*c^3*e^2*g + 3744*a^12*b^8*c^4*e^2*g - 25200*a^13*b^6*c
^5*e^2*g - 72*a^13*b^8*c^3*e*g^2 + 81984*a^14*b^4*c^6*e^2*g + 1296*a^14*b^6
*c^4*e*g^2 - 128256*a^15*b^2*c^7*e^2*g - 7872*a^15*b^4*c^5*e*g^2 + 19200*a^
16*b^2*c^6*e*g^2 - 24*a^13*b^8*c^3*f^2*g + 336*a^14*b^6*c^4*f^2*g + 24*a^14
*b^7*c^3*f*g^2 - 960*a^15*b^4*c^5*f^2*g - 672*a^15*b^5*c^4*f*g^2 - 2304*a^1
6*b^2*c^6*f^2*g + 4224*a^16*b^3*c^5*f*g^2 - 306176*a^15*b*c^8*d*e*g + 21504
*a^16*b*c^7*e*f*g - 1200*a^10*b^10*c^4*d*e*f + 20240*a^11*b^8*c^5*d*e*f - 1
30656*a^12*b^6*c^6*d*e*f + 394368*a^13*b^4*c^7*d*e*f - 528896*a^14*b^2*c^8*
d*e*f + 720*a^10*b^11*c^3*d*e*g - 12816*a^11*b^9*c^4*d*e*g + 89264*a^12*b^7
*c^5*d*e*g - 302400*a^13*b^5*c^6*d*e*g + 493824*a^14*b^3*c^7*d*e*g - 240*a^
11*b^10*c^3*d*f*g + 3872*a^12*b^8*c^4*d*f*g - 22368*a^13*b^6*c^5*d*f*g + 51
840*a^14*b^4*c^6*d*f*g - 25088*a^15*b^2*c^7*d*f*g + 144*a^12*b^9*c^3*e*f*g
- 2256*a^13*b^7*c^4*e*f*g + 12480*a^14*b^5*c^5*e*f*g - 28416*a^15*b^3*c^6*e
*f*g))*((25*b^6*d^2*(-(4*a*c - b^2)^9)^(1/2) - 9*a^2*b^13*e^2 - 25*b^15*d^2
- a^4*b^11*f^2 - a^6*b^9*g^2 + a^6*g^2*(-(4*a*c - b^2)^9)^(1/2) + 80640*a^
7*b*c^7*d^2 + 213*a^3*b^11*c*e^2 - 26880*a^8*b*c^6*e^2 + 27*a^5*b^9*c*f^2 +
3840*a^9*b*c^5*f^2 - 9*a^5*c*f^2*(-(4*a*c - b^2)^9)^(1/2) + 768*a^10*b*c^4
*g^2 + 30*a*b^14*d*e - 6366*a^2*b^11*c^2*d^2 + 35767*a^3*b^9*c^3*d^2 - 1169
28*a^4*b^7*c^4*d^2 + 219744*a^5*b^5*c^5*d^2 - 215040*a^6*b^3*c^6*d^2 + 9*a^
2*b^4*e^2*(-(4*a*c - b^2)^9)^(1/2) - 49*a^3*c^3*d^2*(-(4*a*c - b^2)^9)^(1/2
) - 2077*a^4*b^9*c^2*e^2 + 10656*a^5*b^7*c^3*e^2 - 30240*a^6*b^5*c^4*e^2 +
44800*a^7*b^3*c^5*e^2 + a^4*b^2*f^2*(-(4*a*c - b^2)^9)^(1/2) + 25*a^4*c^2*e
^2*(-(4*a*c - b^2)^9)^(1/2) - 288*a^6*b^7*c^2*f^2 + 1504*a^7*b^5*c^3*f^2 -

```
3840*a^8*b^3*c^4*f^2 + 96*a^8*b^5*c^2*g^2 - 512*a^9*b^3*c^3*g^2 + 615*a*b^1
3*c*d^2 - 10*a^2*b^13*d*f - 35840*a^8*c^7*d*e - 10*a^3*b^12*d*g + 6*a^3*b^1
2*e*f + 6*a^4*b^11*e*g + 7168*a^9*c^6*d*g + 15360*a^9*c^6*e*f - 2*a^5*b^10*
f*g - 3072*a^10*c^5*f*g - 30*a*b^5*d*e*(-(4*a*c - b^2)^9)^(1/2) - 724*a^2*b
^12*c*d*e + 258*a^3*b^11*c*d*f - 43520*a^8*b*c^6*d*f + 168*a^4*b^10*c*d*g -
 152*a^4*b^10*c*e*f - 98*a^5*b^9*c*e*g + 1536*a^9*b*c^5*e*g + 2*a^5*b*f*g*(
-(4*a*c - b^2)^9)^(1/2) - 10*a^5*c*e*g*(-(4*a*c - b^2)^9)^(1/2) + 36*a^6*b^
8*c*f*g + 246*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^9)^(1/2) - 165*a*b^4*c*d^2*(-
(4*a*c - b^2)^9)^(1/2) + 7278*a^3*b^10*c^2*d*e - 39132*a^4*b^8*c^3*d*e + 11
9616*a^5*b^6*c^4*d*e - 201600*a^6*b^4*c^5*d*e + 161280*a^7*b^2*c^6*d*e + 10
*a^2*b^4*d*f*(-(4*a*c - b^2)^9)^(1/2) - 2706*a^4*b^9*c^2*d*f + 14784*a^5*b^
7*c^3*d*f - 44352*a^6*b^5*c^4*d*f + 69120*a^7*b^3*c^5*d*f + 10*a^3*b^3*d*g*
(-(4*a*c - b^2)^9)^(1/2) - 6*a^3*b^3*e*f*(-(4*a*c - b^2)^9)^(1/2) + 42*a^4*
c^2*d*f*(-(4*a*c - b^2)^9)^(1/2) - 1044*a^5*b^8*c^2*d*g + 1548*a^5*b^8*c^2*
e*f + 2688*a^6*b^6*c^3*d*g - 8064*a^6*b^6*c^3*e*f - 1152*a^7*b^4*c^4*d*g +
22400*a^7*b^4*c^4*e*f - 6144*a^8*b^2*c^5*d*g - 30720*a^8*b^2*c^5*e*f - 6*a^
4*b^2*e*g*(-(4*a*c - b^2)^9)^(1/2) + 576*a^6*b^7*c^2*e*g - 1344*a^7*b^5*c^3
*e*g + 512*a^8*b^3*c^4*e*g - 192*a^7*b^6*c^2*f*g + 128*a^8*b^4*c^3*f*g + 15
36*a^9*b^2*c^4*f*g - 51*a^3*b^2*c*e^2*(-(4*a*c - b^2)^9)^(1/2) + 12*a^4*b*c
*d*g*(-(4*a*c - b^2)^9)^(1/2) + 44*a^4*b*c*e*f*(-(4*a*c - b^2)^9)^(1/2) + 1
84*a^2*b^3*c*d*e*(-(4*a*c - b^2)^9)^(1/2) - 186*a^3*b*c^2*d*e*(-(4*a*c - b^
2)^9)^(1/2) - 78*a^3*b^2*c*d*f*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 40
96*a^13*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^
11*b^4*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x**6+f*x**4+e*x**2+d)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

3.131 $\int x^2 \left(a+bx^2+cx^4\right)^p \left(3a+b(5+2p)x^2+c(7+4p)x^4\right)$

Optimal. Leaf size=20

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

```
[Out] x^3*(c*x^4+b*x^2+a)^(1+p)
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.024, Rules used = {1588}

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

Rule 1588

```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq
, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

Rubi steps

$$\int x^2 \left(a+bx^2+cx^4\right)^p \left(3a+b(5+2p)x^2+c(7+4p)x^4\right)\,dx = x^3 \left(a+bx^2+cx^4\right)^{1+p}$$

Mathematica [A] time = 0.14, size = 20, normalized size = 1.00

$$x^3 \left(a+bx^2+cx^4\right)^{p+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x^2 + c*x^4)^p*(3*a + b*(5 + 2*p)*x^2 + c*(7 + 4*p)*x^
4),x]
```

```
[Out] x^3*(a + b*x^2 + c*x^4)^(1 + p)
```

fricas [A] time = 1.06, size = 31, normalized size = 1.55

$$\left(cx^7+bx^5+ax^3\right)\left(cx^4+bx^2+a\right)^p$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="fricas")
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

giac [B] time = 0.61, size = 58, normalized size = 2.90

$$\left(cx^4+bx^2+a\right)^p cx^7 + \left(cx^4+bx^2+a\right)^p bx^5 + \left(cx^4+bx^2+a\right)^p ax^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="giac")
```

```
[Out] (c*x^4 + b*x^2 + a)^p*c*x^7 + (c*x^4 + b*x^2 + a)^p*b*x^5 + (c*x^4 + b*x^2
+ a)^p*a*x^3
```

maple [A] time = 0.01, size = 21, normalized size = 1.05

$$x^{3}\left(c\,x^{4}+b\,x^{2}+a\right)^{p+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x)
```

```
[Out] x^3*(c*x^4+b*x^2+a)^(p+1)
```

maxima [A] time = 1.03, size = 31, normalized size = 1.55

$$\left(cx^{7}+bx^{5}+ax^{3}\right)\left(cx^{4}+bx^{2}+a\right)^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)^p*(3*a+b*(5+2*p)*x^2+c*(7+4*p)*x^4),x, algori
thm="maxima")
```

```
[Out] (c*x^7 + b*x^5 + a*x^3)*(c*x^4 + b*x^2 + a)^p
```

mupad [B] time = 1.10, size = 31, normalized size = 1.55

$$\left(c\,x^{7}+b\,x^{5}+a\,x^{3}\right)\,{\left(c\,x^{4}+b\,x^{2}+a\right)}^{p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*a + b*x^2*(2*p + 5) + c*x^4*(4*p + 7))*(a + b*x^2 + c*x^4)^p,x)
```

```
[Out] (a*x^3 + b*x^5 + c*x^7)*(a + b*x^2 + c*x^4)^p
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)**p*(3*a+b*(5+2*p)*x**2+c*(7+4*p)*x**4),x)
```

```
[Out] Timed out
```

3.132 $\int \frac{x^5(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=210

$$-\frac{(d-ex)^{5/2}(d+ex)^{5/2}\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}}+\frac{d^2(d-ex)^{3/2}(d+ex)^{3/2}\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}}-\frac{d^4\sqrt{d-ex}\sqrt{d+}}{}$$

```
[Out] 1/3*d^2*(2*a*e^4+3*b*d^2*e^2+4*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^10-1/5
*(a*e^4+3*b*d^2*e^2+6*c*d^4)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^10+1/7*(b*e^2+4
*c*d^2)*(-e*x+d)^(7/2)*(e*x+d)^(7/2)/e^10-1/9*c*(-e*x+d)^(9/2)*(e*x+d)^(9/2
)/e^10-d^4*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^10
```

Rubi [A] time = 0.31, antiderivative size = 278, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {520, 1251, 897, 1153}

$$-\frac{\left(d^2-e^2x^2\right)^3\left(ae^4+3bd^2e^2+6cd^4\right)}{5e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2\left(d^2-e^2x^2\right)^2\left(2ae^4+3bd^2e^2+4cd^4\right)}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}}-\frac{d^4\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+c\right.}{e^{10}\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^4*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^10*Sqrt[d - e*x]*Sqr
t[d + e*x])) + (d^2*(4*c*d^4 + 3*b*d^2*e^2 + 2*a*e^4)*(d^2 - e^2*x^2)^2)/(3
*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*c*d^4 + 3*b*d^2*e^2 + a*e^4)*(d^2
- e^2*x^2)^3)/(5*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((4*c*d^2 + b*e^2)*(d^
2 - e^2*x^2)^4)/(7*e^10*Sqrt[d - e*x]*Sqrt[d + e*x]) - (c*(d^2 - e^2*x^2)^5
)/(9*e^10*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^5\left(a+bx^2+cx^4\right)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{x^2\left(a+bx+cx^2\right)}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2\left(\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{\left(2cd^2+be^2\right)x^2}{e^4}+\frac{cx^4}{e^4}\right)dx, x, \sqrt{d^2-e^2x^2}\right.}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(\frac{cd^8+bd^6e^2+ad^4e^4}{e^8}-\frac{d^2\left(4cd^4+3bd^2e^2+2ae^4\right)x^2}{e^8}+\frac{\left(6cd^4+3bd^2e^2+ae^4\right)x^4}{e^8}-\frac{(4c}{}\right.\right.}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{d^4\left(cd^4+bd^2e^2+ae^4\right)\left(d^2-e^2x^2\right)}{e^{10}\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2\left(4cd^4+3bd^2e^2+2ae^4\right)\left(d^2-e^2x^2\right)^2}{3e^{10}\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(6cd^4\right.}{}$$

Mathematica [C] time = 1.38, size = 265, normalized size = 1.26

$$-\frac{\frac{630d^{9/2}\sqrt{d+ex}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)\left(ae^4+bd^2e^2+cd^4\right)}{\sqrt{\frac{ex}{d}+1}}-630d^5\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)\left(ae^4+bd^2e^2+cd^4\right)+\sqrt{d-ex}\sqrt{d+ex}\left(21ae^4\left(8d^4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*a*e^4*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^
4*x^4) + 9*b*(16*d^6*e^2 + 8*d^4*e^4*x^2 + 6*d^2*e^6*x^4 + 5*e^8*x^6) + c*(
128*d^8 + 64*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 40*d^2*e^6*x^6 + 35*e^8*x^8)) +
 (630*d^(9/2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x
]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] - 630*d^5*(c*d^4 + b*d^2*e^2 + a*e^
4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^10
```

fricas [A] time = 1.02, size = 138, normalized size = 0.66

$$-\frac{\left(35\,ce^8x^8+128\,cd^8+144\,bd^6e^2+168\,ad^4e^4+5\left(8\,cd^2e^6+9\,be^8\right)x^6+3\left(16\,cd^4e^4+18\,bd^2e^6+21\,ae^8\right)x^4+4\left(16\right.\right.}{315\,e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/315*(35*c*e^8*x^8 + 128*c*d^8 + 144*b*d^6*e^2 + 168*a*d^4*e^4 + 5*(8*c*d
^2*e^6 + 9*b*e^8)*x^6 + 3*(16*c*d^4*e^4 + 18*b*d^2*e^6 + 21*a*e^8)*x^4 + 4*
(16*c*d^6*e^2 + 18*b*d^4*e^4 + 21*a*d^2*e^6)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d)/e^10
```

giac [A] time = 0.89, size = 269, normalized size = 1.28

$$-\frac{1}{315}\left(\left(\left(\left(\left(5\left(\left(7\left((xe+d)ce^{(-9)}-8\,cde^{(-9)}\right)(xe+d)+3\left(68\,cd^2e^{81}+3\,be^{83}\right)e^{(-90)}\right)(xe+d)-2\left(220\,cd^3e^{81}+27\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/315*(((((5*((7*((x*e + d)*c*e^(-9) - 8*c*d*e^(-9))*(x*e + d) + 3*(68*c*d
^2*e^81 + 3*b*e^83)*e^(-90))*(x*e + d) - 2*(220*c*d^3*e^81 + 27*b*d*e^83)*e
^(-90))*(x*e + d) + (3098*c*d^4*e^81 + 729*b*d^2*e^83 + 63*a*e^85)*e^(-90))
*(x*e + d) - 36*(82*c*d^5*e^81 + 31*b*d^3*e^83 + 7*a*d*e^85)*e^(-90))*(x*e
+ d) + 21*(92*c*d^6*e^81 + 51*b*d^4*e^83 + 22*a*d^2*e^85)*e^(-90))*(x*e + d
) - 210*(4*c*d^7*e^81 + 3*b*d^5*e^83 + 2*a*d^3*e^85)*e^(-90))*(x*e + d) + 3
15*(c*d^8*e^81 + b*d^6*e^83 + a*d^4*e^85)*e^(-90))*sqrt(x*e + d)*sqrt(-x*e
+ d)*e^(-1)
```

maple [A] time = 0.01, size = 145, normalized size = 0.69

$$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(35c\,x^8e^8+45b\,e^8x^6+40c\,d^2e^6x^6+63a\,e^8x^4+54b\,d^2e^6x^4+48c\,d^4e^4x^4+84a\,d^2e^6x^2+7\right.}{315e^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(35*c*e^8*x^8+45*b*e^8*x^6+40*c*d^2*e^6
*x^6+63*a*e^8*x^4+54*b*d^2*e^6*x^4+48*c*d^4*e^4*x^4+84*a*d^2*e^6*x^2+72*b*d
^4*e^4*x^2+64*c*d^6*e^2*x^2+168*a*d^4*e^4+144*b*d^6*e^2+128*c*d^8)/e^10
```

maxima [A] time = 1.03, size = 295, normalized size = 1.40

$$-\frac{\sqrt{-e^2x^2+d^2}\,cx^8}{9\,e^2}-\frac{8\,\sqrt{-e^2x^2+d^2}\,cd^2x^6}{63\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,bx^6}{7\,e^2}-\frac{16\,\sqrt{-e^2x^2+d^2}\,cd^4x^4}{105\,e^6}-\frac{6\,\sqrt{-e^2x^2+d^2}\,bd^2x^4}{35\,e^4}-\frac{\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/9*sqrt(-e^2*x^2 + d^2)*c*x^8/e^2 - 8/63*sqrt(-e^2*x^2 + d^2)*c*d^2*x^6/e
^4 - 1/7*sqrt(-e^2*x^2 + d^2)*b*x^6/e^2 - 16/105*sqrt(-e^2*x^2 + d^2)*c*d^4
*x^4/e^6 - 6/35*sqrt(-e^2*x^2 + d^2)*b*d^2*x^4/e^4 - 1/5*sqrt(-e^2*x^2 + d^
2)*a*x^4/e^2 - 64/315*sqrt(-e^2*x^2 + d^2)*c*d^6*x^2/e^8 - 8/35*sqrt(-e^2*x
^2 + d^2)*b*d^4*x^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*a*d^2*x^2/e^4 - 128/315
*sqrt(-e^2*x^2 + d^2)*c*d^8/e^10 - 16/35*sqrt(-e^2*x^2 + d^2)*b*d^6/e^8 - 8
/15*sqrt(-e^2*x^2 + d^2)*a*d^4/e^6
```

mupad [B] time = 1.65, size = 287, normalized size = 1.37

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{128\,c\,d^9+144\,b\,d^7\,e^2+168\,a\,d^5\,e^4}{315\,e^{10}}+\frac{x^7\,\left(40\,c\,d^2\,e^7+45\,b\,e^9\right)}{315\,e^{10}}+\frac{x^2\,\left(64\,c\,d^7\,e^2+72\,b\,d^5\,e^4+84\,a\,d^3\,e^6\right)}{315\,e^{10}}+\frac{x^3\,\left(64\,c\,d^6\,e^3+72\,b\,d^4\,e^5+84\right.}{315\,e^{10}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((128*c*d^9 + 168*a*d^5*e^4 + 144*b*d^7*e^2)/(315*e^10) +
 (x^7*(45*b*e^9 + 40*c*d^2*e^7))/(315*e^10) + (x^2*(84*a*d^3*e^6 + 72*b*d^5
```

```
*e^4 + 64*c*d^7*e^2))/(315*e^10) + (x^3*(84*a*d^2*e^7 + 72*b*d^4*e^5 + 64*c
*d^6*e^3))/(315*e^10) + (c*x^9)/(9*e) + (x^5*(63*a*e^9 + 54*b*d^2*e^7 + 48*
c*d^4*e^5))/(315*e^10) + (x*(168*a*d^4*e^5 + 144*b*d^6*e^3 + 128*c*d^8*e))/
(315*e^10) + (x^6*(40*c*d^3*e^6 + 45*b*d*e^8))/(315*e^10) + (x^4*(54*b*d^3*
e^6 + 48*c*d^5*e^4 + 63*a*d*e^8))/(315*e^10) + (c*d*x^8)/(9*e^2)))/(d + e*x
)^(1/2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
[Out] Timed out
```

3.133 $\int \frac{x^3(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=159

$$\frac{(d-ex)^{3/2}(d+ex)^{3/2}\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8}-\frac{d^2\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^8}-\frac{(d-ex)^{5/2}(d+ex)^{5/2}\left(be\right.}{5e^8}$$

```
[Out] 1/3*(a*e^4+2*b*d^2*e^2+3*c*d^4)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^8-1/5*(b*e^2
+3*c*d^2)*(-e*x+d)^(5/2)*(e*x+d)^(5/2)/e^8+1/7*c*(-e*x+d)^(7/2)*(e*x+d)^(7/
2)/e^8-d^2*(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^8
```

Rubi [A] time = 0.19, antiderivative size = 213, normalized size of antiderivative = 1.34, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {520, 1251, 771}

$$\frac{\left(d^2-e^2x^2\right)^2\left(ae^4+2bd^2e^2+3cd^4\right)}{3e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{d^2\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)^3\left(be^2+3cd^2\right)}{5e^8\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\left(d^2-\right.}{7e^8\sqrt{d-e}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^8*Sqrt[d - e*x]*Sqrt
[d + e*x])) + ((3*c*d^4 + 2*b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2)^2)/(3*e^8*Sq
rt[d - e*x]*Sqrt[d + e*x]) - ((3*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^3)/(5*e^8*S
qrt[d - e*x]*Sqrt[d + e*x]) + (c*(d^2 - e^2*x^2)^4)/(7*e^8*Sqrt[d - e*x]*Sq
rt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^3\left(a+bx^2+cx^4\right)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\ \text{Subst}\left(\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\ \text{Subst}\left(\int \left(\frac{cd^6+bd^4e^2+ad^2e^4}{e^6\sqrt{d^2-e^2x}}+\frac{\left(-3cd^4-2bd^2e^2-ae^4\right)\sqrt{d^2-e^2x}}{e^6}+\frac{\left(3cd^2+be^2\right)\left(d^2-e^2x\right)^{3/2}}{e^6}-\right.\right.}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{d^2\left(cd^4+bd^2e^2+ae^4\right)\left(d^2-e^2x^2\right)}{e^8\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(3cd^4+2bd^2e^2+ae^4\right)\left(d^2-e^2x^2\right)^2}{3e^8\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(3cd^2+b\right.}{5e^8\sqrt{d}}$$

Mathematica [C] time = 1.07, size = 232, normalized size = 1.46

$$-\frac{\frac{210d^{5/2}\sqrt{d+ex}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)\left(ae^4+bd^2e^2+cd^4\right)}{\sqrt{\frac{ex}{d}+1}}+\sqrt{d-ex}\sqrt{d+ex}\left(35ae^4\left(2d^2+e^2x^2\right)+7b\left(8d^4e^2+4d^2e^4x^2+3e^6x^4\right)+3\right.}{105e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*a*e^4*(2*d^2 + e^2*x^2) + 7*b*(8*d^
4*e^2 + 4*d^2*e^4*x^2 + 3*e^6*x^4) + 3*c*(16*d^6 + 8*d^4*e^2*x^2 + 6*d^2*e^
4*x^4 + 5*e^6*x^6)) + (210*d^(5/2)*(c*d^4 + b*d^2*e^2 + a*e^4)*Sqrt[d + e*x
]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x)/d] - 210*d^3*(c*d
^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^8
```

fricas [A] time = 0.86, size = 104, normalized size = 0.65

$$-\frac{\left(15\,ce^6x^6+48\,cd^6+56\,bd^4e^2+70\,ad^2e^4+3\left(6\,cd^2e^4+7\,be^6\right)x^4+\left(24\,cd^4e^2+28\,bd^2e^4+35\,ae^6\right)x^2\right)\sqrt{ex+d}\,\sqrt{-}}{105\,e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/105*(15*c*e^6*x^6 + 48*c*d^6 + 56*b*d^4*e^2 + 70*a*d^2*e^4 + 3*(6*c*d^2*
e^4 + 7*b*e^6)*x^4 + (24*c*d^4*e^2 + 28*b*d^2*e^4 + 35*a*e^6)*x^2)*sqrt(e*x
 + d)*sqrt(-e*x + d)/e^8
```

giac [A] time = 0.75, size = 194, normalized size = 1.22

$$-\frac{1}{105}\left(\left(\left(3\left(\left(5\left((xe+d)ce^{(-7)}-6\,cde^{(-7)}\right)(xe+d)+\left(81\,cd^2e^{49}+7\,be^{51}\right)e^{(-56)}\right)(xe+d)-4\left(31\,cd^3e^{49}+7\,bde^{51}\right)e^{(-5}\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -1/105*(((3*((5*((x*e + d)*c*e^(-7) - 6*c*d*e^(-7))*(x*e + d) + (81*c*d^2*e
^49 + 7*b*e^51)*e^(-56))*(x*e + d) - 4*(31*c*d^3*e^49 + 7*b*d*e^51)*e^(-56)
)*(x*e + d) + 7*(51*c*d^4*e^49 + 22*b*d^2*e^51 + 5*a*e^53)*e^(-56))*(x*e +
```

```
d) - 70*(3*c*d^5*e^49 + 2*b*d^3*e^51 + a*d*e^53)*e^(-56))*(x*e + d) + 105*(
c*d^6*e^49 + b*d^4*e^51 + a*d^2*e^53)*e^(-56))*sqrt(x*e + d)*sqrt(-x*e + d)
*e^(-1)
```

maple [A] time = 0.01, size = 109, normalized size = 0.69

$$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(15c\,x^6e^6+21b\,e^6x^4+18c\,d^2e^4x^4+35a\,e^6x^2+28b\,d^2e^4x^2+24c\,d^4e^2x^2+70a\,d^2e^4+56b\right.}{105e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(15*c*e^6*x^6+21*b*e^6*x^4+18*c*d^2*e^4
*x^4+35*a*e^6*x^2+28*b*d^2*e^4*x^2+24*c*d^4*e^2*x^2+70*a*d^2*e^4+56*b*d^4*e
^2+48*c*d^6)/e^8
```

maxima [A] time = 1.06, size = 217, normalized size = 1.36

$$-\frac{\sqrt{-e^2x^2+d^2}\,cx^6}{7\,e^2}-\frac{6\,\sqrt{-e^2x^2+d^2}\,cd^2x^4}{35\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,bx^4}{5\,e^2}-\frac{8\,\sqrt{-e^2x^2+d^2}\,cd^4x^2}{35\,e^6}-\frac{4\,\sqrt{-e^2x^2+d^2}\,bd^2x^2}{15\,e^4}-\sqrt{-e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/7*sqrt(-e^2*x^2 + d^2)*c*x^6/e^2 - 6/35*sqrt(-e^2*x^2 + d^2)*c*d^2*x^4/e
^4 - 1/5*sqrt(-e^2*x^2 + d^2)*b*x^4/e^2 - 8/35*sqrt(-e^2*x^2 + d^2)*c*d^4*x
^2/e^6 - 4/15*sqrt(-e^2*x^2 + d^2)*b*d^2*x^2/e^4 - 1/3*sqrt(-e^2*x^2 + d^2)
*a*x^2/e^2 - 16/35*sqrt(-e^2*x^2 + d^2)*c*d^6/e^8 - 8/15*sqrt(-e^2*x^2 + d^
2)*b*d^4/e^6 - 2/3*sqrt(-e^2*x^2 + d^2)*a*d^2/e^4
```

mupad [B] time = 1.49, size = 215, normalized size = 1.35

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{48\,c\,d^7+56\,b\,d^5\,e^2+70\,a\,d^3\,e^4}{105\,e^8}+\frac{x^5\,\left(18\,c\,d^2\,e^5+21\,b\,e^7\right)}{105\,e^8}+\frac{c\,x^7}{7\,e}+\frac{x^3\,\left(24\,c\,d^4\,e^3+28\,b\,d^2\,e^5+35\,a\,e^7\right)}{105\,e^8}+\frac{x\,\left(48\,c\,d^6\,e+56\,b\,d^4\,e^3+70\,a\right.}{105\,e^8}\right.}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((48*c*d^7 + 70*a*d^3*e^4 + 56*b*d^5*e^2)/(105*e^8) + (x^
5*(21*b*e^7 + 18*c*d^2*e^5))/(105*e^8) + (c*x^7)/(7*e) + (x^3*(35*a*e^7 + 2
8*b*d^2*e^5 + 24*c*d^4*e^3))/(105*e^8) + (x*(70*a*d^2*e^5 + 56*b*d^4*e^3 +
48*c*d^6*e))/(105*e^8) + (x^4*(18*c*d^3*e^4 + 21*b*d*e^6))/(105*e^8) + (x^2
*(28*b*d^3*e^4 + 24*c*d^5*e^2 + 35*a*d*e^6))/(105*e^8) + (c*d*x^6)/(7*e^2))
)/(d + e*x)^(1/2)
```

sympy [C] time = 135.14, size = 367, normalized size = 2.31

$$-\frac{iad^3G_{6,6}^{6,2}\left(\begin{matrix}-\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1\\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{ad^3G_{6,6}^{2,6}\left(\begin{matrix}-2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e^4}-\frac{ib}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d**3*meijerg(((-5/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4
, -1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**4) - a*d**3*meijerg(((-2
, -7/4, -3/2, -5/4, -1, 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**4) - I*b*d**5*meijerg(((-9/
4, -7/4), (-2, -2, -3/2, 1)), ((-5/2, -9/4, -2, -7/4, -3/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**6) - b*d**5*meijerg(((-3, -11/4, -5/2, -9/4, -2
, 1), ()), ((-11/4, -9/4), (-3, -5/2, -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e
**2*x**2))/(4*pi**(3/2)*e**6) - I*c*d**7*meijerg(((-13/4, -11/4), (-3, -3,
-5/2, 1)), ((-7/2, -13/4, -3, -11/4, -5/2, 0), ()), d**2/(e**2*x**2))/(4*pi
**(3/2)*e**8) - c*d**7*meijerg(((-4, -15/4, -7/2, -13/4, -3, 1), ()), ((-15
/4, -13/4), (-4, -7/2, -7/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*p
i**(3/2)*e**8)
```

3.134 $\int \frac{x(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=109

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(ae^4+bd^2e^2+cd^4\right)}{e^6}+\frac{(d-ex)^{3/2}(d+ex)^{3/2}\left(be^2+2cd^2\right)}{3e^6}-\frac{c(d-ex)^{5/2}(d+ex)^{5/2}}{5e^6}$$

```
[Out] 1/3*(b*e^2+2*c*d^2)*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^6-1/5*c*(-e*x+d)^(5/2)*(
e*x+d)^(5/2)/e^6-(a*e^4+b*d^2*e^2+c*d^4)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6
```

Rubi [A] time = 0.12, antiderivative size = 149, normalized size of antiderivative = 1.37, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {520, 1247, 698}

$$-\frac{\left(d^2-e^2x^2\right)\left(ae^4+bd^2e^2+cd^4\right)}{e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(d^2-e^2x^2\right)^2\left(be^2+2cd^2\right)}{3e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^4 + b*d^2*e^2 + a*e^4)*(d^2 - e^2*x^2))/(e^6*Sqrt[d - e*x]*Sqrt[d +
 e*x])) + ((2*c*d^2 + b*e^2)*(d^2 - e^2*x^2)^2)/(3*e^6*Sqrt[d - e*x]*Sqrt[d
 + e*x]) - (c*(d^2 - e^2*x^2)^3)/(5*e^6*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x\left(a+bx^2+cx^4\right)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int\left(\frac{cd^4+bd^2e^2+ae^4}{e^4\sqrt{d^2-e^2x}}+\frac{\left(-2cd^2-be^2\right)\sqrt{d^2-e^2x}}{e^4}+\frac{c\left(d^2-e^2x\right)^{3/2}}{e^4}\right)dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{\left(cd^4+bd^2e^2+ae^4\right)\left(d^2-e^2x^2\right)}{e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{\left(2cd^2+be^2\right)\left(d^2-e^2x^2\right)^2}{3e^6\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)^3}{5e^6\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [C] time = 0.69, size = 194, normalized size = 1.78

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5\left(3ae^4+2bd^2e^2+be^4x^2\right)+c\left(8d^4+4d^2e^2x^2+3e^4x^4\right)\right)+\frac{30\sqrt{d}\sqrt{d+ex}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)\left(ae^4+bd^2e^2+cd^4\right)}{\sqrt{\frac{ex}{d}+1}}}{15e^6}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(5*(2*b*d^2*e^2 + 3*a*e^4 + b*e^4*x^2) +
 c*(8*d^4 + 4*d^2*e^2*x^2 + 3*e^4*x^4)) + (30*Sqrt[d]*(c*d^4 + b*d^2*e^2 +
a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[1 + (e*x
)/d] - 30*d*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]]
)/e^6
```

fricas [A] time = 1.03, size = 71, normalized size = 0.65

$$-\frac{\left(3\,ce^4x^4+8\,cd^4+10\,bd^2e^2+15\,ae^4+\left(4\,cd^2e^2+5\,be^4\right)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{15\,e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] -1/15*(3*c*e^4*x^4 + 8*c*d^4 + 10*b*d^2*e^2 + 15*a*e^4 + (4*c*d^2*e^2 + 5*b
*e^4)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/e^6
```

giac [A] time = 0.63, size = 121, normalized size = 1.11

$$-\frac{1}{15}\left(\left(\left(3\left((xe+d)ce^{(-5)}-4\,cde^{(-5)}\right)(xe+d)+\left(22\,cd^2e^{25}+5\,be^{27}\right)e^{(-30)}\right)(xe+d)-10\left(2\,cd^3e^{25}+bde^{27}\right)e^{(-30)}\right)(xe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] -1/15*(((3*((x*e + d)*c*e^(-5) - 4*c*d*e^(-5))*(x*e + d) + (22*c*d^2*e^25 +
 5*b*e^27)*e^(-30))*(x*e + d) - 10*(2*c*d^3*e^25 + b*d*e^27)*e^(-30))*(x*e
+ d) + 15*(c*d^4*e^25 + b*d^2*e^27 + a*e^29)*e^(-30))*sqrt(x*e + d)*sqrt(-x
*e + d)*e^(-1)
```

maple [A] time = 0.01, size = 73, normalized size = 0.67

$$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(3c\,x^{4}e^{4}+5b\,e^{4}x^{2}+4c\,d^{2}e^{2}x^{2}+15a\,e^{4}+10b\,d^{2}e^{2}+8c\,d^{4}\right)}{15e^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(3*c*e^4*x^4+5*b*e^4*x^2+4*c*d^2*e^2*x^2
+15*a*e^4+10*b*d^2*e^2+8*c*d^4)/e^6
```

maxima [A] time = 1.03, size = 139, normalized size = 1.28

$$-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,cx^{4}}{5\,e^{2}}-\frac{4\,\sqrt{-e^{2}x^{2}+d^{2}}\,cd^{2}x^{2}}{15\,e^{4}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,bx^{2}}{3\,e^{2}}-\frac{8\,\sqrt{-e^{2}x^{2}+d^{2}}\,cd^{4}}{15\,e^{6}}-\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,bd^{2}}{3\,e^{4}}-\frac{\sqrt{-e^{2}x^{2}}}{e^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/5*sqrt(-e^2*x^2 + d^2)*c*x^4/e^2 - 4/15*sqrt(-e^2*x^2 + d^2)*c*d^2*x^2/e
^4 - 1/3*sqrt(-e^2*x^2 + d^2)*b*x^2/e^2 - 8/15*sqrt(-e^2*x^2 + d^2)*c*d^4/e
^6 - 2/3*sqrt(-e^2*x^2 + d^2)*b*d^2/e^4 - sqrt(-e^2*x^2 + d^2)*a/e^2
```

mupad [B] time = 1.38, size = 143, normalized size = 1.31

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{8\,c\,d^{5}+10\,b\,d^{3}\,e^{2}+15\,a\,d\,e^{4}}{15\,e^{6}}+\frac{x^{3}\,\left(4\,c\,d^{2}\,e^{3}+5\,b\,e^{5}\right)}{15\,e^{6}}+\frac{c\,x^{5}}{5\,e}+\frac{x^{2}\,\left(4\,c\,d^{3}\,e^{2}+5\,b\,d\,e^{4}\right)}{15\,e^{6}}+\frac{x\,\left(8\,c\,d^{4}\,e+10\,b\,d^{2}\,e^{3}+15\,a\,e^{5}\right)}{15\,e^{6}}+\frac{c\,d\,x^{4}}{5\,e^{2}}\right)}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*((8*c*d^5 + 10*b*d^3*e^2 + 15*a*d*e^4)/(15*e^6) + (x^3*(5
*b*e^5 + 4*c*d^2*e^3))/(15*e^6) + (c*x^5)/(5*e) + (x^2*(4*c*d^3*e^2 + 5*b*d
*e^4))/(15*e^6) + (x*(15*a*e^5 + 10*b*d^2*e^3 + 8*c*d^4*e))/(15*e^6) + (c*d
*x^4)/(5*e^2)))/(d + e*x)^(1/2)
```

sympy [C] time = 90.67, size = 350, normalized size = 3.21

$$-\frac{iadG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{d^{2}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}e^{2}}-\frac{adG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{d^{2}e^{-2i\pi}}{e^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}e^{2}}-\frac{ibd^{3}G_{6,6}^{6,2}\left(\begin{matrix} \\ -\frac{3}{2},\end{matrix}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] -I*a*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0)
, ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - a*d*meijerg(((-1, -3/4, -1/2,
 -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I
*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*b*d**3*meijerg(((-5/4, -3/4), (-1,
 -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(e**2*x**2))/(4
*pi**(3/2)*e**4) - b*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()), ((-7
/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi
**(3/2)*e**4) - I*c*d**5*meijerg(((-9/4, -7/4), (-2, -2, -3/2, 1)), ((-5/2,
 -9/4, -2, -7/4, -3/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**6) - c*d*
*5*meijerg(((-3, -11/4, -5/2, -9/4, -2, 1), ()), ((-11/4, -9/4), (-3, -5/2,
 -5/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**6)
```

3.135 $\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\,\sqrt{d+ex}}\,dx$

Optimal. Leaf size=93

$$-\frac{a\tanh^{-1}\left(\frac{\sqrt{d-ex}\,\sqrt{d+ex}}{d}\right)}{d}-\frac{\sqrt{d-ex}\,\sqrt{d+ex}\,\left(be^2+cd^2\right)}{e^4}+\frac{c(d-ex)^{3/2}(d+ex)^{3/2}}{3e^4}$$

```
[Out] 1/3*c*(-e*x+d)^(3/2)*(e*x+d)^(3/2)/e^4-a*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/
2)/d)/d-(b*e^2+c*d^2)*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.62, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1251, 897, 1153, 208}

$$-\frac{a\sqrt{d^2-e^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(be^2+cd^2\right)}{e^4\sqrt{d-ex}\,\sqrt{d+ex}}+\frac{c\left(d^2-e^2x^2\right)^2}{3e^4\sqrt{d-ex}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(((c*d^2 + b*e^2)*(d^2 - e^2*x^2))/(e^4*Sqrt[d - e*x]*Sqrt[d + e*x])) + (c
*(d^2 - e^2*x^2)^2)/(3*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - (a*Sqrt[d^2 - e^2
*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(d*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \left(b+\frac{cd^2}{e^2}-\frac{cx^2}{e^2}+\frac{a}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right)dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{(cd^2+be^2)(d^2-e^2x^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(a\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\right.}{e^2\sqrt{d-ex}\sqrt{d+}}$$
$$= -\frac{(cd^2+be^2)(d^2-e^2x^2)}{e^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{c(d^2-e^2x^2)^2}{3e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [B] time = 0.88, size = 217, normalized size = 2.33

$$\frac{-\frac{3a\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d\sqrt{d-ex}} + \frac{(e^2x^2-d^2)(3be^2+2cd^2+ce^2x^2)}{e^4\sqrt{d-ex}} + \frac{6d\sqrt{d+ex}(be^2+cd^2)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^4} - \frac{6d^{3/2}\sqrt{\frac{ex}{d}+1}(be^2+cd^2)\sin^{-1}\left(\frac{\sqrt{d-}}{\sqrt{2}}\right.}{e^4}}{3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (((-d^2 + e^2*x^2)*(2*c*d^2 + 3*b*e^2 + c*e^2*x^2))/(e^4*Sqrt[d - e*x]) - (
6*d^(3/2)*(c*d^2 + b*e^2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*S
qrt[d])])/e^4 + (6*d*(c*d^2 + b*e^2)*Sqrt[d + e*x]*ArcTan[Sqrt[d - e*x]/Sqr
t[d + e*x]])/e^4 - (3*a*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])
/(d*Sqrt[d - e*x]))/(3*Sqrt[d + e*x])
```

fricas [A] time = 0.96, size = 80, normalized size = 0.86

$$\frac{3\,ae^4\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right)-\left(cde^2x^2+2\,cd^3+3\,bde^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{3\,de^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/3*(3*a*e^4*log((sqrt(e*x + d)*sqrt(-e*x + d) - d)/x) - (c*d*e^2*x^2 + 2*c
*d^3 + 3*b*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d*e^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746
43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Franci
s algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996
9.6644,20804885.8013]Bad conditionned root j= 2 value 38.9905751966 ratio 0
.000133135092941 mindist 0.00241522618125-a*ln(abs(2*sqrt(d+x*exp(1))/(2*sq
rt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*e
xp(1)+2*d))/sqrt(d+x*exp(1))+2))/d+a*ln(abs(2*sqrt(d+x*exp(1))/(2*sqrt(2)*s
qrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2
*d))/sqrt(d+x*exp(1))-2))/d+2*((-24*c*exp(1)^12/144/exp(1)^16*sqrt(d+x*exp(
1))*sqrt(d+x*exp(1))+48*c*exp(1)^12*d/144/exp(1)^16)*sqrt(d+x*exp(1))*sqrt(
d+x*exp(1))+(-72*c*exp(1)^12*d^2-72*exp(1)^14*b)/144/exp(1)^16)*sqrt(d+x*ex
p(1))*sqrt(-d-x*exp(1)+2*d)
```

maple [C] time = 0.04, size = 143, normalized size = 1.54

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(\sqrt{-e^2x^2+d^2}\,cd\,e^2x^2\mathrm{csgn}(d)+3a\,e^4\ln\left(\frac{2\left(d+\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)\right)d}{x}\right)+3\sqrt{-e^2x^2+d^2}\,bd\,e^2\mathrm{csgn}(}{3\sqrt{-e^2x^2+d^2}\,d\,e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d*(csgn(d)*x^2*c*d*e^2*(-e^2*x^2+d^2)^(1/
2)+3*csgn(d)*(-e^2*x^2+d^2)^(1/2)*b*d*e^2+2*csgn(d)*(-e^2*x^2+d^2)^(1/2)*c*
d^3+3*ln(2*d*((-e^2*x^2+d^2)^(1/2)*csgn(d)+d)/x)*a*e^4)*csgn(d)/(-e^2*x^2+d
^2)^(1/2)/e^4
```

maxima [A] time = 1.00, size = 105, normalized size = 1.13

$$-\frac{\sqrt{-e^2x^2+d^2}\,cx^2}{3\,e^2}-\frac{a\log\left(\frac{2\,d^2}{|x|}+\frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d}-\frac{2\,\sqrt{-e^2x^2+d^2}\,cd^2}{3\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,b}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/3*sqrt(-e^2*x^2 + d^2)*c*x^2/e^2 - a*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2
+ d^2)*d/abs(x))/d - 2/3*sqrt(-e^2*x^2 + d^2)*c*d^2/e^4 - sqrt(-e^2*x^2 + d
^2)*b/e^2
```

mupad [B] time = 2.95, size = 161, normalized size = 1.73

$$\frac{a\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d}-\frac{\sqrt{d-e\,x}\,\left(\frac{2\,c\,d^3}{3\,e^4}+\frac{c\,x^3}{3\,e}+\frac{c\,d\,x^2}{3\,e^2}+\frac{2\,c\,d^2\,x}{3\,e^3}\right)}{\sqrt{d+e\,x}}-\frac{\left(\frac{b\,d}{e^2}+\frac{b\,x}{e}\right)\,\sqrt{d-e\,x}}{\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] (a*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - ((d - e*
x)^(1/2)*((2*c*d^3)/(3*e^4) + (c*x^3)/(3*e) + (c*d*x^2)/(3*e^2) + (2*c*d^2*
x)/(3*e^3)))/(d + e*x)^(1/2) - (((b*d)/e^2 + (b*x)/e)*(d - e*x)^(1/2))/(d +
 e*x)^(1/2)
```

sympy [C] time = 91.28, size = 304, normalized size = 3.27

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix} \middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix} \middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibdG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle|\right.}{4\pi^{\frac{3}{2}}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
d**2/(e**2*x**2))/(4*pi**(3/2)*d) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()
), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*
pi**(3/2)*d) - I*b*d*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0
, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e**2) - b*d*meijerg(((-
1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), d**2*
exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e**2) - I*c*d**3*meijerg(((-5/
4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), d**2/(
e**2*x**2))/(4*pi**(3/2)*e**4) - c*d**3*meijerg(((-2, -7/4, -3/2, -5/4, -1,
 1), ()), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), d**2*exp_polar(-2*I*pi)/(e**
2*x**2))/(4*pi**(3/2)*e**4)
```

3.136 $\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=99

$$-\frac{\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)}{2d^3}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{2d^2x^2}-\frac{c\sqrt{d-ex}\sqrt{d+ex}}{e^2}$$

```
[Out] -1/2*(a*e^2+2*b*d^2)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/d^3-c*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/e^2-1/2*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^2
```

Rubi [A] time = 0.25, antiderivative size = 155, normalized size of antiderivative = 1.57, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {520, 1251, 897, 1157, 388, 208}

$$-\frac{\sqrt{d^2-e^2x^2}\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{c\left(d^2-e^2x^2\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((c*(d^2 - e^2*x^2))/(e^2*Sqrt[d - e*x]*Sqrt[d + e*x])) - (a*(d^2 - e^2*x^
2))/(2*d^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((2*b*d^2 + a*e^2)*Sqrt[d^2 -
 e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(2*d^3*Sqrt[d - e*x]*Sqrt[d + e*x
])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^3\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^3\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\ \mathrm{Subst}\left(\int \frac{a+bx+cx^2}{x^2\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\ \mathrm{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\ \mathrm{Subst}\left(\int \frac{-a-\frac{2(cd^4+bd^2e^2)}{e^4}+\frac{2cd^2x^2}{e^4}}{\frac{d^2}{e^2}-\frac{x^2}{e^2}}\,dx, x, \sqrt{d^2-\right.}{2d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{c\left(d^2-e^2x^2\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(e^2\left(\frac{2cd^4}{e^6}+\frac{-a-\frac{2(cd^4+bd^2e^2)}{e^4}}{e^2}\right)\sqrt{d^2-\right.}{2d^2\sqrt{}}$$

$$= -\frac{c\left(d^2-e^2x^2\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{a\left(d^2-e^2x^2\right)}{2d^2x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(2bd^2+ae^2\right)\sqrt{d^2-e^2x^2}\ \tanh^{-1}}{2d^3\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [B] time = 0.21, size = 233, normalized size = 2.35

$$\frac{-e^2x^2\sqrt{d^2-e^2x^2}\left(ae^2+2bd^2\right)\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)-ad^3e^2+ade^4x^2-4cd^{9/2}x^2\sqrt{d-ex}\sqrt{\frac{ex}{d}+1}\ \sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)}{2d^3e^2x^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^3*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(a*d^3*e^2) - 2*c*d^5*x^2 + a*d*e^4*x^2 + 2*c*d^3*e^2*x^4 - 4*c*d^(9/2)*x
^2*Sqrt[d - e*x]*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])]
+ 4*c*d^4*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x
]] - e^2*(2*b*d^2 + a*e^2)*x^2*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x
^2]/d])/(2*d^3*e^2*x^2*Sqrt[d - e*x]*Sqrt[d + e*x])
```

fricas [A] time = 0.80, size = 98, normalized size = 0.99

$$-\frac{2\,cd^4x^2 - \left(2\,bd^2e^2 + ae^4\right)x^2\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right) + \left(2\,cd^3x^2 + ade^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{2\,d^3e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*c*d^4*x^2 - (2*b*d^2*e^2 + a*e^4)*x^2*log((sqrt(e*x + d)*sqrt(-e*x
+ d) - d)/x) + (2*c*d^3*x^2 + a*d*e^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^3*e
^2*x^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746
43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Franci
s algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996
9.6644,20804885.8013]Bad conditionned root j= 2 value 38.9905751966 ratio 0
.000133135092941 mindist 0.002415226181251/exp(1)*(-(2*a*(2*sqrt(d+x*exp(1)
)/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt
(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^3+8*a*(2*sqrt(d+x*exp(1))/(2*
sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x
*exp(1)+2*d))/sqrt(d+x*exp(1)))*exp(1)^3)/d^3/((2*sqrt(d+x*exp(1))/(2*sqrt(
2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(
1)+2*d))/sqrt(d+x*exp(1)))^2-4)^2-1/2*(a*exp(1)^3+2*b*d^2*exp(1))*ln(abs(2*
sqrt(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)
*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))+2))/d^3+1/2*(a*exp(1)^3+
2*b*d^2*exp(1))*ln(abs(2*sqrt(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*ex
p(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))
-2))/d^3-2*c*exp(1)/2/exp(1)^2*sqrt(d+x*exp(1))*sqrt(-d-x*exp(1)+2*d))
```

maple [C] time = 0.02, size = 163, normalized size = 1.65

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(a\,e^4x^2\ln\left(\frac{2\left(d+\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)\right)d}{x}\right) + 2b\,d^2e^2x^2\ln\left(\frac{2\left(d+\sqrt{-e^2x^2+d^2}\,\operatorname{csgn}(d)\right)d}{x}\right) + 2\sqrt{-e^2x^2+d^2}\,c\,c}{2\sqrt{-e^2x^2+d^2}\,d^3e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^3*(2*csgn(d)*x^2*c*d^3*(-e^2*x^2+d^2)^(
1/2)+ln(2*(d+(-e^2*x^2+d^2)^(1/2)*csgn(d))*d/x)*x^2*a*e^4+2*ln(2*(d+(-e^2*x
^2+d^2)^(1/2)*csgn(d))*d/x)*x^2*b*d^2*e^2+csgn(d)*a*d*e^2*(-e^2*x^2+d^2)^(1
/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/e^2/x^2
```

maxima [A] time = 1.02, size = 123, normalized size = 1.24

$$-\frac{b\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d} - \frac{ae^2\log\left(\frac{2\,d^2}{|x|} + \frac{2\,\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{2\,d^3} - \frac{\sqrt{-e^2x^2+d^2}\,c}{e^2} - \frac{\sqrt{-e^2x^2+d^2}\,a}{2\,d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -b*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*a*e^2*log(2*
d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - sqrt(-e^2*x^2 + d^2)*c/
e^2 - 1/2*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^2)
```

mupad [B] time = 5.15, size = 422, normalized size = 4.26

$$\frac{b\left(\ln\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-1\right)-\ln\left(\frac{\sqrt{d+e\,x}-\sqrt{d}}{\sqrt{d-e\,x}-\sqrt{d}}\right)\right)}{d}-\frac{\left(\frac{c\,d}{e^2}+\frac{c\,x}{e}\right)\sqrt{d-e\,x}}{\sqrt{d+e\,x}}-\frac{\frac{a\,e^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{a\,e^2}{2}+\frac{15\,a\,e^2\left(\sqrt{d+e\,x}-\cdots\right.}{2\left(\sqrt{d-e\,x}-\cdots\right.}}{\frac{16\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}-\frac{32\,d^3\left(\sqrt{d+e\,x}-\sqrt{d}\right)^4}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^4}+\frac{16\,d^3\left(\cdots\right.}{\left(\sqrt{\cdots}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^3*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] (b*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1) -
log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (((c*d)/
e^2 + (c*x)/e)*(d - e*x)^(1/2))/(d + e*x)^(1/2) - ((a*e^2*((d + e*x)^(1/2)
- d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (a*e^2)/2 + (15*a*e^2*((d + e
*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^(1/2))^4))/((16*d^3*((d + e
*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2 - (32*d^3*((d + e*x)^
(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1/2))^4 + (16*d^3*((d + e*x)^(1/2
) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))^6) - (a*e^2*log(((d + e*x)^(1/2
) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (a*e^2*log(((d + e*x)^
(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(2*d^3) + (a*e^2*((d
 + e*x)^(1/2) - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)
```

sympy [C] time = 133.79, size = 270, normalized size = 2.73

$$\frac{iae^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{ae^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**3/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0
,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**3) - a*e**2*meijerg(((1, 5/4, 3/2, 7
/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), d**2*exp_polar(-2*I*pi)/(e*
*2*x**2))/(4*pi**(3/2)*d**3) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((
1/2, 3/4, 1, 5/4, 3/2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d) - b*meijer
g(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), d**2*exp_
polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d) - I*c*d*meijerg(((-1/4, 1/4), (
0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), d**2/(e**2*x**2))/(4*pi
**(3/2)*e**2) - c*d*meijerg(((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/
4), (-1, -1/2, -1/2, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)
*e**2)
```

3.137 $\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=126

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d-ex}\sqrt{d+ex}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{8d^5}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(3ae^2+4bd^2\right)}{8d^4x^2}-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{4d^2x^4}$$

```
[Out] -1/8*(3*a*e^4+4*b*d^2*e^2+8*c*d^4)*arctanh((-e*x+d)^(1/2)*(e*x+d)^(1/2)/d)/
d^5-1/4*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^4-1/8*(3*a*e^2+4*b*d^2)*(-e*x+
d)^(1/2)*(e*x+d)^(1/2)/d^4/x^2
```

Rubi [A] time = 0.28, antiderivative size = 182, normalized size of antiderivative = 1.44, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {520, 1251, 897, 1157, 385, 208}

$$-\frac{\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(3ae^4+4bd^2e^2+8cd^4\right)}{8d^5\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(3ae^2+4bd^2\right)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(4*d^2*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((4*b*d^2 +
3*a*e^2)*(d^2 - e^2*x^2))/(8*d^4*x^2*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*c*d
^4 + 4*b*d^2*e^2 + 3*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]
/d])/(8*d^5*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^5\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^5\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^3\sqrt{d^2-e^2x}}\,dx, x, x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx, x, \sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-3a-\frac{4(cd^4+bd^2e^2)}{e^4}+\frac{4cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^2}\,dx, x, \sqrt{d^2-\right.}{4d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{(4bd^2+3ae^2)(d^2-e^2x^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(4b+\frac{8cd^2}{e^2}+\frac{3ae^2}{d^2}\right)\sqrt{d^2-\right.}{8d}$$

$$= -\frac{a(d^2-e^2x^2)}{4d^2x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{(4bd^2+3ae^2)(d^2-e^2x^2)}{8d^4x^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{(8cd^4+4bd^2e^2+3ae^4)\sqrt{}}{8d^5\sqrt{d-}}$$

Mathematica [A] time = 0.16, size = 134, normalized size = 1.06

$$\frac{-\left(x^4\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)(3ae^4+4bd^2e^2+8cd^4)\right)-d(d^2-e^2x^2)(2ad^2+3ae^2x^2+4bd^2x^2)}{8d^5x^4\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^5*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(d*(d^2 - e^2*x^2)*(2*a*d^2 + 4*b*d^2*x^2 + 3*a*e^2*x^2)) - (8*c*d^4 + 4*
b*d^2*e^2 + 3*a*e^4)*x^4*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d]
)/(8*d^5*x^4*Sqrt[d - e*x]*Sqrt[d + e*x])
```

fricas [A] time = 0.80, size = 102, normalized size = 0.81

$$\frac{\left(8\,cd^4+4\,bd^2e^2+3\,ae^4\right)x^4\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right)-\left(2\,ad^3+\left(4\,bd^3+3\,ade^2\right)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{8\,d^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/8*((8*c*d^4 + 4*b*d^2*e^2 + 3*a*e^4)*x^4*log((sqrt(e*x + d)*sqrt(-e*x + d
) - d)/x) - (2*a*d^3 + (4*b*d^3 + 3*a*d*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x +
 d))/(d^5*x^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746
43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Franci
s algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996
9.6644,20804885.8013]Bad conditionned root j= 2 value 38.9905751966 ratio 0
.000133135092941 mindist 0.002415226181251/exp(1)*(1/2*(-5*a*(2*sqrt(d+x*ex
p(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*
sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^7*exp(1)^5-4*b*d^2*(2*sqrt(d+x*exp
(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*s
qrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^7*exp(1)^3-12*a*(2*sqrt(d+x*exp(1))
/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(
-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5*exp(1)^5+16*b*d^2*(2*sqrt(d+x*exp(1))
/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(
-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5*exp(1)^3-48*a*(2*sqrt(d+x*exp(1))/(2*
sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x
*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^5+64*b*d^2*(2*sqrt(d+x*exp(1))/(2*
sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x
*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^3-320*a*(2*sqrt(d+x*exp(1))/(2*sqr
t(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*ex
p(1)+2*d))/sqrt(d+x*exp(1)))*exp(1)^5-256*b*d^2*(2*sqrt(d+x*exp(1))/(2*sqrt
(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp
(1)+2*d))/sqrt(d+x*exp(1)))*exp(1)^3)/d^5/((2*sqrt(d+x*exp(1))/(2*sqrt(2)*s
qrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2
*d))/sqrt(d+x*exp(1)))^2-4)^4-1/8*(3*a*exp(1)^5+4*b*d^2*exp(1)^3+8*c*d^4*ex
p(1))*ln(abs(2*sqrt(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))
-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))+2))/d^5+1
/8*(3*a*exp(1)^5+4*b*d^2*exp(1)^3+8*c*d^4*exp(1))*ln(abs(2*sqrt(d+x*exp(1))
/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(
-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))-2))/d^5)
```

maple [C] time = 0.03, size = 222, normalized size = 1.76

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(3ae^4x^4\ln\left(\frac{2\left(d+\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)\right)d}{x}\right)+4bd^2e^2x^4\ln\left(\frac{2\left(d+\sqrt{-e^2x^2+d^2}\,\mathrm{csgn}(d)\right)d}{x}\right)+8cd^4x^4\ln\left(\frac{2\left(d+\cdots\right.}{}\right.\right.}{8\sqrt{-\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^5*(3*ln(2*(d+(-e^2*x^2+d^2)^(1/2)*csgn(
d))*d/x)*x^4*a*e^4+4*ln(2*(d+(-e^2*x^2+d^2)^(1/2)*csgn(d))*d/x)*x^4*b*d^2*e
^2+8*ln(2*(d+(-e^2*x^2+d^2)^(1/2)*csgn(d))*d/x)*x^4*c*d^4+3*csgn(d)*x^2*a*d
*e^2*(-e^2*x^2+d^2)^(1/2)+4*csgn(d)*x^2*b*d^3*(-e^2*x^2+d^2)^(1/2)+2*csgn(d
)*a*d^3*(-e^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^4
```

maxima [A] time = 1.03, size = 193, normalized size = 1.53

$$-\frac{c\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{d}-\frac{be^2\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{2d^3}-\frac{3ae^4\log\left(\frac{2d^2}{|x|}+\frac{2\sqrt{-e^2x^2+d^2}\,d}{|x|}\right)}{8d^5}-\frac{\sqrt{-e^2x^2+d^2}\,b}{2d^2x^2}-\frac{3\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -c*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d - 1/2*b*e^2*log(2*
d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*a*e^4*log(2*d^2/abs
(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 1/2*sqrt(-e^2*x^2 + d^2)*b/(d^
2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^2) - 1/4*sqrt(-e^2*x^2 + d^2
)*a/(d^2*x^4)
```

mupad [B] time = 10.82, size = 932, normalized size = 7.40

$$\frac{\frac{ae^4}{4}+\frac{6ae^4\left(\sqrt{d+ex}-\sqrt{d}\right)^2}{\left(\sqrt{d-ex}-\sqrt{d}\right)^2}-\frac{53ae^4\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{2\left(\sqrt{d-ex}-\sqrt{d}\right)^4}-\frac{87ae^4\left(\sqrt{d+ex}-\sqrt{d}\right)^6}{\left(\sqrt{d-ex}-\sqrt{d}\right)^6}+\frac{657ae^4\left(\sqrt{d+ex}-\sqrt{d}\right)^8}{4\left(\sqrt{d-ex}-\sqrt{d}\right)^8}-\frac{121ae^4\left(\sqrt{d+ex}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-ex}-\sqrt{d}\right)^{10}}}{\frac{256d^5\left(\sqrt{d+ex}-\sqrt{d}\right)^4}{\left(\sqrt{d-ex}-\sqrt{d}\right)^4}-\frac{1024d^5\left(\sqrt{d+ex}-\sqrt{d}\right)^6}{\left(\sqrt{d-ex}-\sqrt{d}\right)^6}+\frac{1536d^5\left(\sqrt{d+ex}-\sqrt{d}\right)^8}{\left(\sqrt{d-ex}-\sqrt{d}\right)^8}-\frac{1024d^5\left(\sqrt{d+ex}-\sqrt{d}\right)^{10}}{\left(\sqrt{d-ex}-\sqrt{d}\right)^{10}}+\frac{256d^5\left(\sqrt{d+ex}-\sqrt{d}\right)^{12}}{\left(\sqrt{d-ex}-\sqrt{d}\right)^{12}}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^5*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((a*e^4)/4 + (6*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(
1/2))^2 - (53*a*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^
(1/2))^4) - (87*a*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(
1/2))^6 + (657*a*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2) - d
^(1/2))^8) - (121*a*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) -
d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^
(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1
/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2
))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2)
)^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))
^12) - ((b*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2
 - (b*e^2)/2 + (15*b*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2)
 - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1
/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))
^6) + (c*(log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 -
 1) - log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2)))))/d - (3
*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(8*d^5
) - (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2
*d^3) + (3*a*e^4*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/
2))^2 - 1))/(8*d^5) + (b*e^2*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(
1/2) - d^(1/2))^2 - 1))/(2*d^3) + (7*a*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/(
256*d^5*((d - e*x)^(1/2) - d^(1/2))^2) + (a*e^4*((d + e*x)^(1/2) - d^(1/2))
```

```
^4)/(1024*d^5*((d - e*x)^(1/2) - d^(1/2))^4) + (b*e^2*((d + e*x)^(1/2) - d^
(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**5/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.138 $\int \frac{a+bx^2+cx^4}{x^7 \sqrt{d-ex} \sqrt{d+ex}} dx$

Optimal. Leaf size=212

$$-\frac{e^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^6x^2}-\frac{\sqrt{d-ex}}{}$$

```
[Out] -1/6*a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x^6-1/24*(5*a*e^2+6*b*d^2)*(-e*x+d)
^(1/2)*(e*x+d)^(1/2)/d^4/x^4-1/16*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*(-e*x+d)^(1
/2)*(e*x+d)^(1/2)/d^6/x^2-1/16*e^2*(5*a*e^4+6*b*d^2*e^2+8*c*d^4)*arctanh((-
e^2*x^2+d^2)^(1/2)/d)*(-e^2*x^2+d^2)^(1/2)/d^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 248, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {520, 1251, 897, 1157, 385, 199, 208}

$$-\frac{\left(d^2-e^2x^2\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^6x^2\sqrt{d-ex}\sqrt{d+ex}}-\frac{e^2\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(5ae^4+6bd^2e^2+8cd^4\right)}{16d^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(5\right.}{24d^4x^4\sqrt{d-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(6*d^2*x^6*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((6*b*d^2 +
5*a*e^2)*(d^2 - e^2*x^2))/(24*d^4*x^4*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((8*c*
d^4 + 6*b*d^2*e^2 + 5*a*e^4)*(d^2 - e^2*x^2))/(16*d^6*x^2*Sqrt[d - e*x]*Sqr
t[d + e*x]) - (e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a*e^4)*Sqrt[d^2 - e^2*x^2]*Ar
cTanh[Sqrt[d^2 - e^2*x^2]/d])/(16*d^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
```

```
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^7\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^7\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{a+bx+cx^2}{x^4\sqrt{d^2-e^2x}}\,dx,x,x^2\right)}{2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{\frac{cd^4+bd^2e^2+ae^4}{e^4}-\frac{(2cd^2+be^2)x^2}{e^4}+\frac{cx^4}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^4}\,dx,x,\sqrt{d^2-e^2x^2}\right)}{e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} + \frac{\sqrt{d^2-e^2x^2}\,\text{Subst}\left(\int \frac{-5a-\frac{6(cd^4+bd^2e^2)}{e^4}+\frac{6cd^2x^2}{e^4}}{\left(\frac{d^2}{e^2}-\frac{x^2}{e^2}\right)^3}\,dx,x,\sqrt{d^2-\right.}{6d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(6bd^2+5ae^2\right)\left(d^2-e^2x^2\right)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(6b+\frac{8cd^2}{e^2}+\frac{5ae^2}{d^2}\right)\sqrt{d^2-\right.}{8}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(6bd^2+5ae^2\right)\left(d^2-e^2x^2\right)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(8cd^4+6bd^2e^2+5ae^4\right)\left(d\right.}{16d^6x^2\sqrt{d-ex}\sqrt{d-}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{6d^2x^6\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(6bd^2+5ae^2\right)\left(d^2-e^2x^2\right)}{24d^4x^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(8cd^4+6bd^2e^2+5ae^4\right)\left(d\right.}{16d^6x^2\sqrt{d-ex}\sqrt{d-}}$$

Mathematica [A] time = 0.19, size = 173, normalized size = 0.82

$$\frac{-3e^2x^6\sqrt{d^2-e^2x^2}\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)\left(5ae^4+6bd^2e^2+8cd^4\right)-d\left(d^2-e^2x^2\right)\left(a\left(8d^4+10d^2e^2x^2+15e^4x^4\right)+6\right.}{48d^7x^6\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^7*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] (-(d*(d^2 - e^2*x^2)*(6*(2*b*d^4*x^2 + 4*c*d^4*x^4 + 3*b*d^2*e^2*x^4) + a*(
8*d^4 + 10*d^2*e^2*x^2 + 15*e^4*x^4))) - 3*e^2*(8*c*d^4 + 6*b*d^2*e^2 + 5*a
*e^4)*x^6*Sqrt[d^2 - e^2*x^2]*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/(48*d^7*x^6*S
qrt[d - e*x]*Sqrt[d + e*x])
```

fricas [A] time = 0.81, size = 137, normalized size = 0.65

$$\frac{3\left(8\,cd^4e^2+6\,bd^2e^4+5\,ae^6\right)x^6\log\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{x}\right)-\left(8\,ad^5+3\left(8\,cd^5+6\,bd^3e^2+5\,ade^4\right)x^4+2\left(6\,bd^5+5\,a\right.\right.}{48\,d^7x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(8*c*d^4*e^2 + 6*b*d^2*e^4 + 5*a*e^6)*x^6*log((sqrt(e*x + d)*sqrt(-
e*x + d) - d)/x) - (8*a*d^5 + 3*(8*c*d^5 + 6*b*d^3*e^2 + 5*a*d*e^4)*x^4 + 2
*(6*b*d^5 + 5*a*d^3*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^7*x^6)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: schur
 row 1 1.55494e-10Francis algorithm not precise enough for[1.0,-220.8624746
43,10162.5484803,-174574.213802,1032773.91614]schur row 1 3.66198e-10Franci
s algorithm not precise enough for[1.0,-467.909596927,45612.3731035,-165996
9.6644,20804885.8013]Bad conditionned root j= 2 value 38.9905751966 ratio 0
.000133135092941 mindist 0.002415226181251/exp(1)*(-1/12*(33*a*(2*sqrt(d+x*
exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-
2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^11*exp(1)^7+30*b*d^2*(2*sqrt(d+x
*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)
-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^11*exp(1)^5+24*c*d^4*(2*sqrt(d+
x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d
)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^11*exp(1)^3+20*a*(2*sqrt(d+x*e
xp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2
*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^9*exp(1)^7-168*b*d^2*(2*sqrt(d+x*
exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-
2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^9*exp(1)^5-288*c*d^4*(2*sqrt(d+x
*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)
-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^9*exp(1)^3+1440*a*(2*sqrt(d+x*e
xp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2
*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^7*exp(1)^7+192*b*d^2*(2*sqrt(d+x*
exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-
2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^7*exp(1)^5+768*c*d^4*(2*sqrt(d+x
*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)
-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^7*exp(1)^3+5760*a*(2*sqrt(d+x*e
xp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2
*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5*exp(1)^7+768*b*d^2*(2*sqrt(d+x*
exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-
2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5*exp(1)^5+3072*c*d^4*(2*sqrt(d+
x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d
)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^5*exp(1)^3+1280*a*(2*sqrt(d+x*
exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-
2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^7-10752*b*d^2*(2*sqrt(d
+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(
d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^5-18432*c*d^4*(2*sqr
t(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sq
rt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^3*exp(1)^3+33792*a*(2*sqrt
(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqr
t(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))*exp(1)^7+30720*b*d^2*(2*sqr
t(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sq
rt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))*exp(1)^5+24576*c*d^4*(2*sq
rt(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*s
qrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))*exp(1)^3)/d^7/((2*sqrt(d+
x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d
)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1)))^2-4)^6-1/16*(5*a*exp(1)^7+6*b*
d^2*exp(1)^5+8*c*d^4*exp(1)^3)*ln(abs(2*sqrt(d+x*exp(1))/(2*sqrt(2)*sqrt(d)
-2*sqrt(-d-x*exp(1)+2*d))-1/2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/s
qrt(d+x*exp(1))+2))/d^7+1/16*(5*a*exp(1)^7+6*b*d^2*exp(1)^5+8*c*d^4*exp(1)^
3)*ln(abs(2*sqrt(d+x*exp(1))/(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))-1/
2*(2*sqrt(2)*sqrt(d)-2*sqrt(-d-x*exp(1)+2*d))/sqrt(d+x*exp(1))-2))/d^7)
```

maple [C] time = 0.04, size = 306, normalized size = 1.44

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(15a\,e^{6}x^{6}\ln\left(\frac{2\left(d+\sqrt{-e^{2}x^{2}+d^{2}}\,\mathrm{csgn}(d)\right)d}{x}\right)+18b\,d^{2}e^{4}x^{6}\ln\left(\frac{2\left(d+\sqrt{-e^{2}x^{2}+d^{2}}\,\mathrm{csgn}(d)\right)d}{x}\right)+24c\,d^{4}e^{2}x^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^7*(15*ln(2*(d+(-e^2*x^2+d^2)^(1/2)*csg
n(d))*d/x)*x^6*a*e^6+18*ln(2*(d+(-e^2*x^2+d^2)^(1/2)*csgn(d))*d/x)*x^6*b*d^
2*e^4+24*ln(2*(d+(-e^2*x^2+d^2)^(1/2)*csgn(d))*d/x)*x^6*c*d^4*e^2+15*(-e^2*
x^2+d^2)^(1/2)*csgn(d)*d*x^4*a*e^4+18*(-e^2*x^2+d^2)^(1/2)*csgn(d)*d^3*x^4*
b*e^2+24*(-e^2*x^2+d^2)^(1/2)*csgn(d)*d^5*x^4*c+10*csgn(d)*x^2*a*d^3*e^2*(-
e^2*x^2+d^2)^(1/2)+12*csgn(d)*x^2*b*d^5*(-e^2*x^2+d^2)^(1/2)+8*csgn(d)*a*d^
5*(-e^2*x^2+d^2)^(1/2))*csgn(d)/(-e^2*x^2+d^2)^(1/2)/x^6
```

maxima [A] time = 1.03, size = 271, normalized size = 1.28

$$-\frac{ce^{2}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{2\,d^{3}}-\frac{3\,be^{4}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{8\,d^{5}}-\frac{5\,ae^{6}\log\left(\frac{2\,d^{2}}{|x|}+\frac{2\,\sqrt{-e^{2}x^{2}+d^{2}}\,d}{|x|}\right)}{16\,d^{7}}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,c}{2\,d^{2}x^{2}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/2*c*e^2*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^3 - 3/8*b*
e^4*log(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^5 - 5/16*a*e^6*lo
g(2*d^2/abs(x) + 2*sqrt(-e^2*x^2 + d^2)*d/abs(x))/d^7 - 1/2*sqrt(-e^2*x^2 +
 d^2)*c/(d^2*x^2) - 3/8*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^2) - 5/16*sqrt(-e
^2*x^2 + d^2)*a*e^4/(d^6*x^2) - 1/4*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^4) - 5/24
*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^4) - 1/6*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^6
)
```

mupad [B] time = 20.05, size = 1621, normalized size = 7.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^7*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((b*e^4)/4 + (6*b*e^4*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(
1/2))^2 - (53*b*e^4*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2) - d^
(1/2))^4) - (87*b*e^4*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(
1/2))^6 + (657*b*e^4*((d + e*x)^(1/2) - d^(1/2))^8)/(4*((d - e*x)^(1/2) - d
^(1/2))^8) - (121*b*e^4*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) -
d^(1/2))^10)/((256*d^5*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^
(1/2))^4 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1
/2))^6 + (1536*d^5*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2) - d^(1/2
))^8 - (1024*d^5*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/2) - d^(1/2)
)^10 + (256*d^5*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(1/2) - d^(1/2))
^12) - ((c*e^2*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) - d^(1/2))^2
 - (c*e^2)/2 + (15*c*e^2*((d + e*x)^(1/2) - d^(1/2))^4)/(2*((d - e*x)^(1/2)
 - d^(1/2))^4))/((16*d^3*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2) -
d^(1/2))^2 - (32*d^3*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) - d^(1
/2))^4 + (16*d^3*((d + e*x)^(1/2) - d^(1/2))^6)/((d - e*x)^(1/2) - d^(1/2))
^6) + ((a*e^6)/6 + (4*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/((d - e*x)^(1/2)
```

```
 - d^(1/2))^2 + (71*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/((d - e*x)^(1/2) -
 d^(1/2))^4 - (1558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/(3*((d - e*x)^(1/2
) - d^(1/2))^6) - (540*a*e^6*((d + e*x)^(1/2) - d^(1/2))^8)/((d - e*x)^(1/2
) - d^(1/2))^8 + (4248*a*e^6*((d + e*x)^(1/2) - d^(1/2))^10)/((d - e*x)^(1/
2) - d^(1/2))^10 - (7683*a*e^6*((d + e*x)^(1/2) - d^(1/2))^12)/((d - e*x)^(
1/2) - d^(1/2))^12 + (5558*a*e^6*((d + e*x)^(1/2) - d^(1/2))^14)/((d - e*x)
^(1/2) - d^(1/2))^14 - (3643*a*e^6*((d + e*x)^(1/2) - d^(1/2))^16)/(2*((d -
 e*x)^(1/2) - d^(1/2))^16))/((4096*d^7*((d + e*x)^(1/2) - d^(1/2))^6)/((d -
 e*x)^(1/2) - d^(1/2))^6 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^8)/((d -
e*x)^(1/2) - d^(1/2))^8 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^10)/((d -
e*x)^(1/2) - d^(1/2))^10 - (81920*d^7*((d + e*x)^(1/2) - d^(1/2))^12)/((d -
 e*x)^(1/2) - d^(1/2))^12 + (61440*d^7*((d + e*x)^(1/2) - d^(1/2))^14)/((d
- e*x)^(1/2) - d^(1/2))^14 - (24576*d^7*((d + e*x)^(1/2) - d^(1/2))^16)/((d
 - e*x)^(1/2) - d^(1/2))^16 + (4096*d^7*((d + e*x)^(1/2) - d^(1/2))^18)/((d
 - e*x)^(1/2) - d^(1/2))^18) - (5*a*e^6*log(((d + e*x)^(1/2) - d^(1/2))/((d
 - e*x)^(1/2) - d^(1/2))))/(16*d^7) - (3*b*e^4*log(((d + e*x)^(1/2) - d^(1/
2))/((d - e*x)^(1/2) - d^(1/2))))/(8*d^5) - (c*e^2*log(((d + e*x)^(1/2) - d
^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*d^3) + (5*a*e^6*log(((d + e*x)^(1/
2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(16*d^7) + (3*b*e^4*log
(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 - 1))/(8*d^5)
+ (c*e^2*log(((d + e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 -
1))/(2*d^3) + (197*a*e^6*((d + e*x)^(1/2) - d^(1/2))^2)/(8192*d^7*((d - e*x
)^(1/2) - d^(1/2))^2) + (5*a*e^6*((d + e*x)^(1/2) - d^(1/2))^4)/(4096*d^7*(
(d - e*x)^(1/2) - d^(1/2))^4) + (a*e^6*((d + e*x)^(1/2) - d^(1/2))^6)/(2457
6*d^7*((d - e*x)^(1/2) - d^(1/2))^6) + (7*b*e^4*((d + e*x)^(1/2) - d^(1/2))
^2)/(256*d^5*((d - e*x)^(1/2) - d^(1/2))^2) + (b*e^4*((d + e*x)^(1/2) - d^(
1/2))^4)/(1024*d^5*((d - e*x)^(1/2) - d^(1/2))^4) + (c*e^2*((d + e*x)^(1/2)
 - d^(1/2))^2)/(32*d^3*((d - e*x)^(1/2) - d^(1/2))^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**7/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
[Out] Timed out
```

3.139 $\int \frac{x^2(a+bx^2+cx^4)}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=216

$$\frac{d^2\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+6bd^2e^2+5cd^4)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{x\sqrt{d-ex}\sqrt{d+ex}\,(8ae^4+6bd^2e^2+5cd^4)}{16e^6}-\frac{x^3\sqrt{d-ex}}{}$$

```
[Out] 1/6*c*x^5*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/16*(8*a*e^4+6*b*d^2*e^
2+5*c*d^4)*x*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^6-1/24*(6*b*e^2+5*c*d^2)*x^3*(-
e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4+1/16*d^2*(8*a*e^4+6*b*d^2*e^2+5*c*d^4)*arcta
n(e*x/(-e^2*x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e^7/(-e*x+d)^(1/2)/(e*x+d)
^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 245, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {520, 1267, 459, 321, 217, 203}

$$-\frac{x(d^2-e^2x^2)(8ae^4+6bd^2e^2+5cd^4)}{16e^6\sqrt{d-ex}\sqrt{d+ex}}+\frac{d^2\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)(8ae^4+6bd^2e^2+5cd^4)}{16e^7\sqrt{d-ex}\sqrt{d+ex}}-\frac{x^3(d^2-e^2x^2)}{24e^4\sqrt{d-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*x*(d^2 - e^2*x^2))/(16*e^6*Sqrt[d - e*x
]*Sqrt[d + e*x]) - ((5*c*d^2 + 6*b*e^2)*x^3*(d^2 - e^2*x^2))/(24*e^4*Sqrt[d
 - e*x]*Sqrt[d + e*x]) - (c*x^5*(d^2 - e^2*x^2))/(6*e^2*Sqrt[d - e*x]*Sqrt[
d + e*x]) + (d^2*(5*c*d^4 + 6*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcT
an[(e*x)/Sqrt[d^2 - e^2*x^2]])/(16*e^7*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+bx^2+cx^4\right)}{\sqrt{d-ex}\,\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{x^2\left(a+bx^2+cx^4\right)}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{cx^5\left(d^2-e^2x^2\right)}{6e^2\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{x^2\left(-6ae^2-\left(5cd^2+6be^2\right)x^2\right)}{\sqrt{d^2-e^2x^2}}\,dx}{6e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

$$= -\frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{cx^5\left(d^2-e^2x^2\right)}{6e^2\sqrt{d-ex}\,\sqrt{d+ex}} + \frac{\left(\left(5cd^4+6bd^2e^2+8ae^4\right)\sqrt{d^2\right.}{8e^4\sqrt{d-ex}\,\sqrt{}}$$

$$= -\frac{\left(5cd^4+6bd^2e^2+8ae^4\right)x\left(d^2-e^2x^2\right)}{16e^6\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{cx^5\left(d^2-e\right.}{6e^2\sqrt{d-ex}\,\sqrt{}}$$

$$= -\frac{\left(5cd^4+6bd^2e^2+8ae^4\right)x\left(d^2-e^2x^2\right)}{16e^6\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{cx^5\left(d^2-e\right.}{6e^2\sqrt{d-ex}\,\sqrt{}}$$

$$= -\frac{\left(5cd^4+6bd^2e^2+8ae^4\right)x\left(d^2-e^2x^2\right)}{16e^6\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{\left(5cd^2+6be^2\right)x^3\left(d^2-e^2x^2\right)}{24e^4\sqrt{d-ex}\,\sqrt{d+ex}} - \frac{cx^5\left(d^2-e\right.}{6e^2\sqrt{d-ex}\,\sqrt{}}$$

Mathematica [A] time = 0.79, size = 202, normalized size = 0.94

$$-\frac{ex\sqrt{d-ex}\,\sqrt{d+ex}\left(6\left(4ae^4+3bd^2e^2+2be^4x^2\right)+c\left(15d^4+10d^2e^2x^2+8e^4x^4\right)\right)+96d^2\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)\left(ae^4+bd\right.}{48e^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/48*(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(6*(3*b*d^2*e^2 + 4*a*e^4 + 2*b*e^4*
x^2) + c*(15*d^4 + 10*d^2*e^2*x^2 + 8*e^4*x^4)) - (6*d^(3/2)*(11*c*d^4 + 10
*b*d^2*e^2 + 8*a*e^4)*Sqrt[d + e*x]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])]
)/Sqrt[1 + (e*x)/d] + 96*d^2*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d - e*
x]/Sqrt[d + e*x]])/e^7
```

fricas [A] time = 0.85, size = 134, normalized size = 0.62

$$-\frac{\left(8\,ce^5x^5+2\left(5\,cd^2e^3+6\,be^5\right)x^3+3\left(5\,cd^4e+6\,bd^2e^3+8\,ae^5\right)x\right)\sqrt{ex+d}\,\sqrt{-ex+d}+6\left(5\,cd^6+6\,bd^4e^2+8\,a\right.}{48\,e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/48*((8*c*e^5*x^5 + 2*(5*c*d^2*e^3 + 6*b*e^5)*x^3 + 3*(5*c*d^4*e + 6*b*d^
2*e^3 + 8*a*e^5)*x)*sqrt(e*x + d)*sqrt(-e*x + d) + 6*(5*c*d^6 + 6*b*d^4*e^2
 + 8*a*d^2*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)))/e^7
```

giac [A] time = 0.75, size = 208, normalized size = 0.96

$$\frac{1}{48}\left(6\left(5\,cd^6+6\,bd^4e^2+8\,ad^2e^4\right)\arcsin\left(\frac{\sqrt{2}\,\sqrt{xe+d}}{2\,\sqrt{d}}\right)e^{(-6)}-\left(\left(2\left(\left(4\left((xe+d)ce^{(-6)}-5\,cde^{(-6)}\right)(xe+d)+3\left(1\right.\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/48*(6*(5*c*d^6 + 6*b*d^4*e^2 + 8*a*d^2*e^4)*arcsin(1/2*sqrt(2)*sqrt(x*e +
 d)/sqrt(d))*e^(-6) - ((2*((4*((x*e + d)*c*e^(-6) - 5*c*d*e^(-6))*(x*e + d)
 + 3*(15*c*d^2*e^36 + 2*b*e^38)*e^(-42))*(x*e + d) - (55*c*d^3*e^36 + 18*b*
d*e^38)*e^(-42))*(x*e + d) + (85*c*d^4*e^36 + 54*b*d^2*e^38 + 24*a*e^40)*e^
(-42))*(x*e + d) - 3*(11*c*d^5*e^36 + 10*b*d^3*e^38 + 8*a*d*e^40)*e^(-42))*
sqrt(x*e + d)*sqrt(-x*e + d))*e^(-1)
```

maple [C] time = 0.04, size = 273, normalized size = 1.26

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(8\sqrt{-e^2x^2+d^2}\,c\,e^5x^5\mathrm{csgn}(e)+12\sqrt{-e^2x^2+d^2}\,b\,e^5x^3\mathrm{csgn}(e)+10\sqrt{-e^2x^2+d^2}\,c\,d^2e^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/48*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(8*csgn(e)*x^5*c*e^5*(-e^2*x^2+d^2)^(1/2
)+12*csgn(e)*x^3*b*e^5*(-e^2*x^2+d^2)^(1/2)+10*csgn(e)*x^3*c*d^2*e^3*(-e^2*
x^2+d^2)^(1/2)+24*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^5*x*a+18*(-e^2*x^2+d^2)^(1
/2)*csgn(e)*e^3*x*b*d^2+15*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e*x*c*d^4-24*arctan
(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*a*d^2*e^4-18*arctan(csgn(e)*e*x/(-e^2*x^
2+d^2)^(1/2))*b*d^4*e^2-15*arctan(csgn(e)*e*x/(-e^2*x^2+d^2)^(1/2))*c*d^6)*
csgn(e)/e^7/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 1.11, size = 190, normalized size = 0.88

$$-\frac{\sqrt{-e^2x^2+d^2}\,cx^5}{6\,e^2}-\frac{5\,\sqrt{-e^2x^2+d^2}\,cd^2x^3}{24\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,bx^3}{4\,e^2}+\frac{5\,cd^6\arcsin\left(\frac{ex}{d}\right)}{16\,e^7}+\frac{3\,bd^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^5}+\frac{ad^2\arcsin\left(\right.}{2\,e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -1/6*sqrt(-e^2*x^2 + d^2)*c*x^5/e^2 - 5/24*sqrt(-e^2*x^2 + d^2)*c*d^2*x^3/e
^4 - 1/4*sqrt(-e^2*x^2 + d^2)*b*x^3/e^2 + 5/16*c*d^6*arcsin(e*x/d)/e^7 + 3/
```

```
8*b*d^4*arcsin(e*x/d)/e^5 + 1/2*a*d^2*arcsin(e*x/d)/e^3 - 5/16*sqrt(-e^2*x^
2 + d^2)*c*d^4*x/e^6 - 3/8*sqrt(-e^2*x^2 + d^2)*b*d^2*x/e^4 - 1/2*sqrt(-e^2
*x^2 + d^2)*a*x/e^2
```

mupad [B] time = 23.12, size = 1132, normalized size = 5.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*a*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
a*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*a*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - ((175*c*d^6*(
(d + e*x)^(1/2) - d^(1/2))^3)/(12*((d - e*x)^(1/2) - d^(1/2))^3) + (311*c*d
^6*((d + e*x)^(1/2) - d^(1/2))^5)/(4*((d - e*x)^(1/2) - d^(1/2))^5) - (8361
*c*d^6*((d + e*x)^(1/2) - d^(1/2))^7)/(4*((d - e*x)^(1/2) - d^(1/2))^7) + (
42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^9)/(6*((d - e*x)^(1/2) - d^(1/2))^9
) - (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1/2) - d^(1
/2))^11) + (25295*c*d^6*((d + e*x)^(1/2) - d^(1/2))^13)/(2*((d - e*x)^(1/2)
 - d^(1/2))^13) - (42259*c*d^6*((d + e*x)^(1/2) - d^(1/2))^15)/(6*((d - e*x
)^(1/2) - d^(1/2))^15) + (8361*c*d^6*((d + e*x)^(1/2) - d^(1/2))^17)/(4*((d
 - e*x)^(1/2) - d^(1/2))^17) - (311*c*d^6*((d + e*x)^(1/2) - d^(1/2))^19)/(
4*((d - e*x)^(1/2) - d^(1/2))^19) - (175*c*d^6*((d + e*x)^(1/2) - d^(1/2))^
21)/(12*((d - e*x)^(1/2) - d^(1/2))^21) - (5*c*d^6*((d + e*x)^(1/2) - d^(1/
2))^23)/(4*((d - e*x)^(1/2) - d^(1/2))^23) + (5*c*d^6*((d + e*x)^(1/2) - d^
(1/2)))/(4*((d - e*x)^(1/2) - d^(1/2))))/(e^7*(((d + e*x)^(1/2) - d^(1/2))^
2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^12) - ((23*b*d^4*((d + e*x)^(1/2) - d^
(1/2))^3)/(2*((d - e*x)^(1/2) - d^(1/2))^3) - (333*b*d^4*((d + e*x)^(1/2) -
 d^(1/2))^5)/(2*((d - e*x)^(1/2) - d^(1/2))^5) + (671*b*d^4*((d + e*x)^(1/2
) - d^(1/2))^7)/(2*((d - e*x)^(1/2) - d^(1/2))^7) - (671*b*d^4*((d + e*x)^(
1/2) - d^(1/2))^9)/(2*((d - e*x)^(1/2) - d^(1/2))^9) + (333*b*d^4*((d + e*x
)^(1/2) - d^(1/2))^11)/(2*((d - e*x)^(1/2) - d^(1/2))^11) - (23*b*d^4*((d +
 e*x)^(1/2) - d^(1/2))^13)/(2*((d - e*x)^(1/2) - d^(1/2))^13) - (3*b*d^4*((
d + e*x)^(1/2) - d^(1/2))^15)/(2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*b*d^4
*((d + e*x)^(1/2) - d^(1/2)))/(2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d +
e*x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*a*d^2*at
an(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*b*d^4
*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5) + (
5*c*d^6*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^(1/2))))/(4*e
^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.140 $\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=128

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{4e^5}-\frac{x\sqrt{d-ex}\,\sqrt{d+ex}\,\left(4be^2+3cd^2\right)}{8e^4}+\frac{cx^3(ex-d)\sqrt{d+ex}}{4e^2\sqrt{d-ex}}$$

```
[Out] -1/4*(8*a*e^4+4*b*d^2*e^2+3*c*d^4)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^5
+1/4*c*x^3*(e*x-d)*(e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-1/8*(4*b*e^2+3*c*d^2)*x
*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/e^4
```

Rubi [A] time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {520, 1159, 388, 217, 203}

$$\frac{\sqrt{d^2-e^2x^2}\,\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)\left(8ae^4+4bd^2e^2+3cd^4\right)}{8e^5\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{x\left(d^2-e^2x^2\right)\left(4be^2+3cd^2\right)}{8e^4\sqrt{d-ex}\,\sqrt{d+ex}}-\frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\,\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((3*c*d^2 + 4*b*e^2)*x*(d^2 - e^2*x^2))/(8*e^4*Sqrt[d - e*x]*Sqrt[d + e*x]
) - (c*x^3*(d^2 - e^2*x^2))/(4*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((3*c*d^4
 + 4*b*d^2*e^2 + 8*a*e^4)*Sqrt[d^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x
^2]])/(8*e^5*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1159

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> Simp[(c^p*x^(4*p - 1)*(d + e*x^2)^(q + 1))/(e*(4*p + 2*q + 1))
```

```
, x] + Dist[1/(e*(4*p + 2*q + 1)), Int[(d + e*x^2)^q*ExpandToSum[e*(4*p + 2
*q + 1)*(a + b*x^2 + c*x^4)^p - d*c^p*(4*p - 1)*x^(4*p - 2) - e*c^p*(4*p +
2*q + 1)*x^(4*p), x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && !LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-4ae^2-\left(3cd^2+4be^2\right)x^2}{\sqrt{d^2-e^2x^2}}\,dx}{4e^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{\left(3cd^2+4be^2\right)x\left(d^2-e^2x^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(-8ae^4+d^2\left(-3cd^2-4be^2\right)\right)\sqrt{\ldots}\right.}{8e^4\sqrt{d-ex}\sqrt{\ldots}}$$

$$= -\frac{\left(3cd^2+4be^2\right)x\left(d^2-e^2x^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(\left(-8ae^4+d^2\left(-3cd^2-4be^2\right)\right)\sqrt{\ldots}\right.}{8e^4\sqrt{\ldots}}$$

$$= -\frac{\left(3cd^2+4be^2\right)x\left(d^2-e^2x^2\right)}{8e^4\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx^3\left(d^2-e^2x^2\right)}{4e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(3cd^4+4bd^2e^2+8ae^4\right)\sqrt{d^2-e\ldots}}{8e^5\sqrt{d-ex}\sqrt{d\ldots}}$$

Mathematica [A] time = 0.56, size = 157, normalized size = 1.23

$$-\frac{16\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)\left(ae^4+bd^2e^2+cd^4\right)+ex\sqrt{d-ex}\sqrt{d+ex}\left(4be^2+3cd^2+2ce^2x^2\right)-\frac{2d^{5/2}\sqrt{\frac{ex}{d}+1}\left(4be^2+5cd^2\right)\sin^{-1}\left(\frac{\sqrt{\ldots}}{\sqrt{\ldots}}\right)}{\sqrt{d+ex}}}{8e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/8*(e*x*Sqrt[d - e*x]*Sqrt[d + e*x]*(3*c*d^2 + 4*b*e^2 + 2*c*e^2*x^2) - (
2*d^(5/2)*(5*c*d^2 + 4*b*e^2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[
2]*Sqrt[d])])/Sqrt[d + e*x] + 16*(c*d^4 + b*d^2*e^2 + a*e^4)*ArcTan[Sqrt[d
- e*x]/Sqrt[d + e*x]])/e^5
```

fricas [A] time = 0.95, size = 100, normalized size = 0.78

$$-\frac{\left(2\,ce^3x^3+\left(3\,cd^2e+4\,be^3\right)x\right)\sqrt{ex+d}\,\sqrt{-ex+d}\,+2\left(3\,cd^4+4\,bd^2e^2+8\,ae^4\right)\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{ex}\right)}{8\,e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/8*((2*c*e^3*x^3 + (3*c*d^2*e + 4*b*e^3)*x)*sqrt(e*x + d)*sqrt(-e*x + d)
+ 2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*arctan((sqrt(e*x + d)*sqrt(-e*x + d)
- d)/(e*x)))/e^5
```

giac [A] time = 0.59, size = 135, normalized size = 1.05

$$\frac{1}{8}\left(2\left(3\,cd^4+4\,bd^2e^2+8\,ae^4\right)\arcsin\left(\frac{\sqrt{2}\,\sqrt{xe+d}}{2\,\sqrt{d}}\right)e^{(-4)}-\left(\left(2\left((xe+d)ce^{(-4)}-3\,cde^{(-4)}\right)(xe+d)+\left(9\,cd^2e^{16}+4\,\ldots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(3*c*d^4 + 4*b*d^2*e^2 + 8*a*e^4)*arcsin(1/2*sqrt(2)*sqrt(x*e + d)/s
qrt(d))*e^(-4) - ((2*((x*e + d)*c*e^(-4) - 3*c*d*e^(-4))*(x*e + d) + (9*c*d
^2*e^16 + 4*b*e^18)*e^(-20))*(x*e + d) - (5*c*d^3*e^16 + 4*b*d*e^18)*e^(-20
))*sqrt(x*e + d)*sqrt(-x*e + d))*e^(-1)
```

maple [C] time = 0.02, size = 191, normalized size = 1.49

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(2\sqrt{-e^2x^2+d^2}\,c\,e^3x^3\mathrm{csgn}(e)-8a\,e^4\arctan\left(\frac{ex\,\mathrm{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right)-4b\,d^2e^2\arctan\left(\frac{ex\,\mathrm{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right)+</math>}{8\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/8*(-e*x+d)^(1/2)*(e*x+d)^(1/2)*(2*csgn(e)*x^3*c*e^3*(-e^2*x^2+d^2)^(1/2)
+4*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*x*b+3*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e*x*
c*d^2-8*arctan(1/(-e^2*x^2+d^2)^(1/2)*e*x*csgn(e))*a*e^4-4*arctan(1/(-e^2*x
^2+d^2)^(1/2)*e*x*csgn(e))*b*d^2*e^2-3*arctan(1/(-e^2*x^2+d^2)^(1/2)*e*x*cs
gn(e))*c*d^4)*csgn(e)/e^5/(-e^2*x^2+d^2)^(1/2)
```

maxima [A] time = 1.02, size = 113, normalized size = 0.88

$$-\frac{\sqrt{-e^2x^2+d^2}\,cx^3}{4\,e^2}+\frac{3\,cd^4\arcsin\left(\frac{ex}{d}\right)}{8\,e^5}+\frac{bd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{a\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{3\sqrt{-e^2x^2+d^2}\,cd^2x}{8\,e^4}-\frac{\sqrt{-e^2x^2+d^2}\,bx}{2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/4*sqrt(-e^2*x^2 + d^2)*c*x^3/e^2 + 3/8*c*d^4*arcsin(e*x/d)/e^5 + 1/2*b*d
^2*arcsin(e*x/d)/e^3 + a*arcsin(e*x/d)/e - 3/8*sqrt(-e^2*x^2 + d^2)*c*d^2*x
/e^4 - 1/2*sqrt(-e^2*x^2 + d^2)*b*x/e^2
```

mupad [B] time = 12.86, size = 651, normalized size = 5.09

$$\frac{\frac{14\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,b\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4}-\frac{4\,a\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}-\frac{23\,c</math>}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/((d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*b*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
b*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*b*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*a*atan((e*
((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(e
^2)^(1/2) - ((23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^3)/(2*((d - e*x)^(1/2) -
 d^(1/2))^3) - (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^5)/(2*((d - e*x)^(1/2
) - d^(1/2))^5) + (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^7)/(2*((d - e*x)^(
1/2) - d^(1/2))^7) - (671*c*d^4*((d + e*x)^(1/2) - d^(1/2))^9)/(2*((d - e*x
)^(1/2) - d^(1/2))^9) + (333*c*d^4*((d + e*x)^(1/2) - d^(1/2))^11)/(2*((d -
```

```
 e*x)^(1/2) - d^(1/2))^11) - (23*c*d^4*((d + e*x)^(1/2) - d^(1/2))^13)/(2*(
(d - e*x)^(1/2) - d^(1/2))^13) - (3*c*d^4*((d + e*x)^(1/2) - d^(1/2))^15)/(
2*((d - e*x)^(1/2) - d^(1/2))^15) + (3*c*d^4*((d + e*x)^(1/2) - d^(1/2)))/(
2*((d - e*x)^(1/2) - d^(1/2))))/(e^5*(((d + e*x)^(1/2) - d^(1/2))^2/((d - e
*x)^(1/2) - d^(1/2))^2 + 1)^8) + (2*b*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/
((d - e*x)^(1/2) - d^(1/2))))/e^3 + (3*c*d^4*atan(((d + e*x)^(1/2) - d^(1/2
))/((d - e*x)^(1/2) - d^(1/2))))/(2*e^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.141 $\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=102

$$-\frac{a\sqrt{d-ex}\sqrt{d+ex}}{d^2x}-\frac{\left(2be^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e^3}+\frac{cx(ex-d)\sqrt{d+ex}}{2e^2\sqrt{d-ex}}$$

```
[Out] -(2*b*e^2+c*d^2)*arctan((-e*x+d)^(1/2)/(e*x+d)^(1/2))/e^3+1/2*c*x*(e*x-d)*(
e*x+d)^(1/2)/e^2/(-e*x+d)^(1/2)-a*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2/x
```

Rubi [A] time = 0.12, antiderivative size = 155, normalized size of antiderivative = 1.52, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 388, 217, 203}

$$-\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}}+\frac{\sqrt{d^2-e^2x^2}\left(2be^2+cd^2\right)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -((a*(d^2 - e^2*x^2))/(d^2*x*Sqrt[d - e*x]*Sqrt[d + e*x])) - (c*x*(d^2 - e^
2*x^2))/(2*e^2*Sqrt[d - e*x]*Sqrt[d + e*x]) + ((c*d^2 + 2*b*e^2)*Sqrt[d^2 -
 e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2*e^3*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
```

```
Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^2\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^2\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-bd^2-cd^2x^2}{\sqrt{d^2-e^2x^2}}\,dx}{d^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}}{2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(2b+\frac{cd^2}{e^2}\right)\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{}{1}\right.}{2\sqrt{d-ex}\sqrt{d+e}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{d^2x\sqrt{d-ex}\sqrt{d+ex}} - \frac{cx\left(d^2-e^2x^2\right)}{2e^2\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(cd^2+2be^2\right)\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{}{\sqrt{d^2}}\right.}{2e^3\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] time = 0.56, size = 135, normalized size = 1.32

$$-\frac{\frac{e\sqrt{d-ex}\sqrt{d+ex}\left(2ae^2+cd^2x^2\right)}{d^2x}+4\left(be^2+cd^2\right)\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)-\frac{2cd^{5/2}\sqrt{\frac{ex}{d}+1}\sin^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{2}\sqrt{d}}\right)}{\sqrt{d+ex}}}{2e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/2*((e*Sqrt[d - e*x]*Sqrt[d + e*x]*(2*a*e^2 + c*d^2*x^2))/(d^2*x) - (2*c*
d^(5/2)*Sqrt[1 + (e*x)/d]*ArcSin[Sqrt[d - e*x]/(Sqrt[2]*Sqrt[d])])/Sqrt[d +
 e*x] + 4*(c*d^2 + b*e^2)*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e^3
```

fricas [A] time = 0.96, size = 90, normalized size = 0.88

$$-\frac{2\left(cd^4+2\,bd^2e^2\right)x\arctan\left(\frac{\sqrt{ex+d}\sqrt{-ex+d}-d}{ex}\right)+\left(cd^2ex^2+2\,ae^3\right)\sqrt{ex+d}\sqrt{-ex+d}}{2\,d^2e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/2*(2*(c*d^4 + 2*b*d^2*e^2)*x*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(
e*x)) + (c*d^2*e*x^2 + 2*a*e^3)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^2*e^3*x)
```

giac [B] time = 1.05, size = 257, normalized size = 2.52

$$\frac{1}{2}\left(\left(\pi+2\arctan\left(\frac{\sqrt{xe+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)^2}{xe+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)}\right)\right)\left(cd^2+2\,be^2\right)e^{(-2)}-\left((xe+d)ce^{(-2)}-cde^{(-2)}\right)\sqrt{xe+d}\sqrt{-xe+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/2*((pi + 2*arctan(1/2*sqrt(x*e + d)*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))^2
/(x*e + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))))*(c*d^2 + 2*b*e^2)*e^(-
2) - ((x*e + d)*c*e^(-2) - c*d*e^(-2))*sqrt(x*e + d)*sqrt(-x*e + d) - 8*a*(
(sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*s
qrt(d) - sqrt(-x*e + d)))*e^2/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*
e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)*d^2))*e^(
-1)
```

maple [C] time = 0.02, size = 148, normalized size = 1.45

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(-2b\,d^2e^2x\arctan\left(\frac{ex\,\mathrm{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right)-c\,d^4x\arctan\left(\frac{ex\,\mathrm{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right)+\sqrt{-e^2x^2+d^2}\,c\,d^2e\,x^2\mathrm{csgn}(e)\right.}{2\sqrt{-e^2x^2+d^2}\,d^2e^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/2*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^2*(csgn(e)*x^2*c*d^2*e*(-e^2*x^2+d^2)^(
1/2)-2*arctan(1/(-e^2*x^2+d^2)^(1/2)*e*x*csgn(e))*x*b*d^2*e^2-arctan(1/(-e^
2*x^2+d^2)^(1/2)*e*x*csgn(e))*x*c*d^4+2*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*a)
*csgn(e)/e^3/(-e^2*x^2+d^2)^(1/2)/x
```

maxima [A] time = 1.05, size = 73, normalized size = 0.72

$$\frac{cd^2\arcsin\left(\frac{ex}{d}\right)}{2\,e^3}+\frac{b\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{\sqrt{-e^2x^2+d^2}\,cx}{2\,e^2}-\frac{\sqrt{-e^2x^2+d^2}\,a}{d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] 1/2*c*d^2*arcsin(e*x/d)/e^3 + b*arcsin(e*x/d)/e - 1/2*sqrt(-e^2*x^2 + d^2)*
c*x/e^2 - sqrt(-e^2*x^2 + d^2)*a/(d^2*x)
```

mupad [B] time = 7.00, size = 306, normalized size = 3.00

$$\frac{\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^3}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^3}-\frac{14\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^5}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^5}+\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)^7}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^7}-\frac{2\,c\,d^2\left(\sqrt{d+e\,x}-\sqrt{d}\right)}{\sqrt{d-e\,x}-\sqrt{d}}}{e^3\left(\frac{\left(\sqrt{d+e\,x}-\sqrt{d}\right)^2}{\left(\sqrt{d-e\,x}-\sqrt{d}\right)^2}+1\right)^4}-\frac{4\,b\,\mathrm{atan}\left(\frac{e\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}+\frac{2\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] ((14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^3)/((d - e*x)^(1/2) - d^(1/2))^3 - (
14*c*d^2*((d + e*x)^(1/2) - d^(1/2))^5)/((d - e*x)^(1/2) - d^(1/2))^5 + (2*
c*d^2*((d + e*x)^(1/2) - d^(1/2))^7)/((d - e*x)^(1/2) - d^(1/2))^7 - (2*c*d
^2*((d + e*x)^(1/2) - d^(1/2)))/((d - e*x)^(1/2) - d^(1/2)))/(e^3*(((d + e*
x)^(1/2) - d^(1/2))^2/((d - e*x)^(1/2) - d^(1/2))^2 + 1)^4) - (4*b*atan((e*
((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) - d^(1/2)))))/(e
^2)^(1/2) + (2*c*d^2*atan(((d + e*x)^(1/2) - d^(1/2))/((d - e*x)^(1/2) - d^
(1/2))))/e^3 - ((a/d + (a*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*x)^(1/2))
```

sympy [C] time = 104.02, size = 287, normalized size = 2.81

$$\frac{iaeG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2 \\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0 \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{aeG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0 \end{matrix}\middle| \frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + a*e*meijerg(((1/2, 3/4, 1, 5/4, 3/
2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**
2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1
/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2))/(4*pi**(3/2)*e) + b*meijerg(((-
1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), d**2*exp_p
olar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e) - I*c*d**2*meijerg(((-3/4, -1/4)
, (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), d**2/(e**2*x**2)
)/(4*pi**(3/2)*e**3) + c*d**2*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ())
, ((-5/4, -3/4), (-3/2, -1, -1, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(
4*pi**(3/2)*e**3)
```

3.142 $\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=157

$$-\frac{\left(d^2-e^2x^2\right)\left(2ae^2+3bd^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/3*a*(-e^2*x^2+d^2)/d^2/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/3*(2*a*e^2+3*b
*d^2)*(-e^2*x^2+d^2)/d^4/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)+c*arctan(e*x/(-e^2*
x^2+d^2)^(1/2))*(-e^2*x^2+d^2)^(1/2)/e/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 451, 217, 203}

$$-\frac{\left(d^2-e^2x^2\right)\left(2ae^2+3bd^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}}-\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}}+\frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(3*d^2*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((3*b*d^2 +
2*a*e^2)*(d^2 - e^2*x^2))/(3*d^4*x*Sqrt[d - e*x]*Sqrt[d + e*x]) + (c*Sqrt[d
^2 - e^2*x^2]*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(e*Sqrt[d - e*x]*Sqrt[d +
e*x])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 451

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[d/e^n, Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c
, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n*(p + 1) + 1, 0] && (
IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && Gt
Q[m + n, -1]))
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^4\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-3bd^2-2ae^2-3cd^2x^2}{x^2\sqrt{d^2-e^2x^2}}\,dx}{3d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(3bd^2+2ae^2\right)\left(d^2-e^2x^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\int \frac{1}{\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(3bd^2+2ae^2\right)\left(d^2-e^2x^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(c\sqrt{d^2-e^2x^2}\right)\text{Subst}\left(\int \frac{1}{1+e^{\cdots}}\right.}{\sqrt{d-ex}\sqrt{d+\cdots}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{3d^2x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(3bd^2+2ae^2\right)\left(d^2-e^2x^2\right)}{3d^4x\sqrt{d-ex}\sqrt{d+ex}} + \frac{c\sqrt{d^2-e^2x^2}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right.}{e\sqrt{d-ex}\sqrt{d+ex}}$$

Mathematica [A] time = 0.12, size = 81, normalized size = 0.52

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(a\left(d^2+2e^2x^2\right)+3bd^2x^2\right)}{3d^4x^3} - \frac{2c\tan^{-1}\left(\frac{\sqrt{d-ex}}{\sqrt{d+ex}}\right)}{e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/3*(Sqrt[d - e*x]*Sqrt[d + e*x]*(3*b*d^2*x^2 + a*(d^2 + 2*e^2*x^2)))/(d^4
*x^3) - (2*c*ArcTan[Sqrt[d - e*x]/Sqrt[d + e*x]])/e
```

fricas [A] time = 0.83, size = 90, normalized size = 0.57

$$-\frac{6\,cd^4x^3\arctan\left(\frac{\sqrt{ex+d}\,\sqrt{-ex+d}-d}{ex}\right)+\left(ad^2e+\left(3\,bd^2e+2\,ae^3\right)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{3\,d^4ex^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/3*(6*c*d^4*x^3*arctan((sqrt(e*x + d)*sqrt(-e*x + d) - d)/(e*x)) + (a*d^2
*e + (3*b*d^2*e + 2*a*e^3)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d))/(d^4*e*x^3)
```

giac [B] time = 1.47, size = 555, normalized size = 3.54

$$\frac{1}{3}\left(3\left(\pi+2\arctan\left(\frac{\sqrt{xe+d}\left(\frac{\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)^2}{xe+d}-1\right)}{2\left(\sqrt{2}\sqrt{d}-\sqrt{-xe+d}\right)}\right)\right)c-\frac{4\left(3\,bd^2\left(\frac{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\sqrt{d}-\sqrt{-xe+d}}\right)^5e^2+3\,a\left(\frac{\sqrt{2}\sqrt{d}\cdots}{\sqrt{\cdots}}\right.\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(pi + 2*arctan(1/2*sqrt(x*e + d)*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
^2/(x*e + d) - 1)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))))*c - 4*(3*b*d^2*((sqr
t(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(
d) - sqrt(-x*e + d)))^5*e^2 + 3*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(
x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 24*b*d
^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^3*e^2 - 8*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d
))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4
+ 48*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^2 + 48*a*((sqrt(2)*sqrt(d) - sqrt(-
x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))
*e^4)/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(
sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^3*d^4))*e^(-1)
```

maple [C] time = 0.03, size = 146, normalized size = 0.93

$$-\frac{\sqrt{-ex+d}\,\sqrt{ex+d}\,\left(-3c\,d^4x^3\arctan\left(\frac{ex\,\mathrm{csgn}(e)}{\sqrt{-e^2x^2+d^2}}\right)+2\sqrt{-e^2x^2+d^2}\,a\,e^3x^2\mathrm{csgn}(e)+3\sqrt{-e^2x^2+d^2}\,b\,d^2e\,x^2\mathrm{csg}\cdots\right.}{3\sqrt{-e^2x^2+d^2}\,d^4e\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/3*(-e*x+d)^(1/2)*(e*x+d)^(1/2)/d^4*(-3*arctan(1/(-e^2*x^2+d^2)^(1/2)*e*x
*csgn(e))*x^3*c*d^4+2*(-e^2*x^2+d^2)^(1/2)*csgn(e)*e^3*x^2*a+3*(-e^2*x^2+d^
2)^(1/2)*csgn(e)*e*x^2*b*d^2+a*(-e^2*x^2+d^2)^(1/2)*d^2*csgn(e)*e)*csgn(e)/
(-e^2*x^2+d^2)^(1/2)/x^3/e
```

maxima [A] time = 1.00, size = 85, normalized size = 0.54

$$\frac{c\arcsin\left(\frac{ex}{d}\right)}{e}-\frac{\sqrt{-e^2x^2+d^2}\,b}{d^2x}-\frac{2\sqrt{-e^2x^2+d^2}\,ae^2}{3\,d^4x}-\frac{\sqrt{-e^2x^2+d^2}\,a}{3\,d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] c*arcsin(e*x/d)/e - sqrt(-e^2*x^2 + d^2)*b/(d^2*x) - 2/3*sqrt(-e^2*x^2 + d^
2)*a*e^2/(d^4*x) - 1/3*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^3)
```

mupad [B] time = 2.27, size = 138, normalized size = 0.88

$$-\frac{4\,c\,\mathrm{atan}\left(\frac{e\,\left(\sqrt{d-e\,x}-\sqrt{d}\right)}{\sqrt{e^2}\,\left(\sqrt{d+e\,x}-\sqrt{d}\right)}\right)}{\sqrt{e^2}}-\frac{\left(\frac{b}{d}+\frac{b\,e\,x}{d^2}\right)\,\sqrt{d-e\,x}}{x\,\sqrt{d+e\,x}}-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{3\,d}+\frac{2\,a\,e^2\,x^2}{3\,d^3}+\frac{2\,a\,e^3\,x^3}{3\,d^4}+\frac{a\,e\,x}{3\,d^2}\right)}{x^3\,\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] - (4*c*atan((e*((d - e*x)^(1/2) - d^(1/2)))/((e^2)^(1/2)*((d + e*x)^(1/2) -
 d^(1/2)))))/(e^2)^(1/2) - ((b/d + (b*e*x)/d^2)*(d - e*x)^(1/2))/(x*(d + e*
x)^(1/2)) - ((d - e*x)^(1/2)*(a/(3*d) + (2*a*e^2*x^2)/(3*d^3) + (2*a*e^3*x^
3)/(3*d^4) + (a*e*x)/(3*d^2)))/(x^3*(d + e*x)^(1/2))
```

sympy [C] time = 116.43, size = 257, normalized size = 1.64

$$\frac{iae^3G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ae^3G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{d^2e^{-2i\pi}}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{ibeG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{d^2}{e^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] I*a*e**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3),
(0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**4) + a*e**3*meijerg(((3/2, 7/4, 2,
 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), d**2*exp_polar(-2*I*pi)/(
e**2*x**2))/(4*pi**(3/2)*d**4) + I*b*e*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2
)), ((1, 5/4, 3/2, 7/4, 2), (0,)), d**2/(e**2*x**2))/(4*pi**(3/2)*d**2) + b
*e*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)),
d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((1/4
, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), d**2/(e**2*x**2)
)/(4*pi**(3/2)*e) + c*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1
/4), (-1/2, 0, 0, 0)), d**2*exp_polar(-2*I*pi)/(e**2*x**2))/(4*pi**(3/2)*e)
```

3.143 $\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=160

$$-\frac{(d^2-e^2x^2)(8ae^4+10bd^2e^2+15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(4ae^2+5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/5*a*(-e^2*x^2+d^2)/d^2/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(4*a*e^2+5*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/15*(8*a*e^4+10
*b*d^2*e^2+15*c*d^4)*(-e^2*x^2+d^2)/d^6/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {520, 1265, 453, 264}

$$-\frac{(d^2-e^2x^2)(8ae^4+10bd^2e^2+15cd^4)}{15d^6x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(4ae^2+5bd^2)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{a(d^2-e^2x^2)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(5*d^2*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((5*b*d^2 +
4*a*e^2)*(d^2 - e^2*x^2))/(15*d^4*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((15*c
*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*(d^2 - e^2*x^2))/(15*d^6*x*Sqrt[d - e*x]*Sqr
t[d + e*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
```

```
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^6\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-5bd^2-4ae^2-5cd^2x^2}{x^4\sqrt{d^2-e^2x^2}}\,dx}{5d^2\sqrt{d-ex}\sqrt{d+ex}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(5bd^2+4ae^2\right)\left(d^2-e^2x^2\right)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(15cd^4-2e^2\left(-5bd^2-4ae^2\right.\right.\right.}{15d^4\sqrt{d-\cdots}}$$
$$= -\frac{a\left(d^2-e^2x^2\right)}{5d^2x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(5bd^2+4ae^2\right)\left(d^2-e^2x^2\right)}{15d^4x^3\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(15cd^4+10bd^2e^2+8ae^4\right)\left(d\cdots\right.}{15d^6x\sqrt{d-ex}\sqrt{d+e\cdots}}$$

Mathematica [A] time = 0.12, size = 87, normalized size = 0.54

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(a\left(3d^4+4d^2e^2x^2+8e^4x^4\right)+5bd^2x^2\left(d^2+2e^2x^2\right)+15cd^4x^4\right)}{15d^6x^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/15*(Sqrt[d - e*x]*Sqrt[d + e*x]*(15*c*d^4*x^4 + 5*b*d^2*x^2*(d^2 + 2*e^2
*x^2) + a*(3*d^4 + 4*d^2*e^2*x^2 + 8*e^4*x^4)))/(d^6*x^5)
```

fricas [A] time = 0.86, size = 76, normalized size = 0.48

$$-\frac{\left(3\,ad^4+\left(15\,cd^4+10\,bd^2e^2+8\,ae^4\right)x^4+\left(5\,bd^4+4\,ad^2e^2\right)x^2\right)\sqrt{ex+d}\,\sqrt{-ex+d}}{15\,d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/15*(3*a*d^4 + (15*c*d^4 + 10*b*d^2*e^2 + 8*a*e^4)*x^4 + (5*b*d^4 + 4*a*d
^2*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x + d)/(d^6*x^5)
```

giac [B] time = 2.55, size = 1103, normalized size = 6.89

$$-\frac{4\left(15\,cd^4\left(\frac{\sqrt{2}\,\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\,\sqrt{d}-\sqrt{-xe+d}}\right)^9 e^2+15\,bd^2\left(\frac{\sqrt{2}\,\sqrt{d}-\sqrt{-xe+d}}{\sqrt{xe+d}}-\frac{\sqrt{xe+d}}{\sqrt{2}\,\sqrt{d}-\sqrt{-xe+d}}\right)^9 e^4-240\,cd^4\left(\frac{\sqrt{2}\,\sqrt{d}-\sqrt{-xe+\cdots}}{\sqrt{xe+d}}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/15*(15*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*
e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^2 + 15*b*d^2*((sqrt(2)*sqrt(
d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(
```

```
-x*e + d)))^9*e^4 - 240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e
+ d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^2 + 15*a*((sqr
t(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(
d) - sqrt(-x*e + d)))^9*e^6 - 160*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 +
1440*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^2 - 80*a*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))^7*e^6 + 800*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sq
rt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^4 - 3840*c*d^4*((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^3*e^2 + 928*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x
*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 2560*b*
d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt
(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^4 + 3840*c*d^4*((sqrt(2)*sqrt(d) - sqrt(
-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))
)*e^2 - 1280*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e
 + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 3840*b*d^2*((sqrt(2)*sqrt
(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt
(-x*e + d)))*e^4 + 3840*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d)
 - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^6)*e^(-1)/((((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^2 - 4)^5*d^6)
```

maple [A] time = 0.00, size = 82, normalized size = 0.51

$$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(8a\,e^4x^4+10b\,d^2e^2x^4+15c\,d^4x^4+4a\,d^2e^2x^2+5b\,d^4x^2+3a\,d^4\right)}{15d^6x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/15*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(8*a*e^4*x^4+10*b*d^2*e^2*x^4+15*c*d^4*x
^4+4*a*d^2*e^2*x^2+5*b*d^4*x^2+3*a*d^4)/x^5/d^6
```

maxima [A] time = 1.02, size = 148, normalized size = 0.92

$$-\frac{\sqrt{-e^2x^2+d^2}\,c}{d^2x}-\frac{2\sqrt{-e^2x^2+d^2}\,be^2}{3\,d^4x}-\frac{8\sqrt{-e^2x^2+d^2}\,ae^4}{15\,d^6x}-\frac{\sqrt{-e^2x^2+d^2}\,b}{3\,d^2x^3}-\frac{4\sqrt{-e^2x^2+d^2}\,ae^2}{15\,d^4x^3}-\frac{\sqrt{-e^2x^2+d^2}\,a}{5\,d^2x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -sqrt(-e^2*x^2 + d^2)*c/(d^2*x) - 2/3*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x) -
8/15*sqrt(-e^2*x^2 + d^2)*a*e^4/(d^6*x) - 1/3*sqrt(-e^2*x^2 + d^2)*b/(d^2*x
^3) - 4/15*sqrt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*
a/(d^2*x^5)
```

mupad [B] time = 1.73, size = 146, normalized size = 0.91

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{5\,d}+\frac{x^4\,\left(15\,c\,d^5+10\,b\,d^3\,e^2+8\,a\,d\,e^4\right)}{15\,d^6}+\frac{x^5\,\left(15\,c\,d^4\,e+10\,b\,d^2\,e^3+8\,a\,e^5\right)}{15\,d^6}+\frac{x^2\,\left(5\,b\,d^5+4\,a\,d^3\,e^2\right)}{15\,d^6}+\frac{x^3\,\left(5\,b\,d^4\,e+4\,a\,d^2\,e^3\right)}{15\,d^6}+\frac{a\,e}{5\,d^2}\right)}{x^5\,\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(5*d) + (x^4*(15*c*d^5 + 10*b*d^3*e^2 + 8*a*d*e^4))/(1
5*d^6) + (x^5*(8*a*e^5 + 10*b*d^2*e^3 + 15*c*d^4*e))/(15*d^6) + (x^2*(5*b*d
^5 + 4*a*d^3*e^2))/(15*d^6) + (x^3*(4*a*d^2*e^3 + 5*b*d^4*e))/(15*d^6) + (a
*e*x)/(5*d^2)))/(x^5*(d + e*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.144 $\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=226

$$-\frac{2e^2(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(6ae^2+7bd^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}$$

```
[Out] -1/7*a*(-e^2*x^2+d^2)/d^2/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/35*(6*a*e^2+7*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(24*a*e^4+
28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^6/x^3/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-
2/105*e^2*(24*a*e^4+28*b*d^2*e^2+35*c*d^4)*(-e^2*x^2+d^2)/d^8/x/(-e*x+d)^(1
/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 453, 271, 264}

$$-\frac{2e^2(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^8x\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(24ae^4+28bd^2e^2+35cd^4)}{105d^6x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{(d^2-e^2x^2)(6ae^2+7bd^2)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(7*d^2*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((7*b*d^2 +
6*a*e^2)*(d^2 - e^2*x^2))/(35*d^4*x^5*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((35*c
*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^3*Sqrt[d - e*x]
*Sqrt[d + e*x]) - (2*e^2*(35*c*d^4 + 28*b*d^2*e^2 + 24*a*e^4)*(d^2 - e^2*x^
2))/(105*d^8*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
```

```
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^8\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^8\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-7bd^2-6ae^2-7cd^2x^2}{x^6\sqrt{d^2-e^2x^2}}\,dx}{7d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(7bd^2+6ae^2\right)\left(d^2-e^2x^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(35cd^4-4e^2\left(-7bd^2-6ae^2\right.\right.\right.}{35d^4\sqrt{d-}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(7bd^2+6ae^2\right)\left(d^2-e^2x^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(35cd^4+28bd^2e^2+24ae^4\right)\left(\right.}{105d^6x^3\sqrt{d-ex}\sqrt{d+}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{7d^2x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(7bd^2+6ae^2\right)\left(d^2-e^2x^2\right)}{35d^4x^5\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(35cd^4+28bd^2e^2+24ae^4\right)\left(\right.}{105d^6x^3\sqrt{d-ex}\sqrt{d+}}$$

Mathematica [A] time = 0.14, size = 124, normalized size = 0.55

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(3a\left(5d^6+6d^4e^2x^2+8d^2e^4x^4+16e^6x^6\right)+7b\left(3d^6x^2+4d^4e^2x^4+8d^2e^4x^6\right)+35cd^4x^4\left(d^2+2e\right.\right.}{105d^8x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/105*(Sqrt[d - e*x]*Sqrt[d + e*x]*(35*c*d^4*x^4*(d^2 + 2*e^2*x^2) + 7*b*(
3*d^6*x^2 + 4*d^4*e^2*x^4 + 8*d^2*e^4*x^6) + 3*a*(5*d^6 + 6*d^4*e^2*x^2 + 8
*d^2*e^4*x^4 + 16*e^6*x^6)))/(d^8*x^7)
```

fricas [A] time = 0.97, size = 110, normalized size = 0.49

$$-\frac{\left(15\,ad^6+2\left(35\,cd^4e^2+28\,bd^2e^4+24\,ae^6\right)x^6+\left(35\,cd^6+28\,bd^4e^2+24\,ad^2e^4\right)x^4+3\left(7\,bd^6+6\,ad^4e^2\right)x^2\right)\sqrt{ex+}}{105\,d^8x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="fr
icas")
```

```
[Out] -1/105*(15*a*d^6 + 2*(35*c*d^4*e^2 + 28*b*d^2*e^4 + 24*a*e^6)*x^6 + (35*c*d
^6 + 28*b*d^4*e^2 + 24*a*d^2*e^4)*x^4 + 3*(7*b*d^6 + 6*a*d^4*e^2)*x^2)*sqrt
(e*x + d)*sqrt(-e*x + d)/(d^8*x^7)
```

giac [B] time = 4.73, size = 1517, normalized size = 6.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="gi
ac")
```

```
[Out] -4/105*(105*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^4 + 105*b*d^2*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^13*e^6 - 1960*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqr
t(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^4 + 105
*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8 - 1400*b*d^2*((sqrt(2)*sqrt(d) - sqrt(
-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d))
)^11*e^6 + 16240*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^4 - 840*a*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^11*e^8 + 12656*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sq
rt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 806
40*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/
(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^4 + 14448*a*((sqrt(2)*sqrt(d) - sqr
t(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d
)))^9*e^8 - 69888*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
 sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 259840*c*d^4*((s
qrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqr
t(d) - sqrt(-x*e + d)))^5*e^4 - 40704*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 +
202496*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e +
 d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^6 - 501760*c*d^4*((sqrt(2)*sqrt
(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt
(-x*e + d)))^3*e^4 + 231168*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e
+ d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 358400*b*d
^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^3*e^6 + 430080*c*d^4*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))*e^4 - 215040*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 430080*b*d^2*((sqrt(2)
*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) -
 sqrt(-x*e + d)))*e^6 + 430080*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x
*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^8)*e^(-1)/(((
(sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*s
qrt(d) - sqrt(-x*e + d)))^2 - 4)^7*d^8)
```

maple [A] time = 0.01, size = 118, normalized size = 0.52

$$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(48a\,e^{6}x^{6}+56b\,d^{2}e^{4}x^{6}+70c\,d^{4}e^{2}x^{6}+24a\,d^{2}e^{4}x^{4}+28b\,d^{4}e^{2}x^{4}+35c\,d^{6}x^{4}+18a\,d^{4}e^{2}x^{2}\right.}{105d^{8}x^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/105*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(48*a*e^6*x^6+56*b*d^2*e^4*x^6+70*c*d^4
*e^2*x^6+24*a*d^2*e^4*x^4+28*b*d^4*e^2*x^4+35*c*d^6*x^4+18*a*d^4*e^2*x^2+21
*b*d^6*x^2+15*a*d^6)/x^7/d^8
```

maxima [A] time = 1.01, size = 226, normalized size = 1.00

$$-\frac{2\sqrt{-e^{2}x^{2}+d^{2}}\,ce^{2}}{3\,d^{4}x}-\frac{8\sqrt{-e^{2}x^{2}+d^{2}}\,be^{4}}{15\,d^{6}x}-\frac{16\sqrt{-e^{2}x^{2}+d^{2}}\,ae^{6}}{35\,d^{8}x}-\frac{\sqrt{-e^{2}x^{2}+d^{2}}\,c}{3\,d^{2}x^{3}}-\frac{4\sqrt{-e^{2}x^{2}+d^{2}}\,be^{2}}{15\,d^{4}x^{3}}-\frac{8\sqrt{-e^{2}x^{2}+}}{35\,d^{6}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="ma
xima")
```

```
[Out] -2/3*sqrt(-e^2*x^2 + d^2)*c*e^2/(d^4*x) - 8/15*sqrt(-e^2*x^2 + d^2)*b*e^4/(
d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x) - 1/3*sqrt(-e^2*x^2 + d^2
)*c/(d^2*x^3) - 4/15*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^3) - 8/35*sqrt(-e^2*
x^2 + d^2)*a*e^4/(d^6*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^5) - 6/35*sq
rt(-e^2*x^2 + d^2)*a*e^2/(d^4*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^7)
```

mupad [B] time = 1.82, size = 218, normalized size = 0.96

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{7\,d}+\frac{x^2\,\left(21\,b\,d^7+18\,a\,d^5\,e^2\right)}{105\,d^8}+\frac{x^4\,\left(35\,c\,d^7+28\,b\,d^5\,e^2+24\,a\,d^3\,e^4\right)}{105\,d^8}+\frac{x^7\,\left(70\,c\,d^4\,e^3+56\,b\,d^2\,e^5+48\,a\,e^7\right)}{105\,d^8}+\frac{x^3\,\left(21\,b\,d^6\,e+18\,a\,d^4\,e^3\right)}{105\,d^8}+\right.}{x^7\,\sqrt{d+e\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(7*d) + (x^2*(21*b*d^7 + 18*a*d^5*e^2))/(105*d^8) + (x
^4*(35*c*d^7 + 24*a*d^3*e^4 + 28*b*d^5*e^2))/(105*d^8) + (x^7*(48*a*e^7 + 5
6*b*d^2*e^5 + 70*c*d^4*e^3))/(105*d^8) + (x^3*(18*a*d^4*e^3 + 21*b*d^6*e))/
(105*d^8) + (x^5*(24*a*d^2*e^5 + 28*b*d^4*e^3 + 35*c*d^6*e))/(105*d^8) + (x
^6*(56*b*d^3*e^4 + 70*c*d^5*e^2 + 48*a*d*e^6))/(105*d^8) + (a*e*x)/(7*d^2))
)/(x^7*(d + e*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

3.145 $\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx$

Optimal. Leaf size=292

$$-\frac{8e^4\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(16ae^4+\right.}{105d^6x^5\sqrt{d-e}}$$

```
[Out] -1/9*a*(-e^2*x^2+d^2)/d^2/x^9/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/63*(8*a*e^2+9*
b*d^2)*(-e^2*x^2+d^2)/d^4/x^7/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-1/105*(16*a*e^4+
18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^6/x^5/(-e*x+d)^(1/2)/(e*x+d)^(1/2)-
4/315*e^2*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^2)/d^8/x^3/(-e*x+d)^
(1/2)/(e*x+d)^(1/2)-8/315*e^4*(16*a*e^4+18*b*d^2*e^2+21*c*d^4)*(-e^2*x^2+d^
2)/d^10/x/(-e*x+d)^(1/2)/(e*x+d)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {520, 1265, 453, 271, 264}

$$-\frac{8e^4\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^{10}x\sqrt{d-ex}\sqrt{d+ex}}-\frac{4e^2\left(d^2-e^2x^2\right)\left(16ae^4+18bd^2e^2+21cd^4\right)}{315d^8x^3\sqrt{d-ex}\sqrt{d+ex}}-\frac{\left(d^2-e^2x^2\right)\left(16ae^4+\right.}{105d^6x^5\sqrt{d-e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -(a*(d^2 - e^2*x^2))/(9*d^2*x^9*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((9*b*d^2 +
8*a*e^2)*(d^2 - e^2*x^2))/(63*d^4*x^7*Sqrt[d - e*x]*Sqrt[d + e*x]) - ((21*c
*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(105*d^6*x^5*Sqrt[d - e*x]
*Sqrt[d + e*x]) - (4*e^2*(21*c*d^4 + 18*b*d^2*e^2 + 16*a*e^4)*(d^2 - e^2*x^
2))/(315*d^8*x^3*Sqrt[d - e*x]*Sqrt[d + e*x]) - (8*e^4*(21*c*d^4 + 18*b*d^2
*e^2 + 16*a*e^4)*(d^2 - e^2*x^2))/(315*d^10*x*Sqrt[d - e*x]*Sqrt[d + e*x])
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 271

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x^(m + 1)*(
a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m +
1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
```

```
b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1265

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 +
c*x^4)^p, f*x, x], R = PolynomialRemainder[(a + b*x^2 + c*x^4)^p, f*x, x]},
 Simp[(R*(f*x)^(m + 1)*(d + e*x^2)^(q + 1))/(d*f*(m + 1)), x] + Dist[1/(d*f
^2*(m + 1)), Int[(f*x)^(m + 2)*(d + e*x^2)^q*ExpandToSum[(d*f*(m + 1)*Qx)/x
 - e*R*(m + 2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ
[b^2 - 4*a*c, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d-ex}\sqrt{d+ex}}\,dx = \frac{\sqrt{d^2-e^2x^2}\int \frac{a+bx^2+cx^4}{x^{10}\sqrt{d^2-e^2x^2}}\,dx}{\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\sqrt{d^2-e^2x^2}\int \frac{-9bd^2-8ae^2-9cd^2x^2}{x^8\sqrt{d^2-e^2x^2}}\,dx}{9d^2\sqrt{d-ex}\sqrt{d+ex}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} + \frac{\left(\left(63cd^4-6e^2\left(-9bd^2-8ae\right.\right.\right.}{63d^4\sqrt{d}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(21cd^4+18bd^2e^2+16ae^4\right)\left(\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d\,-}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(21cd^4+18bd^2e^2+16ae^4\right)\left(\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d\,-}}$$

$$= -\frac{a\left(d^2-e^2x^2\right)}{9d^2x^9\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(9bd^2+8ae^2\right)\left(d^2-e^2x^2\right)}{63d^4x^7\sqrt{d-ex}\sqrt{d+ex}} - \frac{\left(21cd^4+18bd^2e^2+16ae^4\right)\left(\right.}{105d^6x^5\sqrt{d-ex}\sqrt{d\,-}}$$

Mathematica [A] time = 0.18, size = 158, normalized size = 0.54

$$-\frac{\sqrt{d-ex}\sqrt{d+ex}\left(a\left(35d^8+40d^6e^2x^2+48d^4e^4x^4+64d^2e^6x^6+128e^8x^8\right)+9b\left(5d^8x^2+6d^6e^2x^4+8d^4e^4x^6+16\right.\right.}{315d^{10}x^9}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^10*Sqrt[d - e*x]*Sqrt[d + e*x]),x]
```

```
[Out] -1/315*(Sqrt[d - e*x]*Sqrt[d + e*x]*(21*c*d^4*x^4*(3*d^4 + 4*d^2*e^2*x^2 +
8*e^4*x^4) + 9*b*(5*d^8*x^2 + 6*d^6*e^2*x^4 + 8*d^4*e^4*x^6 + 16*d^2*e^6*x^
8) + a*(35*d^8 + 40*d^6*e^2*x^2 + 48*d^4*e^4*x^4 + 64*d^2*e^6*x^6 + 128*e^8
*x^8)))/(d^10*x^9)
```

fricas [A] time = 1.02, size = 144, normalized size = 0.49

$$-\frac{\left(35\,ad^8+8\left(21\,cd^4e^4+18\,bd^2e^6+16\,ae^8\right)x^8+4\left(21\,cd^6e^2+18\,bd^4e^4+16\,ad^2e^6\right)x^6+3\left(21\,cd^8+18\,bd^6e^2+16\,a\right.\right.}{315\,d^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] -1/315*(35*a*d^8 + 8*(21*c*d^4*e^4 + 18*b*d^2*e^6 + 16*a*e^8)*x^8 + 4*(21*c
*d^6*e^2 + 18*b*d^4*e^4 + 16*a*d^2*e^6)*x^6 + 3*(21*c*d^8 + 18*b*d^6*e^2 +
16*a*d^4*e^4)*x^4 + 5*(9*b*d^8 + 8*a*d^6*e^2)*x^2)*sqrt(e*x + d)*sqrt(-e*x
+ d)/(d^10*x^9)
```

giac [B] time = 7.35, size = 1931, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4/315*(315*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(
x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^17*e^6 + 315*b*d^2*((sqrt(2)*s
qrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - s
qrt(-x*e + d)))^17*e^8 - 6720*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqr
t(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^15*e^6 + 315
*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^17*e^10 - 5040*b*d^2*((sqrt(2)*sqrt(d) - sqrt
(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)
))^15*e^8 + 76608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
 sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^6 - 3360*a*((sqrt(2
)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d)
- sqrt(-x*e + d)))^15*e^10 + 68544*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d)
)/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^13*e^8
- 580608*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e
 + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^6 + 76608*a*((sqrt(2)*sqrt(d
) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-
x*e + d)))^13*e^10 - 509184*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(
x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^11*e^8 + 28922
88*c*d^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/
(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^6 - 327168*a*((sqrt(2)*sqrt(d) - sq
rt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e +
d)))^11*e^10 + 2363904*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e +
 d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^9*e^8 - 9289728*c*d
^4*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(
2)*sqrt(d) - sqrt(-x*e + d)))^7*e^6 + 2728448*a*((sqrt(2)*sqrt(d) - sqrt(-x
*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^
9*e^10 - 8146944*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) -
sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^8 + 19611648*c*d^4*((
sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sq
rt(d) - sqrt(-x*e + d)))^5*e^6 - 5234688*a*((sqrt(2)*sqrt(d) - sqrt(-x*e +
d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^7*e^1
0 + 17547264*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt
(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^8 - 27525120*c*d^4*((sqrt
(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d
) - sqrt(-x*e + d)))^3*e^6 + 19611648*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))
/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^5*e^10 -
 20643840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*
e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^8 + 20643840*c*d^4*((sqrt(2)
*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) -
 sqrt(-x*e + d)))*e^6 - 13762560*a*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt
(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^3*e^10 + 2064
3840*b*d^2*((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d
)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))*e^8 + 20643840*a*((sqrt(2)*sqrt(d) -
sqrt(-x*e + d))/sqrt(x*e + d) - sqrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e
+ d)))*e^10)*e^(-1)/((((sqrt(2)*sqrt(d) - sqrt(-x*e + d))/sqrt(x*e + d) - s
qrt(x*e + d)/(sqrt(2)*sqrt(d) - sqrt(-x*e + d)))^2 - 4)^9*d^10)
```

maple [A] time = 0.01, size = 154, normalized size = 0.53

$$-\frac{\sqrt{ex+d}\,\sqrt{-ex+d}\,\left(128a\,e^8x^8+144b\,d^2e^6x^8+168c\,d^4e^4x^8+64a\,d^2e^6x^6+72b\,d^4e^4x^6+84c\,d^6e^2x^6+48a\,d^4e^4x\right.}{315d^{10}x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x)
```

```
[Out] -1/315*(e*x+d)^(1/2)*(-e*x+d)^(1/2)*(128*a*e^8*x^8+144*b*d^2*e^6*x^8+168*c*
d^4*e^4*x^8+64*a*d^2*e^6*x^6+72*b*d^4*e^4*x^6+84*c*d^6*e^2*x^6+48*a*d^4*e^4
*x^4+54*b*d^6*e^2*x^4+63*c*d^8*x^4+40*a*d^6*e^2*x^2+45*b*d^8*x^2+35*a*d^8)/
x^9/d^10
```

maxima [A] time = 1.03, size = 304, normalized size = 1.04

$$-\frac{8\sqrt{-e^2x^2+d^2}\,ce^4}{15\,d^6x}-\frac{16\sqrt{-e^2x^2+d^2}\,be^6}{35\,d^8x}-\frac{128\sqrt{-e^2x^2+d^2}\,ae^8}{315\,d^{10}x}-\frac{4\sqrt{-e^2x^2+d^2}\,ce^2}{15\,d^4x^3}-\frac{8\sqrt{-e^2x^2+d^2}\,be^4}{35\,d^6x^3}-\frac{64\sqrt{-e^2}}{315}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^10/(-e*x+d)^(1/2)/(e*x+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] -8/15*sqrt(-e^2*x^2 + d^2)*c*e^4/(d^6*x) - 16/35*sqrt(-e^2*x^2 + d^2)*b*e^6
/(d^8*x) - 128/315*sqrt(-e^2*x^2 + d^2)*a*e^8/(d^10*x) - 4/15*sqrt(-e^2*x^2
 + d^2)*c*e^2/(d^4*x^3) - 8/35*sqrt(-e^2*x^2 + d^2)*b*e^4/(d^6*x^3) - 64/31
5*sqrt(-e^2*x^2 + d^2)*a*e^6/(d^8*x^3) - 1/5*sqrt(-e^2*x^2 + d^2)*c/(d^2*x^
5) - 6/35*sqrt(-e^2*x^2 + d^2)*b*e^2/(d^4*x^5) - 16/105*sqrt(-e^2*x^2 + d^2
)*a*e^4/(d^6*x^5) - 1/7*sqrt(-e^2*x^2 + d^2)*b/(d^2*x^7) - 8/63*sqrt(-e^2*x
^2 + d^2)*a*e^2/(d^4*x^7) - 1/9*sqrt(-e^2*x^2 + d^2)*a/(d^2*x^9)
```

mupad [B] time = 1.87, size = 290, normalized size = 0.99

$$-\frac{\sqrt{d-e\,x}\,\left(\frac{a}{9\,d}+\frac{x^2\,\left(45\,b\,d^9+40\,a\,d^7\,e^2\right)}{315\,d^{10}}+\frac{x^6\,\left(84\,c\,d^7\,e^2+72\,b\,d^5\,e^4+64\,a\,d^3\,e^6\right)}{315\,d^{10}}+\frac{x^7\,\left(84\,c\,d^6\,e^3+72\,b\,d^4\,e^5+64\,a\,d^2\,e^7\right)}{315\,d^{10}}+\frac{x^4\,\left(63\,c\,d^9+54\,b\,d^7\,e^2\right.}{315\,d^{10}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^10*(d + e*x)^(1/2)*(d - e*x)^(1/2)),x)
```

```
[Out] -((d - e*x)^(1/2)*(a/(9*d) + (x^2*(45*b*d^9 + 40*a*d^7*e^2))/(315*d^10) + (
x^6*(64*a*d^3*e^6 + 72*b*d^5*e^4 + 84*c*d^7*e^2))/(315*d^10) + (x^7*(64*a*d
^2*e^7 + 72*b*d^4*e^5 + 84*c*d^6*e^3))/(315*d^10) + (x^4*(63*c*d^9 + 48*a*d
^5*e^4 + 54*b*d^7*e^2))/(315*d^10) + (x^9*(128*a*e^9 + 144*b*d^2*e^7 + 168*
c*d^4*e^5))/(315*d^10) + (x^3*(40*a*d^6*e^3 + 45*b*d^8*e))/(315*d^10) + (x^
5*(48*a*d^4*e^5 + 54*b*d^6*e^3 + 63*c*d^8*e))/(315*d^10) + (x^8*(144*b*d^3*
e^6 + 168*c*d^5*e^4 + 128*a*d*e^8))/(315*d^10) + (a*e*x)/(9*d^2)))/(x^9*(d
+ e*x)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**10/(-e*x+d)**(1/2)/(e*x+d)**(1/2),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)



(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]



(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```